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Inner brane: A D3-brane in Nappi-Witten space from an inner automorphism

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Wess-Zumino-Witten (WZW) models are abstract conformal field theories with an infinite-dimensional symmetry which accounts for their integrability, and at the same time they have a sigma-model description of closed-string propagation on group manifolds which, in turn, endows the models with an intuitive geometric meaning. We exploit this dual algebraic and geometric property of WZW models to construct an explicit example of a field-dependent reflection matrix for open strings in the Nappi-Witten model. Demanding the momentum outflow at the boundary to be zero determines a certain combination of the left and right chiral currents at the boundary. This same reflection matrix is obtained algebraically from an inner automorphism, giving rise to a space-filling D-brane. Half of the infinite-dimensional affine Kac-Moody symmetry present in the closed-string theory is preserved by this unique combination of the left and the right chiral currents. The operator-product expansions of these boundary currents are computed explicitly and they are shown to obey the same current algebra as those of the closed-string chiral currents. Different choices of the inner automorphisms correspond to different background gauge field configurations. Only those B-field configurations, and the corresponding D-branes, that preserve the diagonal part of the infinite-dimensional chiral algebras are allowed. In this way the existence of the D-branes in curved spaces is further constrained by the underlying symmetry of the ambient spacetime.

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Since their discovery [1], studying D-branes has been a very important aspect of research in string theory. While D-branes have simple interpretation as being the hypersurface confining the end points of the open strings from the sigma model point of view, it is much harder to study them in an abstract conformal field theory that does not necessarily have any geometric interpretation for the fields as coordinates of a target spacetime. Important progress has been made throughout the years [2–9] especially in the case of D-branes in Wess-Zumino-Witten (WZW) models [10–34].

WZW models [35] are very special. Not only do they have a sigma-model description in which the fields take values in a group manifold of a particular Lie algebra, they are also conformal field theories with an infinite-dimensional affine Kac-Moody symmetry, where the current algebra is built upon the very same Lie algebra. In this way they nicely relate the algebraic bootstrap approach of the conformal field theory to the more intuitive geometric approach associated with the sigma-model analysis. While closed-string propagation on these group manifolds is much studied, much less is known about the properties of open strings on these manifolds. The known class of D-branes were obtained by gluing the left-moving and the right-moving world sheet currents in the closed-string theory with a field-independent "reflection" matrix, $\mathcal{R}$, i.e. $\mathcal{J}_L + \mathcal{R}\mathcal{J}_R = 0$, with a notable exception of [11]. We would like to propose using inner automorphisms as a general gluing condition, giving rise to field-dependent gluing matrices, $\mathcal{R}$.[1] Different choices of the inner automorphisms are related to different configurations of the background gauge fields coupled to the end points of the open strings. However choosing a consistent background gauge field for the open strings is far from obvious within the conformal field theoretical framework. The intuition gained from the geometric approach serves well in this regard. Sigma-model analysis of the boundary conditions also straightforwardly endows the D-brane with a geometric interpretation. The underlying symmetry of the D-branes is, on the other hand, most easily seen by studying the symmetry preserved by the boundary conformal theory. The algebraic and geometric approaches compliment each other and provide a complete picture of the D-branes.

So far the last aspect of D-branes, namely what symmetry of the underlying curved spacetime is preserved by the D-branes, has been mostly overlooked. We would like to advocate the view that the existence of the D-branes are further constrained by underlying symmetry of the ambient (curved) spacetime. Only those B-field configurations, and the corresponding D-branes, that preserve a subalgebra of the infinite-dimensional chiral algebras at the boundary are allowed. Without a good understanding of how the under-

[1]We speak of inner automorphisms of adjoint action by an arbitrary group element, g. In the case of identity, e, the gluing condition is again $\mathcal{J}_L + \mathcal{J}_R = 0$, coincided with the previously studied case and corresponding to the Neumann boundary condition.

lying infinite-dimensional symmetry constrains the existence of the D-branes, the study of D-branes is curved space is not complete.

