carried out, then the ratios of the coupling constants will have the correct phases.

The choice of phase corresponds to that chosen by Dürr and Wagner[20] for SU(2) (in that reference, the reader will also find an account of the use of diagrammatic techniques for γ algebras).

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PHYSICAL REVIEW D VOLUME 8, NUMBER 4 15 AUGUST 1973

Cabibbo Angle and Rotation Projection*

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(Received 2 April 1973)

The hadron Hamiltonian for strong and nonleptonic weak interactions is regarded as a function of the Cabibbo angle, evolving from a primitive "initial condition" (in which weak and strong hypercharge are identical) to its physical value by a rotation projection. Various existing determinations of the Cabibbo angle are reproduced and a new one is calculated, with good numerical results for an initial condition in which chiral SU(2) is broken only by an octet term in the electromagnetic direction.

The nonleptonic weak, electromagnetic, and semistrong interactions of hadrons define, through their SU(3) breaking,[1] an SU(3) frame orientation for the description of purely hadronic processes. Regarding for the moment the weak hypercharge and electric charge as fixed, "external" field directions which provide a coordinate system with which to probe the strong-interaction Hamiltonian, $H_S(\theta)$, where θ is the Cabibbo angle, let us consider a theory in which $H_S(\theta)$ continuously evolves from $H_S(0)$.

$$H_S(\theta)=\mathcal{K}[H_S(0)]. \tag{1}$$

If $H_S(\theta)=H_S^{(0)}(\theta)+H_S^{(8)}(\theta)$ is octet-broken for all θ, and is on the same SU(3) orbit[2] for all θ, there is an SU(3) transformation

$$U(\theta)\equiv e^{-2i\epsilon_i(\theta)F_i}, \quad U(0)=1$$

such that

$$H_S(\theta)=U(\theta)H_S(0)U^\dagger(\theta). \tag{2}$$

Otherwise (and this is the case of physical interest),

$$\begin{aligned} H_S(\theta)&=U(\theta)[H_S(0)+\tilde{G}(\theta)]U^\dagger(\theta)\\ &=U(\theta)H_S(0)U^\dagger(\theta)+G(\theta)\\ &=\mathcal{K}[H_S(0)], \end{aligned} \tag{3}$$

where $G(\theta)$ is the orbit shift. This allows $H_S(\theta)$ to interpolate between, say, an SU(2)×U(1) orbit for

$H_S(0)$ and a $U(1)\times U(1)$ orbit for $H_S(\theta)$. Now let us assume that the orbit shift operator is a pure SU(3) octet, i.e.,

$$H_S^{(0)}(\theta)=H_S^{(0)}(0) \Longleftrightarrow G^{(0)}(\theta)=0\,. \tag{4}$$

This is equivalent to the assumption that, whatever the obscure interplay of weak, electromagnetic, and semistrong interactions which induce the Cabibbo rotation, the SU(3)-symmetric part of $H_S(\theta)$ is unchanged; the orbit perturbation is only in the octet space in which the interplay occurs; and the operator $\mathcal{K}$ acts like an identity on an SU(3) singlet $H_S(0)$. Now let us require that $H_S(\theta)$ is a $Q\times Y\times \mathcal{C}\times \mathcal{P}$ singlet for all θ, and specify $G(\theta) = G^{(8)}(\theta)$ and $U(\theta)$ by the requirement that $G^{(8)}(\theta)$ and $U(\theta)H_S(0)U^\dagger(\theta)$ separately are $Q\times\mathcal{C}\times\mathcal{P}$ invariant. Then

$$U(\theta)=e^{-2i\epsilon(\theta)F_7} \tag{5}$$

apart from F_3 and F_8 rotations which play no role since $H_S(\theta)$, hence $H_S(0)$, is a $Q\times Y$ singlet. Thus

$$\begin{aligned}H_S(\theta)&=e^{-2i\epsilon(\theta)F_7}H_S(0)e^{2i\epsilon(\theta)F_7}+G^{(8)}(\theta)\\&=\mathcal{K}[H_S(0)],\end{aligned}$$

where $\mathcal{K}$ is specified by a scale parameter $\epsilon(\theta)$ and by the orbit shift operator. Let P be the projection operator on the Y-conserving subspace, $PH_S(\theta) \equiv H_S(\theta)$. The following assumptions are then equivalent: (i) The orbit shift operator $G^{(8)}(\theta)$ has pure

