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$$\alpha m_1 \frac{\Delta m^2}{m_W{}^2} \frac{\sin^2\theta_C}{\sin^2\theta_W},$$

where Δm^2 is the difference between the squared masses of charmed and uncharmed quarks and m_1 is the mass of quark q_1. This is to be compared with the leading-logarithmic divergent term whose coefficient is proportional to $3\alpha(a-\frac{1}{2})m_1/\cos^2\theta_W$. Thus the contribution from F^b to the mass difference is highly suppressed in relation to the leading term.

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Approach to a Complete Bootstrap*

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It is shown in a simple model of quarks and mesons with Yukawa coupling that consistency conditions determine all dimensionless parameters of the model. Similar results are discussed for models also containing baryons (bound states of three quarks) and SU(3) symmetry. We use the static limit of the ladder approximation.

I. INTRODUCTION

Since the bootstrap idea was first formulated[1] no one has succeeded, to the author's knowledge, in constructing a completely bootstrapped model, that is, a model in which all dimensionless constants are uniquely determined by consistency conditions.

In a recent publication[2] (hereafter referred to as II) a model of quarks, mesons, and nucleons was constructed from a four-fermion point interaction, where all particles are bound states. However, as emphasized in II, the four-fermion coupling constant was left as a free parameter, arbitrarily chosen to ensure some simple situation, hence failing to achieve a real bootstrap. We thus see that the condition that all particles be bound states (i.e., that there be no "elementary" particles) is not equivalent to a complete bootstrap.

It is the purpose of this paper to construct a simple soluble model which is completely bootstrapped. While constructing this model, we emphasize mathematical and conceptual simplicity and the bootstrap idea more than the need for precise description of experimental data. From this point of view our model is a mathematical presentation of the bootstrap idea and not a phenomenological physical theory. Nevertheless, we will build our model as much as possible from physical data and intuition.

In Sec. II we define our model, consisting of a spin-$\frac{1}{2}$ quark and a scalar meson interacting via a Yukawa coupling, and we show that in the static limit of the ladder approximation all dimensionless parameters are determined.

In Sec. III we briefly outline some generalizations of our basic model: We include pseudoscalar as well as scalar mesons, we include spin-$\frac{1}{2}$ and -$\frac{3}{2}$ baryons (bound states of three quarks) as well as mesons, and we include SU(3) symmetry.

In Sec. IV we discuss the implication of this paper for other theories of elementary particles and the limitations due to the approximations used in this paper.

II. COMPLETELY BOOTSTRAPPED MODEL OF QUARKS AND MESONS

In an attempt to reach maximum simplicity, at least at the initial steps of this presentation, we will discuss a model consisting only of a spin-$\frac{1}{2}$ quark (all other quantum numbers being zero) and a scalar meson. We will also limit ourselves to the discussion of states of lowest angular momentum. From our physical intuition we will borrow the following ideas.

(1) Quantum field theory can be used to describe hadrons. In particular we will accept the validity of the ladder approximation.

(2) Quarks are much heavier than mesons. That is, $m \gg m_S$, where m is the mass of the quark and m_S the mass of the scalar meson.

(3) Leaving for a minute the limited scope of our initial model, we remark that quarks are very tightly bound into baryons, with binding energies of several BeV. On the other hand we know that nucleons in a nucleus are bound to each other by, at most, a few MeV. Therefore, we conclude that quarks composing one nucleon do not interact with quarks composing a neighbor nucleon. We are thus led to the assumption that the interaction which binds quarks into hadrons has a very short range (much shorter than the range of nuclear forces).

(4) Quarks and mesons interact via a Yukawa coupling

$$g:\bar{q}q\phi: \tag{1}$$

where q, $\bar{q}$, and ϕ are respectively the quark, antiquark, and scalar-meson field operators; g is the quark-scalar meson coupling constant; and : : denotes normal product.

Our consistency conditions follow from the assumption of analyticity of the S matrix; i.e., particles appear as poles in the scattering amplitudes. These conditions are the following.

