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Non-Abelian Compton Scattering on Targets of Arbitrary Spin*

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The scattering of isovector photons on targets of arbitrary spin S is studied up to second order in the frequency of the incident photon. A general proof is given of all the isospin-antisymmetric low-energy theorems that have been conjectured for arbitrary spin, based on the results obtained for low-spin targets.

I. INTRODUCTION

Bég[1] has studied the non-Abelian Compton effect on nucleons to first order in the frequency ω of the incident photon and showed in particular that the well-known Cabibbo-Radicati sum rule follows from the obtained isospin-antisymmetric first-order theorems. Singh[2] studied the nucleon case to second order in ω and derived an isospin-antisymmetric second-order theorem related to the isovector magnetic-moment charge radius. The scattering of isovector photons on spin-1 targets was taken up by Kumar[3] and new results up to second order were obtained. In particular, a zero-order isospin-antisymmetric result, similar to the spin-$\frac{1}{2}$ case,[1] was put in terms of the gyromagnetic ratio and was conjectured to be valid for higher spins. In two previous papers[4] we have studied the case $S=\frac{3}{2}$ and a generalized form for the Cabibbo-Radicati theorem and the magnetic-moment–radius theorem were obtained, as well as other isospin-antisymmetric results, related to the quadrupole and magnetic octupole moments of the system. All these low-energy theorems were put in a form shown to be valid for $S \leqslant \frac{3}{2}$ and conjectured to be valid for higher spins.

In this paper we discuss the scattering of isovector photons on targets with arbitrary spin S. The isospin-antisymmetric part of the scattering amplitude is studied up to second order and a general proof is given of all these low-energy theorems that have been conjectured to be valid for arbitrary spin.

In Sec. II we give a general discussion of the low-energy theorems. Section III is devoted to the expression of the current matrix element for arbitrary spin S. The results are discussed in Sec. IV.

II. THE LOW-ENERGY THEOREMS

We consider the tensor $T^{\alpha\beta}_{\mu\nu}$ given by

$$(2\pi)^4\delta(p'+k'-p-k)\frac{1}{(2\pi)^3}\left(\frac{M^2}{E_pE_{p'}}\right)^{1/2} T^{\alpha\beta}_{\mu\nu}=i\int d^4x\,d^4y\,e^{-ik'\cdot x}e^{ik\cdot y}\langle \vec{p}'|\,[T\{J^\alpha_\mu(x)J^\beta_\nu(y)\}-i\rho^{\alpha\beta}_{\mu\nu}(x)\delta^4(x-y)]|\,\vec{p}\rangle\,. \tag{1}$$

Here α and β are isospin indices, k and k' (p and p') are incident and outgoing photon (target) momenta. Our metric is defined by $k_\mu=(\vec{k}, ik_0)=(\vec{k}, i\omega)$.

Using current conservation $\partial_\mu J^\alpha_\mu=0$, and the basic equal-time commutation relations of the current operators

$J^\alpha_\mu=(J^\alpha_i, iJ^\alpha_0)$:

$$[J^\alpha_0(x), J^\beta_0(y)]\delta(x_0-y_0)=i\epsilon^{\alpha\beta\gamma}J^\gamma_0(x)\delta^4(x-y)\,, \tag{2}$$

$$[J^\alpha_0(x),\ J^\beta_i(y)]\delta(x_0-y_0)=i\epsilon^{\alpha\beta\gamma}J^\gamma_i(x)\delta^4(x-y)+i\,\partial_m[\rho^{\alpha\beta}_{mi}(x)\delta^4(x-y)]\,, \tag{3}$$

one obtains the divergence conditions[1]

$$\begin{aligned}k'_\mu T^{\alpha\beta}_{\mu\nu} &= T^{\alpha\beta}_{\nu\lambda}k_\lambda\\ &=i(2\pi)^3\left(\frac{E_{p'}E_p}{M^2}\right)^{1/2}\epsilon^{\alpha\beta\gamma}\langle\vec{p}'\,|\,J^\gamma_\nu(0)|\,\vec{p}\rangle\,.\end{aligned} \tag{4}$$

