[23]The operators F_1, F_2, and $(\frac{1}{2}H)$ close on the noncompact "spectrum generating" algebra SL$(2, R)$ of the two-dimensional harmonic oscillator. See, e.g., H. J. Lipkin, *Lie Groups for Pedestrians* (North-Holland, Amsterdam, 1966).

[24]F. J. Gilman and H. Harari, Phys. Rev. 165, 1803 (1968); S. Weinberg, *ibid.* 177, 2604 (1969); and in *Lectures on Elementary Particles and Field Theory*, edited by S. Deser *et al.* (MIT Press, Cambridge, Mass., 1971), Vol. I.

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Eikonal Approximation for the Bilocal Vertex Function and νW_2 in a Fermion–Neutral-Vector-Gluon Model*

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The eikonal approximation is used to investigate the forward one-fermion matrix element of the bilocal operator that appears in the most singular term of the canonical light-cone current commutators for the fermion–neutral-vector-gluon model. A relationship is exhibited between this matrix element of the bilocal operator and the leading behavior of νW_2. This relation allows calculations of contributions to the matrix element of the bilocal operator to be applied to the corresponding contribution to νW_2. A simple set of graphs contributing to the matrix element of the bilocal operator is calculated in the eikonal approximation. This gives a contribution to νW_2 in agreement with explicit calculations in perturbation theory for the corresponding set of graphs of νW_2.

I. INTRODUCTION

The determination of the structure functions for deep-inelastic electron-proton scattering is of great theoretical interest. Insight may be gained by examining simple field-theoretic models. In particular, the inelastic structure functions have been studied for a fermion–neutral-vector-gluon model in perturbation theory.[1,2]

In this paper the eikonal contributions to the bilocal operator that appears in the most singular term of the light-cone current commutators in the fermion–neutral-vector-gluon field theory are investigated. Since the forward one-fermion matrix element of this bilocal operator can be related to the leading behavior of νW_2 (as $-q^2 \to \infty$, $\nu \to \infty$), therefore the behavior of νW_2 can also be investigated by the method.[3] In particular, the contribution to this matrix element of the bilocal operator corresponding to structureless gluon graphs can be easily calculated in the eikonal approximation. The results can be related to the contribution from the corresponding graphs of νW_2 by means of the relation found between the bilocal operator and νW_2. The results agree with explicit calculations of these graphs of νW_2 made by Fishbane and Sullivan.[3] The eikonal approximation for these graphs of νW_2 has been obtained by Fried and Moreno[4]; however, the objective of the eikonal calculation we will do here is not a new result for νW_2, but a different approach,[5] i.e., to study the leading bilocal operator and then apply results to νW_2.

In Sec. II, the form of the bilocal operator will be obtained and the relationship between the forward one-fermion matrix element of this bilocal operator and νW_2 will be derived. In Sec. III, the eikonal approximation to the bilocal operator matrix element will be obtained and various approximations made to obtain numerical results. The relationship between the matrix element of the bilocal operator and νW_2 is used to obtain a result for νW_2 within these approximations. In Sec. IV, the physical relevance of these calculations is discussed and further calculations are suggested.

II. THE BILOCAL VERTEX FUNCTION AND ITS RELATION TO νW_2

The electromagnetic current commutators at equal x^+ have been computed canonically for the fermion–neutral-vector-gluon theory.[6] The results are

$$-i\,[J^+(x), J^\nu(0)]_{x^+=0} = \tfrac{1}{4} i\,\partial^x_\alpha \left\{ \left[\bar\psi(x)\gamma^+\gamma^\alpha\gamma^\nu \exp\left(-ig_0\int_0^x dz_\beta A^\beta(z)\right)\psi(0) - \text{H.c.}\right] \epsilon(x^-)\delta^2(\vec{x}_\perp) \right\}.$$

Now the leading term in the operator expansions as $x^2 \to 0$ of the unordered product of currents can be obtained from these light-cone commutators, and can be written

$$J^\mu(x)J^\nu(0) \underset{x^2\to 0}{\sim} O^{\mu\nu\sigma}(x|0)\partial_\sigma^x[\Delta^{(-)}(x)] + \text{less singular terms},$$

where $\Delta^{(-)}(x)$ is the negative-frequency part of the Pauli-Jordan commutator function. The most singular part of $\Delta^{(-)}$ is

$$\frac{-i}{4\pi^2}\,\frac{1}{x^2 - i\epsilon x^-}\,.$$

$O^{\mu\nu\sigma}(x|0)$ is determined by the light-cone commutators to be

$$O^{\mu\nu\sigma}(x|0) = -1\left[\bar\psi(x)\gamma_\alpha \exp\left(-ig_0\int_0^x dz_\beta A^\beta(z)\right)\psi(0) - \text{H.c.}\right](g^{\mu\sigma}g^{\nu\alpha} + g^{\nu\sigma}g^{\mu\alpha} - g^{\mu\nu}g^{\sigma\alpha})$$
$$+ i\epsilon^{\mu\sigma\nu\delta}\left[\bar\psi(x)\gamma_5\gamma_\delta \exp\left(-ig_0\int_0^x dz_\beta A^\beta(z)\right)\psi(0) - \text{H.c.}\right]\,.$$

