PHYSICAL REVIEW D **79,** 125008 (2009)

Renormalization of supersymmetric field theories in loop regularization with string-mode regulators

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(Received 10 December 2008; published 10 June 2009)

By applying the recently developed loop regularization (LR) with string-mode regulators to supersymmetric field theories, we explicitly verify the supersymmetric Ward identities in several supersymmetric models at one-loop level. It is interesting to observe that supersymmetry is such a remarkable symmetry that the supersymmetric Ward identities hold as long as a regularization scheme is realized in the exact four-dimensional space-time with translational invariance for the momentum integration, and the gauge symmetry can be maintained once the regularization scheme preserves supersymmetry and satisfies the consistency condition for logarithmic divergences. As a manifest demonstration, we carry out a complete one-loop renormalization for the massive Wess-Zumino model by adopting the LR method, and it is found that all the quadratic divergences cancel out and the relations among masses and coupling constants hold after renormalization, which agrees with the well-known nonrenormalization theorem. It is concluded that the LR method preserves not only gauge symmetry but also supersymmetry. A simple and definite derivation of Majorana Feynman rules is found to be very useful.

DOI: 10.1103/PhysRevD.79.125008 PACS numbers: 11.30.Pb, 11.10.Gh, 11.15.−q

I. INTRODUCTION

Supersymmetry has attracted physicists for several decades since it was proposed in the 1970s [1]. It is well known that symmetry has played an important role in particle physics, and three of the four basic forces in nature are governed by gauge symmetries and have successfully been described by quantum field theory. However, quantum field theories are bothered by the infinities which must be regularized to be well defined. On the other hand, whether the symmetries of the classical Lagrangian still hold at the quantum level remains an important issue. This is because sometimes it is difficult to distinguish between a real anomaly and an apparent violation of the symmetries due to the use of a symmetry-violating regularization method. In general, when a symmetry of the original Lagrangian is still a symmetry of a full quantum effective action, such a symmetry is regarded as being preserved in the quantum level, but there are several exceptions such as chiral anomaly. Thus one may ask whether supersymmetry is a symmetry of the full quantum theory. This question has been studied in a regularization-independent way in Ref. [2], and the answer is yes. This means when investigating the quantum effects of the supersymmetric theories, one must adopt a supersymmetry-preserving regularization method.

Several regularization methods have been applied to supersymmetric theories, such as dimensional reduction (DRED) [3], differential regularization [4], and the so-called implicit regularization [5]; among them DRED is the most common one. It has been shown that DRED can preserve supersymmetry in several models [6–8]. Strictly speaking, DRED is too mathematically inconsistent [9,10] to be applied to the supersymmetric theories, which is similar to the case when it is applied to the chiral theories. This is because both supersymmetry and the definition of γ_5 require an exact dimension. A consistent regularization method that can be applied to all possible cases in quantum field theories is needed. In this sense, the recently developed loop regularization (LR) with string-mode regulators [11,12] may deserve special attention. It has successfully been applied to the calculations of the triangle anomaly of QED by the clarification of the possible ambiguities caused by γ_5 [13], the evaluation of a consistent coefficient of the CPT and Lorentz symmetry breaking Chern-Simons term [14], the computation of all the one-loop renormalization constants for the non-Abelian gauge theory and the determination for the coefficient of the QCD β function [15], and the derivation of the chiral effective field theory with a dynamically generated spontaneous symmetry breaking [16]. The key concept of this new regularization method is the introduction of the irreducible loop integrals (ILIs) which are evaluated from Feynman integrals by using the Feynman parameter method.

It has been shown that the LR method can preserve the non-Abelian gauge symmetry, while maintaining the divergent behavior of original field theories. In particular, the LR method is realized in the original four-dimensional space-time with translational and Lorentz invariance even if two intrinsic mass scales are introduced; thus it can balance the bosonic and fermionic degrees automatically and there is also no ambiguity about the definition of γ_5. It is then believed that this method will preserve supersymmetry as well. In this paper, we will investigate the applicability of the LR method in supersymmetric theories.

The paper is organized as follows: In Sec. II, we briefly introduce the symmetry-preserving loop regularization with string-mode regulators. In Secs. III and IV, we will verify the supersymmetric Ward identities for the massless Wess-Zumino model and massive Wess-Zumino model [17] separately and show that the LR method indeed respects the Ward identities. Since the Ward identity is the reflection of symmetry in the quantum level, we arrive at the conclusion that the LR method is also a supersymmetry-preserving regularization for supersymmetric models. In Sec. V, we consider the super Yang-Mills theory as the testing ground to explicitly demonstrate the supersymmetric Ward identity and show that the LR method does preserve supersymmetry. Meanwhile the gauge symmetry is maintained only requiring the consistency condition for logarithmic divergences. Note that the conventional dimensional regularization was shown to break the Ward identity in such a model [6], thus an alternative check by using the LR method in our present paper is nontrivial. In particular, we will demonstrate that as long as the Dirac algebra for γ matrices are carried out in four-dimensional space-time and the shift of integration variable can be safely made, the supersymmetric Ward identities are preserved, which is actually independent of any concrete prescription of regularization methods. Namely, as long as the regularization scheme is realized in four-dimensional space-time with translational invariance for momentum integrals, like the LR method, it then preserves supersymmetry. In Sec. VI, as an explicit demonstration, we will carry out the one-loop renormalization for the massive Wess-Zumino model by using the LR method, and all the obtained results agree with the well-known nonrenormalization theorem. Our conclusions and remarks are given in the last section. The detailed derivation of the Majorana Feynman rules is presented in the appendix.

II. SYMMETRY-PRESERVING LOOP REGULARIZATION

It has been shown in [11,12] that all one-loop Feynman integrals can be evaluated into the following onefold ILIs by using the Feynman parametrization method:

$$\begin{aligned} I_{-2\alpha} &= \int \frac{d^4k}{(2\pi)^4} \frac{1}{(k^2 - M^2)^{2+\alpha}}, \\ I_{-2\alpha\mu\nu} &= \int \frac{d^4k}{(2\pi)^4} \frac{k_\mu k_\nu}{(k^2 - M^2)^{3+\alpha}}, \\ \alpha &= -1, 0, 1, 2, \ldots, \\ I_{-2\alpha\mu\nu\rho\sigma} &= \int \frac{d^4k}{(2\pi)^4} \frac{k_\mu k_\nu k_\rho k_\sigma}{(k^2 - M^2)^{4+\alpha}} \end{aligned} \tag{2.1}$$

with I_2 and I_0 corresponding to the quadratic and logarithmic divergent integrals, where the effective mass factor M^2 is a function of the external momenta p_i, the masses of particles m_i, and the Feynman parameters.

In general, the loop momentum independent M^2 can be extended to include the linear term in k, which can be understood as a part of the definition of the ILIs in the LR. The reason is as follows: Let $M^2(k)$ have the following general form including the linear term in k:

$$M^2(k) = M^2 + 2xk.p$$

with x an arbitrary parameter. Then

$$\begin{aligned} k^2 + M^2(k) &= k^2 + 2xp.k + M^2 = (k + xp)^2 + M^2 - p^2 \\ &= (k + xp)^2 + M^2(p) = k'^2 + M^2(p) \end{aligned}$$

with $M^2(p) = M^2 - p^2$ which becomes independent of k, and $k' = k + xp$ via translational invariance. Again, the only thing that must be paid attention to is that one must follow the definition of the ILIs to cancel out the k^2 in the numerator before regularization.

When the regularized onefold ILIs satisfy the following consistency conditions [11,12]:

$$\begin{aligned} I^R_{2\mu\nu} &= \tfrac{1}{2} g_{\mu\nu} I^R_2, \\ I^R_{2\mu\nu\rho\sigma} &= \tfrac{1}{8}(g_{\mu\nu}g_{\rho\sigma} + g_{\mu\rho}g_{\nu\sigma} + g_{\mu\sigma}g_{\rho\nu}) I^R_2, \\ I^R_{0\mu\nu} &= \tfrac{1}{4} g_{\mu\nu} I^R_0, \\ I^R_{0\mu\nu\rho\sigma} &= \tfrac{1}{24}(g_{\mu\nu}g_{\rho\sigma} + g_{\mu\rho}g_{\nu\sigma} + g_{\mu\sigma}g_{\rho\nu}) I^R_0 \end{aligned} \tag{2.2}$$

the resulting loop corrections are gauge invariant. Here the superscript "R" denotes the regularized ILIs.

Note that the introduction on the concept of ILIs is crucial in the loop regularization [11,12], where it has been shown that all Feynman loop integrals can be evaluated to be expressed by the ILIs. From the definition of ILIs, one of the important properties is that there should be no k^2 in the numerator of loop integration. All the ILIs can be classified into the scalar-type ILIs with the following loop integration:

$$\frac{1}{(k^2 - M^2)^\alpha}$$

and the tensor type ILIs with the following loop integration:

$$\frac{k_\mu k_\nu \cdots k_\rho}{(k^2 - M^2)^\alpha}.$$

In evaluating the Feynman loop integrals into ILIs, one should always perform the Dirac algebra and Lorentz index-contraction first to obtain the ILIs defined by the above "simplest" forms for the one-loop case (for the two-loop and higher-loop case, see Ref. [11]). Therefore, for the integration

$$g^{\mu\nu} \cdot k_\mu k_\nu/(k^2 - M^2)^2,$$

which should not be written as

$$g^{\mu\nu} \cdot I_{2\mu\nu},$$

it must be expressed as

$$k^2/(k^2 - M^2)^2,$$

rewriting the k^2 in the numerator into $(k^2 - M^2) + M^2$ so as to cancel out the first term by the denominator. Thus the above Feynman loop integration is regarded to be evaluated into the ILIs and is given by the following form before regularization:

$$g^{\mu\nu} \cdot k_\mu k_\nu/(k^2 - M^2)^2 = I_2 + M^2 * I_0.$$

From the above illustration, it is seen that in the spirit of ILIs, the integration

$$g^{\mu\nu} \cdot k_\mu k_\nu/(k^2 - M^2)^2$$

is not an ILI. One should not regularize such a loop integration in the loop regularization method.

A simple regularization prescription for the ILIs was realized to yield the above consistency conditions. Its procedure is as follows: Rotate to the four-dimensional Euclidean space of momentum, replacing the loop integrating variable k^2 and the loop integrating measure $\int d^4k$ in the ILIs by the corresponding regularized ones $[k^2]_l$ and $\int [d^4k]_l$:

$$k^2 \to [k^2]_l \equiv k^2 + M_l^2,$$
$$\int d^4k \to \int [d^4k]_l \equiv \lim_{N,M_l^2} \sum_{l=0}^{N} c_l^N \int d^4k, \qquad (2.3)$$

where M_l^2 ($l = 0, 1, \cdots$) may be regarded as the regulator masses for the ILIs. The regularized ILIs in the Euclidean space-time are then given by

$$\begin{aligned} I^R_{-2\alpha} &= i(-1)^\alpha \lim_{N,M_l^2} \sum_{l=0}^{N} c_l^N \\ &\quad\times \int \frac{d^4k}{(2\pi)^4} \frac{1}{(k^2 + M^2 + M_l^2)^{2+\alpha}}, \\ I^R_{-2\alpha\mu\nu} &= -i(-1)^\alpha \lim_{N,M_l^2} \sum_{l=0}^{N} c_l^N \\ &\quad\times \int \frac{d^4k}{(2\pi)^4} \frac{k_\mu k_\nu}{(k^2 + M^2 + M_l^2)^{3+\alpha}}, \\ \alpha &= -1, 0, 1, 2, \ldots, \\ I^R_{-2\alpha\mu\nu\rho\sigma} &= i(-1)^\alpha \lim_{N,M_l^2} \sum_{l=0}^{N} c_l^N \\ &\quad\times \int \frac{d^4k}{(2\pi)^4} \frac{k_\mu k_\nu k_\rho k_\sigma}{(k^2 + M^2 + M_l^2)^{4+\alpha}}, \end{aligned} \qquad (2.4)$$

where the coefficients c_l^N are chosen to satisfy the following conditions:

$$\lim_{N,M_l^2} \sum_{l=0}^{N} c_l^N (M_l^2)^n = 0 \qquad (n = 0, 1, \cdots) \qquad (2.5)$$

with the notation $\lim_{N,M_l^2}$ denoting the limit $\lim_{N,M_R^2 \to \infty}$. One may take the initial conditions $M_0^2 = \mu_s^2 = 0$ and $c_0^N = 1$ to recover the original integrals in the limit $M_l^2 \to \infty$ ($l = 1, 2, \cdots$). Such a new regularization is called LR [11,12]. The prescription in the LR method is very similar to Pauli-Villars prescription, but the two concepts are totally different as the prescription in the loop regularization is acting on the ILIs rather than on the propagators in the Pauli-Villars scheme. This is why the Pauli-Villars regularization violates non-Abelian gauge symmetry, while the LR method can preserve non-Abelian gauge symmetry.

As the simplest solution of Eq. (2.5), taking the string-mode regulators

$$M_l^2 = \mu_s^2 + lM_R^2 \qquad (2.6)$$

with $l = 1, 2, \cdots$, the coefficients c_l^N are completely determined

$$c_l^N = (-1)^l \frac{N!}{(N-l)!l!}. \qquad (2.7)$$

Here M_R may be regarded as a basic mass scale of loop regulator. It has been shown in [12] that the above regularization prescription can be understood in terms of Schwinger proper time formulation with an appropriate regulating distribution function.

