Finite size giant magnon

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The quantization of the giant magnon away from the infinite size limit is discussed. We argue that this quantization inevitably leads to string theory on a Z_M orbifold of S^5. This is shown explicitly and examined in detail in the near plane-wave limit.

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A significant amount of work on the AdS/CFT correspondence [1–3] has been inspired by the idea that the planar limit of $\mathcal{N} = 4$ Yang-Mills theory and its string dual might be integrable models which would be completely solvable using a Bethe ansatz [4–6]. Computation of the conformal dimensions of composite operators in $\mathcal{N} = 4$ Yang-Mills theory can be mapped onto the problem of solving an $SU(2, 2|4)$ spin chain. It is known that the spin chain simplifies considerably in the limit of infinite length where dynamics are encoded in the scattering of magnons and integrability would imply a factorized S matrix [7]. Beginning with this limit, a strategy advocated by Staudacher [8], Beisert showed that a residual $SU(2|2)^2$ supersymmetry and integrability determine the $\mathcal{N} = 4$ S matrix up to a phase [9,10]. More recent work constrains [11] and essentially computes this phase [12–17].

An important problem that the integrability program would eventually have to address is that of finite size corrections. In fact, recent four-loop computations of short operators [18,19] suggest that the most advanced form of the integrability ansatz, due to Beisert, Eden, and Staudacher [15], is likely valid only in the infinite size limit and it is spoiled by finite size effects. In the gauge theory, these effects are thought to stem from wrapping interactions [20–22].

A place where some progress has been made in studying finite size effects is the spectrum of a single magnon. The Bethe ansatz implies that the energy spectrum of a single magnon (at least in infinite volume) has the form

$$\Delta - J = \sqrt{1 + \frac{\lambda}{\pi^2}\sin^2\frac{p_{\rm mag}}{2}}. \qquad (1)$$

Here, Δ is the conformal dimension and J is a $U(1) \in SU(4)$ R charge which also dictates the length of the spin chain. $p_{\rm mag}$ is the magnon momentum. The string theory dual of the magnon of the infinite size system, the "giant magnon," was identified by Hofman and Maldacena [23] who showed that it had an energy spectrum of the expected leading large λ limit of (1). Then it was noted that the giant magnon solution could also be found in finite volume [24–31] where an asymptotic expansion of its spectrum is

$$\Delta - J = \frac{\sqrt{\lambda}}{\pi}\left|\sin\frac{p_{\rm mag}}{2}\right| - 4\frac{\sqrt{\lambda}}{\pi}\left|\sin\frac{p_{\rm mag}}{2}\right|^3 \times e^{-2-J\pi/\sqrt{\lambda}|\sin(p_{\rm mag}/2)|} + \cdots. \qquad (2)$$

The finite size corrections are exponentially small with large J. This was first found in the paper by Arutyunov, Frolov, and Zamaklar [24] where, in their approach, it was noted that the spectrum depended on a light-cone gauge fixing parameter. This was not a problem in the strict infinite volume limit, which turned out to be gauge invariant, but it afflicted the exponentially small corrections. Arutyunov *et al.* attributed this gauge variance to the fact that a single magnon with nonzero momentum $p_{\rm mag}$ is not a physical state of either the string theory or its dual, $\mathcal{N} = 4$ Yang-Mills theory. In the gauge theory, a single magnon would be an excitation of the exactly known ferromagnetic ground state of the spin chain, the $\frac{1}{2}$-BPS (Bogomol'nyi-Prasad-Sommerfield) chiral primary operator ${\rm Tr}Z^J$, which has exact conformal dimension of the classical value, $\Delta = J$, protected by supersymmetry. We shall denote the three complex scalar fields of $\mathcal{N} = 4$ Yang-Mills theory as (Z, Ψ, Φ). The 16 states of the magnon multiplet are obtained by a spin flip—a single insertion of $D_\mu Z$ or another scalar or a fermion into the trace. They form a short multiplet of the $SU(2|2) \times SU(2|2)$ subalgebra of $SU(2, 2|4)$ which commutes with ${\rm Tr}Z^J$. Because of cyclicity of the trace, all positions where one could flip one spin in the ground state of the spin chain are equivalent and $\sum_n e^{inp}\,{\rm Tr}Z^n\Psi Z^{J-n} \sim \delta(p)$, the magnon momentum must vanish.[1] Single magnon states with finite magnon momentum do not exist.[2]

The Hofman-Maldacena giant magnon [23] is a soliton solution of the bosonic part of the IIB sigma model prop-

[1] This was not a problem in the limit of the infinite chain discussed in Ref. [23] since there could be other operators present to block cyclicity of the trace and they could be placed infinitely far away from the magnon so that any wave-packet state of the magnon is isolated.

[2] We also note that these states can be obtained by commutators of symmetry generators and ${\rm Tr}Z^J$ so they are all in the same $\frac{1}{2}$-BPS multiplet of the full $SU(2, 2|4)$ algebra and have exact conformal dimensions $\Delta - J = 1$ which agree with (1) and (2) when $p = 0$.

agating on an $R^1 \times S^2$ subspace of $\mathrm{AdS}_5 \times S^5$. They showed that a magnon corresponds to a closed string with an open boundary condition, where the azimuth angle spanned by the two ends of the string corresponds to $p_{\rm mag}$. Arutyunov *et al.* [24] argued that the open boundary condition led to a modification of the level-matching condition and gauge parameter dependence of the spectrum was a result. In Ref. [25] it was suggested that the single magnon is well defined as the twisted state of a closed string on an orbifold—where the orbifold group acts in such a way that it identifies the ends of the string, resulting in a legitimate state of closed string theory. This was advocated as a way to study the spectrum of a single magnon in a setting where it is a physical state and there are no issues with gauge invariance. The giant magnon spectrum was computed there and an asymptotic expansion in the size of the system yields (2) (all results in this picture are identical to what Arutyunov *et al.* [24] obtain if their gauge parameter a is set to zero). In the following we shall develop this idea further. Our main observation will be that if we consider the single magnon state in the IIB string theory with the boundary condition in which the string is open in the direction of magnon motion, we are inevitably led to an orbifold.