$$\Delta Y\neq 0\,,\quad PG^{(8)}(\theta)\equiv 0\,,$$

i.e.,

$$G^{(8)}(\theta)$$

is orthogonal to $H_S(\theta)$, $(1-P)G^{(8)}=G^{(8)}$. (The orbit shift is in this sense maximal.) (ii) The $\mathcal{K}$ operation, while not as simple as an SU(3) rotation, is the projection of an SU(3) rotation on the Y-conserving subspace,

$$\begin{aligned}H_S(\theta)&=PH_S(\theta)\\&=PG^{(8)}(\theta)+PU(\theta)H_S(0)U^\dagger(\theta)\\&=PU(\theta)H_S(0)U^\dagger(\theta)\\&=\mathcal{K}[H_S(0)]\,.\end{aligned} \tag{6}$$

As a result the $\mathcal{K}$ operator is linear and homogeneous in the "initial" condition $H_S(0)$.

Consider now the Hamiltonian $H(\theta)=H_S(\theta) + H_{\mathrm{NLW}}(\theta)$ where $H_{\mathrm{NLW}}(\theta)$ is the nonleptonic $\mathcal{C}\mathcal{P}$-invariant weak contribution to the hadron density. We assume the following transformation properties under chiral[3,4] SU(3):

$$\begin{aligned}&H_S\sim(3,3^*)\oplus(3^*,3)\qquad(\text{spanned by } u_i,\,v_i)\,,\\&H_{\mathrm{NLW}}\sim(1,8)\qquad(\text{spanned by } g_i+h_i)\,.\end{aligned} \tag{7}$$

Thus

$$\begin{aligned}H_S(\theta)&=u_0+c_3(\theta)u_3+c_8(\theta)u_8\\&\quad+c_3'(\theta)(g_3+h_3)+c_6'(\theta)(g_6+h_6)\\&\quad+c_8'(\theta)(g_8+h_8)\\&=Pe^{-2i\epsilon(\theta)F_7}H_S(0)e^{2i\epsilon(\theta)F_7}\\&\quad+e^{-2i\theta F_7}(g_8+h_8)e^{2i\theta F_7}\,,\end{aligned} \tag{8}$$

where by definition (at $\theta=0$, $Y_{\mathrm{weak}}=Y_{\mathrm{strong}}$)

$$H_{\mathrm{NLW}}(\theta)=e^{-2i\theta F_7}(g_8+h_8)e^{2i\theta F_7}\,. \tag{9}$$

The physical input which specifies $\mathcal{K}$ is $\epsilon(\theta)=\theta$, i.e., the rotation component of $\mathcal{K}$ is the same rotation which takes Y_{weak} into Y_{strong}. Since H_{NLW} and H_S have been assigned to different chiral SU(3) representations, we may write

$$H(\theta)=P^{(3,3^*)}e^{-2i\theta F_7}H(0)e^{2i\theta F_7}\,, \tag{10}$$

where $P^{(3,3^*)}$ projects on $Y=0$ as before but acts only in the $(3,3^*+3^*,3)$ subspace. $H(\theta)$ is now specified in a completely geometric (but not covariant) way in terms of the initial condition $H(\theta)$. As a result of (9) we have the selection rules $\Delta S=1$, $\Delta I=\frac{1}{2}$, and ($\Delta S=0$, $\Delta I=0$), ($\Delta S=0$, $\Delta I=1$), ($\Delta S=1$, $\Delta I=\frac{1}{2}$) weak transitions proceed with effective couplings[4,5]

$$(1-\tfrac{3}{2}\sin^2\theta):(\tfrac{3}{4})^{1/2}\sin^2\theta:(\tfrac{3}{4})^{1/2}\sin 2\theta\,.$$

It is not necessary to make the assumption (9) (implying octet dominance) to use the solution (6). It has been invoked to motivate the scale $\epsilon(\theta)=\theta$ of the strong rotation in (6) by identifying it with the rotation in (9).