We exploit this unique gift of WZW theories to uncover a new type of field-dependent gluing conditions, illustrating our techniques using the Nappi-Witten WZW model [36], a pp-wave background supported by a null Neveu-Schwarz (NS) three-form flux. We show, in this note, that it is in fact easy to generalize to field-dependent gluing matrices using the geometric data of the boundary conditions. A very special choice of the inner automorphism, adjoint action by the group element, e^{uJ}, gives rise to a space-filling brane in the Nappi-Witten (NW) model. This gluing condition was first proposed in [11]. Stanciu [11,13,16,17,23] was also the first to stress the importance for the D-branes to preserve part of the underlying affine symmetry. The construction of the D3-brane whose world volume spanning all four directions of the Nappi-Witten space is as follows. In Sec. I we construct the boundary conditions from momentum consideration and show that all four directions are Neumann. The reflection matrix relating the left-moving currents to the right-moving currents is obtained in Sec. II through an algebraic method, resulting in field dependent matrix components. The momenta obtained in Sec. I are reexpressed in terms of chiral currents. This unique combination of the left and the right chiral currents is shown, in Sec. III, to satisfy the same current algebra as the chiral currents of the closed strings. The boundary currents hence preserve half of the infinite-dimensional affine Kac-Moody symmetry present in the closed-string theory. The readers can see for themselves that it is much more straightforward to study D-branes if one fully exploits the dual geometric and algebraic properties of WZW models.

The Nappi-Witten model has received a lot of attention lately because it is one of the solvable string models in the plane-polarized gravitational waves [37–40]. Following this realization D-branes in these backgrounds have been extensively studied (see [41] for a sample of literature). The Nappi-Witten model, describing closed-string propagation in a pp-wave background supported by lightlike NS fluxes, has the added merit of being a WZW model which eventually led to its complete and covariant solution, via a Wakimoto "free-field" realization [42], in which string vertex operators were constructed and scattering amplitudes for an arbitrary number of tachyons were presented. [See also [43] for a different way of deriving the three and four point amplitudes by taking a pp-wave contraction of the $SU(2)\times U(1)$ amplitudes.] In this note we will use a space-filling D3-brane in the NW model to illustrate our techniques. Emphasis is placed on the interplay of the geometric and algebraic approaches. The fact that the free fields obey the Neumann boundary condition at the boundaries is made apparent (in Sec. III) using the covariant free-field realization introduced in [42].

I. SIGMA-MODEL ANALYSIS

We shall begin with the sigma-model side of the story. Starting with a generic string sigma model (in conformal gauge):

$$\mathcal{L} = \int G_{\mu\nu}\partial X^{\mu}\cdot\partial X^{\nu} + \epsilon^{ab}\mathcal{F}_{\mu\nu}\partial_a X^{\mu}\partial_b X^{\nu}. \quad (1)$$

Upon variation of the sigma-model action there are contributions from the boundary terms. Consistency of the open-string background requires that such terms vanish. In the presence of the B-field, $B_{\mu\nu}$, and a background gauge field, $F_{\mu\nu}$, coupling to the end points of the open strings the boundary terms read

$$\delta X^{\mu}[G_{\mu\nu}(X)\partial_{\sigma}X^{\nu} + \mathcal{F}_{\mu\nu}(X)\partial_{\tau}X^{\nu}]|_{\sigma} = 0 \quad (2)$$

in which only the gauge invariant combination, $\mathcal{F} = B + F$, appears. The boundaries of the world sheet are set at $\sigma = 0$ and $\sigma = \pi$. Writing the above using the left-moving and right-moving world sheet derivatives, $\partial_{\pm} = \frac{1}{\sqrt{2}}(\partial_{\tau}\pm\partial_{\sigma})$, we have

$$\partial_{+}X^{\mu} = [(G+\mathcal{F})^{-1}(G-\mathcal{F})]_{\mu\nu}\partial_{-}X^{\nu}, \quad (3)$$

provided that $(G+\mathcal{F})$ is invertible, which it will be for the case at hand.

The Neumann condition is equivalent to the requirement that no world sheet momentum flows out of the boundary of the world sheet. The world sheet momentum flowing across a small element, $d\vec{l} = (d\tau, d\sigma)$, of the boundary of the world sheet is given by

$$dP^{\mu} = P^{\mu}_{\tau}d\sigma - P^{\mu}_{\sigma}d\tau \quad (4)$$

has to vanish in order for the end points of the string to be free. The components of the world sheet momentum are defined canonically by

$$P^{\tau}_{\mu} = \frac{\delta\mathcal{L}}{\delta\partial_{\tau}X^{\mu}} \qquad P^{\sigma}_{\mu} = \frac{\delta\mathcal{L}}{\delta\partial_{\sigma}X^{\mu}}. \quad (5)$$

