term in the unification of strong, weak, and electromagnetic interactions is discussed in Ref. 7.

[24]Of course, we should expect the parameters f, g, v, κ, and λ to be readjusted in order to properly fit the ρ, A_1, and ρ' masses and the ρ width when the present model is embedded in the larger M scheme.

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Proton-Neutron Mass Difference in a Unified Theory of Weak and Electromagnetic Interactions

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The question of cancellation of the logarithmic divergence appearing in the proton-neutron mass difference is studied in a unified theory of weak and electromagnetic interactions. It is concluded that the contribution from weak hadronic currents in an extension of the Salam-Weinberg model for leptons to hadrons does not cancel the logarithmic divergence arising from purely electromagnetic interactions.

I. INTRODUCTION

It has been known for some time that a logarithmic divergence[1] appears in the proton-neutron mass difference when this mass difference is calculated from purely electromagnetic interactions. Recently there has been some interest in unified models[2] of weak and electromagnetic interactions, and in one sense this interest stems from the hope that such models would lead to a finite theory of electromagnetic and weak physical processes. In the version studied by Salam and Weinberg the weak and electromagnetic interactions are mediated by two charged vector bosons $W_\mu^\pm$, a neutral massive vector boson Z_μ, and the massless photon A_μ. The model has been extended by Weinberg[3] to include hadrons; and the bosons A_μ, W_μ, and Z_μ are coupled to the electromagnetic current J_μ^{em}, the weak hadronic charged current J_μ^W, and weak hadronic neutral current J_μ^Z, respectively, with comparable coupling strengths. Thus for q^2 (in the propagators of massive bosons W_μ and Z_μ) $\gg m_W{}^2, m_Z{}^2$ the contributions from J_μ^W and J_μ^Z to the proton-neutron mass difference will be of the same order as that from J_μ^{em}, and it is interesting to see whether the logarithmic divergence to the mass difference arising from J_μ^{em} is canceled from the contributions from J_μ^Z and J_μ^W. The purpose of this paper is to study this question. Our conclusion is that in the Weinberg model the divergence in the mass difference is not canceled and in fact the situation is the same as in ordinary theory with the contribution coming to the mass difference purely from J_μ^{em}; the only difference is that e^2 appearing in the coefficient of the logarithmic divergence for the purely electromagnetic case is replaced by $e^2/(2\cos^2\theta_W)$ in the Weinberg model, θ_W being the parameter appearing in the model (it is defined below).

II. MASS FORMULA

In Weinberg's extension[3] of the Salam-Weinberg model for hadrons the electromagnetic weak hadronic charged and neutral currents in terms of the four quarks q_4, q_1, q_2, q_3 having charges $a, a, a-1, a-1$ are given by

$$
\begin{aligned}
J_\mu^{\text{em}} &= i\bar{q}\gamma_\mu Q q\,,\\
J_\mu^{W} &= i\bar{q}\gamma_\mu(1+\gamma_5)Wq\,, \qquad (2.1\text{a})\\
J_\mu^{Z} &= \tfrac{1}{2}J_\mu^{W(0)} - \frac{4\sin^2\theta_W}{2}J_\mu^{\text{em}}\,,
\end{aligned}
$$

where

$$J_\mu^{W(0)} = i\bar{q}\gamma_\mu(1+\gamma_5)W^0 q\,. \qquad (2.1\text{b})$$

Here q is the column matrix

$$\begin{bmatrix} q_4 \\ q_1 \\ q_2 \\ q_3 \end{bmatrix},$$

Q is the charge matrix, W is the 4×4 matrix

$$\begin{bmatrix} 0 & U \\ 0 & 0 \end{bmatrix},$$

with

$$U=\begin{bmatrix} -\sin\theta_C & \cos\theta_C \\ \cos\theta_C & \sin\theta_C \end{bmatrix},$$

