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# Unified Lepton-Hadron Symmetry and a Gauge Theory of the Basic Interactions*

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An attempt is made to unify the fundamental hadrons and leptons into a common irreducible representation $F$ of the same symmetry group $G$ and to generate a gauge theory of strong, electromagnetic, and weak interactions. Based on certain constraints from the hadronic side, it is proposed that the group $G$ is $SU(4')\times SU(4'')$, which contains a Han-Nambu-type $SU(3')\times SU(3'')$ group for the hadronic symmetry, and that the representation $F$ is $(4,4^*)$. There exist four possible choices for the lepton number $L$ and accordingly four possible assignments of the hadrons and leptons within the $(4,4^*)$. Two of these require nine Han-Nambu-type quarks, three "charmed" quarks, and the observed quartet of leptons. The other two also require the nine Han-Nambu quarks, plus heavy leptons in addition to observed leptons and only one or no "charmed" quark. One of the above four assignments is found to be suitable to generate a gauge theory of the weak, electromagnetic, and $SU(3'')$ gluonlike strong interactions from a selection of the gauges permitted by the model. The resulting gauge symmetry is $SU(2')_L\times U(1)\times SU(3'')_{L+R}$. The scheme of all three interactions is found to be free from Adler-Bell-Jackiw anomalies. The normal strong interactions arise effectively as a consequence of the strong gauges $SU(3'')_{L+R}$. Masses for the gauge bosons and fermions are generated suitably by a set of 14 complex Higgs fields. The neutral neutrino and $\Delta S=0$ hadron currents have essentially the same strength in the present model as in other $SU(2)_L\times U(1)$ theories. The mixing of strong- and weak-gauge bosons (a necessary feature of the model) leads to parity-violating nonleptonic amplitudes, which may be observable depending upon the strength of $SU(3'')$ symmetry breaking. The familiar hadron symmetries such as $SU(3')$ and chiral $SU(3')_L\times SU(3')_R$ are broken only by quark mass terms and by the electromagnetic and weak interactions, not by the strong interactions. The difficulties associated with generating gauge interactions in the remaining three assignments are discussed in Appendix A. Certain remarks are made on the question of proton and quark stability in these three schemes.

## I. INTRODUCTION

Several past attempts[1] including some of the recently proposed schemes of unification[2,3] of the weak and electromagnetic interactions have emphasized the parallelism between the hadrons and leptons in the sense that both types of matter belong to similar *(but distinct)* multiplets of the same group structure[1,4] [e.g., $SU(2)\times U(1)$ or $SU(4)$, etc.]. As a further step toward a unified theory of matter, one may inquire whether the fundamental hadrons and the leptons could be considered as belonging to the *same* irreducible representation of a *common* symmetry group for all matter. It of course goes without saying that such a supersymmetry group has to be badly broken to correspond to the huge mass gap between the hadrons and the leptons and to their asymmetric response toward the strong interactions.

Nevertheless, if we postulate that such a symmetry group is the basis for the existence of hadrons *and* leptons, it may in the first place lead to a tighter classification scheme for the leptons not available at present. This could provide a rationale for the existence of known leptons and also throw some light on further leptons to be expected. The existence of the lepton number $L$ (or baryon number $B$) would derive an interpretation similar to that of $I_3$ and $Y$, being part of a hierarchy of higher symmetry rather than due to an *ad hoc* $U(1)$ symmetry. Such a combined symmetry is also expected to provide a well-defined basis for hadron-lepton universality in the weak and electromagnetic currents.

In the second place, the full dynamical content of such a supersymmetry group may provide new insights. For example, a *full* gauge theory of interactions based on such a symmetry group may lead to a unified treatment of the strong as well as weak electromagnetic interactions in the sense that a single coupling constant may govern the non-Abelian part of the interactions. The empirical difference between the value of the electromagnetic charge and the strong interaction coupling constant may, in such a scheme, be expected to arise from differing charge renormalizations produced by the spontaneous symmetry breaking mechanisms, which are a necessary feature of such theories. A related consequence is the possibility of a whole new class of interactions involving gauge mesons which carry baryon and lepton number. The absence of such interactions in the present energy domain may be attributable to superheavy masses of such gauge mesons, consistent with current ideas of spontaneous symmetry breaking.

With these to serve as motivation, we attempt to make a beginning toward a unified treatment of the hadrons and leptons. In Sec. II we discuss the question of the choice of the symmetry group $G$ and the representation $F$ for fundamental hadronic and leptonic matter. Based on certain constraints from the hadron's side, it is proposed that the relevant group for the *classification* of fundamental hadrons and leptons is $SU(4')\times SU(4'')$, which contains a Han-Nambu-type[5] $SU(3')\times SU(3'')$ group for the hadronic symmetry, and that the representation $F$ is $(4, 4^*)$. It is then pointed out that there exist four distinct possibilities for the assignment of the leptons within the $(4, 4^*)$. Two of these require nine Han-Nambu-type "normal" quarks, three "charmed" quarks, and just the observed quartet of leptons (but no heavy leptons). The other two also require nine Han-Nambu quarks, but they require extra leptons and correspondingly one or no charmed quark.

In Sec. III, we construct a gauge theory of weak, electromagnetic, and $SU(3'')$ gluonlike strong interactions for *one* of the four assignments mentioned above. We restrict ourselves in this note, however, to only a selected set of gauges so as to reproduce primarily the conventional phenomena of interactions for this assignment. (The other three assignments seem to be unsuitable for a variety of reasons discussed in Appendix A.) The symmetry group for the gauge interactions thus obtained is $SU(2')_L\times U(1)\times SU(3'')_{L+R}$. *The scheme, including the strong interactions, is found to be free from Adler-Bell-Jackiw anomalies.* The existence of the two SU(4) degrees of freedom for the $(4, 4^*)$ is found to be essential in realizing this scheme.

The model thus generated is conservative in the sense that it does not exploit all the gauge symmetries permitted by the model. By the same token, no attempt is made to build in universality of the strong, weak, and electromagnetic interactions in the sense mentioned before. We hope to return to these considerations in subsequent work.

The question of generating masses for the gauge mesons and the $(4, 4^*)$ fermions via spontaneous symmetry breaking is briefly discussed in Sec. IV. A mass matrix with desired properties is constructed with 14 complex Higgs fields. It is pointed out that the mass matrix must necessarily mix some of the strong-gauge mesons with the weak ones in order to generate the photon. This may lead to observable effects in parity-violating nuclear transitions depending upon a few factors. The neutral neutrino and $\Delta S=0$ hadron currents are found to have essentially the same strength in the present model as in other $SU(2)_L\times U(1)$ theories[3,4] of weak and electromagnetic interactions. Remarks on the nature of various symmetries in the model are made in Sec. IV. Section V contains a summary of a few peculiar features of the scheme. The difficulties associated with generating gauge interactions in three out of the four lepton assignments are discussed in Appendix A. The possibility of baryon- and lepton-number–violating transitions in these cases and their implications on quark and proton stability are briefly mentioned. In Appendix B we consider the consequences of assuming that the idea of Cabibbo rotation applies to the $(e, \mu)$ leptons as it does to the $(\mathfrak{N}, \lambda)$ hadrons. It is remarked that the two-neutrino experiment (rather than absence of $\mu\to e+\gamma$ decay) would provide a test of whether the universality of hadrons and leptons should be carried this far.

## II. CHOICE OF THE SYMMETRY GROUP AND THE BASIC REPRESENTATION

We shall start with the basic assumption that the representation $F$ of the fundamental entities consists of quarks and at least the known leptons. This excludes the hypothesis that the observed leptons may be composites of something even more fundamental.[6] So far as quarks are concerned, it appears that there are several considerations, which favor nine degrees of freedom for the quarks rather than three. Apart from integral charges, these are (a) the experimental value for the magnitude of the $\pi^0\to 2\gamma$ amplitude[7]; (b) the desirability of having Fermi statistics for the 56-plet of observed baryons, with quarks in relative $s$ states; and (c) the question of the $\Delta T=\frac{1}{2}$ rule[8] for the nonleptonic decays. To be more specific, we will assume[9] that the basic hadrons are the Han-Nambu[5] quarks transforming as $(3, 3^*)$ under the $SU(3')\times SU(3'')$ group; for our discussion the familiar SU(3) group may be identified with $SU(3')$. The quarks will carry integral charges if we assume

$$Q=(I_3'+\tfrac{1}{2}Y')+(I_3''+\tfrac{1}{2}Y'')\,. \tag{1}$$

The low-lying baryons and mesons are assumed to be singlets under SU(3″), thus they have $I_3''=Y''=0$.