To read off the gluing condition for the Neumann directions, say in the "open-string picture" i.e. an open segment terminating at $\sigma = 0$ and $\sigma = \pi$ at the D-brane ($d\sigma = 0$), we demand no momentum outflow at all values of τ. We therefore must have

$$P^{\sigma}_{\mu} = G_{\mu\nu}(X)\partial_{\sigma}X^{\nu} + \mathcal{F}_{\mu\nu}(X)\partial_{\tau}X^{\nu} = 0. \quad (6)$$

This is nothing but the familiar boundary condition, $\partial_{\sigma}X^{\mu} = 0$, in flat space. Similarly the Dirichlet boundary condition, $\partial_{\tau}X^{p} = \delta X^{p} = 0$, should be replaced by $P^{\tau}_{p} = 0$ in curved backgrounds.

Thus the generalized boundary conditions, in the *open-string channel*[2] in a curved spacetime are

[2]In the closed-string channel, a pair of D-branes exchange one closed string. The boundaries of the world sheet are at $\tau = \tau_1$ and $\tau = \tau_2$. The closed-string channel is useful only when one analyzes the boundary states. To avoid possible confusion we will discuss physics solely in the open-string picture.

$$\text{Neumann: } P^{\mu}_{\sigma} = 0 \quad (7)$$

$$\text{Dirichlet: } P^{p}_{\tau} = 0. \quad (8)$$

We will see in the next section that these two boundary conditions are reinterpreted as two special linear combinations of currents at the boundary, namely,

$$P^{\mu}_{\sigma} \leftrightarrow \mathcal{J}_{\text{Neumann}} \quad (9)$$

$$P^{p}_{\tau} \leftrightarrow \mathcal{J}_{\text{Dirichlet}}. \quad (10)$$

As we shall see in Sec. III below when expressed in terms of the free fields [42], the Neumann boundary for the free field indeed reduces to those in flat space. At this point we would like to stress that the Neumann boundary conditions in curved space can only be realized by gluing the left-moving currents to the right-moving currents by an *inner* automorphism of the group which inevitably leads to a gluing matrix with nontrivial field dependence. This is to be contrast with the much studied D-branes arisen from field-independent gluing conditions using group outer automorphisms of the group, notably, $-\mathbb{I}$.

The action of the Nappi-Witten model is given by [36]

$$\mathcal{S} = \frac{1}{2\pi}\int \partial_+ u\partial_- v + \partial_+ v\partial_- u + \partial_+ a\partial_- \bar{a} + \partial_+ \bar{a}\partial_- a + iH(\bar{a}\partial_- a\partial_+ u - a\partial_- \bar{a}\partial_+ u), \quad (11)$$

where the background metric, G, and the gauge invariant combination of the Neveu-Schwarz B-field and gauge field, $\mathcal{F} = B + F$, are given by

$$ds^2 = dudv + dad\bar{a} + 2iH(\bar{a}da - ad\bar{a})du \quad (12)$$

$$\mathcal{F} = 2iH(\bar{a}da - ad\bar{a}) \wedge du, \quad (13)$$

H being a constant. Notice however that we are using a different gauge from that of [36]. In going from the closed-string theory to the open-string theory, one has to specify a background gauge field, F, in addition to the NS background, in order to fully specify an open-string background. It is the gauge invariant combination, $\mathcal{F} = B + F$, that couples to the end points of the open strings. Compare [Eq. (2.6)] in [42], used here for the open strings, and [Eq. (3.4)] in [42], for the closed strings: we had introduced a constant gauge field, $F = du \wedge dv + da \wedge d\bar{a}$. This necessity of using a different gauge from the original one adopted by Nappi and Witten in studying open string was also noticed by the authors of [44,45] when they studied other aspects of the Nappi-Witten model. We will also set $H = 1$ in the later part of the paper.

We now vary the above action and study the boundary conditions at the end points of the string, $\sigma = 0$ and $\sigma = \pi$:

$$\delta v(-\partial_+ u + \partial_- u) = 0 \quad (14)$$

$$\delta u(-\partial_+ v + \partial_- v + iH(\bar{a}\partial_- a - a\partial_- \bar{a})) = 0 \quad (15)$$

$$\delta \bar{a}(-\partial_+ \bar{a} + \partial_- \bar{a} - iH\bar{a}\partial_+ u) = 0 \quad (16)$$

$$\delta a(-\partial_+ a + \partial_- a + iHa\partial_+ u) = 0. \quad (17)$$

We are looking for a solution that satisfies the Neumann boundary condition in all four directions. So we demand that the four expressions inside the parentheses vanish.