To get the gist of our argument, consider the following (drastically oversimplified) example of the closed bosonic string on flat Minkowski spacetime where we legislate that one of the string coordinates is not periodic, but obeys the "magnon" boundary condition $X^1(\tau, \sigma = 2\pi) = X^1(\tau, 0) + p_{\rm mag}$, and all other variables, including $\partial_\sigma X^1(\tau, \sigma)$, are periodic. Then, a solution of the world-sheet equation of motion $(\partial_\tau^2 - \partial_\sigma^2)X^1 = 0$ with the appropriate boundary condition is [32] $X^1 = x^1 + \alpha' p^1 \tau + \frac{\sigma}{2\pi} p_{\rm mag}$ + oscillators. One of the Virasoro constraints is the level-matching condition $L_0 - \tilde{L}_0 = 0$ which takes the form

$$N - \tilde{N} + p^1 \frac{p_{\rm mag}}{2\pi} = 0, \tag{3}$$

where $N = \sum_{n=1}^{\infty} \alpha_{-n} \cdot \alpha_n$ and $\tilde{N} = \sum_{n=1}^{\infty} \tilde{\alpha}_{-n} \cdot \tilde{\alpha}_n$. Since the spectra of the operators N and $\tilde{N}$ are integers, there is no solution of the level-matching condition unless $p^1 p_{\rm mag} = 2\pi \cdot$ integer, i.e. the momentum p^1 is quantized in units of integer $\cdot\, 2\pi/p_{\rm mag}$. This is identical to (and indistinguishable from) the situation where the dimension X^1 is compactified with radius $R = \frac{p_{\rm mag}}{\rm integer}$ and where we consider a wrapped string with fixed momentum which is then quantized in units of $\frac{2\pi}{R}$. We see that the magnon boundary condition leads us to string theory on a simple orbifold, a periodic identification of the direction in which the magnon boundary condition was taken. We shall observe a similar fact for the more complicated case of a single magnon on $\mathrm{AdS}_5 \times S^5$ background.

The bosonic part of the IIB sigma model on $\mathrm{AdS}_5 \times S^5$ and in the conformal gauge is

$$\mathcal{L} = -\frac{\sqrt{\lambda}}{4\pi}\left\{-\left(\frac{1+\frac{Z^2}{4}}{1-\frac{Z^2}{4}}\right)^2 \partial_a T \partial^a T + \left(\frac{1}{1-\frac{Z^2}{4}}\right)^2 \partial_a \vec{Z} \cdot \partial^a \vec{Z} + \left(\frac{1-\frac{Y^2}{4}}{1+\frac{Y^2}{4}}\right)^2 \partial_a \chi \partial^a \chi + \left(\frac{1}{1+\frac{Y^2}{4}}\right)^2 \partial_a \vec{Y} \cdot \partial^a \vec{Y}\right\} \tag{4}$$

supplemented by Virasoro constraints. The eight fields $\vec{Z}$ and $\vec{Y}$ transform as 4-vectors under $SO(4) \times SO(4) \sim SU(2)^4$. We will impose the magnon boundary condition on the angle coordinate

$$\chi(\tau, \sigma = 2\pi) = \chi(\tau, \sigma = 0) + p_{\rm mag}. \tag{5}$$

If $\chi(\tau, \sigma) = \tilde{\chi}(\tau, \sigma) + p_{\rm mag}\sigma/2\pi$ with $\tilde{\chi}$ periodic,

$$\mathcal{L}[T, \vec{Z}, \chi, \vec{Y}] = \mathcal{L}[T, \vec{Z}, \tilde{\chi}, \vec{Y}] - \frac{\sqrt{\lambda}}{4\pi}\left(\left(\frac{p_{\rm mag}}{2\pi}\right)^2 + \frac{p_{\rm mag}}{\pi}\tilde{\chi}'\right)\left(\frac{1-\frac{Y^2}{4}}{1+\frac{Y^2}{4}}\right)^2. \tag{6}$$

The effect of the magnon boundary condition is to add terms to the action. These, as well as similar terms which appear in the Virasoro constraints, will break some of the (super-)symmetries of the background. The last term in (6) has the symmetries $SU(2)^2 \times SU(2)^2 \times R^2$ where the R^2 are translations of T and $\tilde{\chi}$. The bosonic part of the level-matching condition is

$$0 = \int_0^{2\pi} d\sigma \{\Pi_T T' + \Pi_Z Z' + \Pi_{\tilde{\chi}} \tilde{\chi}' + \Pi_Y Y'\} + \frac{p_{\rm mag}}{2\pi} J, \tag{7}$$

where $\Pi_\mu \equiv \partial\mathcal{L}/\partial\dot{X}^\mu$ are the canonical momenta conjugate to coordinates X^μ and the charge J is the generator of translations of $\tilde{\chi}$, $\chi \to \chi + \text{const}$

$$J = \int_0^{2\pi} d\sigma \Pi_{\tilde{\chi}} = \frac{\sqrt{\lambda}}{2\pi}\int_0^{2\pi} d\sigma \left(\frac{1-\frac{Y^2}{4}}{1+\frac{Y^2}{4}}\right)^2 \dot{\tilde{\chi}}. \tag{8}$$

Since $\chi \sim \chi + 2\pi$, the eigenvalues of J must be integers.[3] Furthermore, being generators of translations of the world-sheet σ argument of the fields, and the fields involved being periodic in σ, the first four terms in (7) must be integers plus a possible constant.[4] Since the theory has a symmetry under $\sigma \to 2\pi - \sigma$, the constant must be either zero or one-half. Thus, the spectrum of the first terms in (7) is either integers or integers $+\frac{1}{2}$. To eliminate the second possibility, we shall see that, in the plane-wave limit, we

[3] When fermions are included, they could be half-integers.

