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Skyrmions with holography and hidden local symmetry

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We study baryons as Skyrmions in holographic QCD with $D4/D8/\overline{D8}$ multi-D brane system in type IIA superstring theory, and also in the nonlinear sigma model with hidden local symmetry. Comparing these two models, we find that the extra dimension and its nontrivial curvature can largely change the role of (axial) vector mesons for baryons in four-dimensional space-time. In the hidden local symmetry approach, the ρ-meson field as a massive Yang-Mills field has a singular configuration in Skyrmion, which gives a strong repulsion for the baryon as a stabilizer. When the a_1 meson is added in this approach, the stability of Skyrmion is lost by the cancellation of ρ and a_1 contributions. On the contrary, in holographic QCD, the ρ-meson field does not appear as a massive Yang-Mills field due to the extra dimension and its nontrivial curvature. We show that the ρ-meson field has a regular configuration in Skyrmion, which gives a weak attraction for the baryon in holographic QCD. We argue that Skyrmion with π, ρ, and a_1 mesons become stable due to the curved extra dimension and also the presence of the Skyrme term in holographic QCD. From this result, we also discuss the features of our truncated-resonance analysis on baryon properties with π and ρ mesons below the cutoff scale $M_{\mathrm{KK}} \sim 1$ GeV in holographic QCD, which is compared with other 5D instanton analysis.

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I. INTRODUCTION

In 1961, Skyrme proposed an idea that the baryon can be described as a classical soliton in nonlinear meson field theories, called "Skyrmion" [1]. Later, in the 1970's, Skyrmion was revived in the development of large-N_c QCD, where the baryon mass M_B is found to be proportional to the inverse of the meson-meson coupling g_{eff} as $M_B \propto 1/g_{\mathrm{eff}}$ like a "soliton" [2,3]. Because of the Derrick theorem [4], a stable soliton does not appear only with a two-derivative term of the chiral field. In the Skyrme model, a four-derivative term is supplemented as a stabilizer. Alternately, if one includes vector mesons like the ρ meson, there appear ρ-2π coupling to give a nonlinear coupling of the chiral field via ρ-meson propagation, which can stabilize the soliton [5]. In this way, the importance of (axial) vector mesons for the Skyrmion is naturally expected.

One of convenient approaches for meson phenomenologies is the "hidden local symmetry (HLS)" [6], where ρ meson is introduced as the dynamical gauge boson of the hidden local symmetry in the nonlinear sigma model with $\mathrm{SU}(2)_L \times \mathrm{SU}(2)_R \times \mathrm{SU}(2)_V^{\mathrm{local}}$. Skyrmions with π, ρ, a_1, and ω mesons are discussed in HLS and its extended models [5,7–9].

Recently, Skyrmion has been revived in holographic QCD [10]. The solitonic description for a baryon naturally appears in a holographic dual of $D4/D8/\overline{D8}$ multi-D brane system in type IIA superstring theory [11–13]. Today, holographic QCD is known to reproduce many phenomenological aspects in hadron physics *semiquantitatively*, from dual classical supergravity calculations. However, important aspects in the *difference* between flat space phenomenologies and holographic approach have not been so much investigated.

In this paper, we compare the Skyrmion in HLS and that in holographic QCD, and find that they have significantly different soliton solutions, particularly in the vector-meson profiles, due to the existence of the extra dimension and its nontrivial curvature. As a result, we also discuss the features of our truncated-resonance analysis for baryons with π and ρ mesons below the cutoff scale $M_{\mathrm{KK}} \sim 1$ GeV in holographic QCD [11], which is compared with other 5D instanton approaches.

II. SKYRMIONS WITH HIDDEN LOCAL SYMMETRY

First, we study the Skyrmion with ρ meson as the dynamical gauge boson of the HLS in the nonlinear sigma model. The nonlinear sigma model with scalar manifold on coset space G/H can be generally formulated by the gauge theory with linear symmetry $G \times H_{\mathrm{local}}$ as "gauge equivalence" [6]. In the case of the global chiral symmetry $G = \mathrm{SU}(2)_L \times \mathrm{SU}(2)_R$ spontaneously broken into the isospin subgroup $H = \mathrm{SU}(2)_V$, the chiral field $U(x) \in \mathrm{SU}(2)_L \times \mathrm{SU}(2)_R/\mathrm{SU}(2)_V \simeq \mathrm{SU}(2)_A$ can be divided into two pieces with hidden variables $\xi_L(x)$ and $\xi_R(x)$ as

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$$U(x) \equiv \xi_L^\dagger(x) \cdot \xi_R(x). \tag{1}$$