Using the definition of the canonical momentum, P^{σ}, one can show that the vanishing P^{σ} is identical to requiring all four expressions inside the parentheses in (14)–(17) vanish. This unambiguously established the "Neumannity" of the above boundary conditions.[3]

II. CURRENTS AND THEIR GLUING CONDITIONS

The Nappi-Witten model is also a WZW model in which case there are infinite-dimensional conserved currents on the closed-string world sheet. In fact there are two independent sets of such currents, one for the left movers and one for the right movers, generating the $\mathbf{G}(z) \times \mathbf{G}(\bar{z})$ isometry of the loop group. The insertion of boundary implies that the left-moving and right-moving currents are no longer independent. They are related to each other at the boundary by a "reflection matrix," $\mathcal{R}$:

$$J^a_L(z) + \mathcal{R}^a_b J^b_R(\bar{z}) = 0. \quad (18)$$

$\mathcal{R}^a_b$ are the matrix components of a z dependent linear map acting on the Lie algebra. The gluing matrix, $\mathcal{R}$, encodes both Dirichlet and Neumann boundary conditions. In flat space $+1$ corresponds to a Neumann direction and -1 is Dirichlet. Comparing with the expression of boundary condition (3) from the sigma-model analysis and using $\mathcal{J}_L = -\partial g g^{-1}$ and $\mathcal{J}_L = g^{-1}\bar{\partial} g$, we obtain

$$\mathcal{R}^{\ a}_{b} = [(G + \mathcal{F})^{-1}(G - \mathcal{F})C]^a_b, \quad (19)$$

where G_{ij}, B_{ij} denotes the components of the metric and the B-field in terms of left invariant form fields, $ds^2 = G_{ij}(gdg^{-1})^i(gdg^{-1})^j$, $B = B_{ij}(gdg^{-1})^i \wedge (gdg^{-1})^j$. C^a_b denotes the adjoint map

$$T_a C^a_b \equiv Ad(g) \cdot T_b = gT_b g^{-1}. \quad (20)$$

This relation between the background field, $\mathcal{F}$, and the $\mathcal{R}$ field is invertible as long as $C + \mathcal{R}$ does not have a kernel, in which case we can write

$$\mathcal{F} = G(C - \mathcal{R})(C + \mathcal{R})^{-1}. \quad (21)$$

In other words, this equation serves to relate the algebraic data with the geometric data of WZW models. If a metric G and a background field $\mathcal{F}$ are given, then $\mathcal{R}$ can be determined from it; and vice versa. The insight gained

[3] This is usually called "mixed Neumann Dirichlet" boundary condition in the literature, which we found misleading.

from the sigma-model analysis will enable us to find a consistent background for open strings easily. This gives us considerable advantage over the purely algebraic manipulation as the consistency of a particular open-string background is not obvious in the closed-string picture.

Using this recipe we can directly compute the reflection matrix $\mathcal{R}$ in the Nappi-Witten model. For the space-filling D3-brane[4] it turns out to be [11]

$$\vec{\mathcal{J}}_L + \mathcal{R}\vec{\mathcal{J}}_R = 0 \to \mathcal{R} = Ad(e^{uJ}), \tag{22}$$

where $Ad(g)$ denotes the adjoint action by the group element g, $Ad(g)X = gXg^{-1}$. The boundary conditions (18) are then explicitly given by

$$\mathcal{J}_L^{\pm} + e^{\pm iu}\mathcal{J}_R^{\pm} = 0 \qquad \mathcal{J}_L + \mathcal{J}_R = 0 \qquad \mathcal{T}_L + \mathcal{T}_R = 0. \tag{23}$$

The gluing matrices were usually taken to be outer automorphisms of the algebra, which give rise to a large body of D-branes we know and love. Although the possibility of using inner automorphisms, and hence field-dependent gluing matrices, has been noticed by many researchers [12,16,21,28,31] and the constraint equation that $\mathcal{R}$ has to satisfy in order to be an inner automorphism has been written down in [31], in many conformal field analyses, due to the lack of geometric intuition, the components of the gluing matrices are quickly taken to be field independent. Adjoint action by a group element is of course an inner automorphism of the algebra. The element J lives in the Cartan of the Nappi-Witten algebra and it is the only element in the Cartan subalgebra that has nontrivial action on the rest of the algebra. In a sense what we have proposed is the most natural generalization of the constant gluing matrix.