θ_C being the Cabibbo angle, and $W^0=[W, W^\dagger]_-$. The parameter θ_W appearing in Eq. (2.1a) is defined below. The interaction Lagrangian, as far as the interaction of the above currents with the vector bosons is concerned, is

$$\mathcal{L}_{\rm int}=eJ_\mu^{\rm em}A_\mu+\tfrac{1}{2}(g^2+g'^2)^{1/2}J_\mu^Z Z_\mu + \frac{g}{2\sqrt{2}}(J_\mu^W \overline{W}_\mu+\overline{J}_\mu^W W_\mu), \tag{2.2}$$

where

$$\tan\theta_W=\frac{g'}{g},\quad \frac{g^2g'^2}{g^2+g'^2}=e^2,$$

$$\frac{g^2+g'^2}{16m_Z{}^2}=\frac{g^2}{16m_W{}^2} =\frac{G_W}{2\sqrt{2}}, \tag{2.3a}$$

$$\frac{g^2}{8}=\frac{e^2}{8\sin^2\theta_W},\quad \frac{g^2+g'^2}{4}=\frac{e^2}{4\sin^2\theta_W\cos^2\theta_W}, \tag{2.3b}$$

$$\frac{m_Z}{m_W}=\frac{(g^2+g'^2)^{1/2}}{g}=\frac{1}{\cos\theta_W},$$

$$m_W\geq 37\ \text{GeV},\quad m_Z\geq 75\ \text{GeV}, \tag{2.3c}$$

where e is the electric charge and G_W is the Fermi coupling constant of the weak interaction.

We now derive an expression for the proton-neutron mass difference by using the Lagrangian (2.2). For this purpose we follow a method discussed previously by us[4] and define the isospin currents

$$V_\mu^\pm=i\overline{q}(x)\gamma_\mu T^\pm q(x),$$
$$V_\mu^3=i\overline{q}(x)\gamma_\mu T^3 q(x), \tag{2.4a}$$

where the matrices

$$T^+=\begin{pmatrix} 0&0&0&0\\0&0&1&0\\0&0&0&0\\0&0&0&0\end{pmatrix},\quad T^-=(T^+)^\dagger,$$

$$T^3=[\overline{T}^+, T^-]_-=\begin{pmatrix} 0&0&0&0\\0&1&0&0\\0&0&-1&0\\0&0&0&0\end{pmatrix}. \tag{2.4b}$$

The corresponding isospin charges are

$$I^{\pm,3}(t)=-i\int V_4^{\pm,3}(\vec{x},t)d^3x. \tag{2.4c}$$

Then

$$\partial_\mu V_\mu^+=i[I^+,\mathcal{L}] =ie[I^+,J_\mu^{\rm em}]A_\mu+i\frac{(g^2+g'^2)^{1/2}}{2}[I^+,J_\mu^Z]Z_\mu +i\frac{g}{2\sqrt{2}}[I^+,\overline{J}_\mu^W]W_\mu, \tag{2.5}$$

where we have assumed that I^+ commutes with A_μ, Z_μ, and W_μ. There may be additional terms in Eq. (2.5) arising from quark masses (more explicitly from the mass difference of q_1 and q_2 quarks) in the Lagrangian. Such terms give rise to tadpole-type contributions which we are not considering here.

Taking the matrix elements of Eq. (2.5) between a neutron (initial) and a proton (final) state, using the property of I^+ as an isospin raising operator and taking the spin sum on both sides, we have

$$-\frac{i}{(2\pi)^3}(m_n-m_p)2=-ie[\langle p|A_\mu J_\mu^{\rm em}|p\rangle-\langle n|A_\mu J_\mu^{\rm em}|n\rangle]_{\text{spin sum}}-i\frac{(g^2+g'^2)^{1/2}}{2}[\langle p|Z_\mu J_\mu^Z|p\rangle-\langle n|Z_\mu J_\mu^Z|n\rangle]_{\text{spin sum}} -i\frac{g}{2\sqrt{2}}[\langle p|W_\mu\overline{J}_\mu^W|p\rangle-\langle n|W_\mu\overline{J}_\mu^W|n\rangle]_{\text{spin sum}}. \tag{2.6}$$