Then one can introduce chiral-symmetry transformation by $g_{L(R)} \in \mathrm{SU}(2)_{L(R)}$ and hidden local symmetry transformation by $h(x) \in \mathrm{SU}(2)_V^{\mathrm{local}}$ as

$$\xi_{L(R)}(x) \to h(x)\xi_{L(R)}(x)g_{L(R)}^\dagger, \tag{2}$$

$$V_\mu(x) \to ih(x)\partial_\mu h^\dagger(x) + h(x)V_\mu(x)h^\dagger(x), \tag{3}$$

where $V_\mu(x) = eV_{\mu a}(x)\frac{\tau_a}{2}$ is the "gauge field" of $\mathrm{SU}(2)_V^{\mathrm{local}}$, regarded as the ρ-meson field in HLS. Here, e is the "gauge coupling," later corresponding to the Skyrme parameter [5]. By using the covariant derivatives

$$D_\mu\xi_{L(R)}(x) = \partial_\mu\xi_{L(R)}(x) - iV_\mu(x)\xi_{L(R)}(x), \tag{4}$$

several currents can be introduced as

$$\hat{\alpha}_{\mu\parallel} = (D_\mu\xi_L \cdot \xi_L^\dagger + D_\mu\xi_R \cdot \xi_R^\dagger)/2i = \alpha_{\mu\parallel} - V_\mu, \tag{5}$$

$$\hat{\alpha}_{\mu\perp} = (D_\mu\xi_L \cdot \xi_L^\dagger - D_\mu\xi_R \cdot \xi_R^\dagger)/2i = \alpha_{\mu\perp}, \tag{6}$$

$$\alpha_{\mu\parallel,\perp} = (\partial_\mu\xi_L \cdot \xi_L^\dagger \pm \partial_\mu\xi_R \cdot \xi_R^\dagger)/2i, \tag{7}$$

where $\hat{\alpha}_{\mu\parallel}$ and $\hat{\alpha}_{\mu\perp}$ are parallel and perpendicular components for $\mathrm{SU}(2)_V^{\mathrm{local}}$, respectively. By using these currents (5) and (6), we can construct the chiral Lagrangian with HLS in Euclidean space-time, invariant for $\mathrm{SU}(2)_L \times \mathrm{SU}(2)_R \times \mathrm{SU}(2)_V^{\mathrm{local}}$ as

$$\mathcal{L}_{\mathrm{HLS}} = \mathcal{L}_A + a\mathcal{L}_V + \mathcal{L}_{\mathrm{kin}}, \tag{8}$$

$$\mathcal{L}_A = f_\pi^2 \,\mathrm{tr}(\hat{\alpha}_{\mu\perp}^2) = \tfrac{1}{4}f_\pi^2 \,\mathrm{tr}(\partial_\mu U\partial^\mu U^\dagger), \tag{9}$$

$$\mathcal{L}_V = f_\pi^2 \,\mathrm{tr}(\hat{\alpha}_{\mu\parallel}^2), \tag{10}$$

$$\mathcal{L}_{\mathrm{kin}} = \frac{1}{2e^2}\,\mathrm{tr}(F_{\mu\nu}^2), \tag{11}$$

where a is an arbitrary parameter and f_π is the pion decay constant. π and ρ mesons can interact with each other only through the part $a\mathcal{L}_V$, where the symmetry is gauged. Therefore, the a parameter controls the coupling strength between π and ρ. In this framework, $a = 2$ is favored to reproduce empirically successful relations such as the Kawarabayashi-Suzuki-Riazuddin-Fayyazuddin relation [14] and the vector-meson dominance [15]. We have also introduced the kinetic term of ρ mesons (11) with $F_{\mu\nu} = \partial_\mu V_\nu - \partial_\nu V_\mu + i[V_\mu, V_\nu]$, as a "dynamical pole generation" of HLS due to the quantum effects [6].

By taking the unitary gauge with $\xi_L = \xi_R^{-1} \equiv \xi$, $\mathcal{L}_V$ can be written as

$$\mathcal{L}_V = f_\pi^2 \,\mathrm{tr}(J_\mu - V_\mu)^2, \tag{12}$$

where $J_\mu \equiv \alpha_{\mu\parallel} = (1/2i)(\partial_\mu UU^\dagger + \partial_\mu U^\dagger U)/\det(1 + U)$ is the vector current of the chiral field.

For a baryon, we now take the hedgehog Ansatz for the chiral field $U(x)$ and the ρ-meson field $V_\mu(x)$ as a Skyrmion

$$U^\star(x) = e^{i\tau_a\hat{x}_aF(r)} \qquad (\hat{x}_a \equiv x_a/r,\, r \equiv |\mathbf{x}|). \tag{13}$$

$$V_0^\star(x) = 0, \qquad V_i^\star = eV_{ia}^\star(x)\frac{\tau_a}{2} = \{\varepsilon_{iab}\hat{x}_bG(r)/r\}\frac{\tau_a}{2}, \tag{14}$$

with the topological boundary condition to ensure the baryon number $B = 1$ as

$$F(0) = \pi, \qquad F(\infty) = 0. \tag{15}$$

By substituting the Ansatz (13) and (14) into the action, we get the hedgehog mass of the Skyrmion [5] as follows:

$$E[F,G] = 4\pi\int dr\bigg[\frac{1}{2}f_\pi^2(r^2F'^2 + 2\sin^2F) + af_\pi^2\{G - (1-\cos F)\}^2 + \frac{1}{2e^2}\{2G'^2 + G^2(G-2)^2/r^2\}\bigg]. \tag{16}$$