[4] Consider the operator ξ which has the property $[\xi, \varphi(\sigma)] = i\frac{d}{d\sigma}\varphi(\sigma)$. Consider eigenstates $|\alpha\rangle$ and $|\alpha'\rangle$ where $\xi|\alpha\rangle = \alpha|\alpha\rangle$. If $\langle\alpha'|\varphi(\sigma)|\alpha\rangle = \langle\alpha'|e^{-i\sigma\xi}\varphi(0)e^{i\sigma\xi}|\alpha\rangle = e^{i(\alpha-\alpha')\sigma}\langle\alpha'|\varphi(0)|\alpha\rangle$, the matrix element obeys $\langle\alpha'|\varphi(\sigma)|\alpha\rangle = \langle\alpha'|\varphi(\sigma+2\pi)|\alpha\rangle$ only when $\alpha - \alpha' =$ integers. The eigenvalues are equal to integers plus a constant which is common to all eigenvalues. If, there is a reflection symmetry $\sigma \to 2\pi - \sigma$ under which $\xi \to -\xi$, the constant must be either an integer or half-integer.

can solve for the spectrum explicitly and there we find that it is integers. Then, since the spectrum should not change discontinuously as the plane-wave limit is taken, we conclude that it should always be integers.

Since J comes in units of integers, and the first four terms in (7) are integers, (7) will only have a solution if $\frac{p_{\rm mag}}{2\pi}$ is a rational number, $\frac{m}{M}$. Then, J is quantized in units of M. This is identical to what should occur for a m-times wrapped string on a Z_M orbifold of $AdS_5 \times S^5$ where the orbifold group Z_M makes the identification $\chi \to \chi + 2\pi \frac{m}{M}$.

To get the superstring, we must include the fermions. For this, we must decide what their boundary conditions will be. It is clear that, at large J, we will obtain the correct magnon supermultiplet if we add them in such a way that, in the modification of the Virasoro constraint (7), J also contains the appropriate fermionic contribution $J \to \tilde{J} = \int (\Pi_{\tilde{\chi}} + \Pi_\psi \Sigma \psi')$. This gives the magnon boundary condition for the fermions

$$\begin{aligned}\psi(\tau, \sigma = 2\pi) &= e^{ip_{\rm mag}\tilde{J}} \psi(\tau, \sigma = 0) e^{-ip_{\rm mag}\tilde{J}} \\ &= e^{ip_{\rm mag}\Sigma} \psi(\tau, \sigma = 0), \end{aligned} \tag{9}$$

where $\Sigma = {\rm diag}(\frac{1}{2}, -\frac{1}{2}, \frac{1}{2}, -\frac{1}{2})$ and the orbifold identification is

$$(\chi, \psi) \sim (\chi + p_{\rm mag}, e^{ip_{\rm mag}\Sigma} \psi). \tag{10}$$

All of the fermions have a twist in their boundary condition. With this identification, all supercharges transform nontrivially under the orbifold group and all of the supersymmetries will be broken (in fact, the supercharges are set to zero by the orbifold projection). This twist in the fermion boundary condition and concomitant breaking of supersymmetry is well known from orbifold constructions in string theory [33] and was outlined in detail in a context similar to ours in Ref. [34].

Some supersymmetry can be saved if we impose a slightly more elaborate identification:

$$\begin{aligned}&(\chi, Y_1 + iY_2, \psi) \\ &\quad \sim (\chi + p_{\rm mag}, e^{-ip_{\rm mag}}(Y_1 + iY_2), e^{ip_{\rm mag}\tilde{\Sigma}} \psi), \end{aligned} \tag{11}$$

where, now $\tilde{\Sigma} = {\rm diag}(0, 0, 1, -1)$. This contains the previous identification of the angle χ as well as a simultaneous rotation of the transverse Y coordinates. Half of the fermions are untwisted and this identification preserves half of the supersymmetries. The giant magnon can still be considered a wrapped state of this orbifold where the identified Y coordinates are not excited.

The gauge theory duals of both of these models are well-known orbifold projections of $\mathcal{N} = 4$ theory [33]. They are obtained by beginning with the parent theory, $\mathcal{N} = 4$ super Yang-Mills with gauge group $SU(MN)$ and coupling constant $g_{\rm YM}$. Then, we consider a simultaneous R-symmetry transformation by a generator of the Z_M orbifold group and a gauge transform by a constant $SU(MN)$ matrix $\gamma = {\rm diag}(1, \omega, \omega^2, \ldots, \omega^{M-1})$ where ω is the Mth root of unity. Each diagonal element of the $MN \times MN$-matrix γ is multiplied by the $N \times N$ unit matrix. The projection throws away all fields which are not invariant under the simultaneous transformation. This reduces a typical field which was an $MN \times MN$ matrix in the parent theory to M $N \times N$ blocks embedded in that matrix in the orbifold theory.

For example, consider a field Z of the parent theory which is charged under the orbifold group and transforms as $Z \to \omega Z$. The orbifold projection reduces it to a matrix which obeys

$$Z\gamma = \omega\gamma Z. \tag{12}$$

By similar reasoning, a field Φ which was neutral in the parent theory commutes with γ once the orbifold projection is imposed,

$$\Phi\gamma = \gamma\Phi. \tag{13}$$

Given any single-trace operator of the parent $\mathcal{N} = 4$ theory, for example, a single magnon state such as ${\rm Tr}Z^J\Phi$, there are a family of M states of the orbifold theory ${\rm Tr}\gamma^m Z^J \Phi$ with $m = 0, 1, \ldots, M-1$. The operator must be neutral under the orbifold group transformation in the parent theory. To see this we could insert $1 = \gamma^{M-1}\gamma$ into the trace and use the commutators such as (12) and (13) and cyclicity of the trace to show that the trace of any operator which is not a singlet under the orbifold group must vanish. In our example, if Φ is neutral, this requires quantization of J in units of M, $J = kM$, in the state ${\rm Tr}\gamma^m Z^J \Phi$. This is the gauge dual of the quantization of the momentum J in units of $M \cdot$ integers, rather than integers after the orbifold projection is imposed in the sigma model, discussed after Eq. (8). In addition, the single-trace operator of the parent theory descends to a family of M operators which are distinguished as an additional quantum number, m. It is easy to see that moving the position where Φ was inserted into ${\rm Tr}\gamma^m Z^J \Phi$ changes the operator by an overall factor of ω^m. This implies that this trace is already an eigenstate of magnon momentum, $p_{\rm mag} = 2\pi \frac{m}{M}$. The integer m is the gauge theory dual of the wrapping number of the string state on the orbifold cycle.