Recall that the Nappi-Witten algebra[5] is

$$[J, J_+] = iJ_+, \qquad [J, J_-] = -iJ_-, \qquad [J_+, J_-] = iT. \tag{25}$$

We shall parametrize a generic group element by

$$g = e^{aJ_+ + \bar{a}J_-} e^{uJ + vT} \tag{26}$$

and use the definitions of $\mathcal{J}_+ = -\partial_+ g g^{-1}$ and $\mathcal{J}_- = g^{-1}\partial_- g$ to obtain the following sets of chiral currents:

$$\begin{aligned}
-\mathcal{J}_L^+ &= \partial_+\bar{a} + i\bar{a}\partial_+ u \qquad \mathcal{J}_R^+ = e^{+iu}\partial_-\bar{a}\\
-\mathcal{J}_L^- &= \partial_+ a - ia\partial_+ u \qquad \mathcal{J}_R^- = e^{-iu}\partial_- a\\
-\mathcal{J}_L &= \partial_+ v + i[a\partial_+\bar{a} - \bar{a}\partial_+ a] - a\bar{a}\partial_+ u\\
\mathcal{J}_R &= \partial_- v + i[\bar{a}\partial_- a - a\partial_-\bar{a}]\\
-\mathcal{T}_L &= \partial_+ u \qquad \mathcal{T}_R = \partial_- u.
\end{aligned} \tag{27}$$

Let us remark that the minus sign in the definition of the left-moving current is not arbitrary: it is to ensure that the left-moving and right-moving currents obey the same Kac-Moody algebra.

Now it is a straightforward exercise to plug in the definitions of the left and right currents (27) above into the gluing conditions (22) and obtain the following boundary conditions:

$$-\partial_+ u + \partial_- u = 0 \tag{28}$$

$$-\partial_+ a + \partial_- a + ia\partial_+ u = 0 \tag{29}$$

$$-\partial_+\bar{a} + \partial_-\bar{a} - i\bar{a}\partial_+ u = 0 \tag{30}$$

$$-\partial_+ v + \partial_- v + a\bar{a}\partial_+ u + \frac{i}{2}[\bar{a}(\partial_+ + \partial_-)a - a(\partial_+ + \partial_-)\bar{a}] = 0. \tag{31}$$

Comparing these with the boundary conditions obtained earlier (14)–(17), one sees that they indeed describe a D3-brane! We have made use of (16) and (17) to write (15) into the form of (31). So by now we have succeeded in showing that if we allow more general gluing conditions than those have been considered in the literature we can have a space-filling D-brane in the Nappi-Witten model. The geometry of this D-brane is completely determined by the boundary conditions imposed on the open-string end points. In our case it is that of the plane-polarized gravitational waves supported by the null Neveu-Schwarz three-from fluxes. Whereas the consistency of the open-string background is apparent from the sigma-model analysis, the symmetry preserved by the underlying D-brane is most easily seen from the algebraic study which we shall turn to in the following section. This concludes the classical analysis. We shall turn our attention to the quantum algebra, the operator-product expansions (OPEs), of the currents at the boundary.

III. CURRENT ALGEBRA

The gluing matrix relates the left-moving currents with the right-moving ones as in (23). This particular linear combination of the chiral currents, dictated by the Neumann boundary condition, will be shown in this section, to obey the same Kac-Moody algebra as the original closed-string theory in the bulk. This combination of chiral currents form a close algebra: an OPE of any two such "boundary" currents does not take one outside of this set

[4]The currents are vectors on the tangent space of the group manifold, $\vec{\mathcal{J}} \equiv \mathcal{J}^+\hat{J}^- + \mathcal{J}^-\hat{J}^+ + \mathcal{T}\hat{J} + \mathcal{J}\hat{T}$, where $\hat{J}^-$, $\hat{J}^+$, $\hat{J}$, $\hat{T}$, the generators of the algebra, act as the basis for the tangent vectors of the group manifold. To save typing we will drop the hats for the generators and only use the calligraphy letters to denote chiral current components.

