$$G^{(k)}(x)=(-1)^k k!\left\{b\delta_{k0}+\sum_i \frac{q_i}{(x-a_i)^{k+1}}\right\}, \qquad (C7)$$

and the derivatives of $F(x)$ can be recursively obtained through Eq. (C5) by using Eqs. (C6) and (C7). Hence the problem of differentiation has been reduced to an algebraic one of summation and multiplication.

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Some Consequences of an SU(2)⊗U(1) Gauge Model*

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The experimental consequences of an SU(2)⊗U(1) model in which the intermediate-boson mass m_W is bounded below by 13 GeV are discussed. For the special choice of $m_W = 18$ GeV neutral-current effects essentially disappear. The model includes a heavy neutral lepton of the muon type whose mass is bounded above and below by ~1.2 GeV and 390 MeV, respectively. The model is thus relatively more accessible to experimental tests than other gauge models. Other aspects of the model are discussed.

I. INTRODUCTION

The construction of gauge models of weak and electromagnetic interactions, initiated by Weinberg[1] and Salam,[2] has now mushroomed into a booming industry.[3] The available models more often than not contain intermediate bosons too massive to be produced easily with present-day machines and/or leptons with mass in the range of several GeV. We have considered an SU(2)⊗U(1) model[4] which contains bosons and leptons of relatively low mass and which thus may have the (dubious) distinction of being an early casualty in a confrontation of gauge theories with experiment. (Our previous paper will be referred to as I.)

We list those features of this model that are relevant for experimental investigations.

(a) The lower bound for m_W, the mass of the charged intermediate bosons, is 13 GeV. This may be contrasted with the value of 39 GeV for Weinberg's model[1] and a number of other models. While 13 GeV is the smallest mass allowed in this theory, it turns out that there are reasons that strongly suggest a value of ~18 GeV for m_W.

(b) In order not to upset the present agreement between the muon $(g-2)$ experiment and theory we must not allow the mass of the exotic lepton associated with the muon (called M^0 in I) to be greater than ~1.2 GeV.

(c) In contrast to Weinberg's model the model in I is consistent with present experimental bounds on the possible contribution of a weak neutral current. In particular, if $m_W \cong 18$ GeV the familiar hadrons do not couple[5] to Z, the neutral intermediate boson. In this case, the Z boson may have a very small mass and yet can easily escape experimental detection. These points will be elaborated and discussed in this paper. We begin with a brief summary of the model (Sec. II).

II. SALIENT FEATURES OF THE MODEL

We start with three notions of simplicity in mind. While simplicity is certainly in the eye of the beholder it appears to us that the following considerations are reasonable.

(a) We choose to work with the simple possibility SU(2)⊗U(1) not only because it is adequate but also because it may be embedded in any Lie algebra except SU(2). Our model may be a piece of a more grandiose construction chosen by nature.

(b) We demand that the inclusion of hadrons in the model be carried out within the framework of an acceptable model of hadron structure. We define a hadron model to be acceptable if

(i) it accounts for the spectroscopy of low-mass hadrons keeping intact the spin-statistics connection,[6]

(ii) it accounts for the $\pi^0 \to 2\gamma$ amplitude in magnitude and in sign,[7] and

(iii) it pays whatever respect is due to static SU(6) and/or semiempirical quark-model relations between masses, cross sections, couplings, magnetic moments, etc.[8]

While three triplets of quarks appear to be an extravagance, the simplest acceptable models known to us involve three triplets. These are the so-called RWB (red-white-blue) model[9] and the Han-Nambu model,[10] both based on the hadronic symmetry group SU(3)⊗SU(3)′. (For the sake of definiteness we will write our formulas in a form appropriate to the RWB model. The reader may verify that most of our conclusions hold for both models.) We presume that the familiar hadrons are all SU(3)′ singlets to a high degree of accuracy, say one part in 10^3. The origin of this symmetry, seemingly broken only on the electromagnetic and/or weak level, is at the moment shrouded in mystery, as is the whole problem of hadronic symmetry in gauge models.