There is a theorem to the effect that, in the planar limit of the orbifold gauge theory, untwisted operators (with $m = 0$ in the above examples) have the same correlation functions with each other as those in the planar parent $\mathcal{N} = 4$ gauge theory—with the only difference being a rescaling of the coupling constant by the order of the orbifold group [35]. For this reason, in the planar limit, the gauge theory resulting from either of the orbifold projections (10) or

(11) is a conformal field theory. In the nonsupersymmetric case (10) nonplanar corrections would give a beta function, whereas in the $\mathcal{N} = 2$ supersymmetric case (11) the beta function would vanish in the full theory.

On the orbifold, the spectrum of states in the $\mathcal{N} = 4$ magnon supermultiplet is expected to be split according to the residual symmetries. In the two cases we considered, the first (10) has no supersymmetry but has $SU(2)^4 \times R^2$ bosonic symmetry. We would expect that the fermionic states gain different energies than the bosonic states and that the $SU(2)$ multiplets within the bosonic states also split. In the other case (11), there remains $\mathcal{N} = 2$ supersymmetry and the spectrum should represent the superalgebra $SU(2|1)^2 \times R^2$. The $\mathcal{N} = 4$ magnon supermultiplet becomes

$$\mathrm{Tr}\, \gamma^m D_\mu Z Z^{kM-1}, \tag{14}$$

$$\begin{matrix} \mathrm{Tr}\, \gamma^m \Phi Z^{kM}, & \mathrm{Tr}\gamma^m \bar{\Phi} Z^{kM}, \\ \mathrm{Tr}\gamma^m \bar{\Psi} Z^{kM+1}, & \mathrm{Tr}\gamma^m \Psi Z^{kM-1}, \end{matrix} \tag{15}$$

$$\begin{matrix} \mathrm{Tr}\, \gamma^m \chi_{1\alpha} Z^{kM}, & \mathrm{Tr}\gamma^m \chi_{3\alpha} Z^{kM-1}, \\ \mathrm{Tr}\gamma^m \bar{\chi}^2_{\dot\alpha} Z^{kM}, & \mathrm{Tr}\gamma^m \bar{\chi}^4_{\dot\alpha} Z^{kM+1}. \end{matrix} \tag{16}$$

Here m gives the number of units of magnon momentum $p_{\mathrm{mag}} = \frac{2\pi}{M} m$ and k is the number of units of spacetime momentum $J = kM$. There are two limits where the operators in the set (14)–(16) are degenerate and have energies $\Delta - J = 1$: One is when we turn off the 't Hooft coupling $\lambda = g^2_{\mathrm{YM}} MN \to 0$ so that the operators have their classical conformal dimension. The other is when magnon momentum vanishes, $m = 0$. In the latter, the "untwisted operator" with $m = 0$ is known to have identical correlation functions with the operators in the parent $\mathcal{N} = 4$ theory and therefore have exact conformal dimension $\Delta = J + 1$. The spectrum away from these limits will depend on both λ and m. It would be interesting to check the splitting of the supermultiplet in perturbative gauge theory, a task which we reserve for a later publication. In particular, it would be interesting to study the orbifold Bethe ansatz [36,37].

To conclude, we examine the plane-wave limit of $AdS_5 \times S^5$ where the string theory sigma model is exactly solvable [38]. We redefine the string coordinates as $T = X^+$, and $\chi = \frac{1}{\sqrt{\lambda}} X^- - X^+$. This has been chosen so that $\Delta - J = \frac{1}{i}(\frac{\partial}{\partial T} - \frac{\partial}{\partial \chi}) = \frac{1}{i}\frac{\partial}{\partial X^+}$. In addition we rescale the transverse coordinates $\vec{Y} \to \vec{Y}/\lambda^{1/4}$, and $\vec{Z} \to \vec{X}/\lambda^{1/4}$. The appropriate plane-wave limit [39] then takes $\lambda \to \infty$ simultaneously with $\Delta \to \infty$ and $J \to \infty$ with $\Delta - J$ and $\frac{J}{\sqrt{\lambda}}$ finite. From (7) we see that the limit should be taken so that $p_{\mathrm{mag}} J$ is finite. This implies that

$$p_{\mathrm{mag}} \sim \frac{1}{\sqrt{\lambda}}. \tag{17}$$

The magnon boundary condition (5) implies

$$X^-(\sigma = \pi) = X^-(\sigma = 0) + p_{\mathrm{mag}}\sqrt{\lambda}. \tag{18}$$

The scaling (17) then gives a finite radius for X^-.

We have already argued that $J = \frac{1}{i}\frac{\partial}{\partial \chi} = \sqrt{\lambda}\frac{1}{i}\frac{\partial}{\partial X^-}$ should be quantized in integral units. In fact, in the magnon sector, we have argued that the level-matching condition (7) has a solution only when $p_{\mathrm{mag}} = 2\pi \frac{m}{M}$ where m and M are integers and J is quantized in units of M, $J = kM$ with k an integer. To get the correct scaling of p_{mag} we must therefore take the plane-wave limit by taking M to be large so that $\frac{M}{\sqrt{\lambda}}$ is held finite.