(1) Because $q\bar{q}$ have the quantum numbers of ϕ, the $q\bar{q}$ scattering amplitude has a pole at $s = m_S^2$ (s being the four-momentum squared of the system) and the residue at the pole is the square of the $q\bar{q}\phi$ coupling.

(2) Because $q\phi$ have the quantum numbers of q, the $q\phi$ scattering amplitude has a pole at $s = m^2$ and the residue at the pole is again the square of the $\bar{q}q\phi$ coupling.

(3) Because $\phi\phi$ have the quantum numbers of ϕ, the $\phi\phi$ scattering amplitude has a pole at $s = m_S^2$ and the residue at the pole determines the $\phi\phi\phi$ coupling constant. (The existence of such an interaction thus follows the consistency conditions.)

Let us start with a discussion of the first condition. From the second and third assumptions of our model, and the first consistency condition, it follows that mesons are very tightly bound states of $q\bar{q}$, i.e., they have very high binding energy and a very-short-range interaction between q and $\bar{q}$. The only interaction at our disposal is a Yukawa-type interaction [assumption (4)]. This interaction tempts us to propose that the $q\bar{q}$ interaction is generated by exchanges of mesons. However, since mesons are much lighter than quarks [assumption (2)] this mechanism will generate relatively long-range interactions (with a range given by the Compton wavelength of the meson), in contradiction with assumption (3). Thus we are led to the conclusion that in the simple ladder approximation $q\bar{q}$ are bound through exchanges of quarks, as illustrated in Fig. 1(a). Because the quark is assumed to be heavy, we go the static limit where the propagator of an exchanged quark is replaced by $1/m$. In this case Fig. 1(a) reduces to Fig. 1(b), where the effective $\bar{q}q\phi\phi$ interaction constant is given by

$$g_{\bar{q}q\phi\phi} = g^2/m\,. \tag{2}$$

For a given value of s, e.g., $s = m_S^2$, the $\phi\phi$ bubble is a mere number. Figure 1(b) thus reduces to Fig. 1(c), where

$$g_{\bar{q}q\bar{q}q} = g_{\bar{q}q\phi\phi}{}^2 J_{\phi\phi}(s = m_S^2)\,; \tag{3}$$

here

$$J_{\phi\phi}(S) = \int_{4m_S^2}^{\Lambda'^2} \frac{\rho_{\phi\phi}(s')ds'}{s'-s} \tag{4}$$

and

$$\rho_{\phi\phi}(s) = (1 - 4m_S^2/s)^{1/2}/4\pi^2\,. \tag{5}$$

The sum of ladders such as illustrated by Fig. 1(c) is given in II:

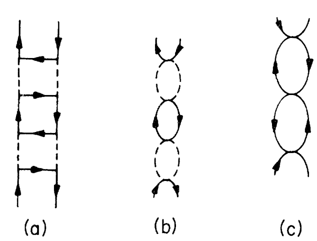


FIG. 1. (a) A contribution to the $q\bar{q}$ scattering amplitude in the ladder approximation; (b) the static limit of (a); (c) an equivalent representation of (b). Solid lines represent quarks; broken lines represent mesons.

$$A^{(s)}_{q\bar{q}} = \frac{g_{\bar{q}q\bar{q}q}}{1 - g_{\bar{q}q\bar{q}q}J_{q\bar{q}}(s, \Lambda)}\,, \tag{6}$$

where

$$J_{\bar{q}q}(s, \Lambda) = \frac{1}{4\pi^2}\int_{4m^2}^{\Lambda^2} \frac{(s' - 4m^2)(1 - 4m^2/s')^{1/2}\,ds'}{s' - s} \tag{7}$$

is the $\bar{q}q$ bubble. The first consistency condition then requires that

$$g_{\bar{q}q\bar{q}q}J_{\bar{q}q}(s = m_S{}^2, \Lambda) = 1 \quad \text{(pole condition)} \tag{8}$$

and

$$\frac{g^2}{4\pi} = 2\pi\left[\int_{4m^2}^{\Lambda^2} \frac{(s - 4m^2)(1 - 4m^2/s)^{1/2}\,ds}{(s - m_S{}^2)^2}\right]^{-1} \quad \text{(residue condition).} \tag{9}$$

Next we consider the second consistency condition. The $q\phi$ scattering amplitude is given in the ladder approximation by a sum over diagrams such as illustrated in Fig. 2(a). In the static limit Fig. 2(a) reduces to Fig. 2(b), where $g_{\bar{q}q\phi\phi}$ is defined by Eq. (2).