(c) We treat the right-handed and left-handed fermions in as symmetrical a manner as is allowed by the $V-A$ structure of weak decays. It is the insistence on this symmetrical treatment that enables us to avoid some of the undesirable features of Weinberg's model.[1] It will be seen below that our model is an avatar of Weinberg's original model.

The covariant derivative appropriate to the group U(1)⊗SU(2) is $\partial_\mu - ig\vec{W}_\mu \cdot \vec{T} - ig' B_\mu Y/2$, where $(\vec{T}, \frac{1}{2}Y)$ are the four matrix generators of the group. $T_3 + \frac{1}{2}Y = Q$ is the electric charge. It is convenient to introduce the Salam-Ward-Weinberg angle[1,2] by the expression $\tan\xi = g'/g$; the neutral massive field will then be

$$Z_\mu = W_{3\mu}\cos\xi - B_\mu \sin\xi \,.$$

The left-handed quarks are assigned to two doublets: $(\mathcal{P}_1, \mathfrak{N}_1(\theta))_L$ and $(\mathcal{P}_2, \lambda_1(\theta))_L$. The second doublet is necessary to ensure the absence of strangeness-changing neutral current. The symmetrical treatment discussed above suggests that we have two right-handed doublets. These may be taken to be $(\mathcal{P}_3, \mathfrak{N}_2)_R$ and $(\mathcal{P}_2, \lambda_3)_R$. The remaining quarks are assigned to ten singlets:

$$\mathcal{P}_{1R},\ \mathfrak{N}_{1R},\ \lambda_{1R},\ \mathfrak{N}_{2L},\ \lambda_{2L},$$
$$\lambda_{2R},\ \mathcal{P}_{3L},\ \mathfrak{N}_{3L},\ \mathfrak{N}_{3R},\ \lambda_{3L}.$$

Here we have used the conventional definition $\mathfrak{N}_1(\theta) = \mathfrak{N}_1\cos\theta + \lambda_1\sin\theta$ and $\lambda_1(\theta) = -\mathfrak{N}_1\sin\theta + \lambda_1\cos\theta$ with θ = the Cabibbo angle.

The assignment of the right-handed quarks is constrained to some extent by the structure of nonleptonic decay. With a given assignment the nonleptonic decay Lagrangian in general would be composed of three terms,

$$\mathcal{L}_{\text{nonleptonic}} = \mathcal{L}_{LL} + \mathcal{L}_{LR} + \mathcal{L}_{RR}\,, \qquad (2.1)$$

where

$$\mathcal{L}_{LL} \sim (V-A)(V-A)\,,$$
$$\mathcal{L}_{LR} \sim (V-A)(V+A)\,,$$

and

$$\mathcal{L}_{RR} \sim (V+A)(V+A)\,.$$

Since we do not wish to lose the elegant current-algebra results for K decay and S-wave hyperon decay,[12] we have arranged our right-handed quarks in such a way that $\mathcal{L}_{\text{nonleptonic}} = \mathcal{L}_{LL}$. To be sure, certain current-algebra results such as the $\Delta I = \frac{1}{2}$ rule for S-wave Ξ and Λ decay would be preserved even with the general $\mathcal{L}_{\text{nonleptonic}}$ in Eq. (2.1). On the other hand, other relations, such as the well-satisfied relationship between[13] $K_L \to \pi^+\pi^-\pi^0$ and $K_S \to \pi^+\pi^-$, become a mystery unless $\mathcal{L}_{\text{nonleptonic}} = \mathcal{L}_{LL}$.

Some years ago, Dashen *et al.*[14] argued that the ratio

$$r \equiv \frac{\text{weak amp}(\Delta I=1, \Delta Y=0)}{\text{weak amp}(\Delta I=0, \Delta Y=0)}$$

would be of order $\sin^2\theta$ in the then-standard current-current theory with the $\Delta I = \frac{1}{2}$ rule understood as a consequence of dynamical octet enhancement, but would be of order ~ 1 if the $\Delta I = \frac{1}{2}$ rule were "built in."[15] We remark that in the type of model discussed here the ratio r would be of order ~ 1 even if the $\Delta I = \frac{1}{2}$ rule is a consequence of dynamical octet enhancement. In other words, an experimental measurement[16] that $r \sim 1$ is not necessarily an evidence against dynamical octet enhancement. On the other hand, a small r (i.e., not of order 1) would be difficult to understand in the framework of the type of model considered here.