Clearly the supergroup $G$ we are seeking must contain the hadronic SU(3′)×SU(3″) group as a subgroup while the lepton number must be a diagonal generator within $G$ outside of SU(3′)×SU(3″). The basic representation $F$ of the supergroup $G$ must contain at least the Han-Nambu nonet of hadrons and the observed quartet of leptons. It is easy to convince oneself that the simplest group structure[10] meeting these constraints is provided by

$$G=\mathrm{SU}(4')\times\mathrm{SU}(4'')\,, \tag{2}$$

with the fundamental fermion (hadron and lepton) representation being given by

$$F=(4,4^*)\,. \tag{3}$$

Denote the components of $F$ by $\psi_{(\alpha,i)}$; the group SU(4′) acts on the index $\alpha=\mathcal{P},\mathfrak{N},\lambda,\chi$, and the group SU(4″) acts on the index $i=a,b,c,d$. The traceless matrices $I_3'$, $Y'$, and $C'$ for the diagonal generators of SU(4′) with respect to the indices $\alpha$ may then be defined by

$$I_3'=\tfrac{1}{2}\begin{array}{c}\begin{array}{cccc}\mathcal{P}&\mathfrak{N}&\lambda&\chi\end{array}\\ \begin{pmatrix}1&&&\\&-1&&\\&&0&\\&&&0\end{pmatrix}\end{array},\quad Y'=\tfrac{1}{3}\begin{array}{c}\begin{array}{cccc}\mathcal{P}&\mathfrak{N}&\lambda&\chi\end{array}\\ \begin{pmatrix}1&&&\\&1&&\\&&-2&\\&&&0\end{pmatrix}\end{array},$$

$$C'=\tfrac{1}{4}\begin{array}{c}\begin{array}{cccc}\mathcal{P}&\mathfrak{N}&\lambda&\chi\end{array}\\ \begin{pmatrix}1&&&\\&1&&\\&&1&\\&&&-3\end{pmatrix}\end{array}, \tag{4}$$

where $C'$ is the charm quantum number of SU(4′). The matrices $I_3''$, $Y''$, and $C''$ for the diagonal generators of SU(4″) *(with respect to the indices i)* may be defined similarly. They are *negatives* of $I_3'$, $Y'$, and $C'$, since they correspond to the 4* representation. Thus,

$$(I_3'')_i=-(I_3')_\alpha\,,\quad (Y'')_i=-(Y')_\alpha$$

and (5)

$$(C'')_i=-(C')_\alpha\,.$$

A suitable extension of the charge formula (1), which gives integer charges to all constituent particles, is given by

$$Q=(I_3'+\tfrac{1}{2}Y'-\tfrac{2}{3}C')+(I_3''+\tfrac{1}{2}Y''-\tfrac{2}{3}C'')\,. \tag{6}$$

One may exhibit $\psi$ by a (4×4) array, the superscript on each component designating the charge:

$$\Psi=\begin{bmatrix}\mathcal{P}_a^0&\mathcal{P}_b^+&\mathcal{P}_c^+&\mathcal{P}_d^0\\ \mathfrak{N}_a^-&\mathfrak{N}_b^0&\mathfrak{N}_c^0&\mathfrak{N}_d^-\\ \lambda_a^-&\lambda_b^0&\lambda_c^0&\lambda_d^-\\ \chi_a^0&\chi_b^+&\chi_c^+&\chi_d^0\end{bmatrix}. \tag{7}$$

The 3×3 top left array denotes the Han-Nambu nonet of hadronic quarks, which we denote subsequently by $\mathcal{K}$.

There appear, in general, four distinct possibilities[11] for the choice of the lepton number $L$:

$$\begin{aligned}L&=(C''+\tfrac{1}{4}) &&\text{(A)}\\ &=(C'-\tfrac{1}{4}) &&\text{(B)}\\ &=(C''+\tfrac{1}{4})+(C'-\tfrac{1}{4}) &&\text{(C)}\\ &=(C''+\tfrac{1}{4})-(C'-\tfrac{1}{4})\,. &&\text{(D)}\end{aligned} \tag{8}$$

Accordingly there are four possible assignments of the hadrons and leptons in the (4, 4*). These are

$$\Psi_A=\begin{bmatrix}\begin{matrix}\mathcal{K}\\(L=0)\end{matrix} & \left.\begin{matrix}\nu_e\\e^-\\\mu^-\\\nu_\mu\end{matrix}\right\}(L=1)\\ \begin{matrix}h^0\ h^+\ h'^+\\(L=0)\end{matrix} & \end{bmatrix},$$

$$\Psi_B=\begin{bmatrix}\begin{matrix}\mathcal{K}\\(L=0)\end{matrix} & \left.\begin{matrix}H^0\\H^-\\H'^-\end{matrix}\right\}(L=0)\\ \begin{matrix}\bar\nu_e\ e^+\ \mu^+\\(L=-1)\end{matrix} & \bar\nu_\mu\end{bmatrix},$$

$$\Psi_C=\begin{bmatrix}\begin{matrix}\mathcal{K}\\(L=0)\end{matrix} & \left.\begin{matrix}\nu_\mu\\\mu^-\\M^-\end{matrix}\right\}(L=1)\\ \begin{matrix}\bar\nu_e\ e^+\ E^+\\(L=-1)\end{matrix} & \begin{matrix}R^0\\(L=0)\end{matrix}\end{bmatrix}, \tag{9}$$

$$\Psi_D=\begin{bmatrix}\begin{matrix}\mathcal{K}\\(L=0)\end{matrix} & \left.\begin{matrix}\nu_\mu\\\mu^-\\M^-\end{matrix}\right\}(L=1)\\ \begin{matrix}\bar\nu_e\ e^+\ E^+\\(L=-1)\end{matrix} & \begin{matrix}\delta^0\\(L=2)\end{matrix}\end{bmatrix}.$$

In each case $\mathcal{K}$ stands for the 3×3 array of the Han-Nambu quarks. Note that:

(1) All four schemes accommodate at least the

known quartet of leptons and allow for the distinction between $\nu_e$ and $\nu_\mu$, and $e$ and $\mu$ via SU(4′) and/or SU(4″) quantum numbers.

(2) Schemes (A) and (B) contain *just the observed quartet of leptons* and no further leptons, while (C) and (D) contain new leptons in addition.

(3) Schemes (A) and (B) look so similar that one may not regard them as distinct. However, once we choose the convention that the low-lying hadrons are singlets with respect to SU(3″) and nonsinglet in general with respect to SU(3′), then (A) and (B) lead to physically distinct schemes of gauge interactions (compare Sec. III and Appendix A).

(4) In addition to the basic Han-Nambu nonet, schemes (A) and (B) consist of three charmed hadrons, called $(h^0, h^+, h'^+)$ and $(H^0, H^-, H'^-)$, respectively, while scheme (C) contains one such charmed hadron, the $R^0$. These objects are presumably much heavier than their uncharmed partners and do not play any role in the formation of the low-lying hadrons.

(5) In each of these four schemes, one may introduce a U(1) symmetry to extend the SU(4′) or the SU(4″) to a U(4′) or U(4″) group, respectively. The corresponding U(1) symmetry may be identified with *fermion-number conservation*. One may assign the fermion number of the basic 16-plet $\psi_{(\alpha,i)}$ to be +1 and their antiparticles to be −1. The theory permits separate conservation of the fermion number $F$ and the lepton number $L$. All the traditional selection rules usually following from baryon-number and lepton-number conservations are now given by the conservations of the fermion number and the lepton number. Baryon number may, of course, be defined in these schemes in terms[12] of $F$ and $L$.