What is effectively the same limit was discussed in Ref. [40] where it was shown to result in a plane-wave background with a periodically identified null direction, $X^- \sim X^- + 2\pi R^-$ where $R^- = \frac{\sqrt{\lambda}}{M}$. [To be consistent with (18), the integer m which appears in p_{mag} is interpreted is a wrapping number.] The resulting discrete light-cone quantization of the string on the plane-wave background is a simple generalization of Metsaev's original solution [38]. Here, we are interested in a wrapped sector where $X^-(\sigma = 2\pi) = X^-(\sigma = 0) + 2\pi R^- m$. In Ref. [40] the spectrum of the IIB string theory in this plane-wave limit was matched with the appropriate generalization of the Berenstein-Maldacena-Nastase limit of the $\mathcal{N} = 2$ Yang-Mills theory which is obtained from $\mathcal{N} = 4$ by the orbifold projection corresponding to (11). It was also used to study nonplanar corrections [41] and finite size corrections at weak coupling [42].

Together with the limit, we take the light-cone gauge, $X^+ = p^+\tau$. Periodicity of X^- quantizes $p^+ = k/R^-$. We obtain the sigma model as a free massive world-sheet field theory

$$\mathcal{L} = -\frac{1}{4\pi}\{\partial_a \vec{Y}\cdot\partial^a\vec{Y} + \partial_a\vec{Z}\cdot\partial^a\vec{Z} + (p^+)^2(Y^2 + Z^2)\} - \frac{ip^+}{2\pi}(\bar{\psi}\partial_-\bar{\psi} + \psi\partial_-\psi + 2ip^+\bar{\psi}\Pi\psi) \tag{19}$$

with $\Pi = \mathrm{diag}(1, 1, 1, 1, -1, -1, -1, -1)$. In this limit, the magnon parameter p_{mag} does not appear in the Lagrangian or the mass-shell condition which determines the light-cone Hamiltonian:

$$p^- = \frac{1}{p^+}\sum_{n=-\infty}^{\infty}\sqrt{n^2 + (p^+)^2}(\alpha_n^{\alpha_1\dot\alpha_1\dagger}\alpha_{n\alpha_1\dot\alpha_1} + \alpha_n^{\alpha_2\dot\alpha_2\dagger}\alpha_{n\alpha_2\dot\alpha_2} + \beta_n^{\alpha_1\dot\alpha_2\dagger}\beta_{n\alpha_1\dot\alpha_2} + \beta_n^{\alpha_2\dot\alpha_1\dagger}\beta_{n\alpha_2\dot\alpha_1}). \tag{20}$$

Its only vestige is in the level-matching condition,

$$km = \sum_{n=-\infty}^{\infty} n(\alpha_n^{\alpha_1\dot\alpha_1\dagger}\alpha_{n\alpha_1\dot\alpha_1} + \alpha_n^{\alpha_2\dot\alpha_2\dagger}\alpha_{n\alpha_2\dot\alpha_2} + \beta_n^{\alpha_1\dot\alpha_2\dagger}\beta_{n\alpha_1\dot\alpha_2} + \beta_n^{\alpha_2\dot\alpha_1\dagger}\beta_{n\alpha_2\dot\alpha_1}), \quad (21)$$

where k are the number of units of $J = kM$ and m is the wrapping number. The bosonic $\alpha_{n\ldots}$ and fermionic $\beta_{n\ldots}$ oscillators have the nonvanishing brackets

$$[\alpha_{m\alpha_1\dot\alpha_1}, \alpha_n^{\beta_1\dot\beta_1\dagger}] = \delta_{mn}\delta^{\alpha_1}_{\beta_1}\delta^{\dot\alpha_1}_{\dot\beta_1}, \quad \{\beta_{m\alpha_1\dot\alpha_2}, \beta_n^{\beta_1\dot\beta_2\dagger}\} = \delta_{mn}\delta^{\alpha_1}_{\beta_1}\delta^{\dot\alpha_2}_{\dot\beta_2}, \quad (22)$$

$$[\alpha_{m\alpha_2\dot\alpha_2}, \alpha_n^{\beta_2\dot\beta_2\dagger}] = \delta_{mn}\delta^{\alpha_2}_{\beta_2}\delta^{\dot\alpha_2}_{\dot\beta_2}, \quad \{\beta_{m\alpha_2\dot\alpha_1}, \beta_n^{\beta_2\dot\beta_1\dagger}\} = \delta_{mn}\delta^{\alpha_2}_{\beta_2}\delta^{\dot\alpha_1}_{\dot\beta_1}, \quad (23)$$

and bispinors of $SO(4)\times SO(4) \sim SU(2)^4$.[5] We confirm in (21), which is the plane-wave limit of (7), there is solution of the level-matching constraint unless $\frac{p_{\rm mag}}{2\pi}J = $ integer. Here, we can think of the null identification as the vestige of the orbifold identification.

The level-matching condition (7) allows 1-oscillator states and the magnon supermultiplet is the 16 states

$$\alpha^\dagger_{km\alpha_1\dot\alpha_1}|p^+\rangle, \quad \alpha^\dagger_{km\alpha_2\dot\alpha_2}|p^+\rangle, \quad \beta^\dagger_{km\alpha_1\dot\alpha_2}|p^+\rangle, \quad \beta^\dagger_{km\alpha_2\dot\alpha_1}|p^+\rangle. \quad (24)$$

These states are degenerate, and their spectrum is given by

$$p^- = \frac{1}{p^+}\sqrt{(km)^2 + (p^+)^2} = \sqrt{1 + (R^-)^2 m^2} = \sqrt{1 + \frac{\lambda'}{M^2}m^2}, \quad (25)$$

which has the form expected from the plane-wave limit of (1) when $p_{\rm mag} = 2\pi\frac{m}{M}$. Note that in this plane-wave limit the finite size corrections that occur in (2) vanish due to the limit of small $p_{\rm mag}$.