The leptons ν_e, e^-, ν_μ, μ^-, together with two heavy neutral leptons E^0 and M^0, are assigned as follows: doublets $(\frac{1}{3}\nu_e \pm \frac{1}{3}\sqrt{8}\, E^0, e^-)_L$, $(E^0, e^-)_R$; singlet $(\pm\frac{1}{3}\sqrt{8}\,\nu_e + \frac{1}{3}E^0)_L$, and similarly for the muon system with $\nu_e \to \nu_\mu$, $e^- \to \mu^-$, $E^0 \to M^0$. The mixing introduced in the left-handed doublet is necessary for universality[17] between β decay and μ decay since the matrix element of the weak current between currently known hadrons H and H' is

$$\langle H' | \mathcal{P}_{1L}\gamma_\mu \mathfrak{N}_{1L}(\theta) | H\rangle = \frac{1}{3}\sum_{i=1}^{3}\langle H' | \mathcal{P}_{iL}\gamma_\mu \mathfrak{N}_{iL}(\theta) | H\rangle \,. \tag{2.2}$$

[As discussed in I, it is the SU(3)′ singlet currents—such as the current in the right-hand side of Eq. (2.2)—which are normalized by the Gell-Mann algebra; the corresponding charges, therefore, have matrix elements close to unity for the $n \to p$ transition.]

In order for μ decay to have the observed magnitude g must be related to the Fermi constant by $(8/\sqrt{2})\, G_F = g^2/9m_W^2$. This differs from the usual expression by a factor of $\frac{1}{9}$. On the other hand, the photon coupling is $e = g\sin\xi$. Hence

$$m_W = m_0 |\sin\xi|^{-1}, \tag{2.3}$$

where $m_0 \equiv \frac{1}{3}(\sqrt{2}\, e^2/8G_F)^{1/2} = 12.6$ GeV. Thus

$$m_W \gtrsim 13 \text{ GeV}. \tag{2.4}$$

The lower limit, reached when $\xi = \pi/2$, is three times smaller than in Weinberg's model.

It is worth noting that, despite the reduction of the semiweak coupling by a factor of three, the cross section for W production in reactions such as

$$\nu_\mu + N(A,Z) \to \mu^- + W^+ + N(A,Z)$$

(when expressed in terms of G_F and m_W^2) is the same as in the usual[18] theory. A similar remark applies to W production in[19] $p + p \to W +$hadrons, so long as the hadrons are SU(3)′ singlets. The situation changes if the energies are high enough to produce SU(3)′ excitations; unfortunately, however, we cannot make more precise statements without going into very specific scaling models.

III. STRUCTURE OF THE NEUTRAL CURRENT AND LIMITS ON INTERMEDIATE BOSON MASS

The gauge theory described here has the interesting feature that a small experimental upper bound on the magnitude of the neutral current is tantamount to a determination of the mass of the charged intermediate boson. The coupling of the neutral boson Z is given by

$$\mathcal{L}_I = \frac{g}{2\cos\xi} Z_\rho \bar{\psi}\gamma^\rho(\tau_3 - 2\sin^2\xi\, Q)\psi, \tag{3.1}$$

where τ_3 is one of the generators of the SU(2) subgroup of the gauge group. [Acting on singlets $\tau_3 = 0$, acting on doublets $\tau_3 = \left(\begin{smallmatrix}1 & 0\\ 0 & -1\end{smallmatrix}\right)$]. Q denotes the charge operator. It is easy to verify that the effective Lagrangian describing the coupling of Z to the *familiar leptons and hadrons* works out to be

$$\mathcal{L}_{\text{eff}} = \frac{g}{2\cos\xi} Z_\rho[\tfrac{1}{9}\bar{\nu}_e\gamma^\rho\tfrac{1}{2}(1-\gamma_5)\nu_e + \tfrac{1}{9}\bar{\nu}_\mu\gamma^\rho\tfrac{1}{2}(1-\gamma_5)\nu_\mu + (1 - 2\sin^2\xi)J^\rho_{\text{em}}]\,. \tag{3.2}$$