From the conditions (4) one has the identity

$$k_i' T_{ij}^{\alpha\beta} k_j = \omega\omega' T_{00}^{\alpha\beta} + i(2\pi)^3 \left(\frac{E_{p'}E_p}{M^2}\right)^{1/2} \epsilon^{\alpha\beta\gamma} \langle \vec{p}'|\,[\tfrac{1}{2}(\omega+\omega')J_0^\gamma(0) + \tfrac{1}{2}(k_i'+k_i)J_i^\gamma(0)]|\,\vec{p}\rangle\,. \tag{5}$$

Next $T_{ij}^{\alpha\beta}$ is divided into two parts:

$$T_{ij}^{\alpha\beta} = U_{ij}^{\alpha\beta} + E_{ij}^{\alpha\beta}\,, \tag{6}$$

where $U_{ij}^{\alpha\beta}$ refers to the unexcited or one-particle (target) pole contribution, and

$$\frac{1}{(2\pi)^6}\left(\frac{M}{E_{p'}}\right)^{1/2} U_{ij}^{\alpha\beta} = \left[\frac{\langle \vec{p}'|J_i^\alpha|\vec{k}\rangle\langle\vec{k}|J_j^\beta|\vec{0}\rangle}{E(\vec{k})-M-\omega} + \frac{\langle \vec{p}'|J_j^\beta|-\vec{k}'\rangle\langle-\vec{k}'|J_i^\alpha|\vec{0}\rangle}{E(\vec{k}')-M+\omega'}\right], \tag{7}$$

where a summation over the intermediate spin states is implied and we have taken the target initially at rest, $\vec{p}=0$. We then recall that the frequency of the outgoing photon is given by the relation

$$M(\omega'-\omega) = \vec{k}\cdot\vec{k}' - \omega\omega'\,. \tag{8}$$

Using Eqs. (5) and (6) and also splitting $T_{00}^{\alpha\beta}$ into its unexcited and excited parts, we have the relation

$$k_i' E_{ij}^{\alpha\beta} k_j = \omega\omega' E_{00}^{\alpha\beta} + \omega\omega' U_{00}^{\alpha\beta} - k_i' U_{ij}^{\alpha\beta} k_j + i(2\pi)^3 (E_{p'}/M)^{1/2} \epsilon^{\alpha\beta\gamma} \langle\vec{p}'|\,[\tfrac{1}{2}(\omega+\omega')J_0^\gamma(0) + \tfrac{1}{2}(k_i'+k_i)J_i^\gamma(0)]|\,\vec{0}\rangle\,, \tag{9}$$

where

$$\frac{1}{(2\pi)^6}\left(\frac{M}{E_{p'}}\right)^{1/2} U_{00}^{\alpha\beta} = \left[\frac{\langle \vec{p}'|J_0^\alpha|\vec{k}\rangle\langle\vec{k}|J_0^\beta|\vec{0}\rangle}{E(\vec{k})-M-\omega} + \frac{\langle \vec{p}'|J_0^\beta|-\vec{k}'\rangle\langle-\vec{k}'|J_0^\alpha|\vec{0}\rangle}{E(\vec{k}')-M+\omega'}\right] \tag{10}$$

and, since $\rho_{00}^{\alpha\beta}(x)=0$, $E_{00}^{\alpha\beta}$ is given by a similar expression containing all but the single-particle intermediate state. As is well known,[5] $E_{00}^{\alpha\beta}$ is of order ω^2 and this statement has been presented in a more precise way by Singh,[2] who has shown that

$$E_{00}^{\alpha\beta} = k_i' k_j \Gamma_{ij}^{\alpha\beta}(k,k')\,, \tag{11}$$

where $\Gamma_{ij}^{\alpha\beta}$ is free of kinematical singularities and symmetric under the interchange $\alpha \leftrightarrow \beta$, $i \leftrightarrow j$, $k \leftrightarrow -k'$, that is, it obeys crossing symmetry.

