# (8,8)-Symmetry Breaking and the Decay Rate $\eta' \to \eta\pi\pi$ †

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The decay $\eta' \to \eta\pi\pi$ is studied, in the framework of the soft-meson current-algebra technique, by assuming that the symmetry-breaking part of the strong-interaction Hamiltonian transforms as the (8, 8) representation of SU(3) ⊗ SU(3). It is found that $\Gamma(\eta' \to \eta\pi\pi)$ varies from 1.2 MeV to 5.9 MeV depending upon the value of the Dalitz-slope parameter $\alpha$.

It has become quite popular to assume that the strong interactions have an approximate Nambu-Goldstone-type SU(3) ⊗ SU(3) symmetry.[1] One could then meaningfully write the strong-interaction Hamiltonian density $H$ as

$$H(x) = H_0(x) + \epsilon H'(x)\,, \tag{1}$$

where $H_0(x)$ is the symmetric part of the Hamiltonian and $|\epsilon| \ll 1$. It is further argued that $\epsilon H'(x)$ may itself be decomposed as

$$\epsilon H'(x) = \epsilon_1 H_1(x) + \epsilon_2 H_2(x)\,, \tag{2}$$

whereby (2) disentangles the two types of symmetry breaking, viz., the SU(3) and SU(3) ⊗ SU(3) breaking jointly characterized by $\epsilon_1 H_1(x)$, and SU(2) ⊗ SU(2) breaking characterized by $\epsilon_2 H_2(x)$. One could, therefore, distinguish two types of theories on the basis of (2): (i) theories which give $\epsilon_1 H_1 \gg \epsilon_2 H_2$ and (ii) those which give $\epsilon_1 H_1 \sim \epsilon_2 H_2$. The smallness of the observed pion mass suggests (i) as the possible chain of SU(3) ⊗ SU(3)-symmetry breaking, but, as pointed out by Dashen,[2] (i) is not required by it. In the first category falls the Gell-Mann–Oakes–Renner[3] (GMOR) model, in which $\epsilon H'$ has the form

$$\epsilon H'(x) = \epsilon_0 U_0(x) + \epsilon_8 U_8(x)\,, \tag{3}$$

where $U_i$ is a scalar-density nonet which, together with the pseudoscalar-density nonet $V_i$, forms the bases of the $(3^*, 3) \oplus (3, 3^*)$ representation of SU(3) ⊗ SU(3). The model suggests that $\epsilon_2 \propto (C + \sqrt{2})$, where $C = \epsilon_0/\epsilon_8$, which together with the value of $C = -1.25$ obtained from the pseudoscalar-meson mass-squared formula leads to $\epsilon_1 \simeq 20\epsilon_2$.[4] This prediction of the GMOR model can be tested experimentally by measuring the matrix elements of a related quantity, the so-called $\Sigma$ term, defined by the commutator

$$\Sigma_{ij} = i[F_i^5, \partial_\mu A_{\mu j}(0)] = \epsilon_2 [F_{i'}^5, [F_j^5, H_2]] \tag{4}$$

[where $A_\mu^i$ is the axial-vector current, $F_j^5 = \int A_{0j}(x)\, d^3x$, and $i, j = 1, 2, 3$].

Such tests have been made by calculating the amplitude in certain meson-meson[5] and meson-baryon[6] scattering processes. Recently, Riazuddin and Oneda[5] have investigated this problem in the strong decay $\eta' \to \eta\pi\pi$ and have shown, using the soft-pion current-algebra technique, that the off-mass-shell decay amplitude

$$T\left(\nu^2 = \nu_0^2 = \left[\frac{m_{\eta'}^2 - m_\eta^2}{2m_{\eta'}}\right]^2,\ \nu_B = 0\right)$$