It is convenient to take the Adkins-Nappi-Witten units for energy and length as $\bar{E} \equiv \frac{1}{E_{\mathrm{ANW}}}E = \frac{2e}{f_\pi}E$ and $\bar{r} \equiv \frac{1}{r_{\mathrm{ANW}}}r = ef_\pi r$ [16]. Then (16) can be written as follows (below, overlines of $\bar{E}$ and $\bar{r}$ are omitted for simplicity):

$$E[F,G] = 4\pi\int dr[(r^2F'^2 + 2\sin^2F) + 2a\{G - (1-\cos F)\}^2 + \{2G'^2 + G^2(G-2)^2/r^2\}], \tag{17}$$

where the parameter dependence appears only in E_{ANW}, r_{ANW}, and coupling strength a.

By solving the Eular-Lagrange equations for $F(r)$ and $G(r)$ in the minimization of the energy (17), we find a stable Skyrmion as shown in Fig. 1. One should note that $G(r)$ has a nonzero value at the origin: $G(0) = 2$. Therefore, to keep the total energy finite, the ρ-meson field (14) appears as a singular configuration with $V_i \propto \frac{1}{r}$ around $r = 0$, having a spatially "broad" extension to gain the kinetic energy in (17). Then such a broad ρ-meson field will pull up the pion field along the radial direction, which is the origin of the *strong repulsion* for the Skyrmion with a finite size (see a schematic plot in Fig. 2). In fact, by increasing the coupling strength a, pion field $F(r)$ is pulled up to outside of baryon, while the ρ-meson field $G(r)$ is pulled down to inside. Furthermore, without the ρ-meson field, the leading term of the chiral field $\mathcal{L}_A$ alone does not support the stable Skyrmion due to the Derrick theorem [4]. Therefore, the ρ-meson field as the broad singular configuration in the massive Yang-Mills sector of HLS plays the essential role as the stabilizer of the Skyrmion.

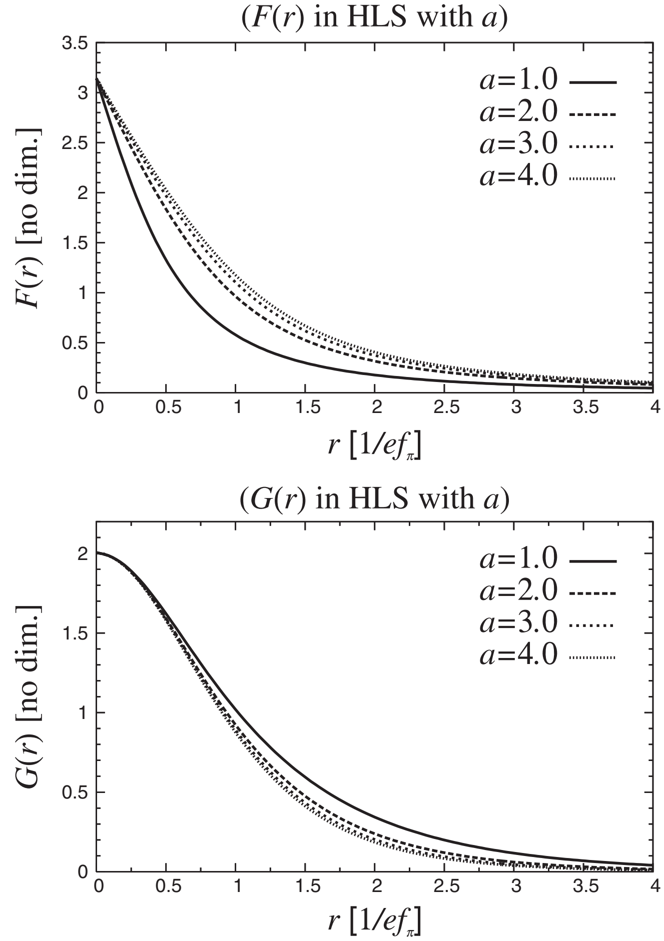

FIG. 1. $F(r)$ and $G(r)$ of Skyrmion in the HLS approach with various values of a parameters in (8).

(Such singular solution was first analyzed in the Yang-Mills theory in Ref. [17].)

Now, dividing the chiral field $U(x)$ in (1) into three hidden pieces as $U(x) = \xi_1 \xi_2 \xi_3$, one can introduce ρ and a_1 mesons for two independent hidden symmetries, which we now call the "HLS_2 model." However, the repulsive effect of the ρ meson was found to be canceled by a_1 meson in HLS_2, giving the catastrophic instability of Skyrmions. Then, the ω meson was additionally introduced as the origin of the strong repulsion for the baryons [7,8].