In Sec. III we present a gauge theory of strong, electromagnetic, and weak interactions in scheme (A) only. The difficulties encountered in generating such interactions in the other three schemes are discussed in Appendix A.

## III. GAUGE INTERACTIONS IN SCHEME (A)

### A. The Weak Gauges

Since the hadrons and leptons in scheme (A) have SU(4′) quartet structures separately, we can obtain the Cabibbo suppression for the $|\Delta S|=1$ charged currents and avoid the $|\Delta S|=1$ neutral currents by following the scheme of Glashow, Iliopoulos, and Maiani.[4] Thus, to start with, we *assume* that the hadronic quarks $\mathfrak{N}_i$ and $\lambda_i$ (with $i=a,b,c$), which enter directly into the currents, *are not physical*, in the sense that the mass matrix is not diagonal with respect to them; they are related to the physical objects $\tilde{\mathfrak{N}}_i$ and $\tilde{\lambda}_i$ $(i=a,b,c)$ by a Cabibbo rotation of angle $\theta$:

$$\begin{pmatrix}\mathfrak{N}_i\\ \lambda_i\end{pmatrix}=\begin{pmatrix}\cos\theta & \sin\theta\\ -\sin\theta & \cos\theta\end{pmatrix}\begin{pmatrix}\tilde{\mathfrak{N}}_i\\ \tilde{\lambda}_i\end{pmatrix}\quad (i=a,b,c). \tag{10}$$

Following Ref. 4, the weak interactions may be generated by a $SU(2')_L$-gauge group with generators given by the sum of the SU(2′) generators acting on the *left-handed* $(p,n)$ and $(\lambda,\chi)$ indices, respectively. These are[13]

$$\begin{aligned} K_1^L&=(F_1'+F_{13}')_L\,,\\ K_2^L&=(F_2'-F_{14}')_L\,,\\ K_3^L&=(F_3'+\tfrac{1}{2}Y'-\tfrac{2}{3}C')_L\,,\end{aligned} \tag{11}$$

where

$$Y'=(\tfrac{4}{3})^{1/2}F_8'$$

and (12)

$$C'=(\tfrac{3}{2})^{1/2}F_{15}'\,.$$

Using the representation of $K_j^L$ with respect to the left-handed basis $\alpha_L^i\equiv(\mathcal{P}_i,\mathfrak{N}_i,\lambda_i,\chi_i)_L$ as

$$[K_j^L]=\tfrac{1}{2}\begin{array}{l}\alpha_L^i\\ \begin{bmatrix}\tau_j & 0\\ 0 & \tilde{\tau}_j\end{bmatrix}\end{array}\quad (i=a,b,c,d;\ j=1,2,3), \tag{13}$$

where $\tau_j$ are the Pauli matrices and $\tilde{\tau}_1=\tau_1$, $\tilde{\tau}_{2,3}=-\tau_{2,3}$, the corresponding fermion–gauge-boson interaction is given by

$$\mathcal{L}_1=g\sum_{j=1}^{3}\sum_{i=a,b,c,d}(\bar{\mathcal{P}}_i\,\bar{\mathfrak{N}}_i\,\bar{\lambda}_i\,\bar{\chi}_i)[K_j^L]\gamma_\mu[\tfrac{1}{2}(1+\gamma_5)]\times\begin{pmatrix}\mathcal{P}_i\\ \mathfrak{N}_i\\ \lambda_i\\ \chi_i\end{pmatrix}W_\mu^j\,. \tag{14}$$

Note that the right-handed fields $(\Psi_A)_R$ are being treated as singlets under the $SU(2')_L$ gauge group. As in Ref. 4, the interaction (14) suppresses $|\Delta S|=1$ neutral currents and has Cabibbo form for charged currents.

### B. The Strong Interactions

The strong interactions (by definition) do not involve the leptons. We shall assume that they respect at least the SU(3′)×SU(3″) symmetry, that they are generated via pure vector and/or axial-vector gauges in order to conserve parity, and that they do not introduce anomalies by themselves or in the presence of other interactions.

*All* these requirements are met easily [in scheme (A)] if we use an SU(3″) octet of vector gauges to generate the basic (gluonlike) strong interactions. Denoting the corresponding gauge mesons by $V_\mu^m$ ($m=1,\ldots,8$), their interactions with fermions are given by

$$\mathcal{L}_2 = f\sum_{m=1}^{8}\sum_{\alpha=\mathcal{P},\mathcal{N},\lambda,\chi}\left[(\bar{\alpha}_a\bar{\alpha}_b\bar{\alpha}_c)(\tfrac{1}{2}\lambda_m)\gamma_\mu\begin{pmatrix}\alpha_a\\ \alpha_b\\ \alpha_c\end{pmatrix}\right]V_\mu^m. \tag{15}$$

The gauge mesons transform as (1, 8) under the SU(4′)×SU(3″) group. The $\lambda_m$'s are the appropriate Gell-Mann matrices for the 3* representation. The interactions as introduced above are, of course, anomaly-free by themselves, since they are vectorial in nature. Remarkably enough, they are found to be anomaly-free also in the presence of the weak $SU(2')_L$ gauge interactions and the U(1) gauge interaction [see Eq. (18)] yet to be introduced to generate electromagnetism. Instead of the SU(3″) octet gauge mesons, however, if we had used a singlet gluon gauge vector meson,[14] coupled symmetrically to the twelve hadrons [in scheme (A)] to generate the strong interactions, the resulting scheme together with the U(1)-gauge interaction needed to generate electromagnetism would have anomalies. In this sense then, the existence of the two (see Ref. 14) SU(4) degrees of freedom in our model seems to play an *essential role* in generating an anomaly-free gauge theory of the strong, weak, and electromagnetic interactions.

### C. Electromagnetism: A New U(1) Gauge

Note that the electric charge operator given by Eq. (6) can be written [with usual notations: $I_3' = (I_3')_L + (I_3')_R \equiv (I_3')_{L+R}$, etc.] as

$$Q=[I_3'+\tfrac{1}{2}Y'-\tfrac{2}{3}C']_L+[I_3''+\tfrac{1}{2}Y'']_{L+R} + \{(I_3'+\tfrac{1}{2}Y'-\tfrac{2}{3}C')_R-\tfrac{2}{3}C''_{L+R}\}. \tag{16}$$

We have so far introduced $SU(2')_L$ and $SU(3'')_{L+R}$ gauges, which generate currents corresponding to the expressions in the square brackets in Eq. (16). No such gauge has been introduced to generate the current corresponding to the piece of the electric charge in the curly bracket in (16). This consists of a part $(I_3'+\frac{1}{2}Y'-\frac{2}{3}C')_R$ belonging to the $SU(2')_R$ group [compare with generators of the $SU(2')_L$ group as in Eq. (11)] and a part $(-\frac{2}{3}C''_{L+R})$ belonging to the SU(4″) group. The corresponding generators commute with the gauge generators already introduced, so that the simplest (though perhaps not the most elegant) possibility to build electromagnetism is to introduce a new U(1) gauge interaction defined by the generator

$$\tilde{Y}=(I_3'+\tfrac{1}{2}Y'-\tfrac{2}{3}C')_R-\tfrac{2}{3}C''_{L+R}. \tag{17}$$

The corresponding gauge meson $U^0$ interacts with the fermions in the following manner:

$$\mathcal{L}_3 = g'U_\mu^0\left\{\tfrac{1}{2}\sum_{i=a,b,c,d}[(\bar{\mathcal{P}}_i\mathcal{P}_i-\bar{\mathcal{N}}_i\mathcal{N}_i)_R+(\bar{\chi}_i\chi_i-\bar{\lambda}_i\lambda_i)_R]+\tfrac{1}{6}\sum_{\alpha=\mathcal{P},\mathcal{N},\lambda,\chi}(\bar{\alpha}_a\alpha_a+\bar{\alpha}_b\alpha_b+\bar{\alpha}_c\alpha_c-3\bar{\alpha}_d\alpha_d)_{L+R}\right\}, \tag{18}$$

where the subscripts $R$ and $(L+R)$ on the currents are defined by the conventional notation

$$\bar{\psi}_1\gamma_\mu[\tfrac{1}{2}(1-\gamma_5)]\psi_2 \equiv (\bar{\psi}_1\psi_2)_R$$

and (19)

$$\bar{\psi}_1\gamma_\mu\psi_2 \equiv (\bar{\psi}_1\psi_2)_{L+R}.$$

The total weak, electromagnetic, and strong interaction of the theory (not including the Higgs bosons to be introduced) is

$$\mathcal{L}'=\mathcal{L}_1+\mathcal{L}_3+\mathcal{L}_2 \tag{20}$$

and the corresponding gauge group is given by

$$G'=SU(2')_L\times U(1)\times SU(3'')_{L+R}. \tag{21}$$

The total number of neutral gauge fields in the theory is four: $W^3$, $V^3$, $V^8$, and $U^0$. The mass matrix arising through spontaneous breaking will in general induce mixing between these fields, which in turn will lead to certain orthogonal combinations of these fields to correspond to the physical particles. One such combination is the massless photon (coupled to the conserved electric-charge gauge). The complexion of the others will depend upon the nature of the mass matrix, which is discussed briefly in Sec. IV.

