

Constraints for weakly interacting light bosons from existence of massive neutron starsM. I. Krivoruchenko,¹ F. Šimkovic,^{2,3} and Amand Faessler⁴¹*Institute for Theoretical and Experimental Physics, B. Chermushkinskaya 25 117218 Moscow, Russia*²*Bogoliubov Laboratory of Theoretical Physics, JINR, 141980 Dubna, Moscow Region, Russia*³*Department of Nuclear Physics and Biophysics, Comenius University, Mlynská dolina F1 SK842 48 Bratislava, Slovakia*⁴*Institut für Theoretische Physik, Tübingen Universität, Auf der Morgenstelle 14 D-72076 Tübingen, Germany*

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Theories beyond the standard model include a number of new particles, some of which might be light and weakly coupled to ordinary matter. Such particles affect the equation of state of nuclear matter and can shift admissible masses of neutron stars to higher values. The internal structure of neutron stars is modified provided the ratio between coupling strength and mass squared of a weakly interacting light boson is above $g^2/\mu^2 \sim 25 \text{ GeV}^{-2}$. We provide limits on the couplings with the strange sector, which cannot be achieved from laboratory experiments analysis. When the couplings to the first family of quarks is considered, the limits imposed by the neutron stars are not more stringent than the existing laboratory ones. The observations on neutron stars give evidence that the equation of state of the β -equilibrated nuclear matter is stiffer than expected from many-body theory of nuclei and nuclear matter. A weakly interacting light vector boson coupled predominantly to the second family of the quarks can produce the required stiffening.

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Dark energy explains the accelerating expansion of the Universe. The density of dark energy $\rho_D \approx 3.8 \text{ keV/cm}^3$ may correspond to a fundamental scale $\lambda_D = \rho_D^{-1/4} \approx 8.5 \times 10^{-5} \text{ m}$ [1–4]. Theoretical schemes with extra dimensions suggest modifications of gravity below λ_D and a multitude of states with masses above $1/\lambda_D$ very weakly coupled to members of multiplets of the standard model. Scales significantly below λ_D represent the interest for supersymmetric extensions of the standard model which include generally a number of new particles, such as the leading dark matter candidate neutralino. Typically, new particles are expected with masses above several hundred GeVs or even higher. However, light particles may exist also, such as a neutral very weakly coupled spin-1 gauge U -boson [5] that can provide annihilation of light dark matter and be responsible for the 511 keV line observed from the galactic bulge [6,7].

Deviations from the inverse-square Newton's law are parametrized often in terms of the exchanges by hypothetical bosons also. Constraints on the deviations from Newton's gravity have been set experimentally in the submillimeter scale [8–13] and down to distances $\sim 10 \text{ fm}$ where effects of light bosons of extensions of the standard model can be expected [14–18]. Constraints on the coupling constants from unobserved missing energy decay modes of ordinary mesons are discussed in Ref. [19].

Bosons with small couplings escape detection in most laboratory experiments. However, bosons interacting with baryons modify the equation of state (EOS) of nuclear matter. Their effect depends on the ratio between the coupling strength and the boson mass squared, so a weakly interacting light boson (WILB) may influence the structure of neutron stars even if its baryon couplings are very small.

The effect of a vector boson on the energy density of nuclear matter can be evaluated by averaging the corresponding Yukawa potential:

$$E_I = \frac{1}{2} \int d\mathbf{x}_1 d\mathbf{x}_2 \rho(\mathbf{x}_1) \frac{g^2}{4\pi} \frac{e^{-\mu r}}{r} \rho(\mathbf{x}_2), \quad (1)$$

where $\rho(\mathbf{x}_1) = \rho(\mathbf{x}_2) \equiv \rho$ is the number density of homogeneously distributed baryons, $r = |\mathbf{x}_2 - \mathbf{x}_1|$, g is the coupling constant with baryons, and μ is the boson mass. A simple integration gives

$$E_I = V \frac{g^2 \rho^2}{2\mu^2}, \quad (2)$$

where V is the normalization volume.

The coherent contribution to the energy density of nuclear matter from vector WILBs should be compared to that from the ordinary ω mesons. In one-boson exchange potential (OBEP) models, the nucleon-nucleon repulsive core at short distances $r \lesssim b = 0.4 \text{ fm}$ is attributed to ω -meson exchanges. Respectively, the ω meson plays a fundamental role in nuclear matter EOS. In the mean-field approximation, the contribution of ω -meson exchanges to the energy has the form of Eq. (2), with g and μ replaced by the ω -meson coupling g_ω and the mass μ_ω .