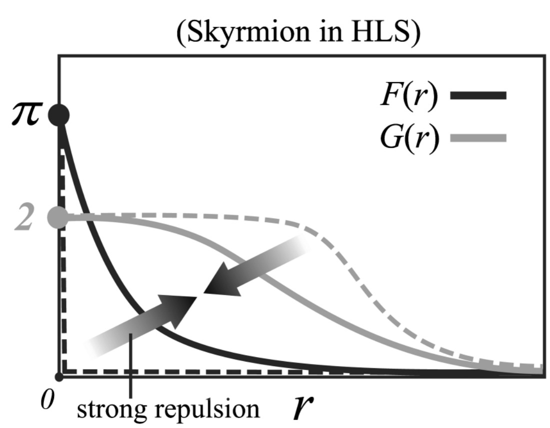

FIG. 2. Schematic figure of Skyrmion in HLS. Dashed black and grey lines show the configurations of $F(r)$ and $G(r)$ without interactions, respectively. $G(r)$ appears as broad singular configuration with the boundary $G(0) = 2$.

III. SKYRMIONS WITH HOLOGRAPHY

Next, we study the Skyrmion in holographic QCD. Let us begin with the non-Abelian Dirac-Born-Infeld (DBI) action of D8 branes with D4 supergravity background, to be dual of strong-coupling large-N_c QCD. After dimensional reductions, one gets the five-dimensional Yang-Mills action [10] as

$$S_{\mathrm{D8}}^{\mathrm{DBI}} = \kappa \int d^4x dz \,\mathrm{tr}\left\{\frac{1}{2} K(z)^{-1/3} F_{\mu\nu}^2 + K(z) F_{\mu z}^2\right\} + O(F^4), \tag{18}$$

with $\kappa = \lambda N_c / 216\pi^3$, and λ is the 't Hooft coupling. $K(z) = 1 + z^2$ expresses the nontrivial curvature in the fifth dimension z. After proper mode expansions for the five-dimensional gauge field $A_M(x, z)$ ($M = 0 \sim 3, z$) with a complete orthogonal basis in the fifth dimension z, one can get an action written by physical meson degrees of freedom with definite parity and G-parity in four-dimensional space-time [11]. In holographic QCD, all the (axial) vector meson fields are naturally introduced so as to obey a homogeneous transformation under the hidden local symmetry transformation [10,11], so that we also adopt such a definition in this paper. The resulting Euclidean effective action for chiral field $U(x)$, ρ-meson field $V_\mu(x)$ and infinite number of (axial) vector mesons is as follows:

$$\begin{aligned} S_{\mathrm{D8}}^{\mathrm{DBI}} = {} & \frac{f_\pi^2}{4} \int d^4x \,\mathrm{tr}(L_\mu^2) - \frac{1}{32e^2} \int d^4x \,\mathrm{tr}[L_\mu, L_\nu]^2 \\ & + \frac{m_\rho^2}{e^2} \int d^4x \,\mathrm{tr}(V_\mu^2) + \frac{1}{2e^2} \int d^4x \,\mathrm{tr}(\partial_\mu V_\nu - \partial_\nu V_\mu)^2 \\ & + i \frac{1}{2e^2} 2g_{3\rho} \int d^4x \,\mathrm{tr}\{(\partial_\mu V_\nu - \partial_\nu V_\mu)[V_\mu, V_\nu]\} \\ & - \frac{1}{2e^2} g_{4\rho} \int d^4x \,\mathrm{tr}[V_\mu, V_\nu]^2 + S_{\pi\text{-}\rho} + S_{a_1, \rho', a_1', \rho'', \ldots}, \end{aligned} \tag{19}$$

where $L_\mu = (1/i) U^\dagger \partial_\mu U$ is one-form of the chiral field, $S_{\pi\text{-}\rho}$ interaction terms between π and ρ, and $S_{a_1, \rho', a_1', \rho'', \ldots}$ contributions from higher excited resonance of (axial) vector mesons (see Ref. [11] for details; $\frac{1}{e} V_\mu$, $e g_{3\rho}$, and $e^2 g_{4\rho}$ in (19) correspond to V_μ, $g_{3\rho}$, and $g_{4\rho}$ in Ref. [11] as just field redefinitions). There appear infinite number of coupling constants like pion decay constant f_π, ρ-meson mass m_ρ, Skyrme parameter e, three-point coupling $g_{3\rho}$, four-point coupling $g_{4\rho}$, and other coupling constants in

$S_{\pi\text{-}\rho}$ and $S_{a_1,\rho',a_1',\rho''},\ldots$. However, holographic QCD has just two independent parameters: $\kappa \propto N_c$ and $M_{\rm KK}$ as the Kaluza-Klein compactification scale of D4 branes. Therefore, by taking two experimental inputs like $f_\pi = 92.4$ MeV and $m_\rho = 776.0$ MeV, all the coupling constants are uniquely determined.