It is easy to see[4] that to the lowest order

$$-(2\pi)^3e\langle N|A_\mu J_\mu^{\rm em}|N\rangle_{\text{spin sum}}=\tfrac{1}{2}e^2\frac{i}{(2\pi)^4}\int d^4q\frac{\delta_{\mu\nu}}{q^2}\phi_{\mu\nu}^e(\nu,q^2), \tag{2.7a}$$

$$-(2\pi)^3\frac{(g^2+g'^2)^{1/2}}{2}\langle N|Z_\mu J_\mu^Z|N\rangle_{\text{spin sum}}=\frac{1}{2}\frac{g^2+g'^2}{4}\frac{i}{(2\pi)^4}\int d^4q\frac{\delta_{\mu\nu}+(q_\mu q_\nu/m_Z{}^2)}{q^2+m_Z{}^2}\phi_{\mu\nu}^Z(\nu,q^2), \tag{2.7b}$$

$$-(2\pi)^3\frac{g}{2\sqrt{2}}\langle N|W_\mu \bar{J}^W_\mu|N\rangle_{\text{spin sum}}=\frac{1}{2}\left(\frac{g^2}{8}\right)\frac{i}{(2\pi)^4}\int d^4q\,\frac{\delta_{\mu\nu}+(q_\mu q_\nu/m_W{}^2)}{q^2+m_W{}^2}\,\phi^W_{\mu\nu}(\nu,q^2)\,, \tag{2.7c}$$

where

$$\phi^e_{\mu\nu}(\nu,q^2)=i(2\pi)^3\int d^4z\,e^{-iq\cdot z}\langle N(p)|\,T(J^{\text{em}}_\mu(z)J^{\text{em}}_\nu(0))|N(p)\rangle\,, \tag{2.8a}$$

$$\phi^Z_{\mu\nu}(\nu,q^2)=i(2\pi)^3\int d^4z\,e^{-iq\cdot z}\langle N(p)|\,T(J^Z_\mu(z)J^Z_\nu(0))|N(p)\rangle\,, \tag{2.8b}$$

$$\phi^W_{\mu\nu}(\nu,q^2)=i(2\pi)^3\int d^4z\,e^{-iq\cdot z}\{\langle N(p)|\,T(\bar{J}^W_\mu(z)J^W_\nu(0))|N(p)\rangle+\bar{J}\leftrightarrow J\}, \tag{2.8c}$$

where the spin sum is understood and will not be written explicitly, N denotes p or n, and $\nu=-(p\cdot q)/m$. Thus from Eqs. (2.6) and (2.7)

$$(m_p-m_n)=[(\Delta m)^{ep}-(\Delta m)^{en}]+[(\Delta m)^{Zp}-(\Delta m)^{Zn}]+[(\Delta m)^{Wp}-(\Delta m)^{Wn}], \tag{2.9a}$$

where

$$(\Delta m)^b=\frac{C^b}{2(2\pi)^4}\frac{i}{2}\int d^4q\,\Delta^b_{\mu\nu}(q)\phi^b_{\mu\nu}(\nu,q^2)\,, \tag{2.9b}$$

where no summation over b is implied here and in what follows and where

$$\begin{aligned}
C^b&=e^2=\frac{g^2+g'^2}{4}=\frac{g^2}{8},\\
\Delta^b_{\mu\nu}&=\frac{\delta_{\mu\nu}}{q^2}\quad(b=e)\\
&=\frac{\delta_{\mu\nu}+(q_\mu q_\nu/m_Z{}^2)}{q^2+m_Z{}^2}\quad(b=Z)\\
&=\frac{\delta_{\mu\nu}+(q_\mu q_\nu/m_W{}^2)}{q^2+m_W{}^2}\quad(b=W)\,.
\end{aligned} \tag{2.10}$$