The electromagnetic (em) current is the familiar one. We emphasize the fact that the coupling $Z_\rho J^\rho_{\text{em}}$ is multiplied by the *universal* factor of $(1 - 2\sin^2\xi)$. This is one of the rewards of the symmetrical treatment of left- and right-handed fermions. In contrast, in Weinberg's model[1] the neutral boson Z couples to a mixture of vector and axial-vector currents, with uncomfortable consequences as pointed out by Lee.[20]

A static-model calculation by Lee estimates the ratio

$$\mathfrak{R} \equiv \frac{(\nu_\mu p \to \nu_\mu p\pi^0) + \sigma(\nu_\mu n \to \nu_\mu n\pi^0)}{2\sigma(\nu_\mu n \to \mu^- p\pi^0)} \tag{3.3}$$

to be

$$\mathfrak{R} = \frac{0.65 + 0.263(1 - 2\sin^2\xi)^2 + 0.46(1 - 2\sin^2\xi)}{0.65 + 0.263 + 0.46} \tag{3.4}$$

for *Weinberg's model*. The numbers 0.65, 0.263, and 0.46 indicate the relative size of contributions to R from the axial-vector–axial-vector, vector-vector, and the axial-vector–vector interference terms, respectively. We note that the relatively small contribution from the vector-vector terms is responsible for translating an experimental upper bound on $\mathfrak{R}$ into an upper bound on $|\xi - \frac{1}{2}\pi|$. On the other hand an experimental upper bound on

$\sigma(\nu_\mu e^- \to \nu_\mu e^-)/\sigma(\nu_e e^- \to \nu_e e^-)$ is equivalent to an upper bound on $|\xi|$ (see below). These two bounds turn out to be mutually incompatible.

In contrast, in our model the Z_ρ couples to a vector (electromagnetic) current only. Thus Lee's calculation would yield

$$\mathcal{R} = \frac{0.263}{1.37} R^2 \tag{3.5}$$

instead of Eq. (3.4). [We find it convenient to introduce the quantity

$$R = \frac{m_W^2}{m_Z^2 \cos^2\xi}(1 - 2\sin^2\xi). \tag{3.6}$$

R is essentially a measure of the size of neutral-current effects in our theory.] The smallness of the vector-vector contribution now implies that the experimental bound[21] $\mathcal{R} < 0.14$ gives only a rather weak bound on R, that $R^2 < 0.72$.

The lepton process $\nu_\mu e^- \to \nu_\mu e^-$ is mediated only by Z exchange in our model. A simple calculation gives

$$\frac{\sigma(\nu_\mu e^- \to \nu_\mu e^-)}{\sigma_{FG}(\nu_e e^- \to \nu_e e^-)} = R^2 . \tag{3.7}$$

Thus an experimental upper bound on $\nu_\mu e^- \to \nu_\mu e^-$ is equivalent to an upper bound on $|\xi - \frac{1}{4}\pi|$. This in turn would bound m_W both above and below [see Eq. (2.3)]:

$$\frac{2m_0^2}{1+|R|} \lesssim m_W^2 \lesssim \frac{2m_0^2}{1-|R|}, \tag{3.8}$$

$$m_0 \sim 13 \text{ GeV}. \tag{3.9}$$

[The upper and lower bounds in Eq. (3.8) above, while attainable in a mathematical sense, correspond to the singular limit in which the vacuum expectation value of the triplet Higgs approaches zero. Hence the bounds are actually better than what is indicated in Eq. (3.8).] The present Gargamelle data[22] set the bound