The degeneracy of the states in (24) can be attributed to an enhancement of the supersymmetry which is well known to occur in the Penrose limit. One would expect, and we shall confirm, that the supersymmetry is broken when corrections to the Penrose limit are taken into account. Before that, we recall that in Refs. [9,10] Beisert argued that magnon states form a 16-dimensional short multiplet of an extended superalgebra $SU(2|2)\times SU(2|2)\times(R^1)^3$ where the spectrum (1) is the shortening condition. The superalgebra $SU(2|2)$ has generators $\mathcal{R}^{\alpha_1}{}_{\beta_1}$ and $\mathcal{L}^{\dot\alpha_2}{}_{\dot\beta_2}$ of $SU(2)\times SU(2)$, supercharges $\mathcal{Q}^{\dot\alpha_2}{}_{\alpha_1}$ and $\mathcal{S}^{\alpha_1}{}_{\dot\alpha_2}$, and the algebra

$$[\mathcal{R}^{\alpha_1}{}_{\beta_1}, \mathcal{J}^{\gamma_1}] = \delta^{\gamma_1}_{\beta_1}\mathcal{J}^{\alpha_1} - \tfrac{1}{2}\delta^{\alpha_1}_{\beta_1}\mathcal{J}^{\gamma_1},$$
$$[\mathcal{L}^{\dot\alpha_2}{}_{\dot\beta_2}, \mathcal{J}^{\dot\gamma_2}] = \delta^{\dot\gamma_2}_{\dot\beta_2}\mathcal{J}^{\dot\alpha_2} - \tfrac{1}{2}\delta^{\dot\alpha_2}_{\dot\beta_2}\mathcal{J}^{\dot\gamma_2},$$
$$\{\mathcal{Q}^{\dot\alpha_2}{}_{\alpha_1}, \mathcal{S}^{\beta_1}{}_{\dot\beta_2}\} = \delta^{\beta_1}_{\alpha_1}\mathcal{L}^{\dot\alpha_2}{}_{\dot\beta_2} + \delta^{\dot\alpha_2}_{\dot\beta_2}\mathcal{R}^{\beta_1}{}_{\alpha_1} + \delta^{\beta_1}_{\alpha_1}\delta^{\dot\alpha_2}_{\dot\beta_2}\mathcal{C},$$
$$\{\mathcal{Q}^{\dot\alpha_2}{}_{\alpha_1}, \mathcal{Q}^{\dot\beta_2}{}_{\beta_1}\} = \epsilon^{\dot\alpha_2\dot\beta_2}\epsilon_{\alpha_1\beta_1}\mathcal{P},$$
$$\{\mathcal{S}^{\alpha_1}{}_{\dot\alpha_2}, \mathcal{S}^{\beta_1}{}_{\dot\beta_2}\} = \epsilon_{\dot\alpha_2\dot\beta_2}\epsilon^{\alpha_1\beta_1}\mathcal{K}.$$

$\mathcal{J}^{\cdots}$ represents any generator with the appropriate index, and $\mathcal{K}$, $\mathcal{P}$, and $\mathcal{C}$ are central charges. In our application, $\mathcal{C} = \Delta - J = p^-$ and

$$\mathcal{R}^{\alpha_1}{}_{\beta_1} = \sum_n\{\alpha_n^{\dagger\alpha_1\dot\gamma}\alpha_{n\beta_1\dot\gamma_1} + \beta_n^{\dagger\alpha_1\gamma_2}\beta_{\beta_1\gamma_2}\} - \frac{1}{2}\delta^{\alpha_1}{}_{\beta_1}\sum_n\{\alpha_n^{\dagger\gamma_1\dot\gamma_1}\alpha_{n\gamma_1\dot\gamma_1} + \beta_n^{\dagger\gamma_1\gamma_2}\beta_{\gamma_1\gamma_2}\},$$

$$\mathcal{L}^{\dot\alpha_2}{}_{\dot\beta_2} = \sum_n\{\alpha_n^{\dagger\gamma_2\dot\alpha_2}\alpha_{n\gamma_2\dot\beta_2} + \beta_n^{\dagger\dot\alpha_2\dot\gamma_1}\beta_{\dot\gamma_1\dot\beta_2}\} - \frac{1}{2}\delta^{\dot\alpha_2}{}_{\dot\beta_2}\sum_n\{\alpha_n^{\dagger\gamma_2\dot\gamma_2}\alpha_{n\gamma_2\dot\gamma_2} + \beta_n^{\dagger\dot\gamma_1\dot\gamma_2}\beta_{\dot\gamma_1\dot\gamma_2}\},$$

$$\mathcal{Q}^{\dot\beta_2}{}_{\alpha_1} = \frac{\bar\eta}{\sqrt{8p^+}}\sum_n\{-e(n)\sqrt{\omega_n + p^+}\alpha^\dagger_{n\alpha_1\dot\gamma_1}\beta_n^{\dot\gamma_1\dot\beta_2} + i\sqrt{\omega_n - p^+}\alpha_{n\alpha_1\dot\gamma_1}\beta_n^{\dagger\dot\gamma_1\dot\beta_2} - i\sqrt{\omega_n - p^+}\beta^\dagger_{n\alpha_1\gamma_2}\alpha_n^{\gamma_2\dot\beta_2} + e(n)\sqrt{\omega_n + p^+}\beta_{n\alpha_1\gamma_2}\alpha_n^{\dagger\gamma_2\dot\beta_2}\},$$

$$\mathcal{S}^{\alpha_1}{}_{\dot\beta_2} = \frac{\bar\eta}{\sqrt{8p^+}}\sum_n\{\sqrt{\omega_n - p^+}\alpha_n^{\dagger\alpha_1\dot\gamma_1}\beta_{n\dot\gamma_1\dot\beta_2} - ie(n)\sqrt{\omega_n + p^+}\alpha_n^{\alpha_1\dot\gamma_1}\beta^\dagger_{n\dot\gamma_1\dot\beta_2} + ie(n)\sqrt{\omega_n + p^+}\beta_n^{\dagger\alpha_1\gamma_2}\alpha_{n\gamma_2\dot\beta_2} - \sqrt{\omega_n - p^+}\beta_n^{\alpha_1\gamma_2}\alpha^\dagger_{n\gamma_2\dot\beta_2}\}, \quad (26)$$

[5] Indices are raised and lowered with $\epsilon^{\alpha_i\beta_i}$ and $-\epsilon_{\alpha_i\beta_i}$, respectively, always operating from the left.