With the string-mode regulators for M_l^2 and c_l^N in the above equations, the regularized ILIs I_2^R and I_0^R can be evaluated to the following explicit forms [11,12]:

$$\begin{aligned} I_2^R &= \frac{-i}{16\pi^2}\left\{M_c^2 - \mu^2\left[\ln\frac{M_c^2}{\mu^2} - \gamma_w + 1 + y_2\left(\frac{\mu^2}{M_c^2}\right)\right]\right\}, \\ I_0^R &= \frac{i}{16\pi^2}\left[\ln\frac{M_c^2}{\mu^2} - \gamma_w + y_0\left(\frac{\mu^2}{M_c^2}\right)\right] \end{aligned} \qquad (2.8)$$

with $\mu^2 = \mu_s^2 + M^2$, and

$$\begin{aligned} \gamma_w &\equiv \lim_N\left\{\sum_{l=1}^{N} c_l^N \ln l + \ln\left[\sum_{l=1}^{N} c_l^N l \ln l\right]\right\} \\ &= \gamma_E = 0.5772\cdots, \\ y_0(x) &= \int_0^x d\sigma \frac{1 - e^{-\sigma}}{\sigma}, \qquad y_1(x) = \frac{e^{-x} - 1 + x}{x}, \\ y_2(x) &= y_0(x) - y_1(x), \qquad \lim_{x\to 0} y_i(x) \to 0, \qquad i = 0, 1, 2, \\ M_c^2 &\equiv \lim_{N,M_R} M_R^2 \sum_{l=1}^{N} c_l^N (l \ln l) = \lim_{N,M_R} M_R^2/\ln N, \end{aligned} \qquad (2.9)$$

which indicates that the μ_s sets an IR "cutoff" at $M^2 = 0$ and M_c provides an UV cutoff. For renormalizable quantum field theories, M_c can be taken to be infinity ($M_c \to \infty$). In a theory without infrared divergence, μ_s can safely run to $\mu_s = 0$. Actually, in the case that $M_c \to \infty$ and $\mu_s = 0$, one recovers the initial integral. Also, once M_R and N are taken to be infinity, the regularized theory becomes independent of the regularization prescription. Note that to evaluate the ILIs, the algebraic computing for multi-γ matrices involving loop momentum $\not{k}$ such as $\not{k}\gamma_\mu\not{k}$ should be carried out to be expressed in terms of the independent components: γ_μ, $\sigma_{\mu\nu}$, $\gamma_5\gamma_\mu$, γ_5.

We shall directly show that loop regularization is manifestly translational invariant in spite of the existence of two energy scales, which is a very important feature in applying to supersymmetric theories in this paper. To see that, we shall verify that the regularized ILIs should arrive at the same results whether the loop regularization prescription is applied before or after shifting the integration variables for momentum. For an explicit illustration, let us examine a simple logarithmic divergent Feynman integral:

$$L = \int \frac{d^4k}{(2\pi)^4} \frac{1}{k^2 - m_1^2} \frac{1}{(k-p)^2 - m_2^2}. \tag{2.10}$$

As the first step of loop regularization, we shall apply the general Feynman parameter formula

$$\frac{1}{a_1^{\alpha_1} a_2^{\alpha_2} \cdots a_n^{\alpha_n}} = \frac{\Gamma(\alpha_1 + \cdots + \alpha_n)}{\Gamma(\alpha_1)\cdots\Gamma(\alpha_n)} \int_0^1 dx_1 \int_0^{x_1} dx_2 \cdots \int_0^{x_{n-2}} dx_{n-1} \times \frac{(1-x_1)^{\alpha_1-1}(x_1-x_2)^{\alpha_2-1}\cdots x_{n-1}^{\alpha_n-1}}{[a_1(1-x_1) + a_2(x_1-x_2) + \cdots + a_n x_{n-1}]^{\alpha_1+\cdots+\alpha_n}} \tag{2.11}$$

to the Feynman integral and obtain the following integral:

$$\begin{aligned} L &= \int \frac{d^4k}{(2\pi)^4} \int_0^1 dx \frac{1}{\{(1-x)(k^2 - m_1^2) + x[(k-p)^2 - m_2^2]\}^2} \\ &= \int \frac{d^4k}{(2\pi)^4} \int_0^1 dx \frac{1}{\{(k-xp)^2 - [(1-x)m_1^2 + xm_2^2 - x(1-x)p^2]\}^2} \\ &= \int_0^1 dx \int \frac{d^4k}{(2\pi)^4} \frac{1}{[(k-xp)^2 - M^2]^2} \end{aligned} \tag{2.12}$$

with $M^2 = (1-x)m_1^2 + xm_2^2 - x(1-x)p^2$.

By making Wick rotation and applying the loop regularization prescription before shifting the integration variable, i.e., rewriting the momentum factor $(k-xp)^2$ into $(k-xp)^2 = k^2 - 2xp.k + x^2p^2$, then replacing k^2 by $k^2 + M_l^2$, namely

$$\begin{aligned} (k-xp)^2 &= k^2 - 2xp.k + x^2p^2 \\ &\to k^2 + M_l^2 - 2xp.k + x^2p^2 \\ &= (k-xp)^2 + M_l^2 \end{aligned} \tag{2.13}$$

we then obtain the regularized Feynman integral

$$L^R = i \lim_{N, M_l^2} \sum_{l=0}^{N} c_l^N \int_0^1 dx \int \frac{d^4k}{(2\pi)^4} \times \frac{1}{[(k-xp)^2 + M^2 + M_l^2]^2}, \tag{2.14}$$

which becomes a well-defined integral, so that we can safely shift the integration variable:

$$L^R = \int_0^1 dx \lim_{N, M_l^2} \sum_{l=0}^{N} c_l^N \int \frac{d^4k}{(2\pi)^4} \frac{1}{(k^2 + M^2 + M_l^2)^2}. \tag{2.15}$$

The same result can be arrived by using the standard procedure of loop regularization with first shifting the integration variable for momentum, which yields the standard scalar-type ILI

$$L_0 = \int_0^1 dx \int \frac{d^4k}{(2\pi)^4} \frac{1}{(k^2 - M^2)^2} = \int_0^1 dx I_0 \tag{2.16}$$

after applying the loop regularization prescription; the same form is reached

$$\begin{aligned} L_0^R &= i \int_0^1 dx \lim_{N, M_l^2} \sum_{l=0}^{N} c_l^N \int \frac{d^4k}{(2\pi)^4} \frac{1}{(k^2 + M^2 + M_l^2)^2} \\ &\equiv L^R, \end{aligned} \tag{2.17}$$

which shows that in the loop regularization method, one can safely shift the integration variables and express all the Feynman integrals in terms of ILIs before applying for the regularization prescription.

From the above explicit demonstration, it is seen that the loop regularization is indeed translational invariant. In fact, this property also allows us to eliminate the ambiguities and make a consistent calculation for the chiral anomaly even in the existence of a linear divergent integral [13,14]. Similar verification of translational invariance can be extended to the linearly and quadratically divergent integrals, which is presented in Appendix A.

The above proof can in general be extended to higher loops based on several theorems proved in Ref. [11], especially based on the theorem I, theorem V, and theorem VI over there. The theorem I is the so-called factorization theorem for overlapping divergences which states that overlapping divergences which contain divergences of subintegrals and overall divergences in the general Feynman loop integrals become completely factorizable in the corresponding ILIs. The theorem V is the so-called reduction theorem for overlapping tensor-type integrals which states that the general overlapping tensor-type Feynman integrals of arbitrary loop graphs are eventually characterized by the overall onefold tensor-type ILIs of the corresponding loop graphs. This theorem is the key theorem for the generalization of treatments and also for the prescriptions from one-loop graphs to arbitrary loop graphs. The theorem VI which is the so-called relation theorem for tensor and scalar-type ILIs which states that for any fold tensor and scalar-type ILIs, as long as their power counting dimension of the integrating loop momentum are the same, the relations between the tensor and scalar-type ILIs are also the same and independent of the fold number of ILIs. This theorem is crucial to extend the consistency conditions of gauge invariance from divergent one-loop ILIs to higher-loop ILIs.

III. WARD IDENTITY IN THE MASSLESS WESS-ZUMINO MODEL

We begin with the massless Wess-Zumino theory which is the simplest supersymmetric model. The Lagrangian is

$$L = -\tfrac{1}{2}(\partial_\mu A)^2 - \tfrac{1}{2}(\partial_\mu B)^2 - \tfrac{1}{2}\bar{\chi}\not{\partial}\chi + \tfrac{1}{2}F^2 + \tfrac{1}{2}G^2 \quad (3.1)$$

$$+ g[-F(A^2 - B^2) + 2GAB + \bar{\chi}(A + i\gamma_5 B)\chi], \quad (3.2)$$

the action is invariant, up to a total derivative, under the global supersymmetric transformation shown below:

$$\begin{aligned} &\delta A = \bar{\epsilon}\chi, \qquad \delta B = -i\bar{\epsilon}\gamma_5\chi, \\ &\delta\chi = -\bar{\epsilon}\not{\partial}(A + i\gamma_5 B) + \bar{\epsilon}(F + i\gamma_5 G), \\ &\delta F = \bar{\epsilon}\not{\partial}\chi, \qquad \delta G = -i\bar{\epsilon}\gamma_5\not{\partial}\chi. \end{aligned} \quad (3.3)$$

Using a functional technique, one can deduce that the one-particle irreducible (1PI) Green functions generating functional Γ is invariant under the supersymmetric transformation [18]. The supersymmetric Ward identity we choose to check is involving two-point irreducible functions:

$$\frac{\delta^2\Gamma}{\delta A(x)\delta A(y)}\delta_{\gamma\alpha} - (\not{\partial}_y)_{\gamma\beta}\frac{\delta^2\Gamma}{\delta\chi_\alpha(x)\delta\bar{\chi}_\beta(y)} = 0. \quad (3.4)$$

This could be obtained from differentiating the equation $\delta\Gamma = 0$ by $A(x)$ and $\bar{\chi}(x)$ [19]. In the momentum space, we can write it as

$$\Gamma_{AA}(p)\delta_{\gamma\alpha} - i(\not{p})_{\gamma\beta}\Gamma_{\chi_\alpha\bar{\chi}_\beta}(p) = 0 \quad (3.5)$$

at one-loop level. Feynman diagrams that contribute to this identity are shown in Fig. 1. In

$$\begin{aligned} \Gamma^{(a)}_{\chi_\alpha\bar{\chi}_\beta}(p) &= 2\times 4g^2\int\frac{d^4k}{(2\pi)^4}\frac{-\gamma_\mu k^\mu}{k^2}\frac{-i}{(k-p)^2} \\ &= -i8g^2\int_0^1 dx\int\frac{d^4l}{(2\pi)^4}\frac{x\gamma_\mu p^\mu}{[l^2 - x(x-1)p^2]^2} \\ &= -i8g^2\int_0^1 dx x\gamma_\mu p^\mu I_0(x(x-1)p^2), \end{aligned} \quad (3.6)$$

the factor 2 appears because the wave line could be A or B, and the factor 4 results from the fact that the fermion is a Majorana particle. We could discern this result more clearly from the Majorana Feynman rules given in the appendix. According to the Feynman rules we should calculate $\langle\chi\chi\rangle$ first, and then obtain the $\langle\chi\bar{\chi}\rangle$ from the relation below:

$$\langle\chi_i\bar{\chi}_j\rangle = \langle\chi_i(C^{-1}\chi)^T_j\rangle = \langle\chi_i\chi_k\rangle(-C^{-1}_{kj}). \quad (3.7)$$

The calculation of Γ_{AA} is straightforward:

$$\begin{aligned} \Gamma^{(b)}_{AA}(p) &= -2g^2\int\frac{d^4k}{(2\pi)^4}\mathrm{Tr}\left[\frac{\gamma_\mu k^\mu}{k^2}\frac{\gamma_\nu(k^\nu - p^\nu)}{(k-p)^2}\right] \\ &= 8g^2\int\frac{d^4l}{(2\pi)^4}\left(\int_0^1 dx\frac{xp^2}{[l^2 - x(x-1)p^2]^2} - \frac{1}{l^2}\right) \\ &= 8g^2\left(\int_0^1 dx x p^2 I_0(x(x-1)p^2) - I_2(0)\right), \end{aligned} \quad (3.8)$$

$$\Gamma^{(c)}_{AA}(p) = 2\times 4g^2\int\frac{d^4l}{(2\pi)^4}\frac{1}{l^2} = 8g^2 I_2(0). \quad (3.9)$$

We can see immediately that the Ward identity (3.5) is satisfied because the integrands cancel out. To arrive at the above results we have only carried out Dirac algebra for γ matrices in the four-dimensional space-time and have made the shift of the integration variables. As these operations are all rational in a four-dimensional well-defined loop regularization method, we conclude that at the one-loop level the LR method indeed preserves the supersymmetric Ward identity in this simple model.

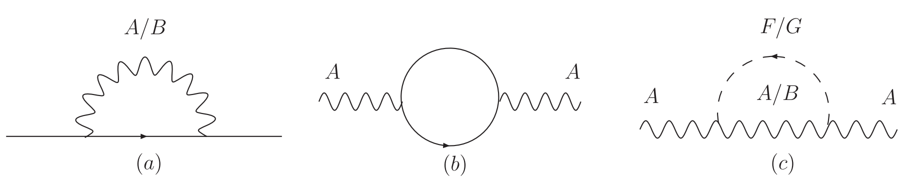

FIG. 1. Three diagrams contribute at the one-loop level.