$$14.3 \lesssim m_W \lesssim 31.7 \text{ GeV}. \tag{3.10}$$

If future experiments should indicate that $R \sim 0$, then Eq. (3.8) predicts that

$$m_W \sim 18 \text{ GeV}. \tag{3.11}$$

In our theory the process $\bar\nu_e + e^- \to \bar\nu_e + e^-$ is mediated by W exchange as well as by Z exchange. The W-exchange diagram is the same as in the standard (Feynman–Gell-Mann) theory. Hence[23]

$$\frac{\sigma(\bar\nu_e + e^- \to \bar\nu_e + e^-)}{\sigma_{FG}} = 1 - R + R^2 . \tag{3.12}$$

As $R \to 0$ the process $\bar\nu_e + e^- \to \bar\nu_e + e^-$ should have the cross section predicted by Feynman and Gell-Mann.

The quantity R may be experimentally measured in hadronic processes as well. For example,

$$\frac{d\sigma}{dt}(\nu_\mu + p \to \nu_\mu + p) \Big/ \frac{d\sigma_{FG}}{dt}(\bar\nu_e + p \to e^+ + n) = \frac{R^2}{2}, \tag{3.13}$$

$$\frac{d\sigma}{dq^2 d\nu}(\nu_\mu + p \to \nu_\mu + X) \Big/ \frac{d\sigma}{dq^2 d\nu}(\mu^- + p \to \mu^- + X) = \left(\frac{\cot 2\xi}{9 \sin 2\xi}\right)^2 \left(\frac{q^2}{q^2 - m_Z^2}\right)^2 \tag{3.14}$$

[with X some SU(3)′ singlet state]. Since in our theory Z effectively couples at low energy directly to the electromagnetic current [Eq. (3.2)] we may dispense with various inequalities discussed in the literature[24] and deal with equalities such as Eq. (3.14). Also processes such as $\pi^0 \to Z \to e^+e^-$ and $\eta \to Z \to e^+e^-$ or $\mu^+\mu^-$ are forbidden by parity considerations.

In summary, the present situation as interpreted by our model is such that there is evidence from the Gargamelle data that R, which provides a measure of neutral-current effects, is small. Because of the relative smallness of the vector-vector contribution in $\nu_\mu + p \to \nu_\mu + p$ the experiment on $\nu_\mu + p \to \nu_\mu + p$ would have to be improved before a useful bound on R may be set. We would like to believe that R is indeed very small. In fact, we are strongly attracted to the possibility $R = 0$ (or equivalently $\xi = \frac{1}{4}\pi$). In this case Z decouples from the familiar hadrons and leptons except for neutrinos. More precisely, as $\xi \to \frac{1}{4}\pi$, our construction shows that

$$\mathcal{L}_{\text{eff}} = \frac{g}{\sqrt{2}} Z_\rho \big[\tfrac{1}{9}(\bar\nu_e \pm \sqrt{8}\,\bar E^0)\gamma^\rho \tfrac{1}{2}(1-\gamma_5)(\nu_e \pm \sqrt{8}\,E^0) + \tfrac{1}{9}(\bar\nu_\mu \pm \sqrt{8}\,\bar M^0)\gamma^\rho \tfrac{1}{2}(1-\gamma_5)(\nu_\mu \pm \sqrt{8}\,M^0) + \text{SU(3)}'\text{nonsinglet current involving hadrons}\big]. \tag{3.15}$$

In this case, neutral-current processes such as $\nu_\mu + p \to \nu_\mu + X$ or heavy-lepton production in[25] $\nu_\mu + p \to M^0 + X$ would be vanishingly small until an SU(3)′ nonsinglet hadron state X is produced. The same remark applies to search experiments like $p + p \to X + \bar M^0 M^0$ or $p + p \to X + M^0 \bar\nu_\mu$. In any case, as remarked in I, when the energy is large enough so that the regime for producing a nonsinglet hadron state X is reached, there will be spectacular changes in neutrino sum rules such as that of Adler and others.[26] This also opens up the amusing possibility that the charged W with $m_W = 18$ GeV may soon be detected while the neutral Z, with a mass possibly as low as a few GeV, could escape detection for years to come. In this case, it would be possible for the processes such as $\nu_e\nu_e \to \nu_e\nu_e$, $\nu_e\nu_\mu \to \nu_e\nu_\mu$, $\nu_\mu\nu_\mu \to \nu_\mu\nu_\mu$, $\nu_e\bar\nu_e \to \nu_e\bar\nu_e$, $\nu_\mu\bar\nu_\mu \to \nu_\mu\bar\nu_\mu$, $\nu_\mu\bar\nu_\mu$