After making the Cottingham[5] rotation, we can write (2.9b) as

$$(\Delta m)^b=\frac{C^b}{2(2\pi)^4}\tfrac{1}{2}(-2\pi)\int_0^\infty dq^2\int_{-1}^{+1}dy\,q^2(1-y^2)\Delta^b_{\mu\nu}\phi^b_{\mu\nu}(q^2,iy)\,, \tag{2.11}$$

where

$$y=\nu/(q^2)^{1/2}\,.$$

III. ANALYSIS OF THE DIVERGENCE IN THE MASS DIFFERENCE

As is well known, $\phi^e_{\mu\nu}$ can be decomposed as

$$\phi^e_{\mu\nu}(q^2;\nu)=\frac{1}{m^2}\left[p_\mu p_\nu-\frac{p\cdot q}{q^2}(q_\mu p_\nu+p_\nu q_\mu)+\frac{(p\cdot q)^2}{q^4}q_\mu q_\nu\right]T^e_2(q^2,\nu)+\left(\delta_{\mu\nu}-\frac{q_\mu q_\nu}{q^2}\right)T^e_1(q^2,\nu)\,, \tag{3.1}$$

while $\phi^Z_{\mu\nu}$ and $\phi^W_{\mu\nu}$ have the decompositions

$$\begin{aligned}
\phi^b_{\mu\nu}(q^2,\nu)=&\frac{1}{m^2}\left[p_\mu p_\nu-\frac{p\cdot q}{q^2}(q_\mu p_\nu+p_\nu q_\mu)+\frac{(p\cdot q)^2}{q^4}q_\mu q_\nu\right]T^b_2(q^2,\nu)\\
&+\left(\delta_{\mu\nu}-\frac{q_\mu q_\nu}{q^2}\right)T^b_1(q^2,\nu)+\frac{1}{2m^2}\epsilon_{\mu\nu\alpha\beta}p_\alpha q_\beta T^b_3(q^2,\nu)\\
&+\frac{q_\mu q_\nu}{m^2}T^b_4(q^2,\nu)+\frac{p_\mu q_\nu+p_\nu q_\mu}{m^2}T^b_5(q^2,\nu)+i\frac{p_\mu q_\nu-p_\nu q_\mu}{m^2}T^b_6(q^2,\nu)\,,\quad b=Z\text{ or }W\,.
\end{aligned} \tag{3.2}$$

In the above equations

$$\text{Abs}\,T^b_i(q^2,\nu)=2\pi W^b_i(q^2,\nu)\,, \tag{3.3}$$

where W_i are the usual structure functions of deep-inelastic lepton-nucleon scattering. In particular

$$W^W_i=W^\nu_i+W^{\bar{\nu}}_i\,. \tag{3.4}$$

The antisymmetric terms in (3.2) do not contribute to $(\Delta m)^b$ in Eq. (2.11) and therefore will not be considered. Now from Eqs. (2.10), (3.1), and (3.2)

$$\Delta^b_{\mu\nu}\phi^b_{\mu\nu}(q^2,\nu)=\Delta^b(q^2)\phi^b(q^2,\nu)+\frac{1}{q^2}\frac{F^b(q^2,\nu)}{m_b^2}, \tag{3.5}$$

where

$$\Delta^b(q^2)=\frac{1}{q^2}\quad (b=e)$$
$$=\frac{1}{q^2+m_Z^2}\quad (b=Z)$$
$$=\frac{1}{q^2+m_W^2}\quad (b=W), \tag{3.6}$$

$$\phi^b(q^2,\nu)=3T^b_1(q^2,\nu)-\left(1+\frac{\nu^2}{q^2}\right)T^b_2(q^2,\nu), \tag{3.7}$$

$$F^b(q^2,\nu)=\frac{1}{m^2}q^2[q^2T^b_4(q^2,\nu)-2m\nu T^b_5(q^2,\nu)], \tag{3.8a}$$

$$m_b=m_Z \text{ or } m_W, \tag{3.8b}$$

where in Eq. (3.7) $b=e$, Z, or W while in Eq. (3.8) $b=Z$ or W only.