The connected spin-averaged (CSA) one-fermion forward matrix element of the unordered product of the electromagnetic currents is therefore

$$\langle p|J^\mu(x)J^\nu(0)|p\rangle_{\text{CSA}} \underset{x^2\to 0}{\sim} (-1)(g^{\mu\sigma}g^{\nu\alpha} + g^{\mu\alpha}g^{\nu\sigma} - g^{\mu\nu}g^{\sigma\alpha})$$
$$\times\left\langle p\left|\bar\psi(x)\gamma_\alpha \exp\left(-ig_0\int_0^x dz_\beta A^\beta(z)\right)\psi(0) - \text{H.c.}\right|p\right\rangle\partial_\sigma^x\Delta^{(-)}(x)\,.$$

Jackiw and Waltz have shown[7] that for the one-fermion forward matrix element, the T product and the unordered product coincide as $x^2\to 0$; thus we may write

$$\langle p|J^\mu(x)J^\nu(0)|p\rangle_{\text{CSA}} \underset{x^2\to 0}{\sim} (-1)(g^{\mu\sigma}g^{\nu\alpha} + g^{\nu\sigma}g^{\mu\alpha} - g^{\mu\nu}g^{\sigma\alpha})$$
$$\times\left\langle p\left|T\left[\bar\psi(x)\gamma_\alpha \exp\left(-ig_0\int_0^x dz_\beta A^\beta(z)\right)\psi(0)\right] - \text{H.c.}\right|p\right\rangle_{\text{CSA}}\partial_\sigma^x\Delta^{(-)}(x)\,. \tag{2.1}$$

The bilocal vertex function is defined by

$$\Gamma^\mu(x^2, x\cdot p) \equiv \left\langle p\left|T\left[\bar\psi(x)\gamma^\mu \exp\left(ig_0\int_0^x dz_\beta A^\beta(z)\right)\psi(0)\right]\right|p\right\rangle_{\text{CSA}}\,, \tag{2.2}$$

and defining $D(x^2, x\cdot p)$ and $C(x^2, x\cdot p)$ by

$$\text{Im}\,\Gamma^\mu(x^2, x\cdot p) = \frac{p^\mu}{2M}D(x^2, x\cdot p) + \tfrac{1}{2}(x^\mu M)C(x^2, x\cdot p), \tag{2.3}$$

the leading behavior of $\langle p|J^\mu(x)J^\nu(0)|p\rangle_{\text{CSA}}$ is given by

$$\langle p|J^\mu(x)J^\nu(0)|p\rangle_{\text{CSA}} \underset{x^2\to 0}{\sim} -\frac{i}{M}D(x^2, x\cdot p)(p^\mu\partial^\nu + p^\nu\partial^\mu - g^{\mu\nu}p\cdot\partial)\,\Delta^{(-)}(x) + \text{less singular terms}. \tag{2.4}$$

Since this matrix element of the unordered product of currents is just the Fourier transform of $W^{\mu\nu}$ for $q^2<0$, we have

$$(2\pi)^2\frac{E_p}{M}\int d^4x\, e^{iq\cdot x}\langle p|J^\mu(x)J^\nu(0)|p\rangle_{\text{CSA}} = -(g^{\mu\nu} - q^\mu q^\nu/q^2)\,W_L(q^2, q\cdot p)$$
$$+\frac{1}{M^2}[p^\mu p^\nu - (q\cdot p/q^2)(p^\mu q^\nu + p^\nu q^\mu) + g^{\mu\nu}(q\cdot p)^2/q^2]W_2(q^2, q\cdot p)\,. \tag{2.5}$$

Comparison of Eqs. (2.4) and (2.5) for $\langle p|J^\mu(x)J^\mu(0)|p\rangle_{\text{CSA}}$ gives the relation, to leading order as $x^2\to 0$,

$$2\pi^2\frac{E_p}{M}\Delta^{(-)}(x)D(x^2, x\cdot p) \underset{x^2\to 0}{\sim} \frac{1}{M}\int\frac{d^4q}{(2\pi)^4}\,e^{-iq\cdot x}\left(\frac{-q\cdot p}{q^2}\right)W_2(q^2, q\cdot p); \tag{2.6a}$$

or in momentum space, the leading behavior of $\nu W_2(q^2, q\cdot p)$ as $-q^2\to\infty$, ω fixed, is ($\omega = -q^2/2q\cdot p$, $\nu = q\cdot p/M$)

$$\nu W_2(q^2, q\cdot p) \underset{\substack{-q^2\to\infty \\ \omega \text{ fixed}}}{\sim} -q^2 \int d^4x\, e^{-iq\cdot x} \Delta^{(-)}(x) D(x^2, x\cdot p)\left(\frac{M}{E_p}\right)^{-1} 2\pi^2 \,. \tag{2.6b}$$

This relation allows the determination of the leading behavior of $(1/M)q\cdot pW_2(q^2, q\cdot p)$ from the behavior of the bilocal vertex function Γ^μ as $x^2 \to 0$.