$\to \nu_e \bar{\nu}_e$ to have anomalously large cross sections proportional to $(e/9m_Z)^4E^2$. This may have interesting bearings on astrophysical problems.

IV. GYROMAGNETIC RATIO OF THE MUON AND THE MASS OF HEAVY LEPTONS

We now discuss the weak contribution to the muon $g-2\equiv 2a_\mu$ factor in our model. As is well known, the agreement between the experimental value of $g-2$ and the theoretical prediction excluding weak-interaction effects is extraordinarily good. Allowing for a two-standard-deviations discrepancy, one deduces that the present data[27] limit the weak contribution to $g-2$ by

$$-3\times 10^{-7} \lesssim a_\mu^{\text{weak}} \lesssim 9\times 10^{-7}. \tag{4.1}$$

This may be translated into an upper bound on the mass of the neutral heavy lepton M^0. Because of the factor of nine in our model we expect that it would give a relatively large contribution to $(g-2)$ and hence that our upper bound on m_{M^0} is more stringent than the corresponding bound in other models. The calculation of $(g-2)$ in gauge models has been performed by a number of authors.[28,29]

The calculation of the Z-field contribution (Fig. 1) in our model is particularly simple since it couples to the muon electromagnetic current. The result reads[30]

$$a_\mu^{(Z)} = \frac{g^2}{48\pi^2\cos^2\xi}(1-2\sin^2\xi)^2\left(\frac{m_\mu}{m_Z}\right)^2\left[1-6\left(\frac{m_\mu}{m_Z}\right)^2\ln\frac{m_Z}{m_\mu}+\frac{25}{4}\left(\frac{m_\mu}{m_Z}\right)^2+\ldots\right] \quad (m_Z > m_\mu)$$
$$= \frac{3}{2\pi^2\sqrt{2}}G_F m_\mu^2 |R\cos 2\xi|\left[1+O\left(\frac{m_\mu^2}{m_Z^2}\ln\frac{m_Z}{m_\mu}\right)\right]. \tag{4.2}$$

Since $|R|<1$, we have the bound

$$a_\mu^{(Z)} < 10^{-8}.$$

The Z-field contribution is, therefore, too small to be of experimental significance as long as $m_\mu^2/m_Z^2 \ll 1$. The contribution from ν_μ [Fig. 2(a)] gives

$$a_\mu^{(\nu_\mu)} \sim 10^{-9},$$

the same value as in standard models,[28,29] and is too small to be relevant here. The small m_W is precisely compensated by the correspondingly smaller $W\bar{\nu}_\mu\mu^-$ coupling.

The heavy lepton or M^0 contribution [Fig. 2(b)] is much more important. Following the standard calculations,[28] as suitably modified for our model, we find

$$a_\mu^{(M^0)} = \mp\left(\frac{6}{\pi^2}\right)\left[\frac{f(y)}{f(0)}\right]G_F m_{M^0} m_\mu + \ldots. \tag{4.3}$$

The dots indicate terms which are not directly proportional to M^0 mass. Here $y\equiv m_{M^0}^2/m_W^2$ and

$$f(y)\equiv 1+\frac{6}{(1-y)^3}\left(\tfrac{1}{2}-2y+\tfrac{3}{2}y^2-y^2\ln y\right).$$

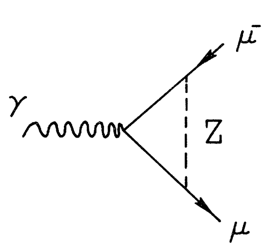

FIG. 1. Z-field contribution.