It is shown in II that the pole and residue conditions are respectively given by

$$\frac{2mg_{\bar{q}q\phi\phi}}{\pi}\int_{(m+m_S)^2}^{\Lambda''^2} \frac{\rho_{q\phi}(s')ds'}{s' - s} = 1\,, \tag{10}$$

where

$$\rho_{q\phi}(s) = s^{-1}[s - (m - m_S)^2]^{1/2}\,[s - (m + m_S)^2]^{1/2} \quad \text{(pole condition)} \tag{11}$$

and

$$\frac{g}{4\pi} = 2\pi\left[\int_{(m+m_S)^2}^{\Lambda''^2} \frac{\rho_{q\phi}(s')ds'}{(s' - m_S{}^2)^2}\right]^{-1} \quad \text{(residue condition).} \tag{12}$$

The last consistency condition can be written in the ladder approximation as a sum over diagrams such as illustrated by Fig. 3(a). From our first consistency condition it follows that this sum is reduced to Fig. 3(b), which in the static limit reduces to Fig. 3(c). Thus, condition (1) ensures that condition (3) is satisfied, and the latter determines the three-meson coupling constant

$$g_{\phi\phi\phi} = \frac{g^3}{m} J_{q\bar{q}}(s = m_S{}^2)\,. \tag{13}$$

It was found in II that Eqs. (2), (8), (9), (10), and (12) give

$m/\Lambda = 10^{-4}$,

$1.1 \times 10^{-5} \leqslant m_S/m \leqslant 1.1 \times 10^{-4}$,

$g_{\bar{q}q\bar{q}q}\,m^2 \approx 0.25 \times 10^{-3}$,

$g^2 = 2.37$,

$\Lambda/\Lambda'' = 0.9 \times 10^{-4}$.

Using these results and Eq. (3) we get

$\Lambda'^2/4m_S{}^2 - 1 \approx 10^{-3}$.

From Eq. (13) we get

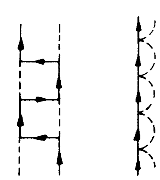
$g_{\phi\phi\phi}/m \approx 1.4 \times 10^4$.

The introduction of three cutoff parameters might seem to undercut the bootstrap. However, we see that the consistency conditions determine all dimensionless parameters of our model, including all such combinations formed with the cutoff parameters. In particular, the cutoff parameters are uniquely numerically determined up to an over-all multiplicative scaling constant. We notice that the quark comes out 10^4 to 10^5 times heavier than the meson, in accordance with our basic assumptions. It cannot be overstressed that this model is too crude to give a precise or even a rough estimate of measurable quantities. The important results are not the specific values of our dimensionless parameters but the fact that they are all determined by consistency conditions.

III. GENERALIZATIONS

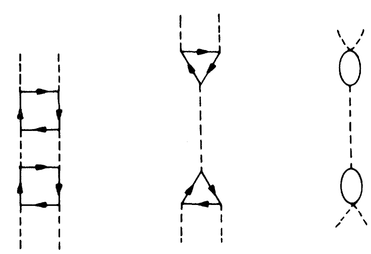
A. A Model with Quarks, Scalar Mesons, and Pseudoscalar Mesons

In addition to scalar mesons we can also include in our model pseudoscalar mesons of mass m_P which couple to quarks via



(a) (b)

FIG. 2. Contributions to the qs scattering amplitude, (a) in the ladder approximation and (b) in the static limit of (a).