III. THE EIKONAL APPROXIMATION FOR THE BILOCAL VERTEX FUNCTION

It is convenient to use functional techniques to rewrite $\Gamma^\mu(x^2, x\cdot p)$ in a manner amenable to the eikonal approximation. For this purpose, additional external potential interactions are introduced:

$$\mathcal{L}(x) = \bar\psi(x)(i\partial\!\!\!/ - m)\psi(x) - \tfrac{1}{4}F^{\mu\nu}(x)F_{\mu\nu}(x) - g_0\bar\psi(x)\gamma^\mu\psi(x)A_\mu(x) + \delta m\bar\psi(x)\psi(x)$$
$$+ \bar\eta(x)\psi(x) + \bar\psi(x)\eta(x) + A^\mu(x)B_\mu(x)\,.$$

Then the vacuum-vacuum transition amplitude is

$$Z(\eta, \bar\eta B) \equiv \langle 0 \text{ out} | 0 \text{ in}\rangle_{\eta\bar\eta B}$$
$$= \left\langle 0 \left| T\left(\exp\left\{i\int d^4x[-g_0\bar\psi^{\text{in}}(x)\gamma^\mu\psi^{\text{in}}(x)A_\mu^{\text{in}}(x) + \delta m\bar\psi^{\text{in}}(x)\psi^{\text{in}}(x) + \bar\eta(x)\psi^{\text{in}}(x) + \eta(x)\bar\psi^{\text{in}}(x) + A^{\mu\,\text{in}}(x)B_\mu(x)]\right\}\right)\right| 0\right\rangle.$$

The LSZ (Lehmann-Symanzik-Zimmermann) reduction formula and functional techniques can now be used to rewrite Γ^μ as

$$\Gamma^\mu(x^2, x\cdot p) = -\frac{1}{Z_2}\frac{M}{(2\pi)^3E_p}\int d^4z\int d^4z'\, e^{ip(z-z')}[\bar u(p,\lambda)(i\overleftarrow{\partial\!\!\!/}_{z'} - m)]_\sigma$$
$$\times\{S_{F\sigma\alpha}(x, z'; ig_0\delta/\delta B^\mu)\gamma^\mu_{\alpha\beta}S_{F\beta\rho}(z, 0; ig_0\delta/\delta B^\mu)$$
$$+\gamma^\mu_{\alpha\beta}S_{F\beta\alpha}(x, 0; ig_0\delta/\delta B^\mu)\; S_{F\sigma\rho}(z', z; ig_0\delta/\delta B^\mu\}$$
$$\times[(-i\overrightarrow{\partial\!\!\!/}_z - m)u(p,\lambda)]_\rho \exp\left[-g_0\int_0^x d\omega_\beta \frac{\delta}{\delta B_\beta(\omega)}\right]\frac{Z(0,0,B)}{Z(0,0,0)}\bigg|_{B=0}^{\text{CSA}},$$

where $S_F(x, 0; ig_0\delta/\delta B^\mu)$ is the Feynman propagator for a fermion interacting with an external potential $ig_0\delta/\delta B^\mu$, i.e.,

$$+iS_F(x, 0; ig_0\delta/\delta B^\mu) \equiv \langle 0 \text{ out}|T\psi(0)\bar\psi(x)|0 \text{ in}\rangle_{\delta/\delta B^\mu}$$
$$= \left\langle 0\left|T\left[\psi^{\text{in}}(0)\bar\psi^{\text{in}}(x)\exp\left(i\int d^4z\,\bar\psi^{\text{in}}(z)\gamma^\mu\psi^{\text{in}}(z)ig_0\frac{\delta}{\delta B^\mu(z)}\right)\right]\right|0\right\rangle.$$