Following Pais[6] we write now the complete minimal basis for $E_{ij}^{\alpha\beta}$. To order ω^2 we have[7] for the isospin-antisymmetric part of the amplitude,

$$\begin{aligned}
E_{ij}^{[\alpha\beta]} &= [I^\alpha, I^\beta]\sum_L a_L(\omega,\omega')E_{ij}^{(L)} \\
&= [I^\alpha, I^\beta]\{(\omega+\omega')a_{1,1}\delta_{ij} + [a_2(0) + M(\omega'-\omega)a_{2,1} + (\omega+\omega')^2 a_{2,2}]\epsilon_{ijm}S_m \\
&\quad + (\omega+\omega')a_{3,1}(S_iS_j + S_jS_i - \tfrac{2}{3}\vec{S}^2\delta_{ij}) + a_4(0)[\delta_{ij}\vec{S}\cdot(\vec{k}'\times\vec{k}) - \vec{k}\cdot\vec{k}'\epsilon_{ijm}S_m] \\
&\quad + a_5(0)\epsilon_{ijm}(k_m'\vec{S}\cdot\vec{k}' + k_m\vec{S}\cdot\vec{k}) + a_6(0)\epsilon_{ijm}(k_m'\vec{S}\cdot\vec{k} + k_m\vec{S}\cdot\vec{k}') \\
&\quad + a_7(0)[k_i'(\vec{S}\times\vec{k}')_j - k_i(\vec{S}\times\vec{k})_j + (i\leftrightarrow j)] \\
&\quad + a_8(0)[k_i(\vec{S}\times\vec{k}')_j - k_i'(\vec{S}\times\vec{k})_j + (i\leftrightarrow j) - 2\vec{k}\cdot\vec{k}'\epsilon_{ijm}S_m] \\
&\quad + a_9(0)[(\langle S_iS_j, S_r\rangle + \langle S_j, S_i, S_r\rangle)(\vec{k}'\times\vec{k})_r - \tfrac{2}{5}(3\vec{S}^2-1)\vec{S}\cdot(\vec{k}'\times\vec{k})\delta_{ij}] \\
&\quad + a_{10}(0)\epsilon_{ijr}(k_m'k_n' + k_mk_n)\langle S_m, S_n, S_r\rangle \\
&\quad + a_{11}(0)[\epsilon_{ijr}(k_m'k_n + k_mk_n')\langle S_m, S_n, S_r\rangle - \tfrac{2}{5}(3\vec{S}^2-1)\vec{k}\cdot\vec{k}'\epsilon_{ijm}S_m] + O(\omega^3)\}\,,
\end{aligned} \tag{12}$$

where

$$\langle A, B, C\rangle = ABC + CAB + BCA\,,$$

$\vec{S}^2 = S(S+1)$, I^α is the αth component of isospin, and the expansion of the coefficients, to order ω^2, is in accordance with crossing symmetry.

Upon contraction with $k_i'k_j$ and making use of Eq. (8), one obtains

$$k_i'E_{ij}^{[\alpha\beta]}k_j=[I^\alpha,I^\beta](\vec{k}\cdot\vec{k}'(\omega+\omega')(a_{1,1}-\tfrac{2}{3}\vec{S}^2a_{3,1}) \\ +\vec{S}\cdot(\vec{k}'\times\vec{k})\{a_2(0)+\vec{k}\cdot\vec{k}'a_{2,1}-\tfrac{2}{5}\vec{k}\cdot\vec{k}'(3\vec{S}^2-1)[a_9(0)+a_{11}(0)]+\omega\omega'[4a_{2,2}-a_{2,1}+2a_7(0)]\} \\ +\{\vec{S}\cdot\vec{k}',\vec{S}\cdot\vec{k}\}(\omega+\omega')a_{3,1}+(k_i'k_j+k_ik_j')(\vec{k}'\times\vec{k})_r\langle S_i,S_j,S_r\rangle[a_9(0)+a_{11}(0)] \\ +(k_i'k_j'+k_ik_j)(\vec{k}'\times\vec{k})_r\langle S_i,S_j,S_r\rangle a_{10}(0)+O(\omega^5))\ . \tag{13}$$

The unknown term $\Gamma_{ij}^{[\alpha\beta]}$ of Eq. (11) can be expanded in the same basis as $E_{ij}^{[\alpha\beta]}$. Therefore, from Eq. (11), one can write

$$\omega\omega'E_{00}^{[\alpha\beta]}=[I^\alpha,I^\beta]\vec{S}\cdot(\vec{k}'\times\vec{k})\omega\omega'b_2(0)+O(\omega^5)\ , \tag{14}$$

where $b_2(0)$ is an unknown coefficient.