We now consider the predictions of (6) for various

$$H_S(0)=u_0+c_3(0)u_3+c_8(0)u_8.$$

There are essentially two inputs $c_3(0)$, $c_8(0)$; specifying both yields two independent relations among c_0, c_3, c_8, and $\sin^2\theta$; specifying one yields one relation among these parameters. The latter case results in general in a quadratic equation for $\sin^2\theta$ which for special input degenerates into a linear equation for $\sin^2\theta$. The specific choice of input rests on one's intuition for the nature of the $\theta=0$ limit of H_S. We consider three cases:

(A) If $c_8(0)=-\sqrt{2}$, $c_3(0)=0$ [chiral SU(2) for $\theta=0$], then

$$c_8+\sqrt{2}\,c_0+\sqrt{3}\,c_3=0\,, \tag{11}$$

$$\begin{aligned}\sin^2\theta&=\frac{\sqrt{2}}{3}\,\frac{c_8+\sqrt{2}\,c_0}{c_0}\\&=-(\tfrac{2}{3})^{1/2}\,\frac{c_3}{c_0}\,.\end{aligned} \tag{12}$$

Both relations have been obtained by Oakes[6] in an operationally equivalent manner. Equation (11) has been obtained by Pegoraro and Rao[7] by looking for nilpotents of a symmetric algebra[2] on $((3,3^*)\oplus(3^*,3))\oplus((1,8)\oplus(8,1))$ space. The solution is characterized by two independent relations and a strong dependence of $\sin^2\theta$ on c_3/c_8. A consequence of (11) and (12) has been obtained by Tanaka and Tarjanne,[8] in the form

$$\tan^2\theta = 2c_3/(c_3+\sqrt{3}\,c_8),$$

by a self-consistency approach using covariant equations with a weak inhomogeneity; with Oakes, they share a large c_6 term, characteristic of covariant solutions involving no orbit change. In (12), the $SU(2)\times SU(2)$ limit of $\sin^2\theta$ is well defined, $\theta\to 0$, but the SU(3) limit is not defined. The estimate $\sqrt{3}\,c_3/c_8=0.03$ of Socolow[9] from the n-p/Λ-p mass-difference ratio leads to $\tan\theta=0.14$, about half its experimental value, but Oakes[10] has argued from an $\eta\to 3\pi$ analysis that $\sqrt{3}\,c_3/c_8$ may be three to four times larger, leading to a good agreement with the Cabibbo angle. The solution depends sensitively on $\sqrt{3}\,c_3/c_8$, known to be small, but not a very accessible parameter. In view of the current development of gauge theories and developments in calculating electromagnetic mass differences, it is conceivable that a c_3 term may not be necessary at all, putting both (11) and (12) in jeopardy. Finally, concerning case A, we wish to note that (11) is equivalent, in quark language, to the lack of a $\bar{\mathcal{P}}\mathcal{P}$ quark mass term[11] in $H(\theta)$ [hence in $H(0)$]; (11) and (12) are equivalent to the assumption that the $\Delta S=0$ parts of (8) (both strong and nonleptonic weak) transform like a *single* member of an octet, $c_3/c_8=c_3'/c_8'$.

(B) If we take as input for $H_S(0)$, motivated by an underlying $SU(2)_L\times U(1)$-gauge-invariant theory, a complete set of $\Delta S=0$, parity invariant, $SU(2)_L\times U(1)$ invariants constructed from $(3,3^*)\oplus(3^*,3)$ elements, and add to it the $SU(2)_L\times U(1)$ $\Delta S=0$, parity-invariant symmetry-breaking terms[12] induced when a $SU(2)_L$ doublet Higgs field ϕ [coupled to

$$\begin{pmatrix} -i\sqrt{2}\,\pi_P^+ \\ \sigma_S+i\pi_P^0\end{pmatrix}$$

and

$$\begin{pmatrix} -i\sqrt{2}\,\pi_S^+ \\ \sigma_P+i\pi_S^0\end{pmatrix}$$

with equal strength] develops a vacuum expectation value, then

$$H_S(0)=2b[(\tfrac{2}{3})^{1/2}u_0+(\tfrac{1}{3})^{1/2}u_8+u_3]\langle\phi\rangle + a(u_0-\sqrt{2}\,u_8) \tag{13}$$

and from (6),

$$\sin^2\theta=\frac{c_8+\sqrt{2}\,c_0-\sqrt{3}\,c_3}{2\sqrt{2}\,c_0-c_8-\sqrt{3}\,c_3} \tag{14}$$

which is the form developed by Cabibbo and Maiani[13] from apparently different assumptions. The input (13), in quark language, simply states that $H_S(0)$ [but not $H_S(\theta)$] lacks an $\bar{\mathfrak{N}}\mathfrak{N}$ quark mass term. [In this sense, the solution is complementary to case (A).] In (14), $\sin^2\theta$ does not depend sensitively on c_3/c_8. Using the Gell-Mann–Oakes–Renner[14] determination for c_8/c_0 and the Socolow[9] determination for c_3/c_8 results in $\sin\theta=0.22$. Equation (14) has well-defined limits for $SU(2)\times SU(2)$ ($\theta\to 0$) and SU(3) ($\theta\to 45°$), the limiting form advocated by Oehme[15] using deformed current algebra.