Finally the LSZ reduction formula for a fermion in an external potential $ig_0\delta/\delta B^\mu(x)$ can be used to rewrite Γ^μ as

$$\Gamma^\mu(x^2, x\cdot p) \equiv \Gamma^\mu_a(x^2, x\cdot p) + \Gamma^\mu_b(x^2, x\cdot p)\,,$$
$$\Gamma^\mu_a(x^2, x\cdot p) = \frac{1}{Z_2}\bar\psi(x; p, \lambda; ig_0\delta/\delta B^\mu)\gamma^\mu\psi(0; p, \lambda; ig_0\delta/\delta B^\mu)\exp\left(-g_0\int_0^x d\omega_\nu\,\delta/\delta B_\nu(\omega)\right)\frac{Z(0,0,B^\mu)}{Z(0,0,0)}\bigg|_{B^\mu=0}^{\text{CSA}},$$
$$\tag{3.1}$$
$$\Gamma^\mu_b(x^2, x\cdot p) = \frac{1}{Z_2}\text{tr}[\gamma^\mu S_F(x, 0; ig_0\delta/\delta B_\mu)]\frac{\langle p,\lambda \text{ out}|p,\lambda \text{ in}\rangle_{\delta/\delta B}}{\langle 0 \text{ out}|0 \text{ in}\rangle_{\delta/\delta B}}\exp\left(-g_0\int_0^x d\omega_\nu\,\delta/\delta B_\nu(\omega)\right)\frac{Z(0,0,B^\mu)}{Z(0,0,0)}\bigg|_{B^\mu=0}^{\text{CSA}},$$

where

$$\psi(0; p, \lambda; ig_0\delta/\delta B^\mu) \equiv \langle 0 \text{ out}|\psi(0)|p,\lambda \text{ in}\rangle_{\delta/\delta B}/\langle 0 \text{ out}|0 \text{ in}\rangle_{\delta/\delta B}$$

and

$$\bar\psi(x; p, \lambda; ig_0\delta/\delta B^\mu) \equiv \langle p,\lambda \text{ out}|\bar\psi(x)|0 \text{ in}\rangle_{\delta/\delta B}/\langle 0 \text{ out}|0 \text{ in}\rangle_{\delta/\delta B}$$

are the wave functions [for a fermion interacting with an external potential $ig_0\delta/\delta B_\mu(x)$] that represent a free fermion of momentum p and spin λ at time $t\to-\infty$ and $t\to+\infty$, respectively. Also

$$\frac{\langle p,\lambda \text{ out}|p,\lambda \text{ in}\rangle_{\delta/\delta B}}{\langle 0 \text{ out}|0 \text{ in}\rangle_{\delta/\delta B}}$$

is the forward one-fermion to one-fermion transition amplitude in the external potential $ig_0\delta/\delta B^\mu(x)$.

Equation (3.1) is an exact formal expression for $\Gamma^\mu(x^2, x\cdot p)$. The contributions Γ_a^μ and Γ_b^μ to Γ^μ are separately gauge-invariant so it is meaningful to discuss these terms separately. The Feynman graphs that contribute to Γ_a^μ and Γ_b^μ in lowest order are shown in Fig. 1.

In order to apply the eikonal approximation to Γ^μ, the frame $p^+\to\infty$, $\vec{p}_\perp = 0$ $(p^2=m^2)$ is chosen and $\Gamma^\mu(x^2, x\cdot p)$ is considered in the region $x^+=0$ (so $x^2 \le 0$). The eikonal approximation is now made by simply replacing $\bar\psi(x;p,\lambda;g_0\delta/\delta B^\mu)$, $\psi(0;p,\lambda;g_0\delta/\delta B^\mu)$, and $\langle p,\lambda \text{ out}|p,\lambda \text{ in}\rangle_{\delta/\delta B}/\langle 0 \text{ out}|0 \text{ in}\rangle_{\delta/\delta B}$ by their eikonal approximations (EA) (see Ref. 8).

$$\bar\psi(x;p,\lambda;g_0\delta/\delta B^\mu) \underset{\text{EA}}{\longrightarrow} -i\left(\frac{m}{E_p}\right)^{1/2}(2\pi)^{-3/2}\bar u(p,\lambda)e^{ip\cdot x}\exp\left[(-1)g_0\int_0^\infty d\tau\,\eta^\mu\frac{\delta}{\delta B^\mu(x+\eta_\xi\tau)}\right], \tag{3.2a}$$

$$\psi(0;p,\lambda;g_0\delta/\delta B^\mu) \underset{\text{EA}}{\longrightarrow} -i\left(\frac{m}{E_p}\right)^{1/2}(2\pi)^{-3/2}\exp\left[(-1)g_0\int_{-\infty}^0 d\sigma\,\eta_\xi^\mu\frac{\delta}{\delta B^\mu(\sigma\eta_\xi)}\right]u(p,\lambda), \tag{3.2b}$$

$$\frac{\langle p',\lambda' \text{ out } p,\lambda \text{ in}\rangle_{\delta/\delta B}}{\langle 0 \text{ out}|0 \text{ in}\rangle_{\delta/\delta B}} \underset{\text{EA}}{\longrightarrow} \delta_{\lambda\lambda'}\delta^3(\vec{p}-\vec{p}\,') - \frac{m}{(E_pE_{p'})^{1/2}}(2\pi)^{-3}\int d^4x\,e^{i(p'-p)\cdot x}g_0\bar u(p',\lambda')\gamma^\mu u(p,\lambda)\delta/\delta B^\mu(x)$$
$$\times\exp\left[(-1)g_0\int_{-\infty}^\infty d\tau\,\eta_\xi^\mu\frac{\delta}{\delta B^\mu(x+\tau\eta_\xi)}\right], \tag{3.2c}$$

where $\eta_\xi = (0,0,1,1)$.