IV. WARD IDENTITY IN THE MASSIVE WESS-ZUMINO MODEL

We are examining another supersymmetric model. The procedure is similar to what we have done in the above massless model. The Lagrangian of the massive Wess-Zumino model is

$$L = -\tfrac{1}{2}(\partial_\mu A)^2 - \tfrac{1}{2}(\partial_\mu B)^2 - \tfrac{1}{2}\bar{\chi}\not\partial\chi + \tfrac{1}{2}F^2 + \tfrac{1}{2}G^2 + m(AF - BG - \tfrac{1}{2}\bar{\chi}\chi) + g[-F(A^2 - B^2) + 2GAB + \bar{\chi}(A + i\gamma_5 B)\chi]. \tag{4.1}$$

It is different from the massless case with the mass term $m(AF - BG - \frac{1}{2}\bar{\chi}\chi)$. In this model bosons and fermions have equal masses as demanded by supersymmetry. In Sec. VI, we will explicitly show that the radiative corrections do not violate such an equality. The supersymmetric transformation of component fields are the same as Eq. (3.3). Following the same procedure, the two-point Ward identity of this model is extended to be [20]

$$\Gamma_{AA}(p)\delta_{\gamma\alpha} - i(\not p)_{\gamma\beta}\Gamma_{\chi_\alpha\bar{\chi}_\beta}(p) + i(\not p)_{\gamma\alpha}\Gamma_{AF}(p) = 0. \tag{4.2}$$

At the one-loop level, the diagrams which contribute to this supersymmetric Ward identity are shown in Figs. 2–4.

It is easy to show that two diagrams in Fig. 4 contribute to Γ_{AF} and their contributions cancel each other:

$$\Gamma_{AF} = 4g^2 \int \frac{d^4k}{(2\pi)^4}\left[\frac{1}{k^2 - m^2}\frac{m}{(k-p)^2 - m^2} + \frac{1}{k^2 - m^2}\frac{-m}{(k-p)^2 - m^2}\right] = 0. \tag{4.3}$$

The calculations of other diagrams are straightforward. We present the final results as follows:

$$\begin{aligned}\Gamma^{(a)}_{\chi_\alpha\bar{\chi}_\beta}(p) &= 4g^2 \int \frac{d^4k}{(2\pi)^4}\left[\frac{-i\gamma_\mu k^\mu + m}{k^2 - m^2}\frac{1}{(k-p)^2 - m^2} + (i\gamma_5)\frac{-i\gamma_\mu k^\mu + m}{k^2 - m^2}(i\gamma_5)\frac{1}{(k-p)^2 - m^2}\right] \\ &= 8g^2 \int \frac{d^4l}{(2\pi)^4}\int_0^1 dx \frac{-ix\gamma_\mu p^\mu}{[l^2 - x(x-1)p^2 - m^2]^2} \\ &= 8g^2 \int_0^1 dx(-ix\gamma_\mu p^\mu) I_0(x(x-1)p^2 + m^2),\end{aligned} \tag{4.4}$$

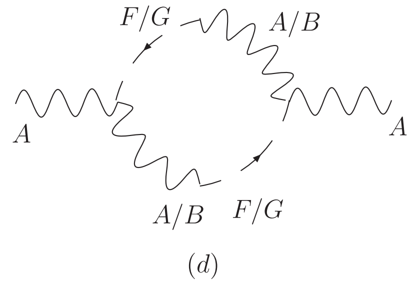

FIG. 3. Additional diagram contributing to Γ_{AA}.

$$\begin{aligned}\Gamma^{(b)}_{AA}(p) &= -2g^2 \int \frac{d^4k}{(2\pi)^4}\mathrm{Tr}\left[\frac{-i\gamma_\mu k^\mu + m}{k^2 - m^2} \times \frac{-i\gamma_v(k^\nu - p^\nu) + m}{(k-p)^2 - m^2}\right] \\ &= 8g^2 \int \frac{d^4k}{(2\pi)^4}\int_0^1 dx\left[\frac{1}{[l^2 - x(x-1)p^2 - m^2]} - \frac{2[m^2 + x(1-x)p^2]}{[l^2 - x(x-1)p^2 - m^2]^2}\right] \\ &= 8g^2 \int_o^1 dx[I_2(x(x-1)p^2 + m^2) - 2[m^2 + x(1-x)p^2]I_0(x(x-1)p^2 + m^2)],\end{aligned} \tag{4.5}$$

$$\begin{aligned}\Gamma^{(c)}_{AA}(p) &= 2 \times 4g^2 \int \frac{d^4k}{(2\pi)^4}\frac{-k^2}{k^2 - m^2}\frac{1}{(k-p)^2 - m^2} \\ &= 8g^2 \int \frac{d^4l}{(2\pi)^4}\int_0^1 dx\left[\frac{-1}{[l^2 - x(x-1)p^2 - m^2]} + \frac{m^2 + x(1-2x)p^2}{[l^2 - x(x-1)p^2 - m^2]^2}\right] \\ &= 8g^2 \int_o^1 dx[-I_2(x(x-1)p^2 + m^2) + [m^2 + x(1-2x)p^2]I_0(x(x-1)p^2 + m^2)],\end{aligned} \tag{4.6}$$

$$\begin{aligned}\Gamma^{(d)}_{AA}(p) &= 2 \times 4g^2 \int \frac{d^4k}{(2\pi)^4}\frac{m}{k^2 - m^2}\frac{m}{(k-p)^2 - m^2} \\ &= 8g^2 \int \frac{d^4l}{(2\pi)^4}\int_0^1 dx \frac{m^2}{[l^2 - x(x-1)p^2 - m^2]^2} \\ &= 8g^2 \int_0^1 m^2 I_0(x(x-1)p^2 + m^2).\end{aligned} \tag{4.7}$$

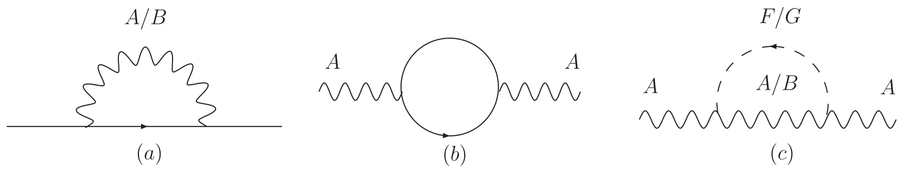

FIG. 2. The same as the massless case.

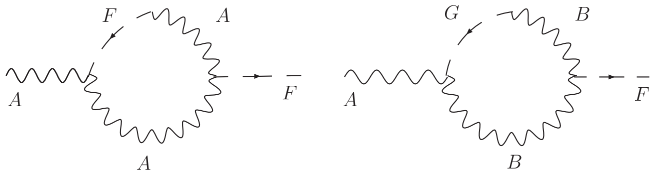

FIG. 4. Two diagrams contributing to Γ_{AF}.

Adding all the contributions together, we can see that the integrands cancel out and the supersymmetric Ward identity holds. Again, to arrive at the above results we have only performed Dirac algebra for γ matrices in the four-dimensional space-time and make the shift of the integration variables. It further shows that in the massive Wess-Zumino model the LR method can preserve supersymmetry as well.

V. WARD IDENTITY IN SUPERSYMMETRIC GAUGE THEORY

Let us consider a more complicated case, i.e., the supersymmetric Yang-Mills theory. This model involves supersymmetry as well as gauge symmetry. In the Wess-Zumino gauge, the Lagrangian (with source terms) can be written as

$$L = -\tfrac{1}{4}(F^a_{\mu\nu})^2 - \tfrac{1}{2}(\partial^\mu A^a_\mu) + C^{*a}\partial \not{D}^{ab} C^b - \tfrac{1}{2}\bar{\lambda}^a \not{D}^{ab}\lambda^b + \tfrac{1}{2}D_a^2 + J^{a\mu}A^a_\mu + \bar{J}^a\lambda^a + j^a_D D^a, \tag{5.1}$$

where λ^a is a Majorana spinor and D^a is the auxiliary field. Similarly, the supersymmetric Ward identity is derived by considering the functional variation of the Green function generating the functional under an infinitesimal supersymmetric transformation. All the fields transform as follows:

$$\delta A^a_\mu = -\bar{\epsilon}\gamma_\mu\lambda^a, \qquad \delta\lambda^a = \sigma^{\mu\nu}F^a_{\mu\nu}\epsilon + i\gamma_5 D^a\epsilon, \qquad \delta D^a = \bar{\epsilon} i\gamma_5 \not{D}^{ab}\lambda^b, \tag{5.2}$$

which leads to the following supersymmetric Ward identity [6,21]:

$$\begin{aligned}
0 = {} & \frac{\delta J^{c'}_{\rho'}(z')}{\delta \hat{A}^c_\rho(z)}\frac{\delta \bar{J}^{b'}(y')}{\delta \hat{\bar{\lambda}}^b(y)}\langle \delta^{c'a}_{\rho'\mu}\delta^4(z'-x)(-\bar{\epsilon}\gamma_\mu\lambda^a(x))i\lambda^{b'}(y')\rangle + \frac{\delta J^{c'}_{\rho'}(z')}{\delta \hat{A}^c_\rho(z)}\frac{\delta \bar{J}^{b'}(y')}{\delta \hat{\bar{\lambda}}^b(y)}\langle \delta^{b'a}\delta^4(y'-x)iA^{c'}_{\rho'}(z')\sigma^{\mu\nu}F^a_{\mu\nu}(x)\epsilon\rangle \\
& + \frac{\delta J^{c'}_{\rho'}(z')}{\delta \hat{A}^c_\rho(z)}\frac{\delta \bar{J}^{b'}(y')}{\delta \hat{\bar{\lambda}}^b(y)}\langle \partial\cdot A^a(\mathrm{x})\bar{\epsilon}\not{\partial}\lambda^a(x)i\lambda^{b'}(y')iA^{c'}_{\rho'}(z')\rangle + \frac{\delta J^{c'}_{\rho'}(z')}{\delta \hat{A}^c_\rho(z)}\frac{\delta \bar{J}^{b'}(y')}{\delta \hat{\bar{\lambda}}^b(y)}\langle i\lambda^{b'}(y')iA^{c'}_{\rho'}(z')(\partial_\mu C^{*a})f^{aef}\bar{\epsilon}\gamma_\mu\lambda^e(\mathrm{x})C^f(x)\rangle \\
& + \frac{\delta^2 \bar{J}^{b'}(y')}{\delta \hat{A}^c_\rho(z)\delta \hat{\bar{\lambda}}^b(y)}\langle \partial\cdot A^a(\mathrm{x})\bar{\epsilon}\not{\partial}\lambda^a(x)i\lambda^{b'}(y')\rangle + \frac{\delta^2 \bar{J}^{b'}(y')}{\delta \hat{A}^c_\rho(z)\delta \hat{\bar{\lambda}}^b(y)}\langle i\lambda^{b'}(y')(\partial_\mu C^{*a})f^{aef}\bar{\epsilon}\gamma_\mu\lambda^e(\mathrm{x})C^f(x)\rangle,
\end{aligned} \tag{5.3}$$

where the notation $\langle ...\rangle$ represents connected Green functions and the integrations over x, y', z' are abbreviated. At the tree level, only the 1st, 2nd, and 3rd terms in the above equation contribute. One can easily verify that the identity holds. At the one-loop level, only the 1st, 2nd, 3rd, and 4th terms contribute; all the diagrams to this order are shown in Figs. 5–7.

We would like to point out that the first line of Eq. (5.3) is exactly the self-energy function of the gauge boson at the one-loop level, as can be seen from the relation below:

$$\frac{\delta J^{c'}_{\rho'}(z')}{\delta \hat{A}^c_\rho(z)} = \Gamma_{A^{c'}_{\rho'}A^c_\rho}(z-z'). \tag{5.4}$$

Gauge symmetry requires this term to be transverse.