It is easy to see that the term F^b ($b=Z$ or W) arises due to the nonconservation of the current J^Z_μ or J^W_μ. Now it has been shown[1] that the deep-inelastic region ($q^2\to\infty$, $\nu\to\infty$, $\xi=q^2/2m_\nu$ finite) is the relevant one in the discussion of possible divergences in Δm. In such a region one may argue that one may effectively neglect the nonconservation of the currents J^Z_μ and J^W_μ so that the contribution from F^b is suppressed in the scaling region relative to the contribution from ϕ^b. Moreover, if one assumes that the divergences of the currents J^Z_μ and J^W_μ are simply given by the quark masses in the Lagrangian, one can show by using the Bjorken-Johnson-Low limit[6] that the term F^b would not give a divergent contribution to the proton-neutron mass difference in the lowest order.[7] For these reasons we shall not consider the term F^b any further and concentrate on ϕ^b. Thus we write from Eq. (2.11), as far as the discussion of possible divergence in the proton-neutron mass difference is concerned,

$$(\Delta m)^b=\frac{C^b}{2(2\pi)^4}\tfrac{1}{2}(-2\pi)\int_{q_m^2}^{\infty}dq^2\Delta^b(q^2)\times\int_{-1}^{+1}dy\,(1-y^2)^{1/2}q^2\phi^b(q^2,iy), \tag{3.9}$$

where q_m denotes the onset of the scaling region and where from Eq. (3.7) we can write ϕ^b as

$$\phi^b(q^2,\nu)=-3T^b_L(q^2,\nu)+2(1+\nu^2/q^2)T^b_2(q^2,\nu). \tag{3.10a}$$

Here

$$T^b_L(q^2,\nu)=(1+\nu^2/q^2)T^b_2(q^2,\nu)-T^b_1(q^2,\nu) \tag{3.10b}$$

$$=T^b_l(q^2,\nu)+T^b_2(q^2,\nu), \tag{3.10c}$$

where

$$T^b_l(q^2,\nu)=\frac{\nu^2}{q^2}T^b_2(q^2,\nu)-T^b_1(q^2,\nu) \tag{3.10d}$$

and the corresponding absorptive parts are

$$\text{Abs}\,T^b_{L,l}=2\pi W^b_{L,l}, \tag{3.11}$$

with the scaling limits

$$\nu W^b_2(q^2,\nu)\to F^b_2(\xi)+O(1/q^2),$$
$$W^b_L(q^2,\nu)\to\frac{1}{m}\left[F^b_L(\xi)+\frac{m^2}{q^2}H^b_L(\xi)\right], \tag{3.12a}$$
$$W^b_l(q^2,\nu)\to\frac{1}{m}\left[F^b_L(\xi)+\frac{m^2}{q^2}H^b_l(\xi)\right],$$

where

$$F^b_L(\xi)=\frac{1}{2\xi}F^b_2(\xi)-F^b_1(\xi),$$
$$H^b_L(\xi)=2\xi F^b_2(\xi)+H^b_l(\xi). \tag{3.12b}$$

Then assuming that (i) T_2 and T_L satisfy unsubtracted dispersion relations in the variable ν for fixed q^2 and (ii) $F_L(\xi)=0$ (this assumption avoids a quadratic divergence in Δm), it has been shown[8] by using Eq. (3.9) and the above equations that

$$(\Delta m)^b_{\text{divergent}}=-\frac{3C^b}{16\pi^2}m\left(\int_{q_m^2}^{\infty}\frac{dq^2}{q^2}\right)\times\left\{\int_0^1\left[F^b_2(\xi)-\frac{H^b_L(\xi)}{\xi}\right]d\xi\right\}. \tag{3.13}$$

Substituting Eq. (3.13) in Eq. (2.9a) and using Eq. (2.3b) and (3.4), one obtains the divergent contribution of the proton-neutron mass difference as follows:

$$(m_p-m_n)_{\text{divergent}}=-\frac{3\alpha}{4\pi}m\left(\int_{q_m^2}^{\infty}\frac{dq^2}{q^2}\right)A, \tag{3.14a}$$

where $\alpha=e^2/4\pi$ and

$$A=\int_0^1\left\{[F^p_2(\xi)-F^n_2(\xi)]-\frac{1}{\xi}[H^p_L(\xi)-H^n_L(\xi)]\right\}d\xi, \tag{3.14b}$$