The NN interactions are described with $g_\omega^2/\mu_\omega^2 = 175 \text{ GeV}^{-2}$ [20]. The relativistic mean-field (RMF) model [21] gives $g_\omega^2/\mu_\omega^2 = 196 \text{ GeV}^{-2}$. The compression modulus of nuclear matter $K = 210 \div 300 \text{ MeV}$ is consistent with $g_\omega^2/\mu_\omega^2 = 125 \div 180 \text{ GeV}^{-2}$ [22]. Stiff RMF models use g_ω^2/μ_ω^2 up to 300 GeV^{-2} [23]. If we wish to stay within current limits and do not want to modify the internal structure of neutron stars qualitatively, as described by realistic models of nuclear matter, one has to require that

vector WILBs fulfill constraint

$$\frac{g^2}{\mu^2} \lesssim \frac{g_\omega^2}{\mu_\omega^2} \approx 200 \text{ GeV}^{-2}. \quad (3)$$

A similar reasoning applies to scalar WILBs which have to compete with the standard σ -meson exchange. In OBEP models, the long-range attraction between nucleons is attributed to σ -meson exchanges. The contribution of the σ mesons to the interaction energy has the form of Eq. (2), with g and μ replaced by the σ -meson coupling g_σ and the mass μ_σ . The sign of the contribution must be negative because of the attraction. Also, ρ should be replaced by the scalar density. In RMF models, the σ -meson mean field decreases the nucleon mass. The effect depends on the ratio g^2/μ^2 also and produces an additional decrease of the energy at fixed volume and baryon number. The empirical values of the ratio g_σ^2/μ_σ^2 are 40% ÷ 60% higher than those of the ω meson [20–23]. The internal structure of neutron stars is not modified significantly provided the coupling strength g and mass μ of scalar WILBs fulfill constraint

$$\frac{g^2}{\mu^2} \lesssim \frac{g_\sigma^2}{\mu_\sigma^2} \approx 300 \text{ GeV}^{-2}. \quad (4)$$

The deviations from the Newton's gravitational potential are usually parametrized in the form

$$V(r) = -\frac{Gm_1m_2}{r}(1 + \alpha_G e^{-r/\lambda}). \quad (5)$$

The second Yukawa term can be attributed to new bosons with $Gm^2\alpha_G = \pm g^2/(4\pi)$ and $\lambda = 1/\mu$, where $+/-$ stands for scalar/vector bosons and m is the proton mass.

On Fig. 1 we show regions in the parameter spaces (g^2, μ) and (α_G, λ) allowed for WILBs by the constraint (3). The constraint for scalar bosons is close to (3). Constraints from other works [10–18] are shown also.

An increase of g (a decrease of μ) of scalar WILBs increases the negative contribution to pressure, makes EOS of nuclear matter softer, and makes neutron stars less stable against gravitational compression. The ratio g^2/μ^2 cannot be increased significantly above the limit (4), since the maximum mass of the neutron star sequence cannot be moved below masses of the observed pulsars.

An increase of g (a decrease of μ) of vector WILBs, conversely, increases the positive contribution to pressure, makes EOS of nuclear matter stiffer, makes neutron stars more stable against gravitational compression, and drives the maximum mass of neutron stars up.

In the case of vector bosons, it is less obvious what kind of the observables confronts to high ratios g^2/μ^2 .

Realistic models of nuclear matter are based on the nucleon-nucleon scattering data. They split into soft and stiff models according to the rate the pressure increases