## IV. PARTICLES AND MASSES

Below we demonstrate a possible scheme[15] of spontaneous symmetry breaking for generating masses of the particles. We sketch the main ideas only briefly, since these are simple extensions of standard ideas in the subject.

All the gauge mesons and the fermions can be given masses appropriately by introducing two sets of scalar fields, i.e.,

(a) a set of 12 complex fields $\sigma_\alpha^i$ transforming as (2+2, 1, 3*) under the gauge group $G'$, where $i = a, b, c$ corresponds to SU(3″) index and $\alpha=1,2,3,4$

corresponds to $SU(2')_L$ indices, plus

(b) a second set of two complex fields $\phi = \binom{\phi^+}{\phi^0}$ transforming as $(2,1,1)$ under $G'$.

We assume that the self-interactions among the scalar fields may be arranged[16] such that their vacuum expectation values are given by

$$\langle \sigma^i_\alpha \rangle_0 = \begin{pmatrix} \sigma & 0 & 0 \\ 0 & \sigma & 0 \\ 0 & 0 & \sigma \\ 0 & 0 & 0 \end{pmatrix} \tag{22}$$

and

$$\langle \phi \rangle_0 = \lambda \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \tag{23}$$

where $\sigma$ and $\lambda$ are real. The three nonzero elements in $\langle \sigma^i_\alpha \rangle$ are taken to be the same in order to obtain (approximately) equal masses for the $SU(3'')$ octet of gauge mesons.

The masses of the gauge mesons are found by substituting the vacuum expectation values for the corresponding fields in the expressions

$$\sum_{(i,\alpha)} |\partial_\mu \sigma^i_\alpha - if(\tfrac{1}{2}\vec{\lambda}(\sigma)\cdot\vec{V}_\mu)_{ij}\sigma^j_\alpha - ig(\vec{t}(\sigma)\cdot\vec{W}_\mu)_{\alpha\beta}\sigma^i_\beta - ig'(t^0(\sigma)U^0_\mu)_{(i,\alpha)(j,\beta)}\sigma^j_\beta|^2 + \sum_{\xi=(+,0)} |\partial_\mu\phi^\xi - ig(\vec{t}(\phi)\cdot\vec{W}_\mu)_{\xi\eta}\phi^\eta - ig'(t^0(\phi)U_\mu)_{\xi\eta}\phi^\eta|^2, \tag{24}$$

where $\vec{\lambda}(\sigma)$ are the Gell-Mann matrices for the $3^*$ representation and

$$(t_K(\sigma))_{\alpha\beta} = \tfrac{1}{2}\begin{pmatrix} \tau_K & 0 \\ 0 & \tilde{\tau}_K \end{pmatrix}_{\alpha\beta} (\tilde{\tau}_1 = \tau_1,\ \tilde{\tau}_{2,3} = -\tau_{2,3}),$$
$$(t^0(\sigma))_{(i,\alpha),(j,\beta)} = \tfrac{1}{6}\delta_{ij}\delta_{\alpha\beta},$$
$$(\vec{t}(\phi))_{\xi\eta} = (\tfrac{1}{2}\vec{\tau})_{\xi\eta}, \tag{25}$$

and

$$(t^0(\phi))_{\xi\eta} = \tfrac{1}{2}\delta_{\xi\eta}.$$

This scheme leads to the following eigenstates and masses after necessary diagonalization of the mixed fields:

$$\begin{aligned}
&V^\pm_{\tilde{\rho}} = V^\pm_\rho \cos\delta + W^\pm \sin\delta, \quad m(V^\pm_{\tilde{\rho}}) = (\tfrac{1}{2})^{1/2} f\sigma + O(\sigma/\lambda);\\
&V^\pm_{K^*} = (\tfrac{1}{2})^{1/2}(V^4 \mp iV^5), \quad m(V^\pm_{K^*}) = (\tfrac{1}{2})^{1/2} f\sigma;\\
&(V^0_{K^*}, \bar{V}^0_{K^*}) = (\tfrac{1}{2})^{1/2}(V^6 \mp iV^7), \quad m(V^0_{K^*}) = (\tfrac{1}{2})^{1/2} f\sigma;\\
&\tilde{W}^\pm = W^\pm \cos\delta - V^\pm_\rho \sin\delta, \quad m(\tilde{W}) = (\tfrac{1}{2})^{1/2} g\lambda + O(\sigma/\lambda);\\
&A = \frac{f(g'W_3 + gU^0) + gg'[V_3 + (\tfrac{1}{3})^{1/2} V_8]}{[f^2(g^2+g'^2) + g^2 g'^2 (\tfrac{4}{3})]^{1/2}}, \quad m_A = 0;\\
&V_{X^0} = \tfrac{1}{2}(V_3 - \sqrt{3}\, V_8), \quad m(V_{X^0}) = (\tfrac{1}{2})^{1/2} f\sigma;\\
&V_{Y^0} = \frac{gg'(g'W_3 + gU^0) - (\tfrac{3}{4}) f(g^2+g'^2)[V_3 + (\tfrac{1}{3})^{1/2} V_8] + O(\sigma/\lambda)}{\{g^2 g'^2 (g^2+g'^2)^2 + \tfrac{3}{4} f^2 (g^2+g'^2)\}^{1/2}}, \quad m(V_{Y^0}) = (\tfrac{1}{2})^{1/2} f\sigma + O(\sigma/\lambda);\\
&Z^0 = \frac{gW_3 - g'U^0}{(g^2+g'^2)^{1/2}} + O(\sigma/\lambda), \quad m(Z^0) = (g^2+g'^2)^{1/2}(\tfrac{1}{2}\lambda) + O(\sigma/\lambda);
\end{aligned} \tag{26}$$

where

$$\tan 2\delta = -2\sqrt{2}\left(\frac{\sigma^2}{\lambda^2}\right)\left(\frac{f}{g}\right)\left[1 - \left(\frac{f^2}{g^2}\right)\left(\frac{\sigma^2}{\lambda^2}\right)\right]^{-1}. \tag{27}$$

We could have exhibited the exact expressions for the diagonal fields. These, however, are complicated and add nothing to the understanding. Thus, in the above equations we have consistently assumed $\sigma/\lambda$ to be a small number of order $10^{-2}$ to $10^{-3}$ and neglected corrections to this order. This is consistent with the assumption of a large $W$ mass $m_W \sim (\tfrac{1}{2})^{1/2} g\lambda \gtrsim 50$ BeV together with the strong-gauge-meson masses $m(V) \sim (\tfrac{1}{2})^{1/2} f\sigma$ of around 3 to 5 BeV.[17] Taking $g/f$ to be nearly equal to the ratio of the electromagnetic charge to typical strong interaction coupling constants (i.e., $g/f$ lying between $\tfrac{1}{10}$ and $\tfrac{1}{30}$), we obtain[18]

$$\left(\frac{\sigma}{\lambda}\right) \simeq \left[\frac{m(V)}{m(W)}\right]\left(\frac{g}{f}\right) \simeq (10^{-2} \text{ to } 10^{-3}). \tag{28}$$

The following points may be noted:

(1) The exchange of $V^\pm_{\tilde{\rho}}$ leads to a $|\Delta S| = 0, 1$ parity-violating amplitude for processes involving

hadrons, which is of order $(\sin\delta)(g/f)\sim(\sigma^2/\lambda^2)$ relative to the parity-conserving amplitude. Such an amplitude however involves a pure SU(3″) nonsinglet transition operator, and therefore vanishes by SU(3″) symmetry for processes involving low-lying hadrons, which are supposed to be SU(3″) singlets. If SU(3″) is not a very strict symmetry (see remark at the end of this section), then for $\sigma^2/\lambda^2\sim 10^{-5}$, the above contribution may still be comparable to the normal weak-interaction contribution and could serve as a distinguishing feature[19] of the model. It should be noted that the additional parity-violating amplitude for hadrons (mentioned above) is a *necessary* feature of our model, since the strong- and weak-gauge mesons must be mixed through the mass matrix in order to generate the photon.