[5]This is a centrally extended two-dimensional Euclidean algebra as it is manifest in the following way of writing it:

$$[J, P_1] = P_2, \qquad [J, P_2] = -P_1, \qquad [P_1, P_2] = T, \qquad [T, *] = 0 \tag{24}$$

In the note we have defined $J_+ = \frac{1}{\sqrt{2}}(P_1 - iP_2)$ and $J_- = \frac{1}{\sqrt{2}} \times (P_1 + iP_2)$.

of four boundary currents. The other linear combination with a relative minus sign between the left and the right chiral currents, corresponding to Dirichlet boundary condition, would not form a close algebra. Alternatively in the closed-string picture, we can construct boundary operators, Q^a, as the conserved charges of these boundary currents. Q^a will have to annihilate the boundary state [46] representing this D3-brane. Any commutators $[Q^a, Q^b]$ will have to annihilate this very boundary state. This again implies that it is consistent to impose the Neumann boundary condition on all of these four directions. The underlying D3-brane, or the boundary state, thus preserves half of infinite-dimensional $\mathbf{G}(z) \times \mathbf{G}(\bar{z})$ symmetries of the WZW model. This is analogous to the Bogomol'nyi-Prasad-Sommerfield (BPS) property of D-branes—preserving half of the spacetime supersymmetry. Given that the gluing condition is actually an inner automorphism of the algebra, it is perhaps not surprising to see part of the original symmetry get preserved after all.

To prove our claims let us recall that the bulk currents satisfy the OPE:

$$\begin{aligned}
&\mathcal{J}(z)\mathcal{J}^{\pm}(w) \sim \pm i \frac{\mathcal{J}^{\pm}(z)}{z-w} \\
&\mathcal{J}^{+}(z)\mathcal{J}^{-}(w) \sim \frac{1}{(z-w)^2} + i\frac{\mathcal{T}(z)}{z-w} \\
&\mathcal{T}(z)\mathcal{J}(w) \sim \frac{1}{(z-w)^2}.
\end{aligned} \tag{32}$$

These OPEs are easily realized using the Wakimoto "free-field" representation introduced in [42]:

$$\begin{aligned}
&-\mathcal{J}_L^{+}(z) = e^{-iu_L}\partial\gamma_L \qquad \mathcal{J}_R^{+}(\bar{z}) = e^{iu_R}\bar{\beta}_R \\
&-\mathcal{J}_L^{-}(z) = e^{iu_L}\beta_L \qquad \mathcal{J}_R^{-}(\bar{z}) = e^{-iu_R}\bar{\partial}\bar{\gamma}_R \\
&-\mathcal{J}_L(z) = \partial v_L \qquad \mathcal{J}_R(\bar{z}) = \bar{\partial} v_R \\
&-\mathcal{T}_L(z) = \partial u_L \qquad \mathcal{T}_R(\bar{z}) = \bar{\partial} u_R,
\end{aligned} \tag{33}$$

where $\beta = \partial\bar{\gamma}$ and $\bar{\beta} = \bar{\partial}\gamma$. Using the following free fields contraction rules,

$$u(z)v(w) \sim \ln(z-w) \qquad \beta(z)\gamma(w) \sim \frac{1}{z-w}, \tag{34}$$

it is a straightforward exercise to verify that the chiral currents (33) obey the OPEs given in (32).

We can now proceed to verify that the linear combinations of the chiral currents in the left-hand side of (23) are consistent, i.e. the algebra closes onto itself:

$$\begin{aligned}
&[\mathcal{J}_L^{+}(z) + e^{iu(z,\bar{z})}\mathcal{J}_R^{+}(\bar{z})][\mathcal{J}_L^{-}(w) + e^{-iu(w,\bar{w})}\mathcal{J}_R^{-}(\bar{w})] \\
&\quad \sim \frac{1}{(z-w)^2} + \frac{1}{(\bar{z}-\bar{w})^2} + \frac{i\mathcal{T}_L(z)}{(z-w)} + \frac{i\mathcal{T}_R(\bar{z})}{(\bar{z}-\bar{w})}
\end{aligned} \tag{35}$$

$$\begin{aligned}
&[\mathcal{J}_L(z) + \mathcal{J}_R(\bar{z})][\mathcal{J}_L^{+}(w) + e^{iu(w,\bar{w})}\mathcal{J}_R^{+}(\bar{w})] \\
&\quad \sim i\left(\frac{\mathcal{J}_L^{+}(z)}{(z-w)} + \frac{e^{iu(z,\bar{z})}\mathcal{J}_R^{+}(\bar{z})}{(\bar{z}-\bar{w})}\right)
\end{aligned} \tag{36}$$