(C) It is characteristic of case (A) that there is no electromagnetic driving term, $c_3(0)=0$, with $c_3(\theta)$ induced completely by the F_7 rotation; in (B), there is a primitive $c_3(0)$ term, presumably of electromagnetic origin, but combined in a SU(2) chiral breaking term $(\frac{2}{3})^{1/2}u_0+(\frac{1}{3})^{1/2}u_8+u_3$ which has an SU(3)-symmetric part. If one requires that the *only breaking of chiral SU(2) when $\theta=0$ is of electromagnetic origin, and that the breaking term transforms like a U-spin singlet member of an octet, then*

$$H_S(0)=b\left(\frac{u_8}{\sqrt{3}}+u_3\right)+a(u_0-\sqrt{2}\,u_8) \tag{15}$$

and

$$\sin^2\theta=\frac{\sqrt{2}\,c_0+c_8-c_3/\sqrt{3}}{2\sqrt{2}\,c_0} \tag{16}$$

resulting in[16] $\sin\theta=0.27$. As in case (B), $\theta\to 45°$ in the SU(3) limit and $\theta\to 0$ in the chiral SU(2) limit. Solutions (B) and (C) for $\sin^2\theta$ are both linear degenerations of equations which in general are quadratic in $\sin^2\theta$, with a $\sin^4\theta$ term which vanishes for special input. Case (C) is unique in that it is the only solution with pure octet $SU(2)\times SU(2)$ breaking in $H_S(0)$ which has this singular characteristic.

Finally, we note that the $H_S(\theta)$ solution, for *any* $Q\times Y$-invariant input $H_S(0)$, has extrema only at $\theta=\frac{1}{2}n\pi$; thus it is $H_S(0)$, not $H_S(\theta)$, which is an extremum, distinguishing this approach from those employing variational principles to induce θ. It is also easy to see that $H_S(\theta)$ is an even[17] function of θ, so that the sign of θ can never be predicted from this approach (hence the Oehme[15] form [$\tan\theta=f(c_8/c_0)$] can never arise from this mechanism). It is interesting to note, however, that most existing "calculations" of the Cabibbo angle or relations between chiral and electromagnetic breaking fall into this class of solutions; and a new solution [case (C)] with a good numerical result, consistent with a small (perhaps vanishing) $c_3(\theta)$ term, can be achieved with an input which is chiral-SU(2)-symmetric up to a U-spin singlet octet term, the F_7 invariant direction of SU(3) breaking by electromagnetism.

ACKNOWLEDGMENT

I would like to thank K. Tanaka for commenting on and suggesting changes in the manuscript.

*Work supported in part by the U.S. Atomic Energy Commission.

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PHYSICAL REVIEW D VOLUME 8, NUMBER 4 15 AUGUST 1973

Dual Models with Global SU(2,2) Symmetry*

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The possibility of enlarging the gauge symmetry of the dual resonance models is considered by studying the structure of SU(2,2)- [or SO(4,2)] invariant dual models. n-point functions based on the *degenerate* representations of SU(2,2) are worked out in detail, and a condition under which these amplitudes are dual is specified. Dual models based on the nondegenerate representations are also discussed. Through a physical interpretation of the characteristics which emerge, a possible connection between the dimension $N-1$ of the hadronic matter and the gauge-symmetry group SO(N,2) is pointed out.

I. INTRODUCTION

Recent developments in the dual resonance models (DRM) have led to a better understanding of the attractive features as well as the limitations of these models. A description of these models in terms of quantized minimal surfaces in space-time has shown[1] that such models arise naturally from the dynamics of one-dimensionally extended objects. The relevance of the gauge conditions in these models was shown in I and II to be related to the coordinate-independent description of the minimal surfaces. Further arguments were given in these works that the well-known tachyon condition on the *external* masses, which comes about because of the requirement of gauge invari-