Here, we remark several interesting features of the meson effective action (19). First, the Skyrme term $\int d^4x\,{\rm tr}[L_\mu, L_\nu]^2$ naturally appears as a stabilizer of the Skyrmion. In this sense, the Skyrme soliton picture is supported by the holographic approach [11]. Second, the relation $g_{3\rho}^2/g_{4\rho} = 1$ does *not* hold. In fact, due to the existence of the fifth dimension z and its nontrivial curvature, the ratio $g_{3\rho}^2/g_{4\rho}$ deviates from unity in holographic QCD as

$$\frac{g_{3\rho}^2}{g_{4\rho}} = \frac{[\kappa\int_{-\infty}^{+\infty} dz K(z)^{-1/3}\psi_1(z)^3]^2}{\kappa\int_{-\infty}^{+\infty} dz K(z)^{-1/3}\psi_1(z)^4} \simeq 0.90, \tag{20}$$

where $\psi_1(z)$ is a proper basis corresponding to the ρ-meson wave function in the z dimension [11]. Note that, with $\psi_1(z) \propto \kappa^{-1/2}$, the value $g_{3\rho}^2/g_{4\rho}$ in (20) is independent of the parameters, κ and $M_{\rm KK}$, in holographic QCD. This implies that the kinetic term of the ρ meson differs from the Yang-Mills type, i.e., the ρ-meson field does *not* appear as a massive Yang-Mills field in four dimensions due to the curved z dimension, which should be compared with the case of HLS with $g_{3\rho}^2/g_{4\rho} = 1$ in Eq. (11). In fact, the masses m_n of (axial) vector mesons (e.g., $m_1 = m_\rho$, $m_2 = m_{a_1}$, $m_3 = m_{\rho'}$, $m_4 = m_{a_1'}$, etc.) are given by the eigenvalues λ_n of the meson wave function $\psi_n(z)$ in z dimension as $m_n^2 = \lambda_n$ [11]. In this sense, the existence of the curvature $K(z)$ in the z dimension is essential to give the *discrete* mass spectra for mesons with $m_\rho < m_{a_1} < m_{\rho'} < m_{a_1'} < \cdots$, similarly to the Kaluza-Klein mechanism with the dimensional reductions (see Fig. 3). [If extra dimensions have no curvatures, all the modes belong to the continuum as plane waves, which contradicts meson properties.] In the following, we propose that any deviation of the ratio $g_{3\rho}^2/g_{4\rho}$ from unity can drastically change the hadron properties in four-dimensional space-time.

Now we take only π and ρ mesons below $M_{\rm KK} \sim 1$ GeV as the ultraviolet cutoff scale in holographic approach. This strategy is called the "truncated-resonance model" for the baryons [11]. By substituting the hedgehog configuration Ansatz (13) and (14) in the action (19), we get the hedgehog mass of the Skyrmion in Adkins-Nappi-Witten unit as

$$E[F,G] = 4\pi\int dr r^2\varepsilon[F,G], \tag{21}$$

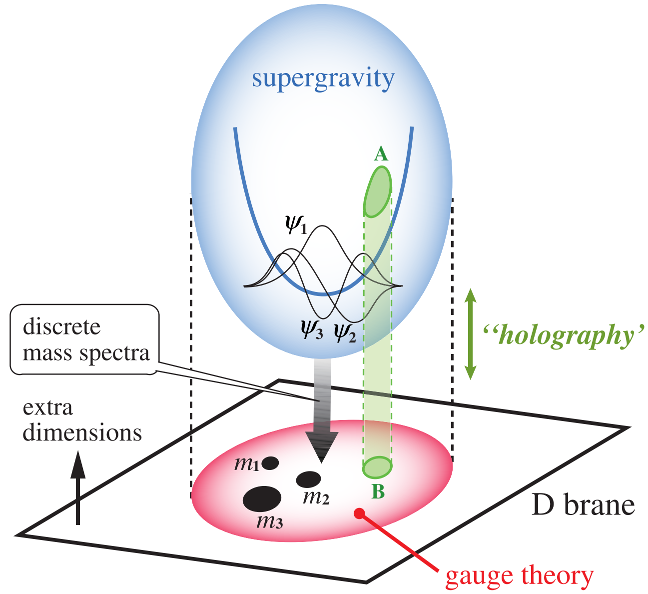

FIG. 3 (color online). Schematic figure of holographic duality between gauge theory as QCD on D brane and supergravity around D brane. Masses of mesons $m_{n=1,2,3,\ldots}$ are given by the oscillation of meson wave functions $\psi_n(z)$ in the extra dimension z [11]. Therefore, the curvature in the extra dimension around the D brane is essential to give the *discrete* meson mass spectra as QCD on the D brane. Then, regions A and B have different curvatures, and the gauge symmetry on A will be distorted on B except for the gauge symmetry of QCD, giving the relation $g_{3\rho}^2/g_{4\rho} \neq 1$.