is related to the matrix element of the $\Sigma$ commutator $\langle \eta | \Sigma^{ij} | \eta' \rangle$ ($\nu$ and $\nu_B$ are the two invariants of the process). Assuming a constant amplitude for the decay over the Dalitz plot, they find that $\Gamma(\eta' \to \eta\pi\pi) \simeq 4 \times 10^{-3}$ MeV. Although the experimental situation regarding $\Gamma(\eta' \to \eta\pi\pi)$ is not very conclusive,[7,8] one naively expects that $\Gamma(\eta' \to \eta\pi\pi) \geq 1$ MeV on the grounds that it is a strong decay which proceeds without angular momentum barrier. In a subsequent paper,[9] the conclusions of Ref. 5 were modified by dropping the zero-Dalitz-slope approximation, and further assuming that the chiral-symmetry limit is also the dilation-symmetry limit and that the latter is realized by a Goldstone boson.[10] Using Gell-Mann's decomposition[11] of the Hamiltonian density

$$H = \hat{H}_0 + (\epsilon_0 U_0 + \epsilon_8 U_8) + \delta \tag{5}$$

[where $\hat{H}_0$ is both scale-invariant and an SU(3) ⊗ SU(3) scalar, and $\delta$ breaks scale invariance but is an SU(3) ⊗ SU(3) scalar] and taking $\delta$ = constant, it is found that $\Gamma(\eta' \to \eta\pi\pi) \lesssim 0.2$ MeV (depending on the scale dimension of $\epsilon H'$). We observe that even this estimate is an order of magnitude smaller than the experimental upper limit.[7,8] Also, there is no convincing experimental evidence for the so-called dilaton.

It is, therefore, tempting to attempt models which correspond to the case (ii) described earlier. In this context, it has been pointed out by Barnes and Isham[12] that a worthy candidate is the representation (8, 8) of the group SU(3) ⊗ SU(3). Apart from predicting $\epsilon_1 H_1 \sim \epsilon_2 H_2$, this particular symmetry-breaking scheme has the added attraction that no new operators, unlike $U_i$ and $V_i$ of the GMOR model, need be introduced in the already existing set of operators of the group SU(3) $\otimes$ SU(3).[13] The purpose of this note is to test the (8, 8) model by obtaining an estimate of the pseudoscalar-meson–pseudoscalar-meson matrix elements of the $\Sigma$ commutator.[14,15] The process we study is again the decay $\eta' \to \eta\pi\pi$. We use the low-energy current-algebra result of Ref. 9 in relating the amplitude $T(\nu, \nu_B)$ at an unphysical point with the matrix elements of the $\Sigma$ commutator, and also assume the same linear extrapolation of the off-shell invariant amplitude $T(\nu, \nu_B)$. Applying the low-energy theorem to the pseudoscalar-meson matrix elements of $\epsilon H'$ and using approximate SU(3) symmetry, applied to the vertices only, we obtain for $\Gamma(\eta' \to \eta\pi\pi)$ a value[16] which varies from 1.2 MeV to 5.9 MeV, depending upon the value of the Dalitz-slope parameter, $\alpha$.[17]

First, we briefly mention the operator algebra of the (8, 8) model. The Barnes-Isham Hamiltonian[18] is

$$\epsilon H'(x) = S_{\alpha\alpha}(x) + 2\sqrt{3}Cd_{8\alpha\beta}S^{\alpha\beta}(x) \qquad (\alpha, \beta = 1, \ldots, 8), \quad (6)$$

where $S_{\alpha\beta}(x)$ are the Schwinger terms which appear in the commutator

$$[L_0^\alpha(x), R_i^\beta(y)]_{x_0=y_0} = iS^{\alpha\beta}(\vec{y})\partial_i\delta^3(\vec{x}-\vec{y}),$$
$$L_\mu^\alpha = \tfrac{1}{2}(V_\mu^\alpha + A_\mu^\alpha), \quad (7)$$
$$R_\mu^\alpha = \tfrac{1}{2}(V_\mu^\alpha - A_\mu^\alpha)$$

($V_\mu^\alpha$ and $A_\mu^\alpha$ are the octet vector and axial-vector currents). Under parity transformation

$$P(S_{\alpha\beta})P^{-1} = S_{\beta\alpha}. \quad (8)$$

The Hamiltonian (6) conserves parity and isospin and exhibits octet dominance in SU(3) breaking.[12] Under SU(3)$\otimes$SU(3), $S_{\alpha\beta}(x)$ transforms as

$$[F_\alpha^+, S_{\beta\gamma}(x)] = if_{\alpha\beta\delta}S^{\delta\gamma}(x),$$
$$[F_\alpha^-, S_{\beta\gamma}(x)] = if_{\alpha\gamma\delta}S^{\beta\delta}(x), \quad (9)$$

where $F_\alpha^\pm = \frac{1}{2}(F_\alpha \pm F_\alpha^5)$.