We now turn to the calculation of each term in the Ward identity, and choose $\xi = 1$ for simplicity:

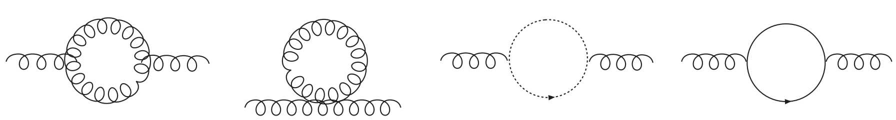

FIG. 5. The 1st term.

$$\Pi_{\mu\nu}^{(1)} = -\frac{1}{2}g^2 C_{acd}C_{bdc}\int\frac{d^4q}{(2\pi)^4}\frac{1}{q^2(q+p)^2}[10q_\mu q_\nu + 5(p_\mu q_\nu + p_\nu q_\mu) - 2p_\mu p_\nu + (5p^2 + 2p\cdot q + 2q^2)g_{\mu\nu}]$$
$$= -\frac{1}{2}g^2 C_{acd}C_{bdc}\int dx\int\frac{d^4l}{(2\pi)^4}\frac{[10l_\mu l_\nu + 10x^2 p_\mu p_\nu - 7p_\mu p_\nu + 4p^2 g_{\mu\nu} + 2(l^2 + x^2p^2)g_{\mu\nu}]}{[l^2 - x(x-1)p^2]^2}$$
$$= -\frac{1}{2}g^2 C_{acd}C_{bdc}\int dx[(10I_{2\mu\nu} + 2g_{\mu\nu}I_2) + (10x^2 p_\mu p_\nu - 7p_\mu p_\nu + 4p^2 g_{\mu\nu} + 2x(2x-1)p^2 g_{\mu\nu})I_0], \quad (5.5)$$

$$\Pi_{\mu\nu}^{(2)} = g^2 C_{acd}C_{bdc}\int\frac{d^4q}{(2\pi)^4}\frac{3g_{\mu\nu}}{q^2}$$
$$= g^2 C_{acd}C_{bdc}\int dx\int\frac{d^4l}{(2\pi)^4}\frac{3g_{\mu\nu}(l^2 + x^2p^2)}{[l^2 - x(x-1)p^2]^2}$$
$$= g^2 C_{acd}C_{bdc}\int dx[3g_{\mu\nu}I_2 + 3x(x-1)p^2 g_{\mu\nu}I_0], \quad (5.6)$$

$$\Pi_{\mu\nu}^{(3)} = g^2 C_{acd}C_{bdc}\int\frac{d^4q}{(2\pi)^4}\frac{p_\mu q_\nu + q_\mu q_\nu}{q^2(q+p)^2}$$
$$= g^2 C_{acd}C_{bdc}\int dx\int\frac{d^4l}{(2\pi)^4}\frac{1}{[l^2 - x(x-1)p^2]^2}\left[-\frac{1}{2}p_\mu p_\nu + l_\mu l_\nu + x^2 p_\mu p_\nu\right]$$
$$= g^2 C_{acd}C_{bdc}\int dx\left[I_{2\mu\nu} + \left(x^2 - \frac{1}{2}\right)p_\mu p_\nu I_0\right], \quad (5.7)$$

$$\Pi_{\mu\nu}^{(1+2+3)} = -g^2 C_{acd}C_{bcd}\int dx[(4x^2 - 3)(p^2 g_{\mu\nu} - p_\mu p_\nu)I_0 - 4I_{2\mu\nu} + 2g_{\mu\nu}I_2]. \quad (5.8)$$

Notice that in supersymmetric Yang-Mills theory, the fermions are massless and belong to the adjoint representation of the gauge group as required by the fermion-boson symmetry. Then,

$$\Pi_{\mu\nu}^{(4)} = -g^2 4\,\mathrm{tr}[T_a T_b]\int\frac{d^4q}{(2\pi)^4}\frac{(p+q)_\mu q_\nu + (p+q)_\nu q_\mu - (q^2 + q\cdot p)g_{\mu\nu}}{q^2(q+p)^2}$$
$$= g^2 C_{acd}C_{bcd}\int dx\int\frac{d^4l}{(2\pi)^4}\left[\frac{(4x^2-2)(p^2 g_{\mu\nu} - p_\mu p_\nu) - 4l_\mu l_\nu}{[l^2 - x(x-1)p^2]^2} + \frac{2g_{\mu\nu}}{[l^2 - x(x-1)p^2]}\right]$$
$$= g^2 C_{acd}C_{bcd}\int dx[(4x^2 - 2)(p^2 g_{\mu\nu} - p_\mu p_\nu)I_0 - 4I_{2\mu\nu} + 2g_{\mu\nu}I_2]. \quad (5.9)$$

Adding the four terms together, we obtain the self-energy of the gauge boson which is gauge covariant:

$$\Pi_{\mu\nu} = g^2 C_{acd}C_{bcd}(p^2 g_{\mu\nu} - p_\mu p_\nu)\int dx I_0. \quad (5.10)$$

It is seen that the transverse condition of $\Pi_{\mu\nu}$ is satisfied in the supersymmetric model with the Feynman $\xi = 1$ gauge.

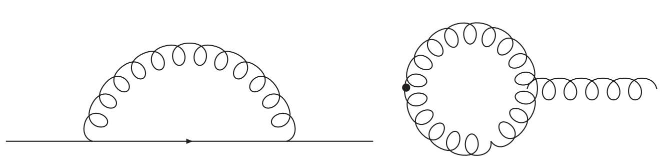

FIG. 6. The 2nd term.

The reason is that the quadratical divergences which will potentially break the transverse condition cancel out in the supersymmetric model. In fact, the cancellation of quadratical divergences is a general feature of supersymmetric field theories. It is also one of the motivations to propose supersymmetry. In other words, if one wants to break supersymmetry but still maintain the gauge symmetry, there are several ways to realize that; for instance, give a mass to the fermion. In this case, the quadratical divergences do not cancel out automatically and they may destroy the transverse condition unless they can be regularized via an appropriate regularization method to satisfy the consistency conditions [11]. As shown in [11,12] the LR method is competent in this case.

Note that here we have carried out the calculation in the Feynman gauge with $\xi = 1$ for simplicity. In the general ξ

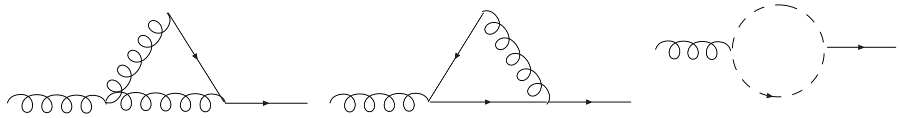

FIG. 7. The 3rd and 4th terms.

gauge, there is a term which could break the transverse condition if the regularization scheme does not satisfy the consistency condition for the logarithmic divergences. The term is in proportion to

$$(\xi - 1) * (a_0 - 1)$$

with a_0 being defined via the logarithmic divergent $I_{0\mu\nu} = \frac{1}{4} a_0 g_{\mu\nu} I_0$. In the Feynman gauge this term vanishes due to $\xi = 1$. In the general ξ gauge, it remains to require that the regularization scheme satisfies the consistency condition for the logarithmic divergent part, i.e., $a_0 = 1$, so that the transverse condition in the gauge boson self-energy can hold.

The fermion self-energy diagram is given by

$$\begin{aligned} 2\sigma^{\alpha\beta} p_\beta \Gamma^{(5)}_{\lambda\lambda} &= -2\sigma^{\alpha\beta} p_\beta \int \frac{d^4 q}{(2\pi)^4} C_{acd} C_{bcd} \times g^2 \gamma^\mu \\ &\quad \times \frac{i}{(\not q + \not p)} \gamma^\nu \frac{-i}{q^2} g_{\mu\nu} \\ &= g^2 C_{acd} C_{bcd} (p^2 \gamma^\alpha - \not p p^\alpha) \int \frac{d^4 l}{(2\pi)^4} \\ &\quad \times \int dx \frac{1}{[l^2 - x(x-1)p^2]^2} \\ &= g^2 C_{acd} C_{bcd} (p^2 \gamma^\alpha - \not p p^\alpha) \int dx I_0. \end{aligned} \tag{5.11}$$

There are two diagrams from the second term of Eq. (5.3). The nonlinear part of $F^a_{\mu\nu}$ (Fig. 6) gives rise to the contribution:

$$\begin{aligned} \not p \Pi^{(6)}_A(p) &= \not p g^2 C_{acd} C_{bcd} \sigma^{\lambda\nu} \int \frac{d^4 q}{(2\pi)^4} \frac{-i g_{\mu\nu}}{q^2} \frac{-i g_{\rho\lambda}}{(q+p)^2} [g^{\tau\rho}(p-q)^\mu + g^{\rho\mu}(2q+p)^\tau - g^{\mu\tau}(2p+q)^\rho] \\ &= \frac{3}{2} g^2 C_{acd} C_{bcd} (\not p p^\tau - p^2 \gamma^\tau) \int \frac{d^4 l}{(2\pi)^4} \int dx \frac{1}{[l^2 - x(x-1)p^2]^2} \\ &= \frac{3}{2} g^2 C_{acd} C_{bcd} (\not p p^\tau - p^2 \gamma^\tau) \int dx I_0. \end{aligned} \tag{5.12}$$

To proceed, we consider the rest diagrams coming from the third and fourth terms of Eq. (5.3).

$$\begin{aligned} \Gamma^{(7)}_\nu &= \int \frac{d^4 q}{(2\pi)^4} [g C_{acd} \gamma^\rho] i(\not p + \not q) \frac{i}{(\not q + \not p)} \left[-i(p+q)^\lambda \frac{-i g_{\mu\lambda}}{(p+q)^2} \right] \\ &\quad \times [g^{\nu k}(q-p)^\mu - g^{k\mu}(2q+p)^\nu + g^{\mu\nu}(2p+q)^k](-i g C_{bdc}) \frac{-i g_{\rho k}}{q^2} \\ &= g^2 C_{acd} C_{bcd} \int dx \int \frac{d^4 l}{(2\pi)^4} \gamma_\mu \left[\frac{g^{\nu\mu}(l^2 + x^2 p^2) - g^{\nu\mu} p^2 - l^\mu l^\nu - x^2 p^\mu p^\nu + p^\mu p^\nu}{[l^2 - x(x-1)p^2]^2} \right] \\ &= g^2 C_{acd} C_{bcd} \int dx \gamma_\mu [g^{\mu\nu} I_2 - I_2^{\mu\nu} + (x(2x-1)p^2 - g^{\mu\nu} p^2 + (1-x^2) p^\mu p^\nu) I_0], \end{aligned} \tag{5.13}$$

$$\begin{aligned} \Gamma^{(8)}_\nu &= \int \frac{d^4 q}{(2\pi)^4} [g C_{dac} \gamma^\mu] \frac{i}{(\not q + \not p)} [g C_{bdc} \gamma^\nu] i \frac{i \not q}{\not q} \frac{-i g_{\mu\lambda}}{q^2} (i q^\lambda) \\ &= -g^2 C_{acd} C_{bcd} \int dx \int \frac{d^4 l}{(2\pi)^4} \gamma^\nu \left[\frac{l^2 + x^2 p^2 - \frac{1}{2} p^2}{[l^2 - x(x-1)p^2]^2} \right] \\ &= -g^2 C_{acd} C_{bcd} \int dx \gamma^\nu \left[I_2 + \left(x(2x-1)p^2 - \frac{1}{2} p^2 \right) I_0 \right], \end{aligned} \tag{5.14}$$

$$\Gamma_\nu^{(9)} = -g^2 C_{acd} C_{bcd} \int \frac{d^4 q}{(2\pi)^4} \frac{i q^\rho \gamma_\rho}{q^2} i(q+p)^\nu \frac{1}{(q+p)^2}$$
$$= g^2 C_{acd} C_{bcd} \int dx \int \frac{d^4 q}{(2\pi)^4} \gamma_\rho \left[\frac{l^\nu l^\rho + x^2 p^\nu p^\rho - \frac{1}{2} p^\nu p^\rho}{[l^2 - x(x-1)p^2]^2} \right]$$
$$= g^2 C_{acd} C_{bcd} \int dx \gamma_\rho \left[I_2^{\nu\rho} + \left(x^2 - \frac{1}{2} \right) p^\nu p^\rho I_0 \right]. \quad (5.15)$$

The total contributions of the three diagrams are found to be

$$-\frac{1}{2} g^2 C_{acd} C_{bcd} (p^2 \gamma^\nu - \not{p} p^\nu) \int dx I_0. \quad (5.16)$$

After taking into account the "i" factors from the formula and adding all the terms together, the integrands cancel out again, which demonstrates that the supersymmetric Ward identity does hold. To arrive at this conclusion, we have only used the properties of four-dimensional γ matrices and translational invariance of momentum integrals. This implies that the LR method can indeed preserve supersymmetry. The gauge symmetry holds only requiring the consistency condition for the logarithmic divergent part due to the cancellation of quadratical divergences in the supersymmetry-preserving regularization method. In general, to preserve gauge symmetry in nonsupersymmetric models, the consistency conditions are needed for both quadratic and logarithmic divergences for the regularized ILIs. So far, we can conclude that the LR method preserves not only non-Abelian gauge symmetry, but also supersymmetry.

VI. RENORMALIZATION OF THE MASSIVE WESS-ZUMINO MODEL

In the previous sections we have shown that the LR method can respect the supersymmetric Ward identities in several models including supersymmetric gauge theory, which implies that the LR method is viable in supersymmetric theories. While in the above applications, we have only used the main features of the LR method, namely, the LR method is realized in four dimensions with translational invariance of momentum. In this section we shall apply the LR method to manifestly perform one-loop renormalization for the massive Wess-Zumino model. We choose such a model as a testing ground because it is fairly simple and well known. The model was shown to be renormalizable to all orders in perturbation theory [18] by using higher derivative regularization. The same conclusion can easily be obtained in the superspace formalism, where supergraph Feynman rules of superfields greatly simplify the calculations. For our purpose, we will use the component fields formalism to renormalize the theory. This is because the superspace formalism maintains supersymmetry in a manifest way, which is not suitable for checking the consistency of a specific regularization scheme in preserving supersymmetry. On the other hand, for the physically interesting case of broken supersymmetry, it is usually preferred to work with component fields.