Equations (13) and (14) are now to be substituted in Eq. (9). It is apparent from Eqs. (13) and (14) that only $a_{2,2}$ and $a_7(0)$ will receive an unknown contribution from $\omega\omega'E_{00}^{[\alpha\beta]}$. All the other amplitudes in Eq. (13) will be completely determined by the known terms of Eq. (9) giving six low-energy theorems.

III. THE CURRENT FOR ARBITRARY SPIN

In this section we shall prepare the way to establish the explicit expression of the low-energy theorems. As we are working on $E_{ij}^{[\alpha\beta]}$ to order $O(\omega^2)$, we need in Eq. (9) $U_{\mu\nu}^{\alpha\beta}$ to order $O(\omega^2)$ and $\langle\vec{p}\,|J_\mu^\alpha(0)|\,\vec{0}\,\rangle$ to order $O(\omega^3)$. To compute these quantities we need the isovector current matrix element for arbitrary spin S. This has been done before[8,9] to order $O(\omega^2)$ and also[10] to order $O(\omega^3)$, in the study of the scattering of physical photons on arbitrary-spin targets. Therefore, we shall quote only the main results, paying attention only to the definition of the various form factors and their relations to the multipole moments, since some confusion of normalization seems to prevail in the literature.[9,10] The expansion of the current matrix element in the Breit frame is[11]

$$(2\pi)^3\langle\tfrac{1}{2}\vec{Q},\sigma'|\{J_0^\alpha,\vec{J}^\alpha\}|-\tfrac{1}{2}\vec{Q},\sigma\rangle \\ =I^\alpha\sum_{l=0}^{2S}\left\{A_l(\vec{Q}^2),-iB_l(\vec{Q}^2)\vec{Q}\times\frac{\partial}{\partial\vec{Q}}\right\}R_l(\vec{S}\cdot\vec{Q})_{\sigma'\sigma}\ , \tag{15}$$

where σ and σ' denote polarization states and

$$R_l(\vec{S}\cdot\vec{Q})=(\vec{S}^2\vec{Q}^2)^{l/2}P_l\left(\frac{\vec{S}\cdot\vec{Q}}{(\vec{S}^2\vec{Q}^2)^{1/2}}\right)\ , \tag{16}$$

where $P_l(x)$ is the Legendre polynominal of degree l. Parity conservation demands $A_l=0$ for odd l and $B_l=0$ for even l. To order $O(Q^3)$, only $A_0(0)=1$, $A_0'=[dF_0(t)/dt]_0$, and $A_2(0)$ will be present in the Breit-frame matrix element of J_0^α, and only $B_1(0)$, $B_1'=[dG_1(t)/dt]_0$, and $B_3(0)$ will appear in the vector part of the matrix element. Before continuing we shall relate these various isovector form factors to the isovector charge mean-square radius $\langle r^2\rangle^V$, electric moment Q^V (in units of $1/M^2$), magnetic dipole moment μ^V (in units of $1/2M$), magnetic dipole mean square radius $\langle R^2\rangle^V$ and the magnetic octupole moment Ω^V (in units of $1/2M^3$), respectively given by the general definitions[12]:

$$\langle r^2\rangle^VI^\alpha=\left\langle\vec{0},S\left|\int J_0^\alpha(\vec{r})r^2d\vec{r}\right|\vec{0},S\right\rangle\ , \tag{17a}$$

$$\frac{Q^V}{M^2}I^\alpha=\left\langle\vec{0},S\left|\int J_0^\alpha(\vec{r})(3z^2-r^2)d\vec{r}\right|\vec{0},S\right\rangle\ , \tag{17b}$$

$$\frac{\mu^V}{2M}I^\alpha=\frac{1}{2}\left\langle\vec{0},S\left|\int(\vec{r}\times\vec{J}^\alpha)_zd\vec{r}\right|\vec{0},S\right\rangle\ , \tag{17c}$$

$$\langle R^2\rangle^VI^\alpha=\frac{1}{2}\left\langle\vec{0},S\left|\int(\vec{r}\times\vec{J}^\alpha)_zr^2d\vec{r}\right|\vec{0},S\right\rangle\ , \tag{17d}$$

$$\frac{\Omega^V}{2M^3}I^\alpha=\frac{1}{4\times6^{1/2}}\left\langle\vec{0},S\left|\int(\vec{r}\times\vec{J}^\alpha)_z(5z^2-r^2)d\vec{r}\right|\vec{0},S\right\rangle\ . \tag{17e}$$