The model reproduces the (meson mass)$^2$ formula and gives a ratio of SU(3) to SU(3)$\otimes$SU(3) breaking almost the same $[\sim -(\sqrt{5}/2)\times 1.25]$ as that given by the GMOR model.[18] However,[19] the divergence of the axial-vector current is given by

$$\partial_\mu A_\mu^\alpha(x) = -2(1-C)f_{\alpha\beta\delta}S^{\beta\delta} - 3Cf_{\alpha p\delta}(S^{p\delta} - S^{\delta p}) + Cf_{\alpha 8\delta}(S^{8\delta} - S^{\delta 8}) \qquad (p = 1, 2, 3). \quad (10)$$

From (10) it is obvious that $A_\mu^i$ is not conserved for any value of $C$. Thus SU(2)$\otimes$SU(2) symmetry is broken the moment SU(3)$\otimes$SU(3) is broken.[20]

The $\Sigma$ commutator is given by

$$\Sigma^{\alpha\beta} = 2(1-C)f_{\beta\gamma\delta}(f_{\alpha\gamma\eta}S^{\eta\delta} - f_{\alpha\delta\eta}S^{\gamma\eta}) + 3Cf_{\beta p\delta}[f_{\alpha p\eta}(S^{\eta\delta} + S^{\delta\eta}) - f_{\alpha\delta\eta}(S^{p\eta} + S^{\eta p})] - Cf_{\beta 8\delta}[f_{\alpha 8\eta}(S^{\eta\delta} + S^{\delta\eta}) - f_{\alpha\delta\eta}(S^{8\eta} + S^{\eta 8})], \quad (11)$$

which reduces to the simple form in the SU(2) $\otimes$ SU(2) case

$$\Sigma^{ij} = \delta_{ij}[(1-C)(S^{\alpha\alpha} - S^{88}) + \tfrac{1}{3}(5+19C)S^{pp}] - 2(1+2C)(\delta_{il}\delta_{jp} + \delta_{ip}\delta_{jl} - \tfrac{2}{3}\delta_{ij}\delta_{lp}). \quad (12)$$

The expression (12) manifests the $I=0$ and $I=2$ components of $\Sigma^{ij}$. However, there is no $I=2$ contribution to the matrix element $\langle\eta|\Sigma^{ij}|\eta'\rangle$ that we are considering.

We now go on to consider the process $\eta'(p') \to \eta(p) + \pi_i(k_1) + \pi_j(k_2)$ with $p' = p + k_1 + k_2$. The invariants of the reaction are

$$\begin{aligned} \nu &= -(k_2 - k_1)\cdot(p'+p)/4m_{\eta'} \\ &= \nu_0 + \frac{k_1\cdot(p+p')}{2m_{\eta'}} \\ &= -\nu_0 - k_2\cdot(p+p')/2m_{\eta'}, \\ \nu_0 &= (m_{\eta'}^2 - m_\eta^2)/4m_{\eta'}, \\ \nu_B &= -k_1\cdot k_2/2m_\eta. \end{aligned} \quad (13)$$