The action of the massive Wess-Zumino model is

$$S_{\rm WZ} = \frac{1}{4} \int d^4 x d^2 \Theta \left(\frac{1}{8} \Phi \bar{D} \Phi - \frac{1}{2} m \Phi^2 - \frac{1}{3} g \Phi^3 \right) + \text{H.c.}, \quad (6.1)$$

where $\Phi(x, \Theta, \bar{\Theta})$ is a chiral superfield. In terms of component fields the Lagrangian can be written as

$$L = \tfrac{1}{2}(\partial_\mu A \partial^\mu A + \partial_\mu B \partial^\mu B + i \bar{\chi} \not{\partial} \chi + F^2 + G^2) - m(AF + BG + \tfrac{1}{2} \bar{\chi} \chi) - g[(A^2 - B^2)F + 2ABG + \bar{\chi}(A - i\gamma_5 B)\chi]. \quad (6.2)$$

The notions used here are slightly different from those in Sec. IV. It is seen that the fields F and G have no dynamical terms. They are auxiliary fields and can be integrated out, which is equivalent to eliminating them from the Lagrangian by using the equations of motion. In fact, in the building of the phenomenological supersymmetric model the auxiliary fields are eliminated by

$$F = mA + g(A^2 - B^2), \quad (6.3)$$

$$G = mB + 2gAB. \quad (6.4)$$

Thus the Lagrangian can be written as

$$L = \tfrac{1}{2}(\partial_\mu A \partial^\mu A - m^2 A^2) + \tfrac{1}{2}(\partial_\mu B \partial^\mu B - m^2 B^2) + \tfrac{1}{2} \bar{\chi}(i \not{\partial} - m)\chi - mgA(A^2 + B^2) - g \bar{\chi}(A - i\gamma_5 B)\chi - \tfrac{1}{2} g^2 (A^2 + B^2)^2, \quad (6.5)$$

which is the Lagrangian to be renormalized by using the LR method. The Lagrangian contains one scalar particle A, one pesudoscalar particle B, and one Majorana fermion χ with equal masses m.

Before proceeding, we will first check what supersymmetry can tell us about the renormalization of the massive Wess-Zumino model. The answer can easily be yielded in the superfield formalism based on the powerful supergraph technique. In the superfield formalism, the nonrenormalization theorem implies that up to any order of the perturbative series only the first term (dynamical term) in Eq. (6.1) needs a counterterm due to the supersymmetry. Namely, after renormalization the action gets the following form:

$$S_{\rm WZ} = \frac{1}{4}\int d^4x d^2\Theta\left(\frac{1}{8}\Phi\bar{D}\Phi - \frac{1}{2}m\Phi^2 - \frac{1}{3}g\Phi^3 + \frac{1}{8}\delta\Phi\bar{D}\Phi\right) + \text{H.c.}$$
$$= \frac{1}{4}\int d^4x d^2\Theta\left(\frac{1}{8}Z\Phi\bar{D}\Phi - \frac{1}{2}\frac{m}{Z}Z\Phi^2 - \frac{1}{3}\frac{g}{Z^{3/2}}Z^{3/2}\Phi^3\right) + \text{H.c.}, \qquad (6.6)$$

where the δ term with $\delta = Z - 1$ is a logarithmically divergent counterterm, and $Z^{1/2}$ is the renormalization constant of the superfield. In terms of component fields, the equations of motion for F and G fields now become

$$F = \frac{1}{Z}[mA + g(A^2 - B^2)], \qquad (6.7)$$

$$G = \frac{1}{Z}(mB + 2gAB). \qquad (6.8)$$

After eliminating the auxiliary fields, it then leads to the renormalized Lagrangian:

$$L = \frac{1}{2}Z\left(\partial_\mu A\partial^\mu A - \left(\frac{m}{Z}\right)^2 A^2\right) + \frac{1}{2}Z\left(\partial_\mu B\partial^\mu B - \left(\frac{m}{Z}\right)^2 B^2\right) + \frac{1}{2}Z\bar{\chi}\left(i\not\partial - \frac{m}{Z}\right)\chi - \frac{m}{Z}\frac{g}{Z^{3/2}}Z^{3/2}A(A^2 + B^2) - \frac{g}{Z^{3/2}}Z^{3/2}\bar{\chi}(A - i\gamma_5 B)\chi - \frac{1}{2}\left(\frac{g}{Z^{3/2}}\right)^2 Z^2(A^2 + B^2)^2, \qquad (6.9)$$

which shows that the renormalizations of fields, mass, and coupling constant must satisfy

$$\phi_{\rm bare} = Z^{1/2}\phi; \qquad m_{\rm bare} = Z^{-1}m; \qquad g_{\rm bare} = Z^{-3/2}g, \qquad (6.10)$$

with $\phi = A, B, \chi$. We may summarize the features of the model: (i) This model is renormalizable, and after renormalization all the vertices remain to be only one coupling constant. (ii) The fields, mass, and coupling constant share a common renormalization constant, which only contains logarithmical divergence. The cancellation of quadratical divergence is a general feature of all supersymmetric theories. (iii) As required by supersymmetry, the masses of bosons still equal the mass of fermion after renormalization.

Let us now make a detailed calculation for one-loop renormalization by using the LR method. The Feynman rules of the Lagrangian (Eq. (6.5)) are listed in the appendix. There are seven types of vertices. What we are going to demonstrate is that after renormalization all these seven types of vertices, we will get the same renormalized coupling constant, and all the renormalization constants satisfy Eq. (6.10). It is easy to verify that one-loop contributions to $\langle A\rangle$, $\langle B\rangle$, $\langle AB\rangle$, $\langle AAB\rangle$, $\langle BBB\rangle$, $\langle AAAB\rangle$, $\langle ABBB\rangle$ are vanishing. The rest of the divergent diagrams at the one-loop level are shown in Fig. 8; the permutation graphs are not presented for simplicity.

The field strength and mass renormalizations of field A can be obtained from the calculations of the two-point Green function $\langle AA\rangle$. Five diagrams can contribute to $\langle AA\rangle$, the total contribution is found to be

$$L_{\langle AA\rangle} = \frac{1}{2}\int\frac{d^4k}{(2\pi)^4}[(-6img)^2 + (-2img)^2]\frac{i}{k^2 - m^2}\times\frac{1}{(k+p)^2 - m^2} + \frac{1}{2}\int\frac{d^4k}{(2\pi)^4}(-12ig^2 - 4ig^2)\times\frac{i}{k^2 - m^2} - \frac{1}{2}\,\text{tr}\int\frac{d^4k}{(2\pi)^4}(2igC^\dagger)\frac{i}{\not k - m}\times C^T(2igC^\dagger)\frac{i}{(\not k + \not p) - m}C^T$$
$$= 4g^2\int_0^1\int\frac{d^4k}{(2\pi)^4}\frac{2(1-x)p^2 + m^2}{[k^2 - m^2 - x(x-1)p^2]^2}$$
$$= 4g^2\int_0^1\int\frac{d^4k}{(2\pi)^4}[2(1-x)p^2 + m^2]I_0, \qquad (6.11)$$

which is only logarithmic divergent as the quadratical divergences cancel out. Using the loop regularization, the regularized I_0 has the following explicit form:

$$I_0^R = \frac{i}{16\pi^2}\left[\ln\frac{M_c^2}{\mu^2} - \gamma_\omega + y_0\left(\frac{\mu^2}{M_c^2}\right)\right]. \qquad (6.12)$$

We shall adopt a subtraction scheme similar to the minimal subtraction scheme in dimensional regularization. For that, it is useful to introduce an arbitrary energy scale parameter μ_s and write I_0^R as

$$I_0^R = \frac{i}{16\pi^2}\ln\frac{M_c^2}{\mu_s^2} + \frac{i}{16\pi^2}\left[\ln\frac{\mu_s^2}{\mu^2} - \gamma_\omega + y_0\left(\frac{\mu^2}{M_c^2}\right)\right], \qquad (6.13)$$

then the divergent terms proportional to $\frac{i}{16\pi^2}\ln\frac{M_c^2}{\mu_s^2}$ for $M_c \to \infty$ in the Feynman integral are canceled by counterterms. As such a divergent term is independent of the Feynman parameters x, we can integrate x easily and obtain the divergent part of these diagrams:

$$L_{\langle AA\rangle;\rm div} = \frac{i}{4\pi^2}g^2(p^2 + m^2)\ln\frac{M_c^2}{\mu_s^2}. \qquad (6.14)$$

The counterterms corresponding to this divergence are

$$\delta L = \tfrac{1}{2}\delta_A(\partial_\mu A\partial^\mu A) - \tfrac{1}{2}\delta_{m_A}m^2A^2, \qquad (6.15)$$

where

$$\delta_A = -\frac{1}{4\pi^2}g^2\ln\frac{M_c^2}{\mu_s^2}; \qquad \delta_{m_A} = \frac{1}{4\pi^2}g^2\ln\frac{M_c^2}{\mu_s^2}. \qquad (6.16)$$

From this we finally get

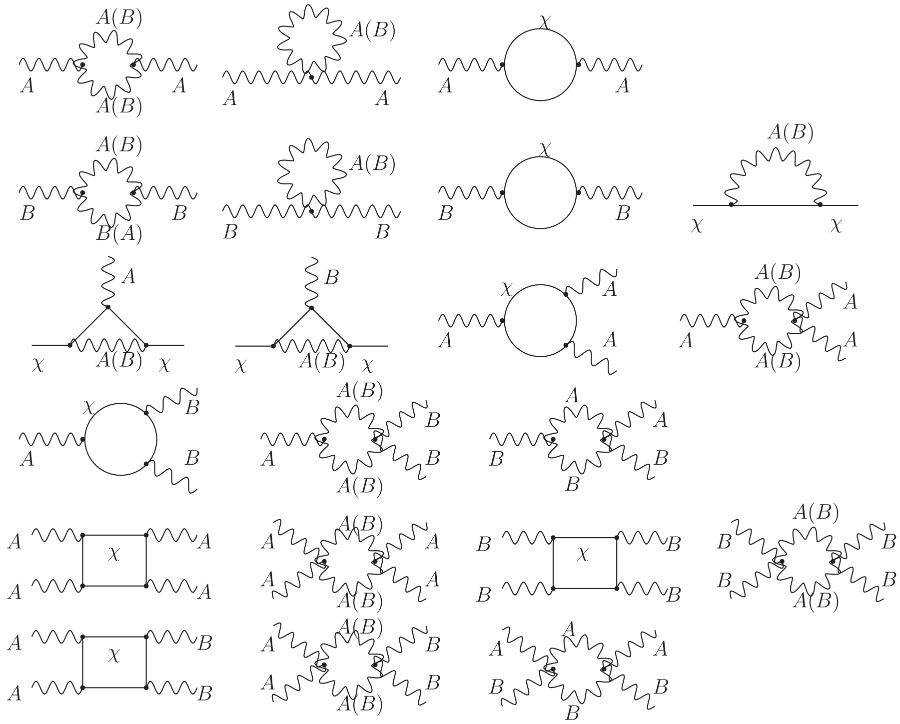

FIG. 8. The nonvanishing one-loop divergent graphs in the massive Wess-Zumino model.

$$A_{\rm bare} = \left(1 - \frac{1}{8\pi^2} g^2 \ln\frac{M_c^2}{\mu_s^2}\right) A = z^{1/2} A;$$
$$m_{A\rm bare} = \left(1 + \frac{1}{4\pi^2} g^2 \ln\frac{M_c^2}{\mu_s^2}\right) m = z^{-1} m, \tag{6.17}$$

where

$$z = 1 - \frac{1}{4\pi^2} g^2 \ln\frac{M_c^2}{\mu_s^2}. \tag{6.18}$$

The calculation for $\langle BB\rangle$ is similar, which gives

$$B_{\rm bare} = \left(1 - \frac{1}{8\pi^2} g^2 \ln\frac{M_c^2}{\mu_s^2}\right) B = z^{1/2} B;$$
$$m_{B\rm bare} = \left(1 + \frac{1}{4\pi^2} g^2 \ln\frac{M_c^2}{\mu_s^2}\right) m = z^{-1} m. \tag{6.19}$$

We now turn to the calculation of $\langle\chi\chi\rangle$. From Fig. 8 we can read directly

$$\begin{aligned} L_{\langle\chi\chi\rangle} &= \int \frac{d^4k}{(2\pi)^4} (2gC^\dagger i) \frac{i}{\not k - m} C^T (2gC^\dagger i) \\ &\quad \times \frac{i}{(k-p)^2 - m^2} + \int \frac{d^4k}{(2\pi)^4} (2gC^\dagger \gamma_5) \\ &\quad \times \frac{i}{\not k - m} C^T (2gC^\dagger \gamma_5) \frac{i}{(k-p)^2 - m^2} \\ &= \int_0^1 dx \int \frac{d^4k}{(2\pi)^4} \frac{-8xg^2 C^\dagger \not p}{(k^2 - m^2 - x(x-1)p^2)^2} \\ &= -\frac{i}{4\pi^2} g^2 C^\dagger \not p \ln\frac{M_c^2}{\mu_s^2} + \ldots. \end{aligned} \tag{6.20}$$