These quantities are computed by the usual relation[13]

$$\left\langle\vec{0},S\left|\int x_{n_1}\cdots x_{n_m}J_\mu^\alpha(\vec{r})d\vec{r}\right|\vec{0},S\right\rangle \\ =(2\pi)^3(-i)^N\lim_{\vec{Q}\to0}\frac{\partial^N}{\partial Q_{n_1}\cdots\partial Q_{n_m}} \\ \times\langle\tfrac{1}{2}\vec{Q},S|J_\mu^\alpha(0)|-\tfrac{1}{2}\vec{Q},S\rangle\ , \tag{18}$$

where $N=n_1+\cdots+n_m$.

Using Eq. (15) one obtains the following relations[12]:

$$\langle r^2\rangle^V=-6A_0'\ , \tag{19a}$$

$$Q^V=-3M^2S(2S-1)A_2(0)\ , \tag{19b}$$

$$\mu^V=2MSB_1(0)\ , \tag{19c}$$

$$\langle R^2\rangle^V=5SB_3(0)-10SB_1'\ , \tag{19d}$$

$$6^{1/2}\Omega^V=-30M^3S(S-1)(2S-1)B_3(0)\ . \tag{19e}$$

With these relations we have the current matrix element in the Breit frame expressed in terms of the multipole moments.

With a Lorentz transformation we can go from the Breit frame to an arbitrary frame.[9] By straightforward calculation one obtains,[9,10] with

$\vec{q}=\vec{p}'-\vec{p}$ and $\vec{P}=\vec{p}'+\vec{p}$,

$$\langle\vec{p}'|J_0^\alpha|\vec{p}\rangle = I^\alpha\left\{1+\vec{q}^{\,2}A_0'+\frac{2MB_1(0)-1}{2M^2}\,i\vec{S}\cdot(\vec{p}'\times\vec{p})+\tfrac{1}{2}A_2(0)[3(\vec{S}\cdot\vec{q})^2-\vec{S}^2\vec{q}^{\,2}]+O(q^4)\right\} \tag{20}$$

and

$$\begin{aligned}\langle\vec{p}'|\vec{J}^\alpha|\vec{p}\rangle = I^\alpha\Bigg[&\frac{\vec{P}}{2M}+i[B_1(0)+\vec{q}^{\,2}B_1'](\vec{S}\times\vec{q})+\tfrac{1}{2}iB_3(0)[5\langle(\vec{S}\times\vec{q}),\vec{S}\cdot\vec{q},\vec{S}\cdot\vec{q}\rangle-3\vec{S}^2\vec{q}^{\,2}(\vec{S}\times\vec{q})]\\&+\frac{\vec{P}}{2M}\left(\vec{q}^{\,2}A_0'+\tfrac{1}{2}A_2(0)[3(\vec{S}\cdot\vec{q})^2-\vec{S}^2\vec{q}^{\,2}]+\frac{i}{2M^2}\vec{S}\cdot(\vec{p}\times\vec{p}')-\frac{\vec{P}^2}{8M^2}-\frac{\vec{q}^{\,2}}{16M^3}+\frac{iB_1(0)}{4M}\vec{S}\cdot(\vec{q}\times\vec{P})\right)\\&-\frac{iB_1(0)}{8M^2}\vec{P}^2(\vec{S}\times\vec{q})-\frac{B_1(0)}{4M^2}\{(\vec{S}\cdot\vec{p}\times\vec{p}'),(\vec{S}\times\vec{q})\}-\frac{iB_1(0)}{8M^2}(\vec{P}\cdot\vec{q})(\vec{S}\times\vec{P})+O(q^5)\Bigg]\,.\end{aligned} \tag{21}$$