(2) The strong interactions are mediated by the following octet of gauge mesons: $V_{\rho}^{\pm}$, $V_{K^*}^{\pm}$, $\overline{V}_{K^*}^{0}$, $V_{K^*}^{0}$, $V_{X^0}$, and $V_{Y^0}$, all with nearly equal mass $(\frac{1}{2})^{1/2}f\sigma$, which is necessary to preserve SU(3″) symmetry. It should be stressed that these mesons are primarily singlets in the conventional SU(3′), but octet under SU(3″); thus they *cannot be identified* with any of the known vector mesons. We have chosen the subscripts $\rho, K^*, \ldots$, etc., for notational simplicity.

(3) The $V_{Y^0}$ interaction, despite the relatively large component of $W_3$ and $U^0$, is parity conserving at least to order $\sigma^2/\lambda^2$ since the combination $(g'W_3+gU^0)$ is coupled to pure vector current. $Z^0$ interaction is, of course, parity violating.

(4) The strengths of neutral current processes[20] such as $\nu_e+\mu\to\nu_e+\mu$, $\nu_\mu+e\to\nu_\mu+e$, and $\nu+p\to\nu+p$, etc., are the same (to order $\sigma/\lambda$) in the present model as in other $SU(2)_L\times U(1)$ theories[3,4] without the strong gauge mesons. The presence of strong interactions in our model and the mass mixing does not alter the situation.

*The Masses of the Fermions.* The fermion masses can be generated appropriately by introducing the following gauge-invariant interaction of the $\phi$ field[21] with the fermions:

$$\mathfrak{L}(\psi,\phi)=\sum_{i=a,b,c,d}\left[(G_{\mathcal{P}_i}\overline{\mathcal{P}}_{iR}\overline{\xi}+G_{\mathfrak{N}_i}\overline{\mathfrak{N}}_{iR}\overline{\phi}+G_{\mathfrak{N}_i\lambda_i}\overline{\lambda}_{iR}\overline{\phi})\begin{pmatrix}\mathcal{P}_i\\ \mathfrak{N}_i\end{pmatrix}_L+(G'_{\mathfrak{N}_i\lambda_i}\overline{\mathfrak{N}}_{iR}\overline{\phi}+G_{\lambda_i}\overline{\lambda}_{iR}\overline{\phi}+G_{\chi_i}\overline{\chi}_{iR}\overline{\xi})\begin{pmatrix}\chi_i\\ \lambda_i\end{pmatrix}_L\right]+\text{H.c.},\tag{29}$$

where[22]

$$\xi=\begin{pmatrix}\phi^{0\dagger}\\ \phi^-\end{pmatrix},\quad \phi=\begin{pmatrix}\phi^{\dagger}\\ \phi^0\end{pmatrix},\tag{30}$$

$$G_{\alpha_a}=G_{\alpha_b}=G_{\alpha_c}\equiv G_\alpha^h\quad(\alpha=\mathcal{P},\mathfrak{N},\lambda\text{ or }\chi),\tag{31}$$

$$G_{\mathfrak{N}_a\lambda_a}=G_{\mathfrak{N}_b\lambda_b}=G_{\mathfrak{N}_c\lambda_c}\equiv\Delta_{\mathfrak{N}\lambda}^h,\tag{32}$$

and

$$G'_{\mathfrak{N}_a\lambda_a}=G'_{\mathfrak{N}_b\lambda_b}=G'_{\mathfrak{N}_c\lambda_c}\equiv\Delta'^h_{\mathfrak{N}\lambda}.\tag{33}$$

The coupling constants are chosen to be real.[23] We also choose

$$G_{\mathfrak{N}_i\lambda_i}=G'_{\mathfrak{N}_i\lambda_i}\tag{34}$$

to avoid parity-violating mass terms, and as usual

$$G_{p_d}=G_{\chi_d}=0\tag{35}$$

to eliminate the appearance of right-handed neutrinos. This ensures masslessness of the neutrinos. Note that:

(1) We have allowed $(\mathfrak{N}\lambda\phi)$-coupling[24] in Eq. (29) to realize Cabibbo rotation of $\mathfrak{N}$ and $\lambda$ quarks in the manner suggested in Sec. III. One may have introduced an analogous ($\mu$-$e$) rotation in the leptonic column. This, however, does not lead to any observable consequence (see Ref. 24) if $m_{\nu_e}=m_{\nu_\mu}=0$; so we choose

$$G_{\mathfrak{N}_d\lambda_d}=G'_{\mathfrak{N}_d\lambda_d}=0.\tag{36}$$

(2) Subject to Eqs. (31)–(36), there are, in all, seven coupling constants to describe the masses of the fourteen fermions (not counting the neutrinos). It is easy to see that, leaving aside the hadronic $\mathfrak{N}$ and $\lambda$ quarks, the masses of the others are given by

$$\begin{aligned}&m_{\mathcal{P}_a}=m_{\mathcal{P}_b}=m_{\mathcal{P}_c}=\lambda G_{\mathcal{P}}^h,\\ &m_{\chi_a}=m_{\chi_b}=m_{\chi_c}=\lambda G_\chi^h,\\ &m_e=\lambda G_{\mathfrak{N}_d},\\ &m_\mu=\lambda G_{\lambda_d}.\end{aligned}\tag{37}$$

To find the masses of the physical $\tilde{\mathfrak{N}}_i$ and $\tilde{\lambda}_i$ hadronic quarks and the Cabibbo angle $\theta$ [see Eq. (10) for definition], one must diagonalize the submass-matrix involving the $(\mathfrak{N}_i,\lambda_i)$ components. This gives

$$\tan 2\theta=\frac{2\Delta_{\mathfrak{N}\lambda}^h}{G_\lambda^h-G_{\mathfrak{N}}^h},\tag{38}$$

$$m_{\tilde{\mathfrak{N}}_i}=\tfrac{1}{2}\lambda(G_{\mathfrak{N}}^h+G_\lambda^h+2\Delta_{\mathfrak{N}\lambda}^h\csc 2\theta),\tag{39}$$

$$m_{\tilde{\lambda}_i}=\tfrac{1}{2}\lambda(G_{\mathfrak{N}}^h+G_\lambda^h-2\Delta_{\mathfrak{N}\lambda}^h\csc 2\theta).\tag{40}$$

Note that one must choose $G_\lambda^h\neq G_{\mathfrak{N}}^h$, otherwise $\theta=45°$.

(3) The observed approximate SU(2) symmetry of hadrons can be satisfied by choosing $G_{\mathcal{P}}^h\simeq G_{\mathfrak{N}}^h\gg\Delta_{\mathfrak{N}\lambda}^h$. The intrinsic mass differences between

the $\mathcal{P}$ and $\tilde{\mathfrak{N}}$ hadronic quarks, which may arise as above, provides a source of nonelectromagnetic isospin breaking.

(4) The familiar "medium strong" SU(3) breaking may be attributed simply to a $\mathfrak{N}_i$-$\lambda_i$ quark mass difference arising due to spontaneous symmetry breaking. It is tempting to correlate the hadronic $\mathfrak{N}-\lambda$ mass splitting to the observed $e$-$\mu$ mass difference, which is on parallel footing and of similar magnitude. However, the analogy is partly spoiled by the masslessness of neutrinos, which apparently have no analogs in the hadronic columns of $\psi_A$.