The standard current-algebra results are[5]

$$\begin{aligned} F^{ij}(\nu=\nu_0, \nu_B=0, k_1^2=0, k_2^2=0) &= F^{ij}(\nu=-\nu_0, \nu_B=0, k_1^2=0, k_2^2=0) \\ &= -\frac{2}{f_\pi^2}\langle\eta(p)|\Sigma^{ij}|\eta'(p')\rangle(2\pi)^3(4p_0p_0')^{-1/2}, \end{aligned} \quad (14)$$

where the first equality corresponds to the limit $k_1 \to 0$, $k_2^2 \to 0$, and the second to $k_2 \to 0$, $k_1^2 \to 0$. Also

$$\begin{aligned} F^{ij}(\nu, \nu_B, k_1^2 = -m_\pi^2 = k_2^2) &= -i\langle\eta(p)\pi^i(k_1)\pi^j(k_2)|S|\eta'(p')\rangle \\ &= (2\pi)^{-3}(4p_0p_0')^{-1/2}T^{ij}(\nu, \nu_B) \\ &= (2\pi)^{-3}(4p_0p_0')^{-1/2}T(\nu, \nu_B)\delta^{ij} \end{aligned} \quad (15)$$

and

$$\partial_\mu A_\mu^i(x) = \frac{f_\pi}{\sqrt{2}}m_\pi^2\phi_\pi^i(x). \quad (16)$$

Note that the Schwinger terms do not contribute in the low-energy limit employed in the derivation of (14). The Adler zeros[21] are

$$\begin{aligned} T(\nu=\nu_0, \nu_B=0, k_1^2=0, k_2^2=-m_\pi^2) &= 0 \\ &= T(\nu=-\nu_0, \nu_B=0, k_1^2=-m_\pi^2, k_2^2=0). \end{aligned} \quad (17)$$

Following Ref. 5, we rewrite Eqs. (14) and (17) as

$$T(\nu^2=\nu_0^2,\ \nu_B=0,\ k_1^2=k_2^2=0)=-\frac{2}{f_\pi^2}\sigma_{\eta\eta'}, \tag{18}$$

$$\begin{aligned} T(\nu^2=\nu_0^2,\ \nu_B=0,\ k_1^2=0,\ k_2^2=-m_\pi^2) &= 0 \\ &= T(\nu^2=\nu_0^2,\ \nu_B=0,\ k_1^2=-m_\pi^2,\ k_2^2=0), \end{aligned} \tag{19}$$

where

$$\sigma_{\eta\eta'}=\langle\eta(p)|[(1-C)(S^{\alpha\alpha}-S^{88}) +\tfrac{1}{3}(5+19C)S^{pp}]|\eta'(p')\rangle. \tag{20}$$

Employing the same extrapolation as used in Ref. 5 in the variables $k_1^2, k_2^2, \nu_B$ (with $k_1^2, k_2^2$ in the region $0 \gtrsim k_1^2, k_2^2 \gtrsim -m_\pi^2$, and with $\nu, \nu_B$ in the neighborhood of the physical region), we write the off-mass-shell amplitude as

$$T(\nu,\nu_B,k_1^2,k_2^2)=a+b(k_1^2+k_2^2)+e\nu_B. \tag{21}$$

Using Eqs. (18) and (19), one gets for the on-mass-shell amplitude

$$T(\nu,\nu_B)=-\frac{2}{f_\pi^2}\sigma_{\eta\eta'}+e\nu_B. \tag{22}$$

From the Dalitz-plot consideration, one may express the physical amplitude as

$$T(\nu,\nu_B)=M(1+\alpha Y), \tag{23}$$

with

$$Y=-1+\frac{2(m_\eta+m_{\eta'})}{m_\pi(m_{\eta'}-m_\eta-2m_\pi)} \times\left[\frac{(m_{\eta'}-m_\eta)^2-2m_\pi^2}{4m_{\eta'}}-\nu_B\right]; \tag{24}$$

we have

$$M=\frac{2}{f_\pi^2}\frac{\sigma_{\eta\eta'}}{1+2\alpha}. \tag{25}$$

Employing the relativistic phase-space calculation of Osborn and Wallace,[22] one gets[9]

$$\Gamma(\eta'\to\eta\pi\pi)=3(1.00+0.24\alpha+0.27\alpha^2)|M|^2\ \text{keV}. \tag{26}$$