Now Γ_a^μ is more easily calculated than Γ_b^μ in the eikonal approximation. For $\Gamma_a^\mu(x^2, x\cdot p)|_{x^+=0;\,p^+\to\infty;\,\vec{p}_\perp=0} = \Gamma_a^\mu(-\vec{x}_\perp^{\,2}, x^-p^+)$, the eikonal approximation gives

$$\Gamma_a^\mu(-\vec{x}_\perp^{\,2}, x^-p^+) \underset{\text{EA}}{\longrightarrow} -\frac{e^{ip\cdot x}}{Z_2(2\pi)^3}\frac{p^\mu}{m}\frac{m}{E_p}\exp\left[(-1)g_0\int_0^\infty d\tau\,\eta_\xi^\mu\frac{\delta}{\delta B^\mu(x+\tau\eta_\xi)}\right]\exp\left[(-1)g_0\int_{-\infty}^0 d\sigma\,\eta_\xi^\mu\frac{\delta}{\delta B^\mu(\sigma\eta_\xi)}\right]$$
$$\times\exp\left[-g_0\int_0^x d\omega_\nu\frac{\delta}{\delta B_\nu(\omega)}\right]\frac{Z(0,0,B^\mu)}{Z(0,0,0)}\bigg|_{B^\mu=0}. \tag{3.3}$$

Using the definition of Z,

$$Z(0,0,B^\mu) = \left\langle 0\left| T\left(\exp\left\{i\int d^4x[-g_0\bar\psi^{\text{in}}(x)\gamma^\mu\psi^{\text{in}}(x)A_\mu^{\text{in}}(x) + \delta m\bar\psi^{\text{in}}(x)\psi^{\text{in}}(x) + A_\mu^{\text{in}}(x)B^\mu(x)]\right\}\right)\right|0\right\rangle.$$

Rewriting $Z(0,0,B^\mu)$ in terms of one-particle connected (OPC) amplitudes and assuming that under charge conjugation, C, $A(x)$ transforms as $CA^\mu(x)C^{-1} = -A^\mu(x)$, we have

$$Z(0,0,B^\mu) = \exp\left\{\sum_{n=0}^\infty\frac{(-1)^n}{(2n)!}g_0^{2n}\left[\prod_{i=1}^{2n}\int d^4x_i\,B^{\mu_i}(x_i)\right]\langle 0 \text{ out}|T(A_{\mu_1}(x_1)\cdots A_{\mu_{2n}}(x_{2n}))|0 \text{ in}\rangle^{\text{OPC}}_{\eta=\bar\eta=B^\mu=0}\right\}.$$

Therefore, using this form for $Z(0,0,B^\mu)$ and evaluating the shift operators and then setting $B^\mu=0$, Eq. (3.3) becomes

$$\Gamma_a^\mu(-\vec{x}_\perp^{\,2}, x^-p^+) \underset{\text{EA}}{\longrightarrow} -\frac{e^{ip\cdot x}}{(2\pi)^3Z_2}\frac{p^\mu}{m}\frac{m}{E_p}\exp\left\{\sum_{n=0}^\infty\frac{(-1)^n}{(2n)!}(g_0)^{2n}\prod_{i=1}^{2n}\left[\int d^4x_i\left(-\int_0^\infty d\tau_i\,\eta_\xi^{\mu_i}\delta^4(x_i-x-\eta_\xi\tau_i)\right.\right.\right.$$
$$-\int_{-\infty}^0 d\sigma_i\,\eta_\xi^{\mu_i}\delta^4(x_i-\sigma_i\eta_\xi)$$
$$\left.\left.-\int_0^1 d\rho_i\,x^{\mu_i}\delta^4(x_i-x\rho_i)\right)\right]$$
$$\left.\times\langle 0 \text{ out}|T(A_{\mu_1}(x_1)\cdots A_{\mu_{2n}}(x_{2n}))|0 \text{ in}\rangle^{\text{OPC}}\right\}. \tag{3.4}$$

In order to obtain a numerical result, the further simplification is made that gluon structure is neglected, so

$$\langle 0 \text{ out}|T(A_{\mu_1}(x_1)\cdots A_{\mu_{2n}}(x_{2n}))|0 \text{ in}\rangle^{\text{PC}} = \delta_{n1}(-ig_{\mu_1\mu_2})D_F(x_1-x_2),$$

where the representation

$$D_F(x) = \frac{-1}{16\pi^2}\int_0^\infty\frac{d\lambda}{\lambda^2}\exp\left[-i\left(x^2\frac{1}{4\lambda}+\mu^2\lambda - i\epsilon\right)\right]$$

is used for the propagator and μ^2 is a gluon mass introduced to eliminate infrared divergences. The contribution to Γ^{μ}_{a} obtained by neglecting gluon structure is denoted by $\tilde{\Gamma}^{\mu}_{a}$.