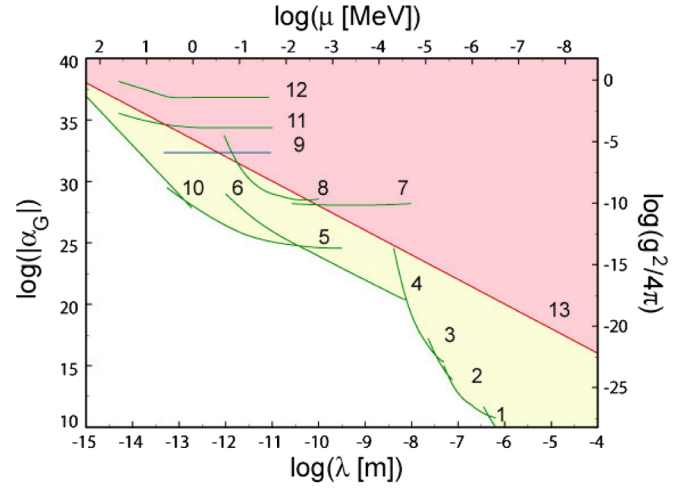


FIG. 1 (color online). Constraints on the coupling strength with nucleons $g^2/(4\pi)$ and the mass μ (equivalently α_G and λ) of hypothetical weakly interacting light bosons: 1 are constraints from Ref. [10], 2 from Ref. [11], 3 from Ref. [12], 4 from Ref. [13], 5 and 10 are constraints from low-energy $n-^{208}\text{Pb}$ scattering [14,16], 6 from Ref. [17], 7 from Ref. [15], 8 and 9 are constraints from spectroscopy of anti-proton atoms [16], 11 and 12 are constraints from near-forward pn scattering for vector and scalar bosons, respectively [18]. The axes are in the \log_{10} scale. The internal structure of neutron stars is not modified qualitatively provided the boson coupling strengths with baryons and masses lie at $g^2/\mu^2 < 200 \text{ GeV}^{-2}$ beneath the highlighted area 13.

with the density. The soft models correspond to low maximum masses of neutron stars $\sim 1.6 M_\odot$, while the stiff models give the upper limit around $\sim 2.6 M_\odot$.

The problem of the softness of nuclear EOS has received new interest due the analysis of strange particle production in heavy-ion collisions. The data at different bombarding energies lead to the conclusion that EOS of nuclear matter must be soft at densities 2 to 3 times of the saturation density [24–26]. Data on the transverse and elliptic flows in heavy-ion collisions suggest a soft EOS around the saturation, too [27].

Last year's observations of pulsars with high masses have been reported. The most massive pulsars are PSR B1516+02B in the globular cluster M5 with the mass of $1.96^{+0.09}_{-0.12} M_\odot$ and PSR J1748-2021B in the globular cluster NGC 6440 with the mass of $2.74 \pm 0.22 M_\odot$ [28]. The mass of rapidly rotating neutron star in the low mass x-ray binary 4U 1636-536 is estimated to be $M = 2.0 \pm 0.1 M_\odot$ [29]. The mass and radius of the x-ray source EXO 0748 – 676 are constrained to $M \geq 2.10 \pm 0.28 M_\odot$ and $R \geq 13.8 \pm 1.8 \text{ km}$ [30]. The observations on neutron stars suggest that EOS of the β -equilibrated nuclear matter is stiff.

The controversy between the conclusions on the softness of nuclear matter as derived from the laboratory experiments and on the stiffness of the β -equilibrated nuclear matter as derived from the astrophysical observations has

been of interest since after the discovery of millisecond pulsars [31,32] and earlier [33].

Current models use to match EOS of neutron matter with a soft EOS at the saturation density and a stiff EOS at higher densities. Such models are in the qualitative agreement with laboratory and astrophysical data [34].

High densities provide favorable conditions for the occurrence of exotic forms of nuclear matter: pion, kaon, and dibaryon condensates, quark matter. New degrees of freedom make EOS softer, pushing the maximum mass of neutron stars down. The recent astrophysical observations seem to exclude the softest EOS, e.g., based on the classical Reid soft core model [35] and make it problematic to accommodate the exotic forms of nuclear matter with masses and radii of the observed pulsars [30] (see, however, [36]).

The in-medium masses of vector mesons depend on the density. Assuming μ is a function of ρ and using Eq. (2), one may evaluate the ω -meson contribution to pressure:

$$P_I = \frac{g^2 \rho^2}{2\mu^2} \left(1 - \frac{2\rho}{\mu} \frac{\partial \mu}{\partial \rho} \right). \quad (6)$$

A positive shift of the ω -meson mass decreases the pressure and leads to a softer EOS, whereas a negative shift leads to a stiffer EOS. The data on the dilepton production in heavy-ion collisions do not give evidence for significant mass shift [37], so the observed stiffness of the β -equilibrated nuclear matter can hardly be attributed to in-medium modifications of the vector mesons.

The realistic models of neutron matter discussed in Ref. [34] neglect hyperon channels, e.g., reactions $\Sigma^- \rightarrow n + e + \bar{\nu}_e$. In RMF models [22,38,39], the β equilibrium of hyperons drops the limiting mass by $0.5 \div 0.8 M_\odot$. This result is in accord with hypernuclear data and other recent calculations [40–42]. The inclusion of the β equilibrium for all baryons brings difficulties in reproducing the observed masses of neutron stars.

Coming back to vector WILBs, we see that their existence is desirable to provide additional stiffening of the β -equilibrated nuclear matter.