We need the following counterterms to cancel this divergence:

$$\delta L = \tfrac{1}{2}\delta_\chi \bar\chi i\not\partial \chi - \tfrac{1}{2}\delta_{m_\chi} m \bar\chi\chi, \tag{6.21}$$

where

$$\delta_\chi = -\frac{1}{4\pi^2} g^2 \ln\frac{M_c^2}{\mu_s^2}; \qquad \delta_{m\chi} = 0, \tag{6.22}$$

which indicates that the renormalization of field and mass is given by

$$\chi_{\rm bare} = \left(1 - \frac{1}{8\pi^2} g^2 \ln\frac{M_c^2}{\mu_s^2}\right) \chi = z^{1/2} \chi;$$
$$m_{\chi\rm bare} = \left(1 + \frac{1}{4\pi^2} g^2 \ln\frac{M_c^2}{\mu_s^2}\right) m = z^{-1} m. \tag{6.23}$$

So far, we have worked out the renormalization constants for the fields A, B, and χ and their masses. The results agree with Eq. (6.10). Let us switch to the renormalization of the coupling constant. As mentioned above, there are seven types of vertices which should be described by only one coupling constant when supersymmetry holds. The contributions from all divergent diagrams shown in Fig. 8 are found to be

$$\begin{aligned} L_{\langle AAA\rangle} &= -\mathrm{tr}\int\frac{d^4k}{(2\pi)^4}(2gC^\dagger i)\frac{i}{\not k - m}C^T(2gC^\dagger i)\frac{i}{(\not k - \not p_1) - m}C^T(2gC^\dagger i)\frac{i}{(\not k + \not p_2) - m}C^T \\ &\quad + \frac{1}{2}\int\frac{d^4k}{(2\pi)^4}(-6img)\frac{i}{k^2 - m^2}\frac{i}{(k+p_1)^2 - m^2}(-12ig^2) + (p_1 \to p_2) + (p_1 \to p_3) \\ &\quad + \frac{1}{2}\int\frac{d^4k}{(2\pi)^4}(-2img)\frac{i}{k^2 - m^2}\frac{i}{(k+p_1)^2 - m^2}(-4ig^2) + (p_1 \to p_2) + (p_1 \to p_3) \\ &= i\frac{3}{2\pi^2}mg^3\ln\frac{M_c^2}{\mu_s^2} + \text{finite terms}, \end{aligned} \tag{6.24}$$

$$\begin{aligned} L_{\langle ABB\rangle} &= -\mathrm{tr}\int\frac{d^4k}{(2\pi)^4}(2gC^\dagger i)\frac{i}{\not k - m}C^T(2gC^\dagger\gamma_5)\frac{i}{(\not k - \not p_1) - m}C^T(2gC^\dagger\gamma_5)\frac{i}{(\not k + \not p_2) - m}C^T \\ &\quad + \frac{1}{2}\int\frac{d^4k}{(2\pi)^4}(-6img)\frac{i}{k^2 - m^2}\frac{i}{(k+p_1)^2 - m^2}(-4ig^2) + \frac{1}{2}\int\frac{d^4k}{(2\pi)^4}(-2img)\frac{i}{k^2 - m^2}\frac{i}{(k+p_1)^2 - m^2}(-12ig^2) \\ &\quad + \frac{1}{2}\int\frac{d^4k}{(2\pi)^4}(-2img)\frac{i}{k^2 - m^2}\frac{i}{(k+p_2)^2 - m^2}(-4ig^2) + (p_2 \to p_3) \\ &= i\frac{1}{2\pi^2}mg^3\ln\frac{M_c^2}{\mu_s^2} + \text{finite terms}, \end{aligned} \tag{6.25}$$

$$\begin{aligned} L_{\langle A\chi\chi\rangle} &= \int\frac{d^4k}{(2\pi)^4}(2gC^\dagger i)\frac{i}{(\not k + \not p_2) - m}C^T(2gC^\dagger i)\frac{i}{(\not k + \not p_1) - m}C^T(2gC^\dagger i)\frac{i}{k^2 - m^2} \\ &\quad + \int\frac{d^4k}{(2\pi)^4}(2gC^\dagger\gamma_5)\frac{i}{(\not k + \not p_2) - m}C^T(2gC^\dagger i)\frac{i}{(\not k + \not p_1) - m}C^T(2gC^\dagger\gamma_5)\frac{i}{k^2 - m^2} \\ &= \text{finite terms}, \end{aligned} \tag{6.26}$$

$$\begin{aligned} L_{\langle B\chi\chi\rangle} &= \int\frac{d^4k}{(2\pi)^4}(2gC^\dagger i)\frac{i}{(\not k + \not p_2) - m}C^T(2gC^\dagger\gamma_5)\frac{i}{(\not k + \not p_1) - m}C^T(2gC^\dagger i)\frac{i}{k^2 - m^2} \\ &\quad + \int\frac{d^4k}{(2\pi)^4}(2gC^\dagger\gamma_5)\frac{i}{(\not k + \not p_2) - m}C^T(2gC^\dagger\gamma_5)\frac{i}{(\not k + \not p_1) - m}C^T(2gC^\dagger\gamma_5)\frac{i}{k^2 - m^2} \\ &= \text{finite terms}, \end{aligned} \tag{6.27}$$

$$\begin{aligned} L_{\langle AAAA\rangle} &= -\mathrm{tr}\int\frac{d^4k}{(2\pi)^4}(2gC^\dagger i)\frac{i}{\not k - m}C^T(2gC^\dagger i)\frac{i}{(\not k + \not p_2) - m}C^T(2gC^\dagger i)\frac{i}{(\not k + \not p_2 + \not p_3) - m}C^T(2gC^\dagger i) \\ &\quad \times\frac{i}{(\not k + \not p_2 + \not p_3 + \not p_4) - m}C^T + (p_2 \leftrightarrow p_3) + (p_3 \leftrightarrow p_4) + \frac{1}{2}(-12ig^2)^2\frac{i}{k^2 - m^2}\frac{i}{(k + p_1 + p_2)^2 - m^2} \\ &\quad + (p_2 \to p_3) + (p_2 \to p_4) + \frac{1}{2}(-4ig^2)^2\frac{i}{k^2 - m^2}\frac{i}{(k + p_1 + p_2)^2 - m^2} + (p_2 \to p_3) + (p_2 \to p_4) \\ &= i\frac{3}{\pi^2}g^4\ln\frac{M_c^2}{\mu_s^2} + \text{finite terms}, \end{aligned} \tag{6.28}$$

$$\begin{aligned} L_{\langle BBBB\rangle} &= -\mathrm{tr}\int\frac{d^4k}{(2\pi)^4}(2gC^\dagger\gamma_5)\frac{i}{\not k - m}C^T(2gC^\dagger\gamma_5)\frac{i}{(\not k + \not p_2) - m}C^T(2gC^\dagger\gamma_5)\frac{i}{(\not k + \not p_2 + \not p_3) - m}C^T(2gC^\dagger\gamma_5) \\ &\quad \times\frac{i}{(\not k + \not p_2 + \not p_3 + \not p_4) - m}C^T + (p_2 \leftrightarrow p_3) + (p_3 \leftrightarrow p_4) + \frac{1}{2}(-12ig^2)^2\frac{i}{k^2 - m^2}\frac{i}{(k + p_1 + p_2)^2 - m^2} \\ &\quad + (p_2 \to p_3) + (p_2 \to p_4) + \frac{1}{2}(-4ig^2)^2\frac{i}{k^2 - m^2}\frac{i}{(k + p_1 + p_2)^2 - m^2} + (p_2 \to p_3) + (p_2 \to p_4) \\ &= i\frac{3}{\pi^2}g^4\ln\frac{M_c^2}{\mu_s^2} + \text{finite terms}, \end{aligned} \tag{6.29}$$

$$\begin{aligned} L_{\langle AABB\rangle} = & -\mathrm{tr}\int \frac{d^4k}{(2\pi)^4}(2gC^\dagger i)\frac{i}{\not k - m}C^T(2gC^\dagger i)\frac{i}{(\not k + \not p_2) - m}C^T(2gC^\dagger\gamma_5)\frac{i}{(\not k + \not p_2 + \not p_3) - m}C^T(2gC^\dagger\gamma_5) \\ & \times \frac{i}{(\not k + \not p_2 + \not p_3 + \not p_4) - m}C^T - \mathrm{tr}\int\frac{d^4k}{(2\pi)^4}(2gC^\dagger i)\frac{i}{\not k - m}C^T(2gC^\dagger i)\frac{i}{(\not k + \not p_2) - m}C^T(2gC^\dagger\gamma_5) \\ & \times \frac{i}{(\not k + \not p_2 + \not p_4) - m}C^T(2gC^\dagger\gamma_5)\frac{i}{(\not k + \not p_2 + \not p_3 + \not p_4) - m}C^T - \mathrm{tr}\int\frac{d^4k}{(2\pi)^4}(2gC^\dagger i)\frac{i}{\not k - m}C^T(2gC^\dagger\gamma_5) \\ & \times \frac{i}{(\not k + \not p_3) - m}C^T(2gC^\dagger i)\frac{i}{(\not k + \not p_3 + \not p_2) - m}C^T(2gC^\dagger\gamma_5)\frac{i}{(\not k + \not p_2 + \not p_3 + \not p_4) - m}C^T \\ & + \frac{1}{2}(-4ig^2)(-12ig^2)\frac{i}{k^2 - m^2}\frac{i}{(k + p_1 + p_2)^2 - m^2} + \frac{1}{2}(-12ig^2)(-4ig^2)\frac{i}{k^2 - m^2}\frac{i}{(k + p_1 + p_2)^2 - m^2} \\ & + (-4ig^2)(-4ig^2)^2\frac{i}{k^2 - m^2}\frac{i}{(k + p_1 + p_3)^2 - m^2} + (p_3 \to p_4) \\ = & \, i\frac{1}{\pi^2}g^4\ln\frac{M_c^2}{\mu_s^2} + \text{finite terms.} \end{aligned} \tag{6.30}$$

We introduce the following counterterms:

$$\delta L = -\delta_1 mgA^3 - \delta_2 mgAB^2 - \delta_3 gA\bar\chi\chi - \delta_4 gB\bar\chi i\gamma_5\chi - \delta_5\tfrac{1}{2}g^2A^4 - \delta_6\tfrac{1}{2}g^2B^4 - \delta_7 g^2A^2B^2 \tag{6.31}$$

with

$$\begin{aligned} \delta_1 = \delta_2 = \frac{1}{4\pi^2}g^2\ln\frac{M_c^2}{\mu_s^2}, \qquad \delta_3 = \delta_4 = 0, \\ \delta_5 = \delta_6 = \delta_7 = \frac{1}{4\pi^2}g^2\ln\frac{M_c^2}{\mu_s^2}. \end{aligned} \tag{6.32}$$

It is easy to check that all the renormalized vertices lead to a single renormalization constant:

$$g_{\mathrm{bare}} = \left(1 + \frac{3}{8\pi^2}g^2\ln\frac{M_c^2}{\mu_s^2}\right)g = z^{-3/2}g. \tag{6.33}$$

This equation, together with Eqs. (6.17), (6.19), and (6.23), shows that the LR method works well in the perturbative theory of the massive Wess-Zumino model.

VII. CONCLUSION

In this paper we have investigated the applicability of the recently developed loop regularization method in supersymmetric theories. By checking several Ward identities in various supersymmetric models, we have explicitly shown that the LR method is applicable to the supersymmetric field theories. We have also directly carried out the calculations for one-loop renormalization of the massive Wess-Zumino model by using the LR method with string-mode regulators. The results are consistent with the general conclusion yielded from the supergraph technique. Once the supersymmetric extensions of the standard model could be discovered at the LHC, such a symmetry-preserving loop regularization method with string-mode regulators can widely be applied to the computations of various supersymmetric processes.

ACKNOWLEDGMENTS

The authors would like to thank A. Cohen for useful discussions. This work was supported in part by the National Science Foundation of China (NSFC) under Grant No. 10821504, No. 10491306, and the Project of Knowledge Innovation Program (PKIP) of Chinese Academy of Science.

APPENDIX A: TRANSLATIONAL INVARIANCE OF LOOP REGULARIZATION

The verification of translational invariance in Sec. II can simply be extended to the linearly and quadratically divergent integrals.

Consider first the quadratically divergent integral

$$L_2 = \int\frac{d^4k}{[(k - xp)^2 + M^2]} \tag{A1}$$

by rewriting the momentum factor $(k - xp)^2$ into $(k - xp)^2 = k^2 - 2xp.k + x^2p^2$, then replacing k^2 by $k^2 + M_l^2$, one has

$$\begin{aligned} (k - xp)^2 &\to k^2 + M_l^2 - 2xp.k + x^2p^2 \\ &= (k - xp)^2 + M_l^2. \end{aligned} \tag{A2}$$

Thus the proof in the manuscript for the scalar type logarithmic loop integration can be easily extended to the scalar type quadratically divergent ILIs, namely

$$L_2 \to L_2^R = \lim_{N,M_l^2}\sum_{l=0}^{N}c_l^N\int\frac{d^4k}{[(k - xp)^2 + M_l^2]}. \tag{A3}$$

The regularized ILI L_2^R is well defined and allows us to shift the momentum. We then have

$$L_2^R = \lim_{N,M_l^2} \sum_{l=0}^{N} c_l^N \int \frac{d^4k}{[(k-xp)^2 + M_l^2]}$$
$$= \lim_{N,M_l^2} \sum_{l=0}^{N} c_l^N \int \frac{d^4k}{[k^2 + M_l^2]} = I_2^R. \quad \text{(A4)}$$

Actually, it is this translational invariance which allows us to clarify the ambiguity caused by the linear divergent in evaluating the triangle anomaly and CPT/Lorentz violating Chern-Simons term, which was shown in Ref. [14]. To be more clear here, we demonstrate it as follows.