(5) The familiar SU(3′) and chiral $SU(3')_L \times SU(3')_R$ symmetries are broken in the present model by quark mass terms (arising through spontaneous symmetry breaking) and by electromagnetic and weak interactions, but *not* by the strong interactions. In fact, it is remarkable that even after spontaneous symmetry breaking, the strong interactions in our model are invariant under the full $U(4')_L \times U(4')_R$ group (neglecting the weak mixing terms of strong and weak gauge mesons). This appears to be essentially a consequence of generating strong interactions together with weak and electromagnetic interactions by a renormalizable gauge principle. We believe this provides at least a partial answer to the often asked question: "Why do weak currents generate the symmetries of the strong interactions?" One, however, does not yet know what, if any, should set the *scale* for the breaking of various symmetries (i.e., SU(4′), SU(3′), SU(2′), chiral $U(4')_L \times U(4')_R$, $U(2')_L \times U(2')_R$, etc.), which, at present, may be introduced with arbitrary strengths through quark mass terms.

(6) Regarding SU(3″) symmetry, we have been able to arrange (because of the choice of the representations of the Higgs scalars, in particular $\sigma^i_\alpha$ and the associated vacuum expectation values) to give symmetric masses to the quarks and almost symmetric masses to the strong gauge mesons (ignoring higher-order loop corrections involving the $\sigma$'s. Furthermore, the quarks do not couple[22] directly to the $\sigma$'s. Thus, after the normal Higgs shifting, global SU(3″) symmetry is still well preserved in the Lagrangian except in the Higgs boson sector, in which it is lost because of the *incompleteness* of the leftover Higgs multiplet $\sigma^i_\alpha$ (in the unitary gauge). In this case, even though the Higgs bosons are coupled strongly to the strong gauge mesons, the SU(3″) breaking, which arises due to the emission and absorption of such Higgs bosons in a hadronic amplitude, is found to be only of order $\alpha$ compared to the symmetric amplitude (for $S$-matrix elements with no external Higgs bosons). This could then account for SU(3″) being an approximate observable symmetry as desired, despite the fact that the gauge symmetry is broken spontaneously. This question will be considered in more detail in a subsequent note.

## V. SUMMARY

We summarize some of the peculiar features of scheme (A):

(1) The scheme generates strong interactions through an SU(3″) octet of gauge mesons, which are singlets under the familiar SU(3′) symmetry.[25] This is nonconventional in the sense that the basic strong interactions seem to owe their "existence" in such a scheme to the unfamiliar hidden SU(3″) symmetry of hadrons and *not* to the manifest SU(3′) symmetry of the low-lying world. Of course, the strong interactions thus generated respect the full $U(4')_L \times U(4')_R \times SU(3'')_{L+R}$ symmetry automatically (see Sec. IV). At the present stage, it is an assumption that the observed (strong) interactions among the known hadrons are *effective* interactions arising due to the exchange of the SU(3″) octet of gauge mesons [which we will refer to as the SU(3″) gluons]. If these gluons are not very heavy ($m \lesssim 5$ BeV, say), it should be feasible to search for their production in pairs[17] through collisions of energetic low-lying hadrons. The discovery of such gluons [which incidentally carry electric charge, even though they are SU(3′) singlets] would of course provide an interesting test of the present ideas. We should emphasize that the introduction of an SU(3″) octet of gluons to generate strong interactions instead of a singlet vector gluon (coupled to the baryonic current) is dictated by our insistence on an *anomaly-free* theory.

(2) The scheme leads to parity-violating amplitudes in hadronic processes on account of mixing of strong and weak gauge mesons. With further calculations and accumulation of experimental data, it might be possible to disentangle these contributions from that of the normal weak interactions, especially if SU(3″) is not a very strict symmetry.

(3) The main virtue of the supersymmetry group $G$, as presented here, has been to provide a classification scheme for the basic hadrons and leptons and thereby a possible rationale for the choice of the fundamental fermions. A special choice of $G$ and representation $F$ has also led to an anomaly-free gauge description of their *presently* known interactions (leaving out gravity and $CP$ violation). On the other hand, as remarked in Sec. I, we have hardly made use of the full dynamical content of the supersymmetry group $G$ in the sense that a whole class of gauge interactions permitted by the

model has not been considered. Most notably, the gauges which transform hadrons into leptons have been left out. Likewise, the left and right SU(4′) gauges have been treated in the most asymmetric manner. It may be of some interest to consider the implications of such gauge degrees of freedom in the model. Furthermore, by gauging the particular subgroup SU(3″) for generating strong interactions, we have built in the basic distinction between the hadrons and the leptons. A part of the future program should be to have this distinction emerge in a more natural manner from the theory.

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## APPENDIX: GAUGE INTERACTIONS IN SCHEMES (B), (C), AND (D)

In attempting to generate a gauge theory of interactions in these three schemes [as opposed to scheme (A)], we encounter two main problems: (1) There appear, in general, baryon- and lepton-number–violating transitions, and (2) it seems difficult, if not impossible, to realize a satisfactory gauge theory of weak, electromagnetic, *and* strong interactions.

In general, (1) leads to an weak amplitude for the decay of quarks into leptons in order $G_F$ and of the proton into leptons plus mesons in order $G_F{}^3$. In schemes (C) and (D) [but not in (B)], depending on further details, this may still be compatible with the observed lower limit on lifetime of the proton. However, in none of these schemes does there seem to be an easy solution to (2).

To illustrate these remarks, consider the generation of weak interactions in scheme (B) by assuming that the weak gauge group is SU(2). For scheme (B), the assignments are [see Eq. (9)]

$$\overline{\psi}_B = \begin{bmatrix} \mathcal{P}_a^0 & \mathcal{P}_b^+ & \mathcal{P}_c^+ & H^0 \\ \mathfrak{N}_a^- & \mathfrak{N}_b^0 & \mathfrak{N}_c^0 & H^- \\ \lambda_a^- & \lambda_b^0 & \lambda_c^0 & H'^- \\ \overline{\nu}_e & e^+ & \mu^+ & \overline{\nu}_\mu \end{bmatrix} .$$

The multiplets for the SU(2) gauge group must be chosen in accordance with the following constraints:

(1) The semileptonic decays of the known hadrons [which are SU(3″) singlets] can arise only through currents of the type $(\overline{\mathcal{P}}_i \mathfrak{N}_i)^{i=a,b,\text{ or } c}$ involving particles in the *same* column of $\Psi_B$. Furthermore, to incorporate the known hadron-lepton universality in a natural manner, we ought to choose multiplets of SU(2) such that the weak current is pure SU(3″) singlet. This suggests that we introduce the multiplets

$$I = \left\{ \begin{pmatrix} \mathcal{P}_a^0 \\ \mathfrak{N}_a^- \end{pmatrix}_L , \begin{pmatrix} \mathcal{P}_b^+ \\ \mathfrak{N}_b^0 \end{pmatrix}_L , \text{ and } \begin{pmatrix} \mathcal{P}_c^+ \\ \mathfrak{N}_c^0 \end{pmatrix}_L \right\}$$

as doublets of the SU(2) gauge group.

(2) To account for the observed $|\Delta S| = 1$ semileptonic decays involving charged currents, we must assume[4] that the $(\mathfrak{N}_i, \lambda_i)_{i=a,b,c}$ quarks are related to the physical $(\tilde{\mathfrak{N}}_i, \tilde{\lambda}_i)_{i=a,b,c}$ quarks by Cabibbo rotation. In this case, in order to cancel the $|\Delta S| = 1$ neutral currents, we must *also* introduce the multiplets

$$J = \left\{ \begin{pmatrix} \overline{\nu}_e \\ \lambda_a^- \end{pmatrix}_L , \begin{pmatrix} e^+ \\ \lambda_b^c \end{pmatrix}_L , \begin{pmatrix} \mu^+ \\ \lambda_c^0 \end{pmatrix}_L \right\}$$

as doublets of the SU(2) group. (It is this[26] that leads to difficulties; see later.)