It is important to note that, unlike the situation in, for example, the Georgi-Glashow model,[31] the sign of the heavy-lepton contribution to a_μ is not prescribed in our model; the minus sign in Eq. (4.3) corresponds to using $\frac{1}{3}\nu_\mu + \frac{1}{3}\sqrt{8}M^0$ as the neutral part of the left-handed doublet, the plus sign to $\frac{1}{3}\nu_\mu - \frac{1}{3}\sqrt{8}M^0$.

In order to get a sensible bound on m_{M^0} we must choose the positive sign in Eq. (4.3); this leads to $m_{M^0} < 1216$ MeV. Since the decay $K^+\to\mu^+M^0$ is not observed, we can also bound m_{M^0} below. Hence the inequality

$$400 \leqslant m_{M^0} \leqslant 1200 \text{ MeV}. \tag{4.4}$$

This contrasts with the situation in other models[28] in which the mass of heavy leptons is typically bounded below by several GeV.

V. PRODUCTION AND DECAY OF M

Gauge models have lent substance to the search for exotic leptons. Our model predicts that the search in e^+e^- annihilation experiments would be in vain since e^+e^- cannot go to $M^0\bar{M}^0$ in leading

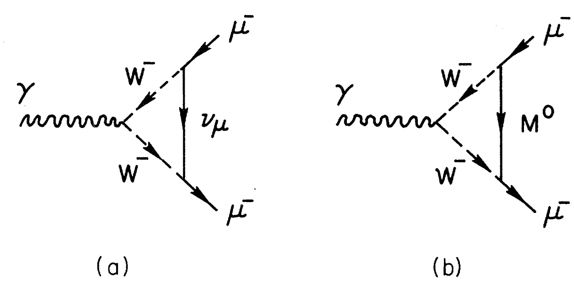

FIG. 2. Contributions from ν_μ and M^0.

order. As we already remarked, if $m_W = 18$ GeV, then the production rate for processes such as $p+p\to X(\mathrm{SU(3)'\,singlet}) + M^0\bar{\nu}_\mu$ and $e^+e^-\to Z\to M^0\bar{M}^0$ would also vanish. In that case M^0 would have to be produced by high-energy muon beams.

At energies sufficiently above threshold, the cross sections are readily estimated in our model:

$$\lim_{E_\mu, E_{\bar{\nu}}\to\infty} \frac{\sigma(\mu^- + p \to M^0 + n)}{\sigma(\bar{\nu}_\mu + p \to \mu^+ + n)} = 8.5\,, \tag{5.1}$$

$$\frac{\sigma(\mu_L^- + p \to M^0 + \cdots)}{\sigma(\nu_\mu + p \to \mu^- + \cdots)} = 8\,, \tag{5.2}$$

$$\frac{\sigma(\mu_R^- + p \to M^0 + \cdots)}{\sigma(\bar{\nu}_\mu + n \to \mu^+ + \cdots)} = 9\,. \tag{5.3}$$

Here μ_L (μ_R) denotes a 100% left (right) polarized μ beam and the dots indicate that one has summed over all hadronic excitations. Also, if neutral-current effects involving the familiar hadrons do not vanish ($\xi \neq \frac{1}{4}\pi$)

$$\frac{\sigma(\nu_\mu + p \to M^0 + p)}{\sigma(\nu_\mu + p \to \nu_\mu + p)} = 8\,, \tag{5.4}$$

$$\frac{\sigma(\nu_\mu + p \to M^0 + \cdots)}{\sigma(\nu_\mu + p \to \nu_\mu + \cdots)} = 8\,. \tag{5.5}$$

We now list the decay modes of M^0 and estimate the partial decay rates (M = mass of M^0):

(i) $\Gamma(M^0 \to \mu^-\pi^+) = 3.4\times 10^{-11}\, m_\pi \left(\frac{m_\pi}{m_N}\right)\left(\frac{M}{m_N}\right)^3$.