with

$$F_2^p(\xi)-F_2^n(\xi)=[F_2^{ep}(\xi)-F_2^{en}(\xi)]+\frac{1}{4\sin^2\theta_W\cos^2\theta_W}[F_2^{Zp}(\xi)-F_2^{Zn}(\xi)]+\frac{1}{8\sin^2\theta_W}[F_2^{(\nu+\bar\nu)p}(\xi)-F_2^{(\nu+\bar\nu)n}(\xi)], \tag{3.14c}$$

$$H_L^p(\xi)-H_L^n(\xi)=[H_L^{ep}(\xi)-H_L^{en}(\xi)]+\frac{1}{4\sin^2\theta_W\cos^2\theta_W}[H_L^{Zp}(\xi)-H_L^{Zn}(\xi)]+\frac{1}{8\sin^2\theta_W}[H_L^{(\nu+\bar\nu)p}(\xi)-H_L^{(\nu+\bar\nu)n}(\xi)]. \tag{3.14d}$$

Regarding the H_L's, two alternatives have been considered in the literature[8]:

$$H_L^b(\xi)=0 \tag{3.15a}$$

or

$$H_l^b(\xi)=0 \Rightarrow H_L^b(\xi)=2\xi F_2^b(\xi) \tag{3.15b}$$

[cf. Eq. (3.12b)]. Then

$$A=\pm\int_0^1[F_2^p(\xi)-F_2^n(\xi)]d\xi, \tag{3.16}$$

where the + or − sign holds according to whether one has alternative (3.15a) or (3.15b). We now show that the second and third terms in Eq. (3.14c) arising from J_μ^Z and J_μ^W do not cancel the first term arising from J_μ^{em} so that the logarithmic divergence remains in the proton-neutron mass difference. To see this we note that it has been shown[9] that in the Weinberg model the following relations hold:

$$F_2^{(\nu+\bar\nu)p}(\xi)=F_2^{(\nu+\bar\nu)n}(\xi), \tag{3.17a}$$

$$F_2^{Zp}(\xi)-F_2^{Zn}(\xi)=-2\sin^2\theta_W(1-2\sin^2\theta_W)\times[F_2^{ep}(\xi)-F_2^{en}(\xi)] \tag{3.17b}$$

which give

$$F_2^p(\xi)-F_2^n(\xi)=\frac{1}{2\cos^2\theta_W}[F_2^{ep}(\xi)-F_2^{en}(\xi)], \tag{3.18}$$

so that from Eqs. (3.16) and (3.18) we have from Eq. (3.14a)

$$(m_p-m_n)_{\text{divergent}}=\mp\frac{3\alpha}{4\pi}m\left(\int_{q_m^2}^\infty\frac{dq^2}{q^2}\right)\frac{1}{2\cos^2\theta_W}\times\int_0^1[F_2^{ep}(\xi)-F_2^{en}(\xi)]d\xi, \tag{3.19}$$

where the − or + sign holds according to whether the alternative (3.15a) or (3.15b) holds. From the above discussion we see that in Weinberg's unified model of weak and electromagnetic interactions, the situation regarding the logarithmic divergence in the proton-neutron mass difference is the same as in the purely electromagnetic case; it is not canceled and only its magnitude is changed by the factor $1/(2\cos^2\theta_W)$.

Finally we discuss the effect of the scalar meson present in the Salam-Weinberg model on the mass difference. The portion of the Lagrangian which gives the coupling of the quarks with such a meson is

$$\mathcal{L}_q(\phi)=-\lambda\bar q\Gamma q-\bar q\Gamma q\phi, \tag{3.20}$$

where Γ is a diagonal 4×4 matrix whose diagonal matrix elements we denote by $\tilde\gamma_i$ $(i=4,1,2,3)$. In Eq. (3.20) $\lambda=\langle\phi_1\rangle_0$ with $\phi_1=\phi+\lambda$, $\langle\phi\rangle_0=0$. By using the first part of the Eq. (2.5), we see that the first term of Eq. (3.20) gives a contribution to the proton-neutron mass difference of the form

$$\tfrac12\lambda(2\pi)^3(\langle p|\bar q\Gamma q|p\rangle-\langle n|\bar q\Gamma q|n\rangle). \tag{3.21}$$