where $\omega_n = \sqrt{(p^+)^2 + n^2}$ and $e(n) = \frac{n}{|n|}$. We have used Metsaev's [38] conventions for the supercharges (those called Q^- and $\bar{Q}^-$) and notation for oscillators as summarized, for example, in Ref. [43]. Computing their algebra, we find that the plane-wave background supercharges indeed satisfy Beisert's extended superalgebra with the central extensions set to the plane-wave limits of those found by Beisert [9]

$$\mathcal{P} = -i\frac{\sqrt{\lambda}p_{\rm mag}}{4\pi} \leftarrow \frac{\sqrt{\lambda}}{4\pi}(e^{-ip_{\rm mag}} - 1),$$
$$\mathcal{K} = i\frac{\sqrt{\lambda}p_{\rm mag}}{4\pi} \leftarrow \frac{\sqrt{\lambda}}{4\pi}(e^{ip_{\rm mag}} - 1). \tag{27}$$

The existence of the central extension follows directly from the fact that the unextended algebra closes up to the level-matching condition and the level-matching condition (7) contains the term with $km = \frac{1}{2\pi}2\pi\frac{m}{M}\cdot kM = \frac{1}{2\pi}p_{\rm mag}J$.

A derivation of Beisert's superalgebra in the context of the $\mathrm{AdS}_3 \times S^5$ sigma model was first given in Ref. [44] and developed in Ref. [45]. They worked with the unorbifolded theory by "relaxing" the level-matching condition. Then, there is a central charge in the superalgebra which depends on the level mismatch. The idea is that, once the resulting algebraic structure is used to study magnon and multimagnon states, the level-matching condition should be reimposed so as to get a physical state of the string theory. They work in the "magnon limit," where $J \to \infty$, but magnon momentum is not necessarily small (in our case it relaxes the plane-wave limit by taking M as not necessarily large). They obtain the full central extension, rather than the form linearized in $p_{\rm mag}$ that we have found in (27). In their work, they use a generalized light-cone gauge $x^+ = \tau = (1-a)T + a\chi$, $x_- = \chi - T$ with a a parameter. They also use the identification, $x_-(\tau, \sigma = 2\pi) - x_-(\tau, \sigma = 0) = p_{\rm ws}$ with $p_{\rm ws}$ an eigenvalue of the level operator and $x_+ = \tau$ trivially periodic in σ. For the variables in (4), this amounts to using the boundary condition $\chi(\tau, \sigma = 2\pi) - \chi(\tau, \sigma = 0) = -(1-a)p_{\rm ws}$ and $T(\tau, \sigma = 2\pi) - T(\tau, 0) = ap_{\rm ws}$ which is different from the one which we use when $a \neq 0$ (they primarily use $a = \frac{1}{2}$), where $T(\tau, \sigma = 2\pi) = T(\tau, \sigma = 0)$ and $\chi(\tau, \sigma = 2\pi) - \chi(\tau, \sigma = 0) = p_{\rm mag}$. This makes no difference at infinite J where the effect of a is diluted by scaling. However, it matters at finite size. In fact, the same gauge fixing was used in Ref. [24] and the a dependence of the one-magnon spectrum found there (away from the infinite J limit) can be attributed to this a dependence of boundary conditions, rather than the gauge variance which is claimed there.

To see how the spectrum will be split in the near plane-wave limit, we must include corrections to the Lagrangian and the Virasoro constraints that are of order $\frac{1}{\sqrt{\lambda}}$. A systematic scheme for including these corrections in the usual $p_{\rm mag} = 0$ sector is outlined in the series of papers [46–48] and nicely summarized in Ref. [49]. There they find that the correction terms to the Hamiltonian add normal ordered terms which are quartic in oscillators. They also adjust the gauge by adjusting the world-sheet metric in such a way that the level-matching condition remains unmodified. We have shown, and will present elsewhere, that the modifications of a procedure in the magnon sector are minimal. The corrections to the free field theory light-cone Hamiltonian are of two types, quartic normal ordered pieces from near-plane-wave limit corrections to the sigma model identical in form to those found in Refs. [46–49] and terms such as the last one in Eq. (6) which arise from the orbifolding.

To leading order in perturbation theory, the normal ordered quartic interaction Hamiltonian cannot shift the spectrum of 1-oscillator states. Furthermore, none of the extra terms displayed in Eq. (6) contribute in the leading order in $1/\sqrt{\lambda}$. However, recall that, to preserve some supersymmetry, the orbifold identification (11) that we have been discussing also acts on the transverse direction and this action must also be taken into account. This generates simple correction terms in the Hamiltonian to order $\frac{1}{\sqrt{\lambda}}$. The relevant part of the interaction Hamiltonian is

$$H_{\rm int} = i\frac{p_{\rm mag}}{2\pi}\frac{1}{2\pi}\int_0^2 \pi d\sigma (Y_{1_1\dot{2}_1}Y'_{2_1\dot{1}_1} + ip^+(\psi\tilde{\Sigma}\psi + \bar{\psi}\,\tilde{\Sigma}\,\bar{\psi})). \tag{28}$$

With this orbifold identification exactly half of the supersymmetries are preserved in the near plane-wave limit. Specifically, out of the 16 supersymmetries $\mathcal{Q}^{\dot{\alpha}_2}_{\alpha_1}$, $\mathcal{S}^{\alpha_1}_{\dot{\alpha}_2}$, only $\mathcal{S}^{\alpha_1}_{\dot{1}_2}/\mathcal{Q}^{\dot{1}_2}_{\alpha_1}$ and $\mathcal{S}^{\dot{2}_2}_{\dot{\alpha}_1}/\mathcal{Q}^{\dot{\alpha}_1}_{2_2}$ survive. This leads to a splitting of the energies of the single impurity states.

The original multiplet had 16 states (8 bosons $\alpha^\dagger_{\alpha_1\dot{\alpha}_1}|0\rangle$, $\alpha^\dagger_{\alpha_2\dot{\alpha}_2}|0\rangle$ and 8 fermions $\beta^\dagger_{\dot{\alpha}_1\alpha_2}|0\rangle$, $\beta^\dagger_{\dot{\alpha}_1\dot{\alpha}_2}|0\rangle$). In the near plane wave, it breaks up into 4 supermultiplets of the residual superalgebra: one with 9 elements (5 bosons and 4 fermions) and two with 3 elements (2 fermions and 1 boson in each) and 1 boson singlet.