Let us first present J. Jauch and F. Rohrlich's discussion on the logarithmically divergent integrals [22]. Considering the following integral:

$$L_0 = \int \frac{d^4k}{[(k-p)^2 + M^2]^2} \quad \text{(A5)}$$

and making use of the identity

$$\frac{1}{\alpha^n} - \frac{1}{\beta^n} = -\int_0^1 \frac{n(\alpha-\beta)dz}{[(\alpha-\beta)z+\beta]^{n+1}} \quad \text{(A6)}$$

for $n = 2$, we can rewrite the above integral as follows:

$$L_0 = \int \frac{d^4k}{(k^2+M^2)^2} - 2\int d^4k \int_0^1 \times \frac{(p^2 - 2p\cdot k)dz}{[k^2 + M^2 + (p^2 - 2p\cdot k)z]^3}$$
$$\equiv I_0 + L_c. \quad \text{(A7)}$$

The second term L_c of the right-hand side is convergent, so we can safely shift the origin of k

$$k_\mu \to k_\mu + p_\mu z \quad \text{(A8)}$$

and the second term reads

$$L_c = -2\int_0^1 dz \int \frac{p^2(1-2z) - 2p\cdot k}{[k^2 + M^2 + p^2 z(1-z)]^3} d^4k. \quad \text{(A9)}$$

The term in the numerator which is odd in k will vanish. Using the identity,

$$\int \frac{(k^2)^{m-2} d^4k}{(k^2+M^2)^n} = \frac{i\pi^2}{(M^2)^{n-m}} B(m, n-m), \quad \text{(A10)}$$

where $B(m, n-m) = \Gamma(m)\Gamma(n-m)/\Gamma(n)$ and $n > m > 0$ is the condition of convergence. So the second term in Eq. (A7) now goes as

$$L_c = -2\frac{i\pi^2}{2} \int_0^1 dz \frac{p^2(1-2z)}{M^2 + p^2 z(1-z)}$$
$$= -i\pi^2 \ln[M^2 + p^2 z(1-z)]|_0^1 = 0. \quad \text{(A11)}$$

Therefore, for the logarithmic divergent integral, we arrive at the following identity:

$$L_0 = \int \frac{d^4k}{[(k-p)^2 + M^2]^2} = \int \frac{d^4k}{[k^2+M^2]^2} = I_0, \quad \text{(A12)}$$

which is independent of the regularization.

Nevertheless, if first applying the loop regularization prescription and then shifting the momentum, the corresponding relation becomes a straightforward consequence

$$L_0 \to L_0^R = \lim_{N,M_l^2} \sum_{l=0}^{N} c_l^N \int \frac{d^4k}{[(k-p)^2 + M_l^2]^2}$$
$$= \lim_{N,M_l^2} \sum_{l=0}^{N} c_l^N \int \frac{d^4k}{[k^2 + M_l^2]^2} = I_0^R. \quad \text{(A13)}$$

Let us now consider the linear divergent integral. When using the identity Eq. (A6), a similar proof can be carried out and shows that a shift of k in a linearly divergent integral will result in a finite additive constant

$$L_{1,\mu} = \int \frac{k^\mu d^4k}{[(k-p)^2 + M^2]^2} = \int \frac{(k+p)^\mu d^4k}{[k^2+M^2]^2} - \frac{i\pi^2}{2} p^\mu$$
$$\equiv I_{1\mu} + p_\mu I_0 + L_{c\mu}, \quad \text{(A14)}$$

which has been shown to cause an ambiguity in evaluating the chiral anomaly if the regularization schemes are not applied appropriately [14]. This is because the results may depend on the procedure of applying the regularization schemes before or after using the identity Eq. (A6).

To be safe, we shall apply the LR prescription before shifting the momentum. It then leads to the following result:

$$L_{1,\mu} \to L_{1,\mu}^R = \lim_{N,M_l^2} \sum_{l=0}^{N} c_l^N \int \frac{k^\mu d^4k}{[(k-p)^2 + M_l^2]^2}$$
$$= \lim_{N,M_l^2} \sum_{l=0}^{N} c_l^N \int \frac{(k+p)^\mu d^4k}{[k^2 + M_l^2]^2}$$
$$= \lim_{N,M_l^2} \sum_{l=0}^{N} c_l^N \int \frac{p^\mu d^4k}{[k^2 + M_l^2]^2} = p_\mu I_0^R, \quad \text{(A15)}$$

where we have shifted the momentum for the well-defined regularized integral, without using the above identity.

On the other hand, when applying the LR prescription before shifting the momentum, using the identity presented above for the integration, we then arrive at the following expression:

$$L_{1,\mu} \to L_{1,\mu}^R = \lim_{N,M_l^2} \sum_{l=0}^{N} c_l^N \int \frac{k^\mu d^4k}{[(k-p)^2 + M_l^2]^2}$$
$$= \lim_{N,M_l^2} \sum_{l=0}^{N} c_l^N \int \frac{(k+p)^\mu d^4k}{[k^2 + M_l^2]^2}$$
$$- \frac{i\pi^2}{2} p^\mu \lim_{N,M_l^2} \sum_{l=0}^{N} c_l^N. \quad \text{(A16)}$$

The second term of the right-hand side actually vanishes due to the following conditions for the coefficients in the LR:

$$\lim_{N,M_l^2}\sum_{l=0}^{N} c_l^N (M_l^2)^n = 0 \quad (n = 0, 1, \cdots). \tag{A17}$$

Thus we finally yield the following relation:

$$\begin{aligned} L_{1,\mu}^R &= \lim_{N,M_l^2}\sum_{l=0}^{N} c_l^N \int \frac{k^\mu d^4k}{[(k-p)^2+M_l^2]^2} \\ &= \lim_{N,M_l^2}\sum_{l=0}^{N} c_l^N \int \frac{(k+p)^\mu d^4k}{[k^2+M_l^2]^2} \\ &= \lim_{N,M_l^2}\sum_{l=0}^{N} c_l^N \int \frac{p^\mu d^4k}{[k^2+M_l^2]^2} = p_\mu I_0^R, \end{aligned} \tag{A18}$$

which just shows that in the LR method the translation of momentum can safely be made for a linearly divergent integral.

Now we turn to the quadratically divergent integral,

$$L_2 = \int \frac{d^4k}{[(k-p)^2+M^2]}, \tag{A19}$$

which can be rewritten as follows when using the previous identity:

$$\begin{aligned} L_2 &= \int \frac{d^4k}{(k^2+M^2)} - 2\int d^4k \int_0^1 \\ &\quad \times \frac{(p^2-2p\cdot k)dz}{[k^2+M^2+(p^2-2p\cdot k)z]^2} \\ &\equiv I_2 + L_{2c}. \end{aligned} \tag{A20}$$

Since the second term involves only linear and logarithmical divergences, we can then use the previous identities for those integrals when shifting the origin of k and get the following result with a finite additive constant:

$$\begin{aligned} L_{2c} &= -2\int d^4k \int_0^1 \frac{(p^2-2p\cdot k)dz}{[k^2+M^2+(p^2-2p\cdot k)z]^2} \\ &= -2\int_0^1 dz \int \frac{p^2 d^4k}{[k^2+M^2+p^2z(1-z)]^2} \\ &\quad + 2\int_0^1 dz \int \frac{2p\cdot(k+xp)d^4k}{[k^2+M^2+p^2z(1-z)]^2} - i\pi^2p^2. \end{aligned} \tag{A21}$$

The term which is odd in k does not contribute, and two integrals of the right-hand side cancel each other due to the relation

$$\begin{aligned} &\int_0^1 dz \int \frac{zp^2 d^4k}{[k^2+M^2+p^2z(1-z)]^2} \\ &= \frac{1}{2}\int_0^1 dz \int \frac{p^2 d^4k}{[k^2+M^2+p^2z(1-z)]^2}. \end{aligned} \tag{A22}$$

Thus we arrive at the following identity:

$$\begin{aligned} L_2 &= \int \frac{d^4k}{[(k-p)^2+M^2]} = \int \frac{d^4k}{[k^2+M^2]} - i\pi^2p^2 \\ &\equiv I_2 + L_{2c}. \end{aligned} \tag{A23}$$

Just like the discussion in the linearly divergent integral, by applying the LR prescription before shifting momentum, we have

$$\begin{aligned} L_2 \to L_2^R &= \lim_{N,M_l^2}\sum_{l=0}^{N} c_l^N \int \frac{d^4k}{[(k-p)^2+M_l^2]} \\ &= \lim_{N,M_l^2}\sum_{l=0}^{N} c_l^N \int \frac{d^4k}{[k^2+M_l^2]} = I_2^R, \end{aligned} \tag{24}$$

where the shift of momentum has been made for the regularized L_2^R. On the other hand, again applying the LR prescription before shifting momentum, but using the identity obtained above, we arrive at the following expression:

$$\begin{aligned} L_2 \to L_2^R &= \lim_{N,M_l^2}\sum_{l=0}^{N} c_l^N \int \frac{d^4k}{[(k-p)^2+M_l^2]} \\ &= \lim_{N,M_l^2}\sum_{l=0}^{N} c_l^N \int \frac{d^4k}{[k^2+M_l^2]} - i\pi^2p^2 \lim_{N,M_l^2}\sum_{l=0}^{N} c_l^N \end{aligned} \tag{A25}$$

accordingly. Because of the vanish of the second term in the right-hand side, we obtain the same regularized result

$$\begin{aligned} L_2^R &= \lim_{N,M_l^2}\sum_{l=0}^{N} c_l^N \int \frac{d^4k}{[(k-p)^2+M_l^2]} \\ &= \lim_{N,M_l^2}\sum_{l=0}^{N} c_l^N \int \frac{d^4k}{[k^2+M_l^2]} = I_2^R. \end{aligned} \tag{A26}$$

So far we have demonstrated that loop regularization can preserve translational invariance not only in the logarithmically, but also in the linearly and quadratically divergent integral.

APPENDIX B: DERIVATION OF MAJORANA FEYNMAN RULES

Here we are going to present a simple and definite derivation of Majorana Feynman rules which are useful for our calculations in this paper. We will begin with the quantization of the free Majorana fermion and figure out the difficulties of formulating the Majorana Feynman rules, then provide a consistent prescription. The unusual Majorana Feynman rules are the result from the Majorana fermion self-conjugacy. Though the two-components formulation of the Majorana field is more fundamental, it is still very useful to work in the four-components formalism because the γ matrices are more convenient for practical calculations.

The Majorana fermion field χ is quantized by stipulating the following equal-time anticommutators:

$$\{\chi_\alpha(\mathbf{x}), \chi_\beta^\dagger(\mathbf{y})\} = \delta_{\alpha\beta}\delta^3(\mathbf{x}-\mathbf{y}),$$
$$\{\chi_\alpha(\mathbf{x}), \chi_\beta(\mathbf{y})\} = \{\chi_\alpha^\dagger(\mathbf{x}), \chi_\beta^\dagger(\mathbf{y})\} = 0. \quad \text{(B1)}$$

The plane wave decomposition of χ is not obvious. In the two-components formalism the difficulty is reflected in the fact that the equation of motion (EOM) is no longer a linear equation since the EOM connects χ to its complex conjugation. In the four-components formalism the difficulty lies in the Majorana condition: $\chi = \chi^c = C\bar{\chi}^T$. But if we use the spinors u and v which satisfy $u_{\mathbf{k},s} = C\bar{v}_{\mathbf{k},s}^T$ and $v_{\mathbf{k},s} = C\bar{u}_{\mathbf{k},s}^T$, then χ can be expanded as

$$\chi = \int \frac{d^3k}{(2\pi)^3}\frac{1}{2E_{\mathbf{k}}}\sum_s[c_{\mathbf{k},s}u_{\mathbf{k},s}e^{-ikx} + c_{\mathbf{k},s}^\dagger v_{\mathbf{k},s}e^{ikx}], \quad \text{(B2)}$$

where c and $c^\dagger$ are the annihilation and creation operators of Majorana fermions. For Majorana fields, we still have

$$\langle 0|T\chi_\alpha(x)\bar{\chi}_\beta(y)|0\rangle = \int \frac{d^4k}{(2\pi)^4}e^{-ik(x-y)}\left(\frac{i}{\not{k}-m}\right)_{\alpha\beta} = S_{F\alpha\beta}(x-y). \quad \text{(B3)}$$

Note that because of the Majorana condition $\chi = C\bar{\chi}^T$ and $\bar{\chi} = \chi^T C$, $\langle 0|T\chi_\alpha(x)\chi_\beta(y)|0\rangle$ and $\langle 0|T\bar{\chi}_\alpha(x)\bar{\chi}_\beta(y)|0\rangle$ do not vanish. It is easy to show that

$$\langle 0|T\chi_\alpha(x)\chi_\beta(y)|0\rangle = S_{F\alpha\gamma}(x-y)C_{\gamma\beta}^T, \quad \text{(B4)}$$

$$\langle 0|T\bar{\chi}_\alpha(x)\bar{\chi}_\beta(y)|0\rangle = C_{\alpha\gamma}^T S_{F\gamma\beta}(x-y). \quad \text{(B5)}$$