(a) (b) (c)

FIG. 3. (a) A contribution to the meson-meson scattering amplitude; (b) the meson-meson scattering amplitude; (c) the static limit of (b).

$$g_P : \bar{q}\gamma_5 q\pi : \tag{14}$$

where π is the pseudoscalar-meson field operator and g_P the $\bar{q}q\pi$ coupling constant.

This model can be developed in the same way as the previous one. The only changes are the following.

(1) When the $\bar{q}q$ scattering amplitude is considered, intermediate states of SS as well as PP contribute to the scalar-meson bound state; hence $J_{\phi\phi}$ of the previous section should be replaced by $J_{\phi\phi}+J_{\pi\pi}$, where $J_{\pi\pi}$ is given by $J_{\phi\phi}$ where m_S has been replaced by m_P.

(2) For the pseudoscalar-meson bound state only SP intermediate states contribute, and $J_{\phi\phi}$ should be replaced by $J_{\phi\pi}$ which is given by $J_{\phi\phi}$ where one m_S has been replaced by m_P.

Notice that we do not get any mixture of the two cases, namely a PP or SS bubble in the same chain with a PS bubble, because the $q\bar{q}$ bubble is identically zero if the two Feynman propagators are contracted via γ_5 at one end of the bubble and via I at the other end.

(3) The meson-quark scattering amplitude has to be considered as a two-by-two matrix, as each of the incoming and the outgoing mesons can be scalar or pseudoscalar. Similar considerations apply to the meson-meson scattering amplitude. The explicit development of this model does not present any conceptual difficulties but involves tedious calculations. Therefore we will not present this model in any more detail but will simply state the results; again, all dimensionless parameters are completely determined, in particular

$$m/\Lambda = 10^{-3}, \quad 0.1 < m_P/m_S < 1$$

and

$$m_P/m = 2.10^{-7} \quad \text{or} \quad m_P/m = 6.10^{-9}$$

Again we notice that the mesons are much lighter than the quark.

B. A Model with Quarks, Mesons, and Baryons

To our initial model (Sec. II) we now add a baryon N of spin $\frac{1}{2}$ and mass M, and a baryon N^* of spin $\frac{3}{2}$ and mass M^*. We assume that they couple to quarks via

$$G_N : q_\alpha C^{\alpha\beta} q_\beta q_\gamma \bar{N}^\gamma : +\text{H.c.}, \tag{15}$$

where α, β, γ are Dirac indices, N^γ is the spin-$\frac{1}{2}$ baryon field operator, G_N is the $qqq\bar{N}$ coupling constant, and $C^{\alpha\beta}$ is the charge conjugation operator. Similarly, for the spin-$\frac{3}{2}$ baryon,

$$G_{N^*} : q_\alpha (C\gamma_\mu)^{\alpha\beta} q_\beta q_\gamma (\bar{N}^*_\mu)^\gamma : +\text{H.c.}, \tag{16}$$

where $N^*_{\mu\alpha}$ is the Rarita-Schwinger field operator, and G_{N^*} is the $qqq\bar{N}^*$ coupling constant. In both cases symmetrization over Dirac indices is assumed.

Now again the model is developed in the same way as in Sec. II, with the same numerical results, but in addition we have to consider the three-quark scattering amplitude. This amplitude is the sum of diagrams such as illustrated in Fig. 4(a), which reduces to Fig. 4(b) in the static limit. In Fig. 4(b), for a given value of s (e.g., $s=M^2$ or M^{*2}) the NS or N^*S bubble is a mere number, and Fig. 4(b) reduces to Fig. 4(c), where the six-fermion coupling constant G is determined by

$$\begin{aligned} G(M^2) &= \frac{G_N^{\,2}}{m^2} J_{N\phi}(s=M^2), \\ G(M^{*2}) &= \frac{G_{N^*}^{\,2}}{m^2} J_{N^*\phi}(s=M^{*2}); \end{aligned} \tag{17}$$

here $J_{N\phi}$ is given by $J_{q\phi}$, where m is replaced by M and $J_{N^*\phi}$ is an analogous expression for the N^*S bubble.