(3) One must of course introduce the leptonic multiplets

$$K = \left\{ \begin{pmatrix} e^+ \\ \overline{\nu}_e \end{pmatrix}_R , \begin{pmatrix} \mu^+ \\ \overline{\nu}_\mu \end{pmatrix}_R \right\}$$

as doublets of the SU(2) group to have leptonic decays.

(4) The combination $(I + J + K)$ is the *minimal set* needed to reproduce the known weak interactions without $|\Delta S| = 1$ neutral currents. The left and right components of fields not included in this set may be regarded as singlets[27] of the SU(2) group. One can introduce an additional U(1) gauge interaction to generate electromagnetism without anomalies.

It is clear that the above scheme leads to baryon- and lepton-number–violating transitions since the same gauge bosons $(W^+, W^-, W^3)$ are exchanged between the sets $I$ and $J$ or $J$ and $K$. Thus physical $\tilde{\mathfrak{N}}_i$ and $\tilde{\lambda}_i$ quarks can decay to leptons in order $g^2$ as follows:

$$\begin{array}{ll} & \text{Amplitude} \\ \tilde{\lambda}_a^- \to \overline{\nu}_e + W^- \to \overline{\nu}_e + (e^- + \overline{\nu}_e) & (g^2 \cos\theta / m_W{}^2) \\ \tilde{\mathfrak{N}}_a^- \to \overline{\nu}_e + W^- \to \overline{\nu}_e + (e^- + \overline{\nu}_e) & (-g^2 \sin\theta / m_W{}^2) . \end{array} \tag{A1}$$

So there would be *no stable quarks* in this scheme.

For possible decays of the observed hadrons to leptons, it is sufficient to examine the possibility of such decays of the proton only. Assuming that the low-lying hadrons are primarily SU(3″) singlets, the proton consists of one each of $(a, b, c)$ quarks. Consider any one of its components; for example $(\mathcal{P}_a^0 \mathcal{P}_b^+ \tilde{\mathfrak{N}}_c^0)$. Assuming that the quarks and diquarks are heavier than the proton, one can easily verify that the lowest order[28] in which the proton can have a real decay is $g^6$; the amplitude for such a decay is proportional to $(g^6/m_W^6)\sin^3\theta$. A possible chain of virtual transitions leading to proton decay is indicated below[29]:

$$p \to \begin{cases} \mathcal{P}_a^0 \to \pi^+ + \tilde{\mathfrak{N}}_a^- \xrightarrow[g\sin\theta]{} \pi^+ + (\bar{\nu}_e + W^-) \\ \qquad\qquad \xrightarrow[g]{} \pi^+ + (\bar{\nu}_e + e^- + \bar{\nu}_e)\ , \\ \mathcal{P}_b^+ \to \pi^+ + \tilde{\mathfrak{N}}_b^0 \xrightarrow[g\sin\theta]{} \pi^+ + (e^+ + W^-) \\ \qquad\qquad \xrightarrow[g]{} \pi^+ + (e^+ + e^- + \bar{\nu}_e)\ , \\ \tilde{\mathfrak{N}}_c^0 \xrightarrow[-g\sin\theta]{} \mu^+ + W^- \xrightarrow[g]{} \mu^+ + (e^- + \bar{\nu}_e)\ . \end{cases} \tag{A2}$$

A crude estimate of the lifetime of proton associated with the above decay is found to be between $10^{15}$ and $10^{19}$ sec. Even though this is of the order of or greater than the age of the universe, it is still ridiculously low compared to the experimental lower limit[30] $\tau_p > 10^{35}$ sec.

Similar considerations apply also to scheme (C); however, there is an essential difference due to the appearance of the unknown lepton $E^+$ in the place of $\mu^+$, which may help remove the conflict with proton lifetime. To see this, note that if we choose the SU(2) multiplets for $\Psi_c$ [given by Eq. (9)] analogous to the *minimal* set $(I+J+K)$ for $\Psi_B$, the doublet

$$\begin{pmatrix} \mu^+ \\ \lambda_c^0 \end{pmatrix}_L$$

for scheme (B) is replaced by the doublet

$$\begin{pmatrix} E^+ \\ \lambda_c^0 \end{pmatrix}_L$$

for scheme (C), the other multiplets remaining the same. On account of this, the heavy lepton $E^+$ would appear as an end product (instead of $\mu^+$) in the conversion of $\tilde{\mathfrak{N}}_c^0$ [see Eq. (A2)]. If $M_{E^+} > m_p$, the decay of proton involving real $E^+$ is forbidden. It is then easy to see that the decay of proton must at least involve the conversion of a "$c$" quark (say, $\tilde{\mathfrak{N}}_c^0$) to an appropriate "$b$" or "$a$" quark, which in turn can decay to light leptons. As the above conversion would take place only via strong interactions and therefore would be accompanied by the emission of an appropriate strongly interacting SU(3″) octet meson, it can be shown, in this case, that the proton lifetime would be higher than the estimate given above depending upon the nature of SU(3″) symmetry.[31] For example, if SU(3″) were broken only weakly, the proton decay amplitude would involve fourth or higher powers of $G_F$; its lifetime may thus become long enough so as not to conflict with the experimental lower limit on $\tau_p$. The quarks could still decay in order $G_F$. These remarks apply to scheme (D) as well as to scheme (C).

We may now illustrate remark (2), mentioned in the beginning of this section. Note that the degree of freedom, which has been used to generate the known weak interactions in the above schemes, does not permit us to build strong interactions with desired symmetry and still preserve the goal of renormalizability. This is apparent simply by noting that the SU(2) gauge symmetry which treats

$$\begin{pmatrix} \bar{\nu}_e \\ \lambda_a^- \end{pmatrix}_L$$

as a doublet must be violated by strong interactions, which affect the hadron $\lambda_a^-$ but not the lepton $\bar{\nu}_e$. In such a case renormalizability is lost. This argument can be further extended. In principle, a different choice of the weak gauge group [such as $O(3)$] and a cleverer choice of the multiplets may alter the situation presented here. We have not explored this possibility in detail; however, it appears difficult to change the situation substantially.

In summary, even though schemes (C) and (D) offer some intriguing possibilities which are worth noting, none of the schemes (B), (C), and (D) seems to be appropriate for realizing the goal we have sought to achieve in this paper, i.e., to generate the weak, electromagnetic, *and* strong interactions by a renormalizable gauge principle. The latter seems to be feasible only for scheme (A). We should also remind[32] the reader that the exotic possibilities pointed out in this appendix apply *exclusively* to schemes (B), (C), and (D) and not to (A), which is discussed in the text.

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[1]See, for example, Z. Maki and T. Maskawa, Prog. Theor. Phys. 49, 1007 (1973) (earlier references

may be found in this paper). Also see A. Gamba, R. E. Marshak, and S. Okubo, Proc. Natl. Acad. Sci. USA 45, 881 (1959); S. L. Glashow, J. Iliopoulos, and L. Maiani, Phys. Rev. D 2, 1285 (1970).

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[6]The possibility of observed leptons being composites of fractionally charged leptonic quarks is suggested by A. Salam and J. C. Pati, Phys. Lett. (to be published).

[7]S. Adler, Phys. Rev. 177, 2426 (1969); S. Okubo, *ibid.* 179, 1629 (1969).

[8]J. C. Pati and C. H. Woo, Phys. Rev. D 3, 2920 (1971).

[9]Certain variations of the Han-Nambu model suit our purpose equally well. See, for example, the version due to Cabibbo, Maiani, and Preparata (Ref. 9) and also the discussion in J. C. Pati and C. H. Woo, Phys. Rev. D 3, 1173 (1971).

[10]Note, if we do not insist on Han-Nambu structure for the hadrons, we may choose the group $G$ to be given by SU(4) rather than SU(4′)×SU(4″), since SU(4) can accommodate three quantum numbers, which may be identified with $I_3$, $Y$, and $L$. In this case, if we wished to maintain a quark structure for the observed hadrons and leptons, we might consider building them "compositely" out of a single quartet of fermions $f=(\mathcal{P},\mathcal{N},\lambda,l)$. However, one may verify that the $4\times4\times4$ representation will not contain the known leptons without bringing in a host of other, too-exotic subjects. The most suitable choice with $G=\mathrm{SU}(4)$ for the basic representation $F$ seems to be 15. This case looks very similar to scheme (C) [see Eq. (9)], except that the baryons in the 15-plet of SU(4) would be more naturally identified with the observed (octet and singlet) baryons than with quarks.