(ii) $\Gamma(M^0 \to \mu^- K^+)$ is a small fraction of $\Gamma(M^0 \to \mu^-\pi^+)$, the suppression factor comes from the Cabibbo angle and phase space.

(iii) $\Gamma(M^0 \to \bar{\nu}_\mu \nu_\mu \nu_\mu) + \Gamma(M^0 \to \bar{\nu}_e \nu_e \nu_\mu)$

$$= \tfrac{3}{2}(7776\pi^3)^{-1} G_F^2 M^5 \left(\frac{m_W^2}{\cos^2\xi\, m_Z^2}\right)^2 \Gamma(\mu^-)$$

$$\gtrsim 3.8\times 10^{-2} \left(\frac{M}{m_\mu}\right)^5 \Gamma(\mu^-)\,.$$

(The inequality comes from the upper bound[4] on m_Z.)

(iv) $\Gamma(M^0 \to \nu_\mu \pi^0) = 0$ in leading order.

(v) $\Gamma(M^0 \to \nu_\mu \mu^+\mu^-) = 17\,(M/m_\mu)^5[1+O(R)]\Gamma(\mu)$.

(vi) $\Gamma(M^0 \to \nu_e e^+ \mu^-) = \Gamma(M^0 \to \nu_\mu \mu^+\mu^-)$.

(vii) $\Gamma(M^0 \to \nu_\mu e^+ e^-) = 4R^2 (M/m_\mu)^5 \Gamma(\mu)$.

(viii) $\Gamma(M^0 \to \nu_\mu \gamma)$ is probably negligible.

(ix) $\Gamma(M^0 \to \pi^+\pi^0\mu^-)$ should be somewhat smaller than $\Gamma(M^0 \to \nu_\mu \mu^+\mu^-)$ due to the pion form factor and phase-space considerations.

(x) $\Gamma(M^0 \to K^+ K^0 \mu^-)$ is smaller than $\Gamma(M^0 \to \pi^+\pi^0\mu^-)$ due to phase space.

These values are to be taken as estimates. We have often set $m_\mu = 0$ in phase-space calculations and have assumed that R is small.

For $m_{M^0} \sim \frac{1}{2} m_N$, the first listed decay mode corresponds to a partial lifetime of 7×10^{-12} sec and is by far the most important decay mode.

For $m_{M^0} \sim 1$ GeV, the rates for the decay modes in (i), (v), (vi), and (ix) are all of the order of 10^{-10} MeV. The rate for the mode in (iii) is

$$\sim 2\times 10^{-13}\ \mathrm{MeV} \left(\frac{m_W}{\cos\xi\, m_Z}\right)^4$$

and would not compete even for unreasonably small m_Z. The lifetime comes out to be $\sim 4\times 10^{-13}$ sec.

VI. CONCLUSION

We have presented a renormalizable gauge model of weak and electromagnetic interactions which is within the framework of a satisfactory model of hadron structure, which is in accord with all the known facts, and which makes a number of interesting experimental predictions. Even if the model passed all experimental tests, one would still have to face the problem of why nature chooses to make use of the seemingly *ad hoc* particle assignments of the type used in Sec. II. It is clear that no satisfactory answer to this type of problem will be forthcoming until one has unravelled the mystery of broken hadron symmetries in the context of gauge theories.[32]

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$$\begin{aligned} U(9)_L \otimes U(9)_R &\rightarrow U(1)_B \otimes SU(9)_L \otimes SU(9)_R \\ &\rightarrow U(1)_B \otimes [SU(3)'_L \otimes SU(3)'_R] \otimes [SU(3)_L \otimes SU(3)_R] \\ &\rightarrow U(1)_B \otimes SU(3)' \otimes U(1)_Y \otimes [SU(2)_L \otimes SU(2)_R] \\ &\rightarrow U(1)_B \otimes SU(3)' \otimes U(1)_Y \otimes SU(2) \\ &\rightarrow U(1)_B \otimes U(1)_Q . \end{aligned}$$