IV. RESULTS AND DISCUSSION

From Eq. (8) one has

$$\frac{1}{E(\vec{k})-M-\omega}+\frac{1}{E(\vec{k}')-M+\omega'}=-\frac{\vec{k}\cdot\vec{k}'}{M\omega\omega'}+\frac{\omega'-\omega}{4M^2}+O(\omega^3)\,, \tag{22}$$

$$\frac{1}{E(\vec{k})-M-\omega}-\frac{1}{E(\vec{k}')-M+\omega'}=-\frac{1}{\omega}-\frac{1}{\omega'}-\frac{\omega+\omega'}{4M^2}+O(\omega^4)\,. \tag{23}$$

Using Eqs. (20)–(23) in Eqs. (7) and (10), and recalling Eq. (14), we have for the isospin-antisymmetric part of Eq. (9),

$$\begin{aligned}k_i'E_{ij}^{[\alpha\beta]}k_j=[I^\alpha,I^\beta]\Bigg\{&\tfrac{1}{2}(\omega+\omega')\vec{k}\cdot\vec{k}'[\vec{S}^2A_2(0)-2A_0']+i\vec{S}\cdot(\vec{k}'\times\vec{k})\bigg[-B_1(0)+\vec{k}\cdot\vec{k}'\left(2B_1'(0)-3S^2B_3(0)+\frac{2MB_1(0)-1}{4M^3}\right)\\&+\omega\omega'[-2B_1'(0)+3\vec{S}^2B_3(0)-ib_2(0)]\bigg]\\&-\tfrac{3}{4}\{\vec{S}\cdot\vec{k},\vec{S}\cdot\vec{k}'\}(\omega+\omega')A_2(0)+\tfrac{5}{2}iB_3(0)(k_i'k_j+k_ik_j')(\vec{k}'\times\vec{k})_r\langle S_i,S_j,S_r\rangle\\&-\tfrac{5}{2}iB_3(0)(k_i'k_j'+k_ik_j)(\vec{k}'\times\vec{k})_r\langle S_i,S_j,S_r\rangle+O(\omega^5)\Bigg\}\,.\end{aligned} \tag{24}$$

By comparing Eqs. (13) and (24), and using Eqs. (19a)–(19e) we obtain the following isospin-antisymmetric low-energy theorems:

$$a_{1,1}=\tfrac{1}{6}(\langle r^2\rangle^V)\,, \tag{25a}$$

$$a_2(0)=\frac{\mu^V}{2iMS}\,, \tag{25b}$$

$$a_{2,1}=i\left(-\frac{\langle R^2\rangle^V}{5S}+\frac{\mu^V}{4M^3S}-\frac{1}{4M^3}\right)\,, \tag{25c}$$

$$a_{3,1}=\frac{Q^V}{4M^2S(2S-1)}\,, \tag{25d}$$

$$a_9(0)+a_{11}(0)=\frac{\sqrt{6}\,\Omega^V}{12iS(S-1)(2S-1)M^3}\,, \tag{25e}$$

$$a_{10}(0)=\frac{i\sqrt{6}\,\Omega^V}{12S(S-1)(2S-1)M^3}\,. \tag{25f}$$

Theorem (25b) confirms a conjecture[2] based on the analysis of the corresponding results for $S=1$.

Theorems (25a) and (25c)–(25f) constitute a proof of the results conjectured before[4] on the basis of the results obtained for $S \leq \frac{3}{2}$. In particular, theorem (25a) is the generalized form of the Cabibbo-Radicati theorem and (25c) is the generalized form of the magnetic moment radius theorem, which were first derived, respectively, by Bég[1] and Singh[2] for the nucleon case, in terms of the usual nucleon isovector form factors.

The low-energy theorems up to second order associated with the isospin-symmetric part of the amplitude give a trivial extension of the results obtained earlier for physical Compton scattering.[2,6,10] A more general analysis of the amplitude for physical Compton scattering on targets of arbitrary spin, up to order $O(\omega^3)$, has been given by Lin.[10] It would be interesting to investigate the implication of this type of analysis on the isospin-antisymmetric part of the amplitude for isovector photons.

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[7]With the last term present in $E_{ij}^{(9)}$ and in $E_{ij}^{(11)}$, the magnetic octupole moment will not appear in the amplitude $A_{2,1}$ when it is expressed in terms of the magnitude radius $\langle R^2\rangle^V$, in Eq. (25c).

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