Finally we give our estimates for $\sigma_{\eta\eta'}$. We define the following decomposition for the pseudoscalar-meson–pseudoscalar-meson matrix elements of the operator $S^{\rho\sigma}$:

$$\begin{aligned} \langle P^\alpha(p)|S^{\rho\sigma}(0)|P^\gamma(p')\rangle = {} & \frac{A_1(t)}{8}\delta_{\rho\sigma}\delta_{\alpha 0}\delta_{\gamma 0}+A_2(t)\delta_{\rho\sigma}\delta_{\alpha\gamma}+A_3(t)d_{\beta\rho\sigma}d_{\beta\alpha\gamma} \\ & +A_4(t)(\delta_{\alpha 0}d_{\gamma\rho\sigma}+\delta_{\gamma 0}d_{\alpha\rho\sigma})+A_5(t)\xi_{\theta\rho\sigma}\xi_{\theta\alpha\gamma} \\ & (\alpha,\gamma=0,\dots,8;\ \beta,\rho,\sigma=1,\dots,8;\ \theta=1,\dots,27). \end{aligned} \tag{27}$$

Here, $t=-(p-p')^2$ and $\xi_{\theta\rho\sigma}$ are the Clebsch-Gordan coefficients of the 27-plet representation of SU(3). With this parametrization and defining the $\eta$ and $\eta'$ states as

$$\eta=p\eta_8+q\eta_0,\quad \eta'=q\eta_8-p\eta_0,$$

with $p^2+q^2=1$, we get the following expression for $\sigma_{\eta\eta'}$:

$$\begin{aligned} \sigma_{\eta\eta'}=-pq\Big[&\left(\frac{3A_1}{2}+2A_3+24(\tfrac{2}{3})^{1/2}A_4+\tfrac{12}{40}A_5\right) \\ &+C(\tfrac{3}{2}A_1+6A_3+24(\tfrac{2}{3})^{1/2}A_4-\tfrac{84}{40}A_5)\Big] \\ &+2\sqrt{3}(q^2-p^2)(1+3C)[(\tfrac{2}{3})^{1/2}A_3+A_4]. \end{aligned} \tag{28}$$

Taking the low-energy limit in (27) and neglecting the $t$ dependence of the invariants, as in Refs. 3 and 15, we have

$$\begin{aligned} A_2\delta_{\rho\sigma}\delta_{\alpha\gamma}+A_3d_{\beta\rho\sigma}d_{\beta\alpha\gamma}+A_5\xi_{\theta\sigma\gamma}\xi_{\theta\alpha\gamma} &= -i\frac{\sqrt{2}}{F_\gamma}\langle P_\alpha(p)|[F_5{}^\gamma,S^{\rho\sigma}(0)]|0\rangle \\ &= -i\frac{\sqrt{2}}{F_\alpha}\langle 0|[F_5{}^\alpha,S^{\rho\sigma}(0)]|p^\gamma(p')\rangle, \end{aligned} \tag{29}$$

where we shall use the low-energy theorem only for $\alpha=1,\dots,7$. Parametrizing the pseudoscalar-meson-to-vacuum matrix elements of $S^{\rho\sigma}$ as

$$\langle P^\alpha|S^{\rho\sigma}|0\rangle=yf^{\alpha\rho\sigma} \tag{30}$$

and comparing the coefficient of $\delta_{\rho\sigma}\delta_{\alpha\gamma}$, etc., we get from Eq. (29)

$$\begin{aligned} A_2 &= -3\sqrt{2}\frac{y}{F}, \\ A_3 &= \frac{-9\sqrt{2}}{5}\frac{y}{F}, \\ A_5 &= 2\sqrt{2}\frac{y}{F}, \end{aligned} \tag{31}$$

where

$$F_\pi\approx F_K=F. \tag{32}$$

Further, using Eq. (6) for $\epsilon H'$ and the decomposition (27), the pseudoscalar meson masses are

$$m_\pi^2=8A_2+\tfrac{10}{3}CA_3,$$

$$m_K^2=8A_2-\tfrac{5}{3}CA_3,$$

$$m_{\eta_8}{}^2 = 8A_2 - \tfrac{10}{3}CA_3 \equiv X\,, \tag{33}$$

$$m_{\eta_0}{}^2 = m_0{}^2 + A_1 + 8A_2 + 16(\tfrac{2}{3})^{1/2}A_4 \equiv Y\,,$$

$$m_{\eta_0-\eta_8}{}^2 = \frac{10C}{\sqrt{3}}[A_4 + (\tfrac{2}{3})^{1/2}A_3] \equiv Z\,.$$