The Compton wavelength of WILBs is assumed to be greater than the radius of nuclei, e.g., $1/\mu > R \approx 7 \text{ fm} \approx (30 \text{ MeV})^{-1}$ for the lead. The contribution of WILBs to the binding energy of nuclei then equals $\sim A^2 g^2/R$, like for photons. Since $g^2/(4\pi)$ is much smaller than the fine structure constant, the effect of WILBs on nuclei is negligible. Above $\sim 10^2 \text{ MeV}$ the coupling constant of WILBs is close to unity, so WILBs there are neither weakly interacting nor light.

WILBs thus do not modify observables in laboratory experiments on hypernuclear physics, nuclear structure, and heavy-ion collisions, since their baryon couplings are very small. The characteristic scale of the parameters of these particles is fixed by the upper limit (3).

The mass-radius relations for nonrotating neutron stars are shown on Fig. 2 for four values of the ratio $g^2/\mu^2 = 0,$

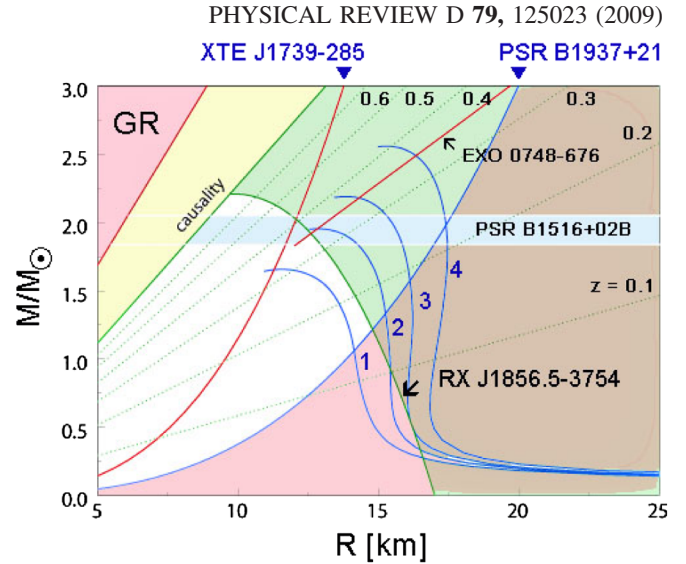


FIG. 2 (color online). Mass of nonrotating neutron stars as a function of radius: 1—RMF model of hyperon matter with the compression modulus $K = 300 \text{ MeV}$ [22]; 2—the same as 1 including a flavor-singlet vector WILB coupled to baryons with $g^2/\mu^2 = 25 \text{ GeV}^{-2}$ [1/8 of the limit (3)]; 3—the same as 2 with $g^2/\mu^2 = 50 \text{ GeV}^{-2}$; 4—the same as 2 with $g^2/\mu^2 = 100 \text{ GeV}^{-2}$. The highlighted area within $M = 1.96^{+0.09}_{-0.12} M_\odot$ shows the mass constraint from PSR B1516+02B. The neutron star sequences should cross the rotation speed limit curves shown for pulsar PSR B1937+21 with the rotation frequency of $\nu = 642 \text{ Hz}$ [53] and the neutron star XTE J1739-285 showing X-ray burst oscillations with frequency of $\nu = 1122 \text{ Hz}$ [54]. The mass-dependent lower bound on radii of neutron stars determined from the blackbody radiation of RX J1856.5-3754 is shown. The dotted straight lines $z = 0.1 \div 0.6$ indicate the red shift at surfaces of neutron stars. The red shift of $z = 0.35$ measured for EXO 0748-676 constrains the radii of neutron stars by $R > 12 \text{ km}$ and, respectively, masses [30].

25, 50 and 100 GeV^{-2} of a flavor-singlet vector WILB. At densities below $\rho_{\text{drip}} = 4.3 \times 10^{11} \text{ g/cm}^3$ the matter represents an atomic lattice. WILBs do not modify properties of nuclei and the Baym-Pethick-Sutherland EOS [43], accordingly. At densities $\rho_{\text{drip}} < \rho \leq \rho_{\text{nucl}} = 2.8 \times 10^{14} \text{ g/cm}^3$, atomic lattice coexists with neutron liquid. The matter at $\rho_{\text{drip}} < \rho \leq \rho_{\text{nucl}}$ is described by the Baym-Bethe-Pethick EOS [44]. Above ρ_{nucl} , nuclei dissolve and the matter is described by the β -equilibrated hyperon liquid with the compression modulus $K = 300 \text{ MeV}$ [22]. WILBs contribute to the energy density and pressure above ρ_{drip} , as described by Eqs. (2) and (6) with $\partial \mu / \partial \rho = 0$, through the spatially extended nucleon and hyperon liquid components of the neutron star matter. The vector WILBs give equal contributions to the chemical potentials of the octet baryons and do not violate the chemical β equilibrium [45]. The inclusion of such vector bosons therefore does not change the composition of the neutron star matter.