The explicit expressions of $u_{\mathbf{k},s}$ and $v_{\mathbf{k},s}$ as well as the spin-sum identities can be found in [23]. We list the results here:

$$u_{\mathbf{k},s} = \begin{pmatrix}\sqrt{k\cdot\sigma}\zeta_s\\ \sqrt{k\cdot\bar{\sigma}}\zeta_s\end{pmatrix},$$
$$\bar{u}_{\mathbf{k},s} = (\zeta_s^\dagger\sqrt{k\cdot\bar{\sigma}}, \zeta_s^\dagger\sqrt{k\cdot\sigma})$$
$$v_{\mathbf{k},s} = \begin{pmatrix}2s\sqrt{k\cdot\sigma}\zeta_{-s}\\ -2s\sqrt{k\cdot\bar{\sigma}}\zeta_{-s}\end{pmatrix},$$
$$\bar{v}_{\mathbf{k},s} = (-2s\zeta_s^\dagger\sqrt{k\cdot\bar{\sigma}}, 2s\zeta_s^\dagger\sqrt{k\cdot\sigma}) \quad \text{(B6)}$$

and $\zeta_{\pm 1/2}$ are defined as below (here θ is the polar angle of $\mathbf{k}$ and ϕ is the azimuthal angle of $\mathbf{k}$.):

$$\zeta_{1/2}(\mathbf{k}) = \begin{pmatrix}\cos\frac{\theta}{2}\\ e^{i\phi}\sin\frac{\theta}{2}\end{pmatrix}, \qquad \zeta_{-1/2}(\mathbf{k}) = \begin{pmatrix}-e^{-i\phi}\sin\frac{\theta}{2}\\ \cos\frac{\theta}{2}\end{pmatrix}. \quad \text{(B7)}$$

The spin-sum identities are

$$\sum_s u_{\mathbf{k},s}\bar{u}_{\mathbf{k},s} = \not{k}+m, \qquad \sum_s v_{\mathbf{k},s}\bar{v}_{\mathbf{k},s} = \not{k}-m,$$
$$\sum_s u_{\mathbf{k},s}v_{\mathbf{k},s}^T = (\not{k}+m)C^T, \qquad \sum_s v_{\mathbf{k},s}u_{\mathbf{k},s}^T = (\not{k}-m)C^T,$$
$$\sum_s \bar{u}_{\mathbf{k},s}^T\bar{v}_{\mathbf{k},s} = C^\dagger(\not{k}-m), \qquad \sum_s \bar{v}_{\mathbf{k},s}^T\bar{u}_{\mathbf{k},s} = C^\dagger(\not{k}+m). \quad \text{(B8)}$$

Before starting the derivation of Majorana Feynman rules we may briefly review the derivation for the usual Dirac fermions. The argument below follows the one in [24]. The calculation of a typical scattering matrix element corresponds to the evaluation of the following expression:

$$\langle 0|b_1\ldots b_m d_1\ldots d_n T[(\bar{\psi}(x_1)\Gamma\psi(x_1))\ldots(\bar{\psi}(x_l)\Gamma\psi(x_l))] \times b_1^\dagger\ldots b_p^\dagger d_1^\dagger\ldots d_q^\dagger|0\rangle. \quad \text{(B9)}$$

First, we should rearrange the interaction terms to make them follow the order of contractions. Since only one type of contraction $\langle\psi\bar{\psi}\rangle$ exists for the Dirac fermion, the internal propagator reads: $\langle\psi\bar{\psi}\rangle = S_F(p)$, where the fermion charge and the momentum flows are well defined from $\bar{\psi}$ to ψ. The Feynman rule for the vertex directly reads as $i\Gamma$. For the Dirac fermion, the fermion charge flow (in fact this is also the momentum flow) of the internal propagator forms a continuous flow. When writing down the analytic expression, one should do it along the opposite direction of the continuous flow. The most important step is to determine the relative sign of interfering Feynman graphs (RSIF). There are in general three types of commutations which can contribute to the RSIF. First, when reordering b_i, d_i, $b_i^\dagger$, and $d_i^\dagger$ to put them in the appropriate places of Wick contractions, it causes a factor $(-1)^P$. Here P is the parity of the permutation of the annihilation and creation operators. This factor can be read from the order of external spinors in the analytic expression with respect to the given reference order. Second, for a closed fermion loop, one needs to exchange the first and the last field operator in the fermion chain, which gives a factor $(-1)^L$, where L is the number of fermion loops. Finally, since $d_i^\dagger$ must contract with $\bar{\psi}$ and d_i must contract with ψ, one needs to move the creation operator d_i to the beginning of the Wick contraction and move the annihilation operator $d_i^\dagger$ to the end, which leads to a factor $(-1)^V$ with V being the total number of spinors v and $\bar{v}$. Since V is universal for all graphs of a given process, this factor can therefore be omitted.

We now turn to investigate the Majorana fermion case. First, we consider the situation that there are no Dirac fermions but only Majorana fermions. As mentioned above, all possible contractions between χ and/or $\bar{\chi}$ do not vanish now. In this case, after rearranging the interaction terms to perform Wick contraction for operators one by one, we need to consider four types of Majorana propagators, i.e. $\langle\chi\chi\rangle$, $\langle\chi\bar{\chi}\rangle$, $\langle\bar{\chi}\chi\rangle$, and $\langle\bar{\chi}\bar{\chi}\rangle$. More seriously, the propagators depend on the sign of its momentum p, but now we cannot define the orientation from $\bar{\chi}$ to χ as the arrow of momentum. That means we need to find out a new method to resign the arrow of momentum. For the Feynman rule of vertex, it raises a new ambiguity. For instance, when contracting an interaction Lagrangian $\bar{\chi}\Gamma\chi$ in the time-order product, one can contract the operator $\bar{\chi}$ with one field operator lain on the left of this vertex and contract χ with another lies on the right, or one can also contract χ with one field operator lain on the left and contract $\bar{\chi}$ with another lies on the right. In the later case an additional (-1) will emerge. Previous discussions [25,26] for the Majorana Feynman rules follow this analysis and try to reduce the number of propagators and vertices, while the resulting consequences are still too obscure and not easy to use. In Ref. [24], the author introduced the charge-conjugate fields ψ^c and $\bar{\psi}^c$ to Feynman rules and tried to give a uniform description of the Dirac and Majorana field. Here we shall provide an alternative and simple description.

First, we may eliminate $\bar{\chi}$ from the interaction Lagrangian by using the Majorana condition $\bar{\chi} = -\chi^T C^\dagger$, so that only one type of propagator $\langle\chi\chi\rangle$ remains. We then use a line without arrow to represent a Majorana propagator. Since Majorana fermions cannot carry any charge, this representation is natural. In the momentum space, the Feynman rule for the Majorana propagator is $\frac{i}{\not{k}-m}C^T$. To obtain the Feynman rule of vertex, we may rewrite $\bar{\chi}\Gamma\chi$ as

$$\bar{\chi}_\alpha\Gamma_{\alpha\beta}\chi_\beta = \chi_\alpha(-C^\dagger_{\alpha\rho}\Gamma_{\rho\beta})\chi_\beta = -\chi_\beta(\Gamma^T_{\beta\rho}C^\dagger_{\rho\alpha})\chi_\alpha$$
$$= \tfrac{1}{2}\chi_\alpha(-C^\dagger\Gamma - \Gamma^T C^\dagger)_{\alpha\beta}\chi_\beta = \tfrac{1}{2}\chi_\alpha\Gamma'_{\alpha\beta}\chi_\beta \tag{B10}$$

with

$$\Gamma' = -C^\dagger\Gamma - \Gamma^T C^\dagger = -\Gamma'^T. \tag{B11}$$

Now the ambiguity mentioned above disappears as Γ' is antisymmetric. The Feynman rule for vertex simply becomes $i\Gamma'$. One can treat the Majorana fermions just like a real scalar boson to obtain the correct symmetric factor of a given graph.

Next, we should determine the direction of momentum in Majorana propagators. Remember that generally a factor e^{-ikx} means momentum k flows in the point x and e^{ikx} means momentum k flows out of the point x. Every contraction between two field operators $O(x)$, $O(y)$ can always be written in the form: $\langle O(x)O(y)\rangle = \int\frac{d^4k}{(2\pi)^4}e^{-ik(x-y)}S(k)$. For example, in our case

$$\langle 0|T\chi_\alpha(x)\chi_\beta(y)|0\rangle = \int\frac{d^4k}{(2\pi)^4}e^{-ik(x-y)}\bigg(\frac{i}{\not{k}-m}C^T\bigg)_{\alpha\beta}, \tag{B12}$$

which indicates that the direction of momentum flow is always opposite to the direction of the contraction for a propagator, and in a fermion chain the momentum flows of propagators form a continuous flow with direction opposite to the direction of the contractions. In [24] such a flow was called "fermion flow." Here we may, more precisely, call it "fermion momentum flow." This comes to the following conclusion: for each fermion chain we fix an arbitrary orientation (fermion momentum flow), the momentums of all fermion propagators follow this orientation, and we should write down the Feynman rules proceeding opposite to the chosen orientation.

Finally, to complete the Majorana Feynman rules, we need to give the rules of external fermion lines and determine the RSIF. The rules of the external fermion lines can easily be obtained from the plan-wave decomposition of χ, see Eq. (B2). Since

$$\langle 0|c_{\mathbf{k},s}\chi_\alpha(x) \rightarrow v_{\alpha\mathbf{k},s}e^{ikx}, \tag{B13}$$

$$\chi_\alpha(x)c^\dagger_{\mathbf{k},s}|0\rangle \rightarrow u_{\alpha\mathbf{k},s}e^{-ikx}, \tag{B14}$$

which implies that the creation of a Majorana fermion corresponds to a spinor $v_{\alpha\mathbf{k},s}$ with momentum k flow out, and the annihilation of a Majorana fermion corresponds to a spinor $u_{\alpha\mathbf{k},s}$ with momentum k flow in. If the spinor is located at the beginning of contraction, we should write it as a row vector, say a u^T or v^T. Now we can give a prescription to fix the RSIF. The factor $(-1)^P$ can be gotten from the permutation parity of the spinors in the obtained analytical expression with respect to some reference order. The factor $(-1)^L$ can be gotten from the number of closed fermion loops. The factor $(-1)^V$ now is a little different from that in Dirac field theory. Since moving any one creation operator arising from the initial state to the beginning of the contraction will contribute a factor -1, and moving any one annihilation operator arising from the final state to the end of the contraction which also contributes a factor -1, it seems that we should count the total number of such an operation. Suppose that there are "a" fermions in the initial sate and "b" fermions in the final state, and we must move the ith fermion creation operators to the beginning and the jth fermion annihilation operators to the end. Then we have $a - i + j = b + i - j$, i.e. $|i - j| = \frac{1}{2}|a - b|$. Namely, $V = \frac{1}{2}|a - b|$. Since a and b are universal for all graphs of a process, we can omit $(-1)^V$ all the time.

Let us consider the situation that a Majorana fermion χ couples to a Dirac fermion ψ. The interaction Lagrangian contains the following terms:

$$\bar{\chi}\Gamma\psi + \bar{\psi}\,\bar{\Gamma}\,\chi \quad (\text{where}: \bar{\Gamma} = \gamma^0\Gamma^\dagger\gamma^0). \tag{B15}$$

When keeping a continuous fermion momentum flow for a fermion internal line, we then need to consider two types of Dirac propagators: $\langle\psi\bar{\psi}\rangle$ and $\langle\bar{\psi}\psi\rangle$ which have the following explicit forms:

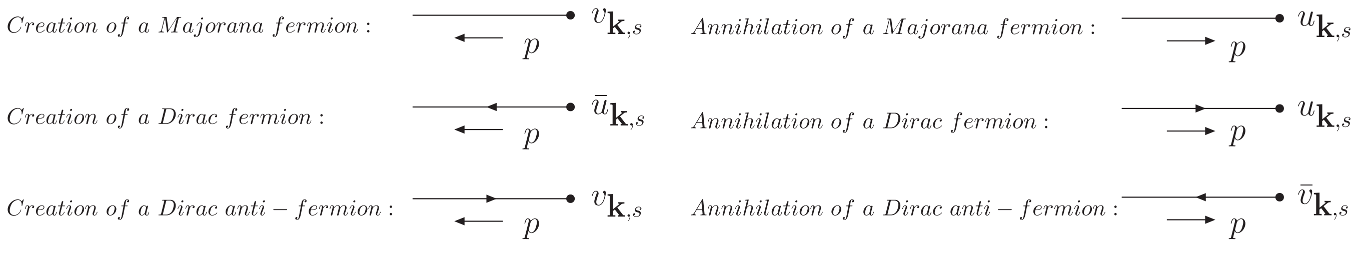

FIG. 9. Feynman rules for external lines.

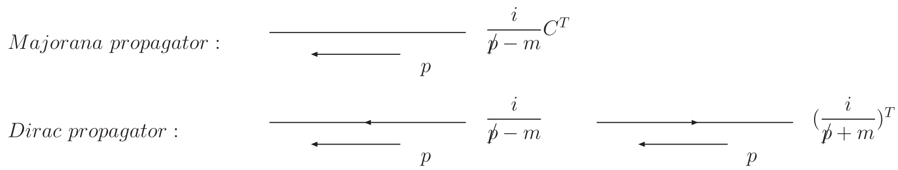

FIG. 10. Feynman rules for propagators.

$$\langle 0|T\psi_\alpha(x)\bar{\psi}