The remaining integrations can be done[9] to obtain the eikonal approximation for the leading behavior[10] of $\tilde{\Gamma}^{\mu}_{a}$ as $\vec{x}_{\perp}{}^2 \to 0$:

$$\tilde{\Gamma}^{\mu}_{a}(-\vec{x}_{\perp}{}^2, x^-p^+) \underset{\substack{\text{EA} \\ \vec{x}_{\perp}{}^2 \to 0}}{\sim} \frac{1}{Z_2}\frac{p^{\mu}}{E_p}\frac{e^{ip\cdot x}}{(2\pi)^3}\exp\left[-\frac{g_0^2}{2\pi^2}\int_0^1 \frac{d\alpha}{\alpha}(e^{-ix\cdot p\alpha}-1)\ln(|\vec{x}_{\perp}|\mu\alpha)\right],$$

or for the renormalized vertex function,

$$\tilde{\Gamma}^{\mu}_{Ra}(-\vec{x}_{\perp}{}^2, x^-p^+) \underset{\substack{\text{EA} \\ \vec{x}_{\perp}{}^2 \to 0}}{\sim} \frac{p^{\mu}}{E_p}\frac{e^{ip\cdot x}}{(2\pi)^3}\exp\left[-\frac{g^2}{2\pi^2}\int_0^1 \frac{d\alpha}{\alpha}(e^{-i\alpha x\cdot p}-1)\ln(|\vec{x}|\alpha\mu)\right]. \tag{3.5}$$

Now the contribution to $D(x^2, x\cdot p)$ from $\tilde{\Gamma}^{\mu\,\text{Eikonal}}_{Ra}$ is given by Eq. (2.3),

$$(2\pi)^3\frac{E_p}{M}D(x^2, x\cdot p) \underset{x^2 \to 0}{\sim} 2\,\text{Im}\,e^{ip\cdot x}\exp\left[-\frac{g^2}{2\pi^2}\int_0^1\frac{d\alpha}{\alpha}(e^{-ix\cdot p\alpha}-1)\ln((-x^2)^{1/2}\mu\alpha)\right].$$

Using the relation (2.6b) between $D(x^2, x\cdot p)$ and $W_2(q^2, q\cdot p)$, the contribution to νW_2 from $\tilde{\Gamma}^{\mu}_{aR}$ can be calculated:

$$\nu W_2(q^2,\omega) \underset{\substack{-q^2\to\infty \\ \omega\ \text{fixed}}}{\sim} -\tfrac{1}{2}\pi q^2\int d^4x\, e^{iq\cdot x}\frac{(-i/4\pi^2)}{x^2-i\epsilon x^-}\,\text{Im}\left\{e^{ix\cdot p}\exp\left[-\frac{g^2}{2\pi^2}\int_0^1\frac{d\alpha}{\alpha}(e^{-ix\cdot p\alpha}-1)\ln((-x^2)^{1/2}\mu\alpha)\right]\right\}.$$

Within the approximation of keeping only leading-order terms in each order of perturbation theory, the Fourier transform can be reduced to

$$\nu W_2(q^2,\omega) \underset{\substack{-q^2\to\infty \\ \omega\ \text{fixed}}}{\sim} \frac{\omega}{\pi}\theta(q\cdot p)\int_{-\infty}^{\infty}d(x\cdot p)[e^{i(x\cdot p)(1-\omega)}-e^{-i(x\cdot p)(1+\omega)}]\exp\left[\frac{g^2}{4\pi^2}\int_0^1\frac{d\xi}{\xi}(e^{-i\xi x\cdot p}-1)\ln\left(\frac{2q\cdot p}{\mu^2}\right)\right]. \tag{3.6}$$

The remaining integration does not give any elementary function. However, as $1-\omega\to 0_+$, the leading behavior in $1-\omega$ may be calculated. The result then becomes

$$\nu W_2(q^2,\omega) \underset{\substack{(1-\omega)\to 0_+ \\ -q^2\to\infty}}{\sim} \frac{g^2}{4\pi^2}\frac{1}{1-\omega}\ln(-q^2/\mu^2)\exp\left[\frac{g^2}{4\pi^2}\ln(-q^2/\mu^2)\ln(1-\omega)\right]. \tag{3.7}$$

The structure of νW_2 has been investigated in perturbation theory by making use of the optical theorem to relate $W^{\mu\nu}$ to the imaginary part of the forward spin-averaged Compton amplitude for virtual photon-fermion scattering. Fishbane and Sullivan[3] have calculated, by this method, the graphs corresponding to the contribution from Γ^{μ}_{aR} due to the graphs with no gluon "structure" (i.e., $\tilde{\Gamma}^{\mu}_{aR}$). Their results are exactly the same as Eq. (3.7) near $\omega = 1$.