The highlighted area at the upper left corner of Fig. 2 excludes within general relativity the radii of neutron stars

below the Schwarzschild radius. The causal limit excludes the area $R \lesssim 3GM_\odot$ [46]. The rotation speed limit curves are constructed using the modified Keplerian rate $\nu_{\max} \approx 1045(M/M_\odot)^{1/2}(10 \text{ km}/R)^{3/2}$ Hz, which accounts for the deformation of rotating neutron stars and effects of general relativity [47].

It is seen from Fig. 2 that, despite selecting EOS with the high compression modulus, the neutron star sequence with $g^2/\mu^2 = 0$ contradicts the mass measurement of PSR B1516+02B. It gives a very low mass of the neutron star from the blackbody radiation radius constraint also, which confronts the lower limit of $\sim 0.85 M_\odot$ for masses of protoneutron stars [48].

The value of $g^2/\mu^2 = 200 \text{ GeV}^{-2}$ gives the maximum mass slightly above $3.0 M_\odot$. However, the neutron star sequence does not cross the rotation speed limits, while the redshift remains always below $z = 0.35$. The upper bound (3) is thus critical for the internal structure of neutron stars [49].

The vector WILBs increase the minimum and maximum mass limits and radii of neutron stars and are able to bring in the agreement models of hyperon matter which are soft with the astrophysical observations on neutron stars which require a stiff EOS. The ratio $g^2/\mu^2 \approx 50 \text{ GeV}^{-2}$ might be reasonable. Such a value, however, clearly contradicts to the laboratory constraints shown on Fig. 1 in the entire mass range $\mu = 10^{-9}$ to 10^2 MeV .

The in-medium modification of masses of vector bosons modify EOS. Vector WILBs can be compared to the ω meson where $|\delta\mu_\omega|/\mu_\omega \lesssim 0.1$ above the saturation density [37]. A vector WILB mass shift can be estimated as $\delta\mu^2 \sim g^2/g_\omega^2 2\mu_\omega \delta\mu_\omega$. The in-medium modification is small provided $|\delta\mu^2| \lesssim \mu^2$, i.e., $g^2/\mu^2 \lesssim 10^3 \text{ GeV}^{-2}$, so in the region of interest (3) holds for the vacuum masses.

The laboratory constraints shown on Fig. 1 do not apply to WILBs coupled to hyperons. A vector WILB coupled predominantly to the second family of the quarks makes hyperon matter EOS stiffer also. It contributes differently to chemical potentials of the octet baryons and suppresses the hyperon content of the neutron star matter due the additional repulsion. One can expect the ratio g^2/μ^2 should be close to or higher than that estimated above ($\sim 50 \text{ GeV}^{-2}$). In such a scenario, nuclear matter without hyperons can be treated as reasonable approximation for

the modeling structure of neutron stars in the β equilibrium also, e.g., in line with Ref. [34] where models with the blocked hyperon channels are shown to be in the qualitative agreement with the laboratory and astrophysical constraints.

Gauge bosons interact with the conserved currents only, but flavor is not conserved. A WILB coupled to the second family of the quarks cannot be a gauge boson, so it does not arise naturally in the current theoretical schemes. Here, we do not have a goal whatsoever to go beyond the phenomenological analysis.

Hypernuclear data restrict NY potentials, whereas the interaction between hyperons YY is not known experimentally. The stiffness of the hyperon matter might also be attributed to the $\phi(1020)$ -meson exchange, whose coupling to the nonstrange baryons is suppressed according to the Okubo-Zweig-Iizuka rule (see, however, [52]).

Summarizing, we have assumed the existence and derived constraints for a new boson that couples to nuclear matter. Such a particle contributes, by its coherent force among nuclear constituents, to a modified EOS and affects the structure of neutron stars. The neutron stars exclude scalar bosons with the coupling strengths and masses above the line $l3$ on Fig. 1, whereas in a narrow band below it and above a vector boson coupled to quarks of the second family could modify the EOS in a direction favored by the observed masses and radii of neutron stars. The astrophysical constraints in the nonstrange sector are less stringent than the most accurate laboratory ones. They are unique, however, for scalar WILBs in the strange sector. The region of validity of the astrophysical constraints extends from $\lambda \sim 10 \text{ fm}$ to about 10 km . Detailed studies of manifestations of new bosons in astrophysics, physics of neutron stars, and hadron decays to energy missing channels can shed more light on the existence of WILBs and their possible effect on the structure of neutron stars.

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