$$\begin{aligned} r^2\varepsilon[F,G] &= (r^2F'^2 + 2\sin^2F) + \sin^2F(2F'^2 + \sin^2F/r^2) \\ &\quad + 2\frac{m_\rho^2}{e^2f_\pi^2}G^2 + (4G^2/r^2 + 2G'^2) \\ &\quad - 2g_{3\rho}(2G^3/r^2) + g_{4\rho}(G^4/r^2) \\ &\quad + r^2\varepsilon_{\pi\text{-}\rho}[F,G], \end{aligned} \tag{22}$$

where $\varepsilon_{\pi\text{-}\rho}$ is the contribution from $S_{\pi\text{-}\rho}$ in (19) [see Ref. [11] for details; $\frac{1}{2e}G$ in (22) corresponds to G in Ref. [11] as just field redefinition with hedgehog Ansatz (14)].

By solving the Eular-Lagrange equations for $F(r)$ and $G(r)$, we find a stable Skyrmion as shown in Fig. 4. One should note that the ρ-meson field appears as a regular configuration with $G(0) = 0$, not as the broad singular one with $G(0) = 2$ in Sec. II. Therefore, the ρ-meson field appearing in the core region of the baryon will pull down the pion field to inside of the baryon, giving the *weak attraction* for the baryon with a slight shrinkage of its total size [11] (see a schematic plot in Fig. 5). This result is clearly different from the case of HLS in Sec. II, where the ρ-meson field gives the strong repulsion for the baryon. We find that such a remarkable difference comes from the value $g_{3\rho}^2/g_{4\rho}$. In fact, substituting $G(r) = \alpha$ (constant) near the origin $r = 0$, the energy density in (22) is domi-

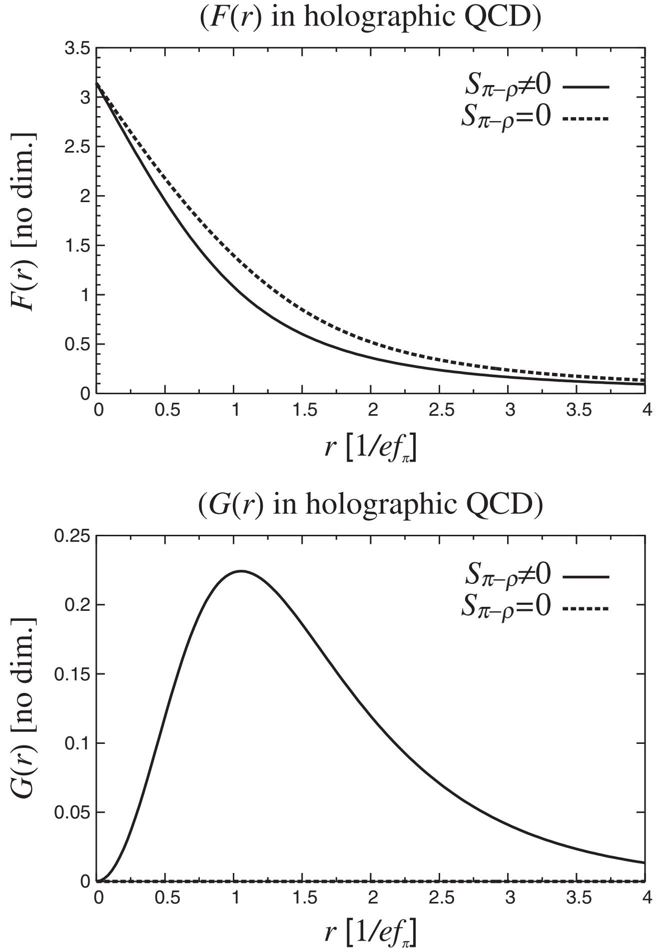

FIG. 4. $F(r)$ and $G(r)$ of Skyrmion in holographic QCD. Dashed lines with label "$S_{\pi\text{-}\rho} = 0$" show the configurations without interactions between π and ρ in (19).