IV. DISCUSSION OF THE ASSUMPTIONS AND RESULTS

The eikonal approximation for Γ^{μ}_{a} was made by eikonally approximating the wave functions $\bar{\psi}$ and ψ in a frame-dependent way, since the eikonal approximation keeps only the leading terms in p^+ in the frame $p^+\to\infty$, $\vec{p}_{\perp}=0$ ($p^2=m^2$). Since Γ^{μ}_{a} depends only on x^2 and $x\cdot p$, dropping terms nonleading in p^+ may not be justified as p^+ comes into Γ^{μ} only through x^-p^+. For $x\cdot p$ large, however, this approximation is meaningful. Since the region $x\cdot p$ large can be seen by Eq. (3.6) to be related to the leading behavior in $1-\omega$ of νW_2 as $1-\omega$ approaches zero, the results of the eikonal approximation should agree with explicit calculations of these graphs νW_2 in the region $-q^2\to\infty$, $1-\omega\to 0_+$. It is seen in Sec. III that this is the case. Further-

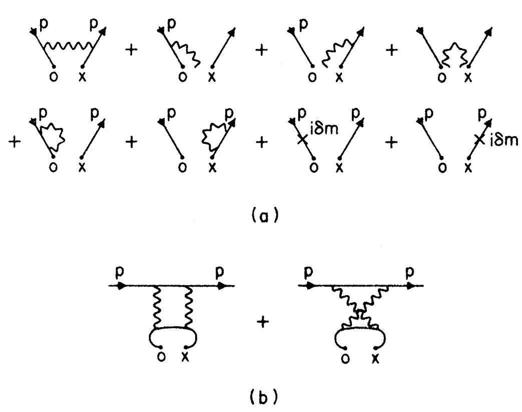

FIG. 1. Figure 1(a) represents the graphs of Γ^{μ}_{a} in order $g_0{}^2$. Figure 1(b) represents the graphs of Γ^{μ}_{b} in order $g_0{}^4$.

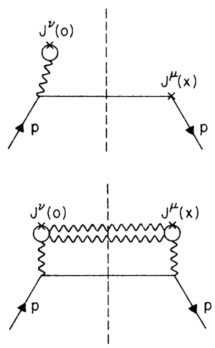

FIG. 2. Examples of unitarity graphs contributing to $W^{\mu\nu}$ in a fermion–neutral-vector-gluon model where the two currents J^{μ} and J^{ν} act on different fermion lines.

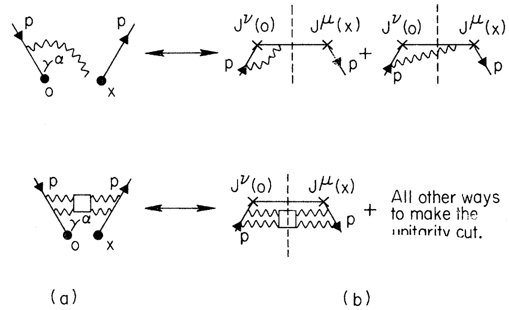

FIG. 3. The correspondence between graphs of Γ_a^{α} and graphs of $W^{\mu\nu}$. Figure 3(a) represents two graphs contributing to Γ_a^{α} and Fig. 3(b) represents the corresponding unitarity graphs of $W^{\mu\nu}$.

more, the explicit calculations of Gribov and Lipatov[1] for $\nu W_2(q^2, \nu)$ show that these structureless gluon graphs dominate in this region of momentum space. More precisely, they have shown that these graphs of νW_2 dominate in the "quasi-elastic" or threshold region; $1-\omega \ll 1$ and $\ln(-q^2/\mu^2) \gg 1$ with $g^2\ln^2(-q^2/\mu^2) \sim 1$.