The following equation illustrates the breaking of the original supermultiplet:

$$
\begin{array}{ccccccccc}
 & & \mathcal{S}^{2_1}_{\dot{1}_2}/\mathcal{Q}^{\dot{1}_2}_{2_1} & & \mathcal{Q}^{\dot{1}_2}_{1_1}/\mathcal{S}^{1_1}_{\dot{1}_2} & & \mathcal{S}^{1_1}_{\dot{2}_2}/\mathcal{Q}^{\dot{2}_2}_{1_1} & & \mathcal{Q}^{\dot{2}_2}_{2_1}/\mathcal{S}^{2_1}_{\dot{2}_2} \\
 & \alpha^\dagger_{2_1\dot{2}_1}|0\rangle & - & \beta^\dagger_{\dot{2}_1\dot{1}_2}|0\rangle & - & \alpha^\dagger_{1_1\dot{2}_1}|0\rangle & & \beta^\dagger_{\dot{2}_1\dot{2}_2}|0\rangle & \\
\mathcal{Q}^{\dot{2}_1}_{2_2}/\mathcal{S}^{2_2}_{\dot{2}_1} & | & & | & & | & & | & \\
 & \beta^\dagger_{2_1 2_2}|0\rangle & - & \alpha^\dagger_{2_2\dot{1}_2}|0\rangle & - & \beta^\dagger_{1_1 2_2}|0\rangle & & \alpha^\dagger_{2_2\dot{2}_2}|0\rangle & \\
\mathcal{S}^{2_2}_{\dot{1}_1}/\mathcal{Q}^{\dot{1}_1}_{2_2} & | & & | & & | & & | & \\
 & \alpha^\dagger_{2_1\dot{1}_1}|0\rangle & - & \beta^\dagger_{\dot{1}_1\dot{1}_2}|0\rangle & - & \alpha^\dagger_{1_1\dot{1}_1}|0\rangle & & \beta^\dagger_{\dot{1}_1\dot{2}_2}|0\rangle & \\
\mathcal{Q}^{\dot{1}_1}_{1_2}/\mathcal{S}^{1_2}_{\dot{1}_1} & & & & & & & & \\
 & \beta^\dagger_{2_1 1_2}|0\rangle & - & \alpha^\dagger_{1_2\dot{1}_2}|0\rangle & - & \beta^\dagger_{1_1 1_2}|0\rangle & & \alpha^\dagger_{1_2\dot{2}_2}|0\rangle. & \\
\mathcal{S}^{1_2}_{\dot{2}_1}/\mathcal{Q}^{\dot{1}_2}_{2_1} & & & & & & & &
\end{array}
\qquad (29)
$$

Here, columns and rows with dashes represent the surviving supersymmetry transformations $\mathcal{S}^{\alpha_1}_{\dot{1}_2}/\mathcal{Q}^{\dot{1}_2}_{\alpha_1}$ and $\mathcal{S}^{2_2}_{\dot{\alpha}_1}/\mathcal{Q}^{\dot{\alpha}_1}_{2_2}$. Columns and rows without dashes represent the broken supersymmetries $\mathcal{S}^{\alpha_1}_{\dot{2}_2}/\mathcal{Q}^{\dot{2}_2}_{\alpha_1}$ and $\mathcal{S}^{1_2}_{\dot{\alpha}_1}/\mathcal{Q}^{\dot{\alpha}_1}_{1_2}$.

The energy degeneracy of the original multiplet is likewise broken by the interaction Hamiltonian in the near plane-wave limit. One of the triplets gets positive energy shift, its energy becoming

$$\sqrt{1+\lambda\frac{m^2}{M^2}}+\frac{1}{2\sqrt{\lambda}}\frac{\lambda\frac{m^2}{M^2}}{\sqrt{1+\lambda\frac{m^2}{M^2}}}.$$

The other triplet gets equal but negative energy shift:

$$\sqrt{1+\lambda\frac{m^2}{M^2}}-\frac{1}{2\sqrt{\lambda}}\frac{\lambda\frac{m^2}{M^2}}{\sqrt{1+\lambda\frac{m^2}{M^2}}}.$$

A singlet and a 9-multiplet are annihilated by the interaction Hamiltonian and thus retain the energy of the original multiplet:

$$\sqrt{1+\lambda\frac{m^2}{M^2}}.$$

In conclusion, we have made a number of observations about the giant magnon solution of string theory. We observed that the previously noted resemblance of the magnon to a wrapped string on a Z_M orbifold of $AdS_5 \times S^5$ seems to be the only solution of the Virasoro constraints in the string sigma model. We argued that this point of view is consistent with AdS/CFT duality as single magnons are physical states of the orbifold projections of $\mathcal{N}=4$ supersymmetric Yang-Mills theory. We also argued that this point of view is consistent with the plane-wave limit, where the sigma model is solvable. In that limit, the orbifold identification appears as a periodic identification of the null coordinate and the magnon is a wrapped string. There, we can see explicitly how the wrapping modifies the supersymmetry algebra and is consistent with the magnon spectrum. The $\mathcal{N}=2$ supersymmetry of the orbifold is enhanced to $\mathcal{N}=4$ supersymmetry in the plane-wave limit, so that the full 16-dimensional magnon supermultiplet appears there. We end with a question. We have shown that the supersymmetry is broken again by near-plane-wave limit corrections to the sigma model by showing that the energies of the magnon multiplet are split. However, there is another limit, the magnon limit, which is similar to the plane wave in that λ and J are taken to infinity but it differs in that p_{mag} remains of order 1, rather than scaling to zero. It would be interesting to understand whether the supersymmetry is also enhanced in this limit so that the orbifold quantization of the infinite volume limit has more supersymmetry than the orbifold itself.

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