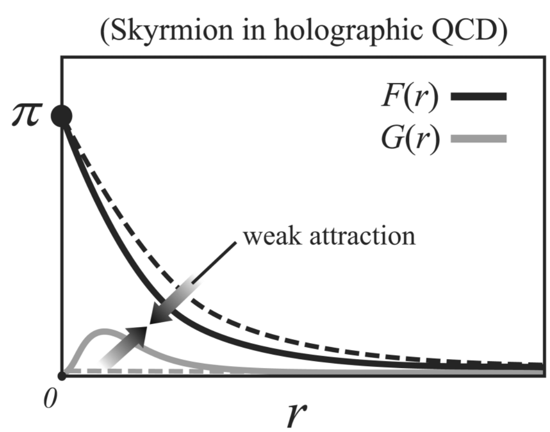

FIG. 5. Schematic figure of Skyrmion in holographic QCD. Dashed black and grey lines show the configurations of $F(r)$ and $G(r)$ without interactions, respectively. $G(r)$ appears as regular configuration in the core region of a baryon with the boundary $G(0) = 0$.

nated by $(\partial_\mu V_\nu - \partial_\nu V_\mu)^2$, three-point and four-point ρ-meson coupling terms as

$$(4\alpha^2 r^{-2}) - 2g_{3\rho}(2\alpha^3 r^{-2}) + g_{4\rho}(\alpha^4 r^{-2}), \tag{23}$$

which is ultraviolet divergent in the integration over r. Therefore, for $G(r) = \alpha$ to be realized near $r = 0$ as a finite energy configuration, the coefficient of r^{-2} in (23) should vanish as

$$\alpha^2(g_{4\rho}\alpha^2 - 4g_{3\rho}\alpha + 4) = 0. \tag{24}$$

Furthermore, the Eular-Lagrange equation of $G(r)$ can also be dominated near $r = 0$ by the terms (23) as

$$\frac{1}{4\pi}\left[\frac{\delta E[F, G]}{\delta G(r)} - \frac{d}{dr}\left\{\frac{\delta E[F, G]}{\delta G'(r)}\right\}\right]_{r\to 0} = 4\alpha(g_{4\rho}\alpha^2 - 3g_{3\rho}\alpha + 2)r^{-1} = 0. \tag{25}$$

Therefore, for $G(r)$ to take a nonzero value near $r = 0$ as a finite energy soliton solution, the following must be satisfied as the necessary condition from (24) and (25) as

$$\alpha \neq 0 \quad \text{and} \quad \alpha = 2/g_{3\rho} \quad \text{and} \quad g_{3\rho}^2/g_{4\rho} = 1, \tag{26}$$

otherwise, $\alpha = 0$. Actually, (26) is the condition of the Yang-Mills field, reproducing the results of HLS in Sec. II with $g_{3\rho} = 1$ and $G(0) = 2$. In the case of holographic QCD, $g_{3\rho}^2/g_{4\rho} \neq 1$, and therefore only the regular ρ-meson configuration with $G(0) = 0$ is allowed. As a whole, even any deviation of the ratio $g_{3\rho}^2/g_{4\rho}$ from unity due to the curved extra dimension can drastically change the baryon profiles: for $g_{3\rho}^2/g_{4\rho} = 1$ the ρ meson with a strong repulsion, and for $g_{3\rho}^2/g_{4\rho} \neq 1$ with a weak attraction for the baryon.

Here, we note several expected features. Even if a_1 meson is included for the Skyrmion analysis in holographic QCD, there is *no* catastrophic instability as in the case of HLS_2 in Sec. II for the following reasons: First, the Skyrme term naturally appears from the DBI action of the D8 brane as a stabilizer of the Skyrmion. Second, because of the relation $g_{3\rho}^2/g_{4\rho} \neq 1$ in holographic QCD, the ρ meson has weak attraction, not strong repulsion for the baryon. Therefore, even if the ρ-meson contributions could be canceled by the a_1 meson, there is no catastrophic instability of Skyrmion into zero size in the presence of the Skyrme term.

Recently, the baryon has also been discussed as the holonomy of an *instanton* in the five-dimensional gauge theory on D8 branes with D4 supergravity background [18–21]. Since the instanton is introduced before the mode expansions of five-dimensional gauge field $A_M(x, z)$, one would expect that it is composed by pion and infinite tower of (axial) vector mesons $\rho, a_1, \rho', a_1', \rho'' \cdots$. Actually, the DBI action of the D8 brane is known to lead the instability of instanton into zero size, so that the inclusion of the Chern-Simons (CS) action is claimed as the stabilizer [18–21], though the CS sector is, in the 't Hooft coupling expansion, higher-order contribution relative to the DBI sector. Then, one might relate the instability of instanton with the instability of Skyrmion with π, ρ, and a_1 mesons as in the case of HLS_2 in Sec. II. However, we can argue that Skyrmion with π, ρ and a_1 mesons have *no* catastrophic instability in holographic QCD due to the curved extra dimension. Once the stability is established, additional inclusion of the CS sector, corresponding to the effects of the ω meson [22], does not affect the present discussions of the stability with the a_1 meson in holographic QCD.