It is shown in II that the sum of diagrams of the type illustrated in Fig. 4(c) will develop poles at $M \ll m$ and $M^* \ll m$ for the three quarks in a state of spin $\frac{1}{2}$ and $\frac{3}{2}$, respectively, if

$$G \to \Lambda^{-5} \quad \text{as } \Lambda \to \infty .$$

We see that this will be the case if $G_N \to \Lambda^{-2}$ and $G_{N^*} \to \Lambda^{-2}$ as $\Lambda \to \infty$. This behavior of G_N and G_{N^*} is indeed expected from their dimensions.

As discussed in II, all dimensionless constants will again be determined, although their exact numerical value will be much more difficult to extract.

C. SU(3)

The model of the previous subsection can be easily generalized to include SU(3), following closely the procedure of II. Thus, we will not repeat this generalization here but simply mention that we obtain the Cutkosky relations, the additive

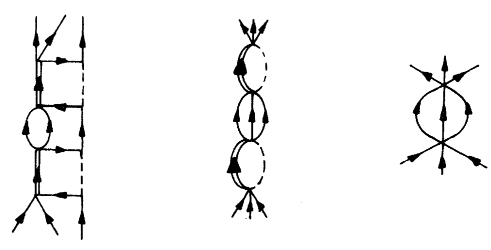


FIG. 4. (a) A contribution to the three-quark scattering amplitude in the ladder approximation; (b) the static limit of (a); (c) an equivalent representation of (b). The double lines represent baryons.

quark model, and the absence of exotic quarks (e.g., of charge $\frac{4}{3}$).

The model of quarks with scalar and pseudoscalar mesons can also be generalized[3] to include SU(3). In particular one obtains the Cutkosky relations for $g_{\overline{q}q\phi}$ and $g_{\overline{q}q\pi}$ separately.

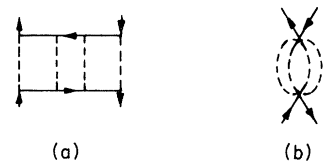


FIG. 5. (a) A many-meson intermediate-state contribution to the $q\overline{q}$ scattering amplitude; (b) the static limit of (a).

IV. CONCLUSIONS

We have seen in the previous sections that for some simple models consistency conditions imply a complete and explicit bootstrap. Unfortunately, the models examined here are too simple and approximate to be seriously considered as physical theories. However, these examples suggest that consistency conditions alone might lead to a complete bootstrap for more sophisticated theories, with fewer approximations than in this paper. Consider for example the approximation in Sec. II where we take into account only the two-meson intermediate state for the $\overline{q}q$ scattering amplitude. Obviously we should also take into account intermediate states involving more mesons, even when we look at the $q\overline{q}$ scattering amplitude near the mass of a single meson. In principle we can replace the two-meson intermediate state by a sum over intermediate states of many mesons, such as illustrated in Fig. 5(a). In the static limit Fig. 5(a) will be reduced to Fig. 5(b) and again the sum over many-meson intermediate states, for a given s, will be a mere number, depending on cutoff. In this case we have to replace $J_{\phi\phi}(s, \Lambda')$ by a more complicated function $F(s, \Lambda')$. As Λ' varies from its threshold to infinity, $F(s, \Lambda')$ varies continuously from zero to infinity [$F(s, \Lambda')$, as well as $J_{\phi\phi}(s, \Lambda')$, behaves like Λ'^2 for $\Lambda' \to \infty$]. Therefore, we will again be able to choose Λ' in such a way that the consistency conditions will be satisfied. However, since $F(s, \Lambda')$ is a complicated function, the explicit calculation of the numerical values of the dimensionless parameters of the model will be more difficult, and probably not practically feasible. It seems from the above argument that in general the approximations used so far are not essential to obtain a bootstrap. Probably even if we do not go to the static limit, and even if we do not restrict ourselves to the ladder approximation, we can still write, at least formally, the consistency conditions, and these conditions will again fix all dimensionless parameters. Obviously these equations will be too complicated to even be written down explicitly. However, it might be interesting, starting with more realistic models than ours, to write down consistency conditions analogous to those discussed here, to solve them if possible, and then to compare the numerical values with experiment.

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