$$\begin{aligned}
&[\mathcal{J}_L(z) + \mathcal{J}_R(\bar{z})][\mathcal{J}_L^{-}(w) + e^{-iu(w,\bar{w})}\mathcal{J}_R^{-}(\bar{w})] \\
&\quad \sim -i\left(\frac{\mathcal{J}_L^{-}(z)}{(z-w)} + \frac{e^{-iu(z,\bar{z})}\mathcal{J}_R^{-}(\bar{z})}{(\bar{z}-\bar{w})}\right)
\end{aligned} \tag{37}$$

$$\begin{aligned}
&[\mathcal{T}_L(z) + \mathcal{T}_R(\bar{z})][\mathcal{J}_L(w) + \mathcal{J}_R(\bar{w})] \\
&\quad \sim \frac{1}{(z-w)^2} + \frac{1}{(\bar{z}-\bar{w})^2}.
\end{aligned} \tag{38}$$

All other OPEs are regular. The right-hand side of the equations are to be evaluated at the boundary, $z = \bar{z}$.

Let us remark that the phase $e^{iu(z,\bar{z})}$ being nonchiral is essential for the closure of the above OPEs. The contribution from $\mathcal{J}_L e^{iu(z,\bar{z})}$ is equal and opposite of that from $\mathcal{J}_R e^{iu(z,\bar{z})}$:

$$\mathcal{J}_L(z)e^{iu(w,\bar{w})} \sim \frac{-ie^{iu(z,\bar{z})}}{z-w} \qquad \mathcal{J}_R(\bar{z})e^{iu(w,\bar{w})} \sim \frac{ie^{iu(z,\bar{z})}}{\bar{z}-\bar{w}} \tag{39}$$

which is the magic that enables the algebra to close.

Notice however that the central charge of the system is the *total* of the left-moving and right-moving parts. One can also prove that the above currents are conserved and indeed generate the symmetry transformations advertised. This is left as an exercise to the readers. An interesting open question is to investigate the implication of this infinite-dimensional symmetry present in the D3-branes on the BPS property of the object. The interplay of these extra infinite-dimensional symmetries with the other (super)symmetries in the target space should also be understood better and will be reported in a separate publication.

Furthermore, using the free fields we can rewrite the boundary conditions (14)–(17) as

$$\partial u_L - \bar{\partial} u_R = 0 \tag{40}$$

$$\partial v_L - \bar{\partial} v_R = 0 \tag{41}$$

$$\partial\bar{\gamma}_L - e^{-iHu_R}\bar{\partial}(e^{iHu_R}\bar{\gamma}_R) = 0 \tag{42}$$

$$e^{-iHu_L}\partial(e^{iHu_L}\gamma_L) - \bar{\partial}\gamma_R = 0, \tag{43}$$

where we have reinstated the dependence on the field strength, H, a real parameter. In the cases of the physical free fields like u and v, they are the familiar Neumann boundary conditions. The twisting (a phase factor) in the other two equations is due to the fact that $\bar{\gamma}_L$ and γ_R are not physical fields. They are related to the physical ones, $\bar{\gamma}_R$ and γ_L, respectively, at the boundary by nontrivial phases, e^{iHu_R} and e^{iHu_L}, respectively. The readers are referred to Sec. 3.2 of [42] for a detailed discussion on how the two sets of fields come about in the NW model. This explains the form of the boundary conditions above.

To conclude. we have successfully shown that it is much more straightforward to study D-branes if one fully exploits the dual geometric and algebraic properties of WZW models. We demonstrate our technique by uncovering a space-filling D-brane in a particular WZW model, that discovered by Nappi and Witten. This D-brane arises when one allows for a field-dependent gluing matrix. The reflection matrix is shown to be an adjoint action by a particular group element, e^{uJ}, on the algebra. These gluing conditions on the chiral currents are shown to be identical to the Neumann boundary condition of the open strings. The merit of the method is that it allows the geometric property D-branes to remain transparent by fully utilizing the geometric aspect of WZW models. In order for it to be useful however the technique should be generalized. On the conformal field theory side, one would like to generalize the boundary states to reflect the field dependence in the gluing matrix. It is also educational to apply our techniques to the better understood models like $SL(2)$ and $SU(2)$ to see if one gets new insight into these theories. It also serves as a cross-check for our techniques.

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