This contribution we have called the "tadpole-type" contribution previously [below Eq. (2.5)] and it is proportional to the quark mass difference. Now since the unified model we are considering is renormalizable, a parameter of the model can be redefined so as to absorb the logarithmic divergence we have found in the proton-neutron mass difference previously. The mass renormalization counter term for this purpose is available by redefining the matrix Γ in Eqs. (3.20) and (3.21) as

$$\Gamma=\Gamma_R+\Gamma_I, \tag{3.22}$$

where the matrix elements of $\bar q\Gamma_R q$ are finite while at least some of matrix elements of $\bar q\Gamma_I q$ are logarithmically divergent so as to cancel the logarithmic divergence we have found previously. Thus we rewrite Eq. (3.21) as

$$\tfrac12\lambda(2\pi)^3[(\langle p|\bar q\Gamma_R q|p\rangle-\langle n|\bar q\Gamma_R q|n\rangle)+(\langle p|\bar q\Gamma_I q|p\rangle-\langle n|\bar q\Gamma_I q|n\rangle)]=(\Delta m)^T_{\text{finite}}+(\Delta m)^T_\infty, \tag{3.23}$$

where the renormalizability of the model implies that $(\Delta m)^T_\infty$ cancels the logarithmic divergence we have found previously. However, the proton-neutron mass difference is then essentially a free parameter.

The term $\bar q\Gamma_R q$ in Eq. (3.20) gives the quark masses so that $\lambda\gamma\sim m_1$, γ_i denoting the diagonal matrix elements of Γ_R and m_1 being a typical quark mass. On the other hand $g\lambda$ is of the order of vector boson mass m_W. Thus γ must be of the order gm_1/m_W. The term $\bar q\Gamma_R q\phi$ in Eq. (3.20) can also contribute to the electromagnetic mass difference due to the ϕ-meson exchange. This contribution would be logarithmically divergent. But since γ is of the order gm_1/m_W, the scalar-meson exchange contribution would be of the order[10] $(m_1/m_W)^2$ and thus highly suppressed compared to the logarithmically divergent contributions from the $\delta_{\mu\nu}$ part of the vector-boson propagators discussed previously. Thus it is impossible to cancel the leading-logarithmic contribution from the Z-boson and photon exchanges by means of the scalar-meson exchange and one has to carry out the mass renormalization in the way indicated above. There is a possibility of the cancellation of the contribution of the scalar-meson exchange with the contribution arising from the $q_\mu q_\nu$ part of the vector-boson propagator which is of the same order and type as mentioned in Ref. 7.

One can also take the point of view that in the Lagrangian

$$\mathcal{L} = eJ_\mu^{\text{em}} A_\mu + \frac{(g^2+g'^2)^{1/2}}{2} J_\mu^Z Z + \frac{g}{2\sqrt{2}}(J_\mu^W \overline{W}_\mu + \overline{J}_\mu^W W) - \lambda \overline{q}\Gamma q - \overline{q}\Gamma q \phi$$

all the parameters are physical and finite. Then the logarithmic divergence cannot be canceled.

Finally we may mention that the nucleon, being a strongly interacting particle, should not be treated as a pointlike Dirac particle. It should not appear in the Lagrangian as a fundamental entity. Strong-interaction effects must be taken into account in any discussion of proton-neutron mass difference. In fact the proton-neutron mass difference calculated in Born approximation comes out to be finite, the necessary convergence factor being supplied by the electromagnetic form factors of the nucleon. One should clearly understand the source of the possible logarithmic divergence in the proton-neutron mass difference in the usual theory. The deep-inelastic region $q^2 \to \infty$, $\nu \to \infty$, $\xi = q^2/2m\nu$ finite is the relevant one in the discussion of possible divergences in Δm. In this region, the experiments on inelastic electroproduction indicate the scaling behavior of inelastic structure functions. Thus the experiments indicate that we are seeing a pointlike structure in this kinematic region, that is to say the nucleon behaves as if it is made up of pointlike constituents (partons). These partons are Dirac particles and may be identified with quarks. Experiments also indicate that spin-$\frac{1}{2}$ partons are coupled to the lepton current via pure $V-A$ currents of type $\overline{q}\gamma_\mu(1+\gamma_5)q$. Thus it is very relevant that the fundamental Lagrangian written from gauge-theory considerations should involve only quark fields; the nucleon, being the composite of the quarks, should not enter in the primitive Lagrangian. The hadronic currents which enter this Lagrangian satisfy light-cone algebra. Light-cone algebra is relevant for the scaling region. In our approach, this basic Lagrangian is used to extract the underlying algebra and the various coupling strengths for calculating the neutron-proton mass difference. It may be mentioned that the real test of gauge theories for hadrons lies in the deep-inelastic region where we are directly probing the pointlike constituents of the nucleon, the pointlike constituents being described by the primitive Lagrangian written from gauge-theory considerations.