Here, it is noted that holographic QCD has the ultraviolet cutoff scale $M_{\mathrm{KK}} \sim 1$ GeV. In fact, there exist an infinite number of non-QCD Kaluza-Klein modes in the S^1 compactification of D4 branes. Therefore, the $\mathrm{D4}/\mathrm{D8}/\overline{\mathrm{D8}}$ multi-D brane system can be regarded as QCD below the M_{KK} scale. Indeed, the appearance of M_{KK} is essential to reproduce the non-SUSY nature in QCD, which breaks the conformal invariance to give color confinement and chiral-symmetry breaking. Now, in the formation of the instanton in the five-dimensional space-time, the infinite number of non-QCD modes above M_{KK} are also included. Therefore, one should be careful to their possible artifacts, which may cause the instability of the instanton. In our study, we truncate the meson resonances at the M_{KK} scale to include only the relevant QCD modes at the level of the classical supergravity.

If one would like to see a fine structure of the baryon and also its excitations, a larger M_{KK} is to be taken. In such a case, heavier meson resonances are also systematically treated in a way consistent with the cutoff scale. In fact, the upper bound of M_{KK} comes from the local approximation of the fundamental strings on the D4 branes, which is compactified with a radius M_{KK}^{-1}. Therefore, if one can, e.g., include the effect of finite string length in the action (18), M_{KK} might be taken sufficiently large with fixed Λ_{QCD} [13,23]. In this sense, the instability of the instanton into zero size might imply a need for more consistent treatment with quantum corrections and string length at short distance.

Such a situation may resemble that of the strong-coupling expansion in QCD with the plaquette lattice gauge action [24]. For a strong coupling with large g^2, the expansion in powers of $1/g^2$ gives a basic picture of quark confinement [25] and chiral-symmetry breaking [26], providing also an analytical method to calculate hadron properties [26]. The strong-coupling QCD, however, corresponds to a large lattice spacing a, and has a limitation on its spatial resolution. Then, if one includes higher-order terms of $1/g^2$ in the cluster expansion [24], a finer structure of the theory becomes gradually visible.

Now, for a larger M_{KK}, the relation $g_{3\rho}^2/g_{4\rho} \neq 1$ still holds due to the curvature in the extra dimensions, and the Skyrme term generally exists as a stabilizer of the Skyrmion in dual of QCD, which is manifest in the classical limit as in the action (18). With these considerations, we can expect the stability of the Skyrmion in the present study even for the larger M_{KK}, which will be more rigorously discussed elsewhere in the near future.

IV. SUMMARY AND OUTLOOK

We have studied baryons as the Skyrmions in HLS and also in holographic QCD. We conclude that the relation $g_{3\rho}^2/g_{4\rho} \neq 1$ due to the extra dimension and its nontrivial curvature can drastically change the roles of (axial) vector mesons for the baryons in four dimensions. In fact, the effective action density of the Yang-Mills shape as $F_{MN}F^{MN}$ (M, $N = 0 \sim 3$, z) on the probe D8 brane can be inevitably distorted through the projection into the flat four-dimensional space-time, giving the relation $g_{3\rho}^2/g_{4\rho} \neq 1$. Then, we also discuss the features of our truncated-resonance analysis for baryons with π and ρ mesons below $M_{\mathrm{KK}} \sim 1$ GeV in holographic QCD.

Our results are not limited in the case of the holographic model with $\mathrm{D4}/\mathrm{D8}/\overline{\mathrm{D8}}$ multi-D brane configurations as the Sakai-Sugimoto model. Even if one starts from any multi-D brane configuration with gauge theory on its surface, and also if one relies on the gauge/gravity duality, there is no reason for the (axial) vector mesons to appear as the Yang-Mills field with the constraint $g_{3\rho}^2/g_{4\rho} = 1$, because of the curved extra dimension. Note here that, in the holographic framework, the curvature in the extra dimension is needed to give the discrete mass spectra of hadrons in the projected four-dimensional theory as QCD. Furthermore, also from the phenomenological point of view in four dimensions, the constants $g_{3\rho}^2$ and $g_{4\rho}$ can be easily shifted at the quantum level, spoiling the condition $g_{3\rho}^2/g_{4\rho} = 1$. As a whole, our results in Sec. III should be more general relative to the case of HLS in Sec. II.

Of course, HLS has an interesting physical meaning. Actually, dividing the chiral field (1) into a sequence of products as $U(x) = \xi_1\xi_2\cdots\xi_{N+1}$, one can introduce the gauge field $A_\mu^k(x)$ ($k = 1, 2, \cdots, N$) for each gauge symmetry acting at each cutting point. If one takes the limit $N \to \infty$ to include an infinite tower of (axial) vector mesons, one can naturally introduce the five-dimensional gauge field $A_\mu(x, z)$ by shifting the discrete index k into a continuous variable z. This strategy is called the dimensional deconstruction model of QCD, or the bottom-up holographic approach [27]. In this sense, a fact that certain local symmetry is *hidden* within the chiral field [6] has implied the existence of a larger gauge symmetry extending to the *extra dimensions*, which corresponds to the theoretical background of HLS today.

However, HLS does not have the concept of curvature or gravity in the extra dimension. In this paper, we suggest that such a curved extra dimension can drastically change the view of our daily life in four-dimensional space-time.

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