[11]This follows by requiring that the Han-Nambu nonet of hadronic quarks carry zero lepton number. Note that other linear combinations $L=a(c'-1.4)+b(c''+\frac{1}{4})$ *not* included in (A)–(D) of Eq. (8) could also be considered as separate possibilities. These are, however, not distinct physically from (A)–(D). Note also that we may drop the terms $\pm\frac{1}{4}$ in Eq. (8) without altering any of the physical consequences in the present paper. These terms would be relevant only if a physical field were coupled to lepton number.

[12]For example, one can choose $B=(F-L)$ and $(F+L)$ in schemes (A) and (B), respectively, thereby assigning $B=1$ for the hadronic quarks ($L=0$) and $B=0$ for the leptons. In schemes (C) and (D), some of the particles must carry *both* baryon and lepton numbers no matter how one chooses to define $B$ in terms of $F$ and $L$.

[13]We follow the notation for SU(4) generators as given by D. Amati, H. Bacry, J. Nuyts, and J. Prentki, Nuovo Cimento 34, 1732 (1964).

[14]Note that in a scheme with only one SU(4) degree of freedom, a singlet gluon gauge is the only available gauge for generating strong interactions once weak interactions have made use of $\mathrm{SU}(2)_L$ gauges.

[15]We are indebted to Dr. R. Mohapatra for a major contribution and most helpful discussions on this section.

[16]For the present, we have not investigated whether $\lambda'\sigma^4$-like terms may be written down to yield the pattern of nonzero vacuum expectation values assumed in Eq. (22).

[17]Note that the strong gauge bosons, being primarily in the SU(3″) octet (apart from weak mixing), cannot be produced singly through collisions of normal hadrons; they have to be produced in pairs. In this case, masses around 3–5 BeV for these mesons are consistent with their absence in production experiments so far. For some discussion on production, decays, and observability of SU(3″) nonsinglet objects, see Pati and Woo (Ref. 9) and G. A. Snow, University of Paris report, 1972 (unpublished).

[18]One may also estimate $\sigma/\lambda$ without assuming values of $m(V)$ and $f$ separately as follows: The ratio of typical weak to strong interaction amplitude is expected to be of order

$$\frac{g^2/m_W{}^2}{f^2/m_V{}^2}\simeq\frac{\sigma^2}{\lambda^2}\,,$$

which on the other hand is known to be of order $10^{-5}$ to $10^{-6}$, consistent with Eq. (28).

[19]But for the SU(3″) suppression, one might be tempted to consider the possible relevance of such an amplitude in understanding the seemingly large discrepancy between certain recent calculations of the parity-violating amplitude in nuclear transitions and their observed values. See, for example, M. Gari, D. Dumitrescu, J. G. Zabotlisky, and H. Kummel, Phys. Lett. 35B, 19 (1971); B. Desplanques and N. Vinh Mau, *ibid.* 35B, 28 (1971).

[20]For a review of theoretical and experimental status on neutral currents see B. W. Lee, in *Proceedings of the XVI International Conference on High Energy Physics, Chicago-Batavia, Ill., 1972,* edited by J. D. Jackson and A. Roberts (NAL, Batavia, Ill. 1973), Vol. 4, p. 249.

[21]$\sigma_\alpha^i$ do not have Yukawa coupling to fermions on account of SU(3″) gauge invariance.

[22]Note that Eqs. (31)–(33) between the coupling constants are required to satisfy SU(3″) gauge invariance. This leads to SU(3″)-symmetric quark mass terms.

[23]If one allows them to be complex, one will obtain parity- and $CP$-violating mass terms. Such possibilities could be relevant for inducing CP violation.

[24]Note that in general a $(\mathcal{P}\chi\phi)$ coupling is also allowed, which would lead to a rotation of the $(\mathcal{P},\chi)$ fields. The net effect of simultaneous rotations of the $(\mathcal{N},\lambda)$ and the $(\mathcal{P},\chi)$ quarks by angles $\theta_1$ and $\theta_2$, respectively, is that (for processes of interest) the Cabibbo angle is $(\theta_1-\theta_2)$. We do not introduce $(\mathcal{P},\chi)$ rotation. *Note added in proof.* A related remark is that if $m_{\nu_e}=m_{\nu_\mu}$ and there is no nondiagonal mass term connecting $\nu_e \leftrightarrow \nu_\mu$, one can

always rotate the two neutrinos so as to counteract the effect of ($\mu$-$e$) rotation. The $\mu \to e + \gamma$- decay experiment and the famous two neutrino experiments are, therefore, insensitive to such rotations. We thank Dr. W. Alles for this last remark.

[25]Note the distinction in this case from the models of A. Salam and J. Strathdee [Nuovo Cimento 11A, 397 (1972) (in particular p. 426)] and I. Bars, M. B. Halpern, and M. Yoshimura [Phys. Rev. Lett. 29, 969 (1972)], who also propose a gauge theory of the strong, electromagnetic, and weak interactions. In these models, hadrons are *not* coupled directly to the weak gauge bosons, and their strong interactions are mediated by a nonet of vector and a nonet of axial-vector gauge mesons, which transform as (8+1) under familiar SU(3). Note that, for low-lying hadrons, in our scheme, SU(3′) and SU(3) are synonymous.

[26]Note that the leptons in set $J$ cannot be replaced by the hadrons ($H^0$, $H^-$, $H'^-$) because of their charges.

[27]On the other hand, for the sake of symmetry, one might have introduced an extended set by treating the four columns of $\Psi_B$ in an identical manner and likewise the four rows. This extended set will be

$$(I+J+K)+\left\{\begin{pmatrix}H^0\\H^-\end{pmatrix}_L,\begin{pmatrix}\bar{\nu}_\mu\\H'^-\end{pmatrix}_L,\begin{pmatrix}\lambda_b^0\\\lambda_a^-\end{pmatrix}_R,\begin{pmatrix}\lambda_c^0\\H'^-\end{pmatrix}_R,\right.$$
$$\left.\begin{pmatrix}\mathfrak{N}_b^0\\\mathfrak{N}_a^-\end{pmatrix}_R,\begin{pmatrix}\mathfrak{N}_c^0\\H^-\end{pmatrix}_R,\begin{pmatrix}\mathcal{P}_b^+\\\mathcal{P}_a^0\end{pmatrix}_R,\begin{pmatrix}\mathcal{P}_c^+\\H^0\end{pmatrix}_R\right\}.$$

For scheme (B), the consequences (discussed in the text) are the same for the extended set as it is for the minimal set. [This is not so for schemes (C) and (D).]

[28]Note that the loop diagram of order $g^4$ cannot contribute in this case, since it cannot get rid of quarks entirely.

[29]Note that the emission of two $\pi^+$ mesons is essential, since it allows the conversion of $\mathcal{P}_a^0$ and $\mathcal{P}_b^+$ to the physical $\tilde{\mathfrak{N}}_a^-$ and $\tilde{\mathfrak{N}}_b^0$ quarks, respectively. The latter contain $\lambda_a^-$ and $\lambda_b^0$ by an amount given by $\sin\theta$; it is the $\lambda$ quarks that can be converted to (lepton$+W$).

[30]Particle Data Group, Phys. Lett. 39B, 1 (1972) and specific references therein.

[31]If SU(3″) were an exact symmetry [in Scheme (C)], one could realize the intriguing possibility that the proton is absolutely stable, even if the quarks are not. In this case, at least one other particle (in addition to the proton) must also be absolutely stable. This may be the $E^+$ or a SU(3″) nonsinglet hadron (distinct from quarks).

[32]As a further side remark, it is worth noting that in each of these schemes [(A) through (D)] one may introduce a singlet gauge particle coupled to the fermion number, which could be absolutely conserved. By demanding that the corresponding interaction be long-range (and very very weak), one could, for example, avoid the transcendence of fermion number in black-hole physics, though baryon and lepton numbers may still be transcended individually.