The first three equations give the Gell-Mann–Okubo mass formula

$$3m_{\eta_8}{}^2 - 4m_K{}^2 + m_\pi{}^2 = 0\,.$$

Diagonalization gives

$$m_\eta{}^2 + m_{\eta'}{}^2 = m_{\eta_8}{}^2 + m_{\eta_0}{}^2$$

$$= X + Y\,,$$

$$Z(q^2 - p^2)(qp)^{-1} = Y - X\,, \tag{34}$$

$$m_{\eta'}{}^2 - m_\eta{}^2 = -Z/pq\,.$$

Also

$$Z^2 = \tfrac{4}{3}(m_K{}^2 - m_\eta{}^2)^2[(\tfrac{2}{3})^{1/2} + A_4/A_3]^2$$

$$= (m_{\eta_8}{}^2 - m_\eta{}^2)(m_{\eta'}{}^2 - m_{\eta_8}{}^2)\,. \tag{35}$$

Putting $A_4/A_3 = -1/\sqrt{6}$, one gets

$$\tfrac{2}{9}(m_K{}^2 - m_\pi{}^2)^2 = (m_{\eta_8}{}^2 - m_\eta{}^2)(m_{\eta'}{}^2 - m_{\eta_8}{}^2)\,,$$

which is satisfied for $m_{\eta'} = 960$ MeV. Combining Eqs. (31) and (33), one gets

$$A_2 = \frac{1}{12C}(m_\pi{}^2 - m_K{}^2)\,,$$

$$A_3 = \frac{1}{5C}(m_\pi{}^2 - m_K{}^2) = -\sqrt{6}A_4\,, \tag{36}$$

$$A_5 = \frac{2}{9C}(m_K{}^2 - m_\pi{}^2)\,,$$

$$C = \frac{2(m_\pi{}^2 - m_K{}^2)}{m_\pi{}^2 + 2m_K{}^2} \approx -0.885\,.$$

Also

$$m_0{}^2 = m_\eta{}^2 + m_{\eta'}{}^2 - \frac{2m_K{}^2 - m_\pi{}^2}{3}$$

$$+ \tfrac{1}{5}(m_\pi{}^2 - 2m_K{}^2) - A_1\,. \tag{37}$$

Putting all these pieces in Eq. (28) one gets

$$\sigma_{\eta\eta'} = \frac{m_\pi{}^2 - m_K{}^2}{15\sqrt{2}\,C(m_{\eta'}{}^2 - m_\eta{}^2)}$$

$$\times\{\tfrac{20}{3}(1-C)(m_K{}^2 - m_\pi{}^2)$$

$$+ 15C(1+C)[(m_{\eta'}{}^2 + m_\eta{}^2) - \tfrac{1}{3}(4m_K{}^2 - 3m_\pi{}^2)]$$

$$+ 6(1+3C)[\tfrac{2}{3}(4m_K{}^2 - m_\pi{}^2) - (m_{\eta'}{}^2 + m_\eta{}^2)]\}\,. \tag{38}$$

Substituting the value of $C$ from Eq. (36) and the masses, one gets

$$\sigma_{\eta\eta'} = 0.019(7.77 + 1.52m_0{}^2)\,. \tag{39}$$

Since $A_1$ would be of the order of mass splittings within the pseudoscalar-meson octets,[5] we have $|A_1| \simeq 0.04$ to $0.1$ GeV$^2$, giving $m_0{}^2 \simeq 1.1$ to $0.9$ GeV$^2$.