If all the graphs of νW_2 are considered, three types may be distinguished: graphs in which the two currents act on different fermion lines, graphs in which the two currents both act on the initial fermion line (this is, of course, also the final fermion line), and graphs in which the two currents act on the same line of fermion propagators, but this line does not connect to the initial or final fermion line. Graphs of νW_2 in which the currents $J^{\mu}(x)$ and $J^{\nu}(0)$ act on different fermion lines, as illustrated in Fig. 2, have no corresponding contributions in $\Gamma^{\mu}(x^2, x\cdot p)$. When graphs of this type contribute to the leading behavior of νW_2 in the Bjorken limit, the relation (2.6) between the leading behavior of νW_2 and the behavior of Γ^{μ} near $x^2=0$ will fail in perturbation theory since Γ^{μ} has no contributions corresponding to this type of graph in νW_2. Graphs of νW_2 in which the two currents act on the line of propagators connecting the initial and final fermion correspond to the graphs of Γ^{μ} contained in Γ_a^{μ}. The relation is illustrated in Fig. 3. It is found in perturbation theory[1,3] that of these graphs, the leading behavior comes from the graphs in which the gluons have no structure. These are just the graphs calculated in $\tilde{\Gamma}_a^{\mu}$.

Finally there are the graphs of νW_2 in which the two currents act on the same line of fermion propagators, but this line does not connect to the initial or final fermion. These "pair production" graphs correspond to the graphs of Γ^{μ} contained in Γ_b^{μ}. The relation is illustrated in Fig. 4. The explicit calculations of Gribov and Lipatov[1] for νW_2 show that these pair-production graphs do contribute to the leading behavior of νW_2 in the Bjorken limit, so Γ_b^{μ} must also be calculated. A simple method to apply the eikonal approximation to calculate the leading behavior of Γ_b^{μ} (as $x^2 \to 0$) has not yet been found.

Note added in proof. The experimental data on elastic electron-proton scattering at high energy[11] indicate that the electric, $G_E(q^2)$, and magnetic, $G_M(q^2) \simeq \mu_p G_E(q^2)$, form factors of the proton have a faster decrease than the standard dipole form factor for $-q^2$ from 5 GeV^2 to the largest measured momentum transfer 25 GeV^2. This behavior suggests[12] that asymptotically $G_E(q^2) \simeq G_M(q^2)/\mu_p$ decreases more rapidly than $(-q^2)^{-2}$.

The vertex function in the quark–neutral-vector gluon model can be calculated for large spacelike momentum transfer using the eikonal approximation.[8] This gives the leading logarithmic behavior of the form factor for $-q^2 \gg m^2$,

$$\begin{aligned}G_E(q^2) &\simeq G_M(q^2)/\mu_p\\ &\sim A\exp\{(-q^2/16\pi^2)\ln^2(-q^2/m^2)\}\\ &= A(m^2/-q^2)^{(q^2/16\pi^2)\ln(-q^2/m^2)}.\end{aligned}$$

Although the present data are not conclusive, the form found in the quark–neutral-vector-gluon model fits the large $-q^2$ data well. This suggests that the form factor may fall off faster than any power but that the power will increase slowly as $\ln(-q^2)$. Such behavior of the elastic form factor has the

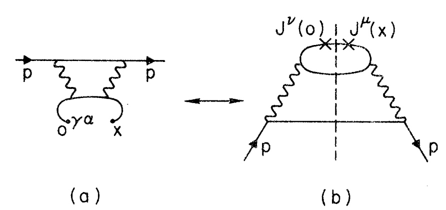

FIG. 4. The correspondence between graphs of Γ_b^{α} and graphs of $W^{\mu\nu}$. Figure 4(a) represents a graph contributing to Γ_b^{α} and Fig. 4(b) represents the corresponding unitarity graphs of $W^{\mu\nu}$.

consequence that νW_2^{ep} will not scale near threshold if a Drell-Yan-West relation[13] holds. It is interesting to note that in the quark–neutral-vector-gluon model this is in fact the case. The Drell-Yan-West relation holds with the simple modification that the power depends on $\ln(-q^2/m^2)$. For if the elastic form factor

$$\frac{G_M(q^2)}{\mu_p} \simeq G_E(q^2) \sim A(m^2/-q^2)^p,$$

where $p = (q^2/16\pi^2)\ln(-q^2/m^2)$, the Drell-Yan-West relation would predict that the threshold behavior of νW_2^{ep} should be

$$\nu W_2^{ep} \sim B(1-\omega)^{2p-1} = B(1-\omega)^{-1}(1-\omega)^{(q^2/8\pi^2)\ln(-q^2/m^2)}.$$

[Here B may also depend on $\ln(-q^2/m^2)$.] This is the form for νW_2 near threshold found in the quark–neutral-vector-gluon model in Eq. (3.6) and also in Refs. 3 and 4.

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[10]By leading behavior of $\tilde{\Gamma}_a^\mu$ is meant that the leading term of the exponent is kept (as $x^2 \to 0$). This is equivalent to keeping only the leading term in x^2 as $x^2 \to 0$ in each order of the perturbative expansion of $\tilde{\Gamma}_a^\mu$.

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