We conclude that as far as the proton-neutron mass difference is concerned the unified model we have discussed does not lead us to a satisfactory situation. The situation regarding this problem is more or less similar to the one in pure electrodynamics since in the latter case one has also in general a logarithmic divergence due to photon exchange in the mass difference which can be removed by introducing a counterterm (which may be taken again as arising from a "tadpole-type" contribution) leaving the mass difference essentially a free parameter.

Note added. While we were writing this paper, we came across a report by A. Love and G. G. Ross [Nucl Phys. (to be published)] reaching essentially the same conclusion as ours by considering the contribution from the neutral weak current J_μ^Z.

Note added in proof. After the submission of the paper we came across a paper by S. Weinberg [Phys. Rev. Letter 29, 388 (1972)] where the problem of mass differences in spontaneously broken gauge theories is discussed. In particular he has discussed another extension of the Salam-Weinberg model of leptons to hadrons where there are six massive intermediate bosons and a photon and the proton-neutron mass difference is finite. The model we have discussed involves only three massive intermediate bosons and a photon.

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$$\alpha m_1 \frac{\Delta m^2}{m_W{}^2}\,\frac{\sin^2\theta_C}{\sin^2\theta_W}\,,$$

where Δm^2 is the difference between the squared masses of charmed and uncharmed quarks and m_1 is the mass of quark q_1. This is to be compared with the leading-logarithmic divergent term whose coefficient is proportional to $3\alpha(a-\frac{1}{2})m_1/\cos^2\theta_W$. Thus the contribution from F^b to the mass difference is highly suppressed in relation to the leading term.

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Approach to a Complete Bootstrap*

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It is shown in a simple model of quarks and mesons with Yukawa coupling that consistency conditions determine all dimensionless parameters of the model. Similar results are discussed for models also containing baryons (bound states of three quarks) and SU(3) symmetry. We use the static limit of the ladder approximation.

I. INTRODUCTION

Since the bootstrap idea was first formulated[1] no one has succeeded, to the author's knowledge, in constructing a completely bootstrapped model, that is, a model in which all dimensionless constants are uniquely determined by consistency conditions.

In a recent publication[2] (hereafter referred to as II) a model of quarks, mesons, and nucleons was constructed from a four-fermion point interaction, where all particles are bound states. However, as emphasized in II, the four-fermion coupling constant was left as a free parameter, arbitrarily chosen to ensure some simple situation, hence failing to achieve a real bootstrap. We thus see that the condition that all particles be bound states (i.e., that there be no "elementary" particles) is not equivalent to a complete bootstrap.

It is the purpose of this paper to construct a simple soluble model which is completely bootstrapped. While constructing this model, we emphasize mathematical and conceptual simplicity and the bootstrap idea more than the need for precise description of experimental data. From this point of view our model is a mathematical presentation of the bootstrap idea and not a phenomenological physical theory. Nevertheless, we will build our model as much as possible from physical data and intuition.

In Sec. II we define our model, consisting of a spin-$\frac{1}{2}$ quark and a scalar meson interacting via a Yukawa coupling, and we show that in the static limit of the ladder approximation all dimensionless parameters are determined.

In Sec. III we briefly outline some generalizations of our basic model: We include pseudoscalar as well as scalar mesons, we include spin-$\frac{1}{2}$ and -$\frac{3}{2}$ baryons (bound states of three quarks) as well as mesons, and we include SU(3) symmetry.