Using a value of $m_0{}^2 = 1$ GeV$^2$ and the slope of the Dalitz plot $\alpha = -0.28$, we get

$$\Gamma(\eta' \to \eta\pi\pi) = 5.9 \text{ MeV}\,. \tag{40}$$

However, the decay rate depends very sensitively on the slope parameter $\alpha$, as can be seen by Eq. (25). Also, the uncertainty on this number is quite large, $\alpha = -0.28 \pm 0.06$, resulting in a considerable variation in the prediction for the $\eta' \to \eta\pi\pi$ decay rate. If one takes the extreme view that $\alpha = 0$, then

$$\Gamma(\eta' \to \eta\pi\pi) \simeq 1.2 \text{ MeV}\,.$$

The experimental situation for this quantity is not very conclusive. The recent experimental number[8] for the total width of $\eta'$ is

$$\Gamma(\eta' \to \text{all}) < 1.9 \text{ MeV}\,,$$

which gives

$$\Gamma(\eta' \to \eta\pi\pi) < 1.3 \text{ MeV}$$

for a branching ratio of

$$\frac{\Gamma(\eta' \to \eta\pi\pi)}{\Gamma(\eta' \to \text{all})} = (68 \pm 2.2)\%\,.$$

A precise determination of $\alpha$ is therefore very desirable. If future experiments do not confirm the appreciable enhancement in the number of events for higher dipion invariant mass (as suggested by the large value $\alpha = -0.28 \pm 0.06$), then the (8, 8)-symmetry breaking has a chance to work. On the other hand, if $\alpha$ remains essentially unchanged, then the (8, 8) model would be an unsatisfactory description of $\eta' \to \eta\pi\pi$. It may be that $\epsilon H'$ has both a $(3, 3^*) \oplus (3^*, 3)$ and (8, 8) admixture.

*Note added in proof.* A recent report on the decay $\eta' \to \eta\pi\pi$ from the Brookhaven group has been brought to our notice. J.S. Danburg *et al.* [in *Experimental Meson Spectroscopy—1972,* Proceedings of the Third International Conference, Philadelphia, 1972, edited by Kwan-Wu Lai and Arthur H. Rosenfeld (A.I.P., New York, 1972)] have reported the following values for the asymmetry parameter $\alpha$:

$$\alpha = -0.046^{+0.040}_{-0.039} \quad (\eta' \to \pi^+\pi^-\eta_N)\,,$$

$$\alpha = -0.08 \pm 0.08 \quad (\eta' \to \pi^+\pi^-\eta_C)\,,$$

i.e., close to uniform distribution. This gives a value for $\Gamma(\eta' \to \eta\pi\pi)$ which is well within the upper bound of Ref. 8. The case of the (8, 8) model as a possible candidate for the chiral symmetry breaking seems to be strengthened.

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[16]Our results depend upon the parameter $m_0^2$ introduced so that the mass of the SU(3)-singlet (ninth) pseudoscalar meson need not go to zero in the SU(3) ⊗ SU(3) symmetry limit. However, the dependence is not crucial to our conclusions. Also, we have used $f_\pi = 0.134$ GeV. The Goldberger-Treiman value $f_\pi = 0.124$ GeV would enhance the predicted width by 40%.

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[18]K. J. Barnes and C. J. Isham use $H_B = (A/\sqrt{8})\, S^{\alpha\alpha} + (\frac{3}{5})^{1/2} B d_{8\alpha\beta} S^{\alpha\beta}$ and the normalization is adjusted so that the ratio of $B$ to $A$ is the ratio of SU(3) to SU(3) ⊗ SU(3) breaking.

[19]Here and subsequently, latin indices will run over only to 1 to 3. Also, we have used $d_{8\alpha\beta} = (1/2\sqrt{3})(-\delta_{\alpha\beta} + 3\delta_{\alpha p}\delta_{\beta p} - \delta_{\alpha 8}\delta_{\beta 8})$.

[20]B. Renner and A. Sudbery, Nucl. Phys. B13, 27 (1967).

[21]See, for example, S. L. Adler and R. F. Dashen, *Current Algebras and Applications to Particle Physics* (Benjamin, New York, 1968), where references to original literature can be found.

[22]H. Osborn and D. J. Wallace, Nucl. Phys. B20, 23 (1970). Earlier references for the decay $\eta' \to \eta\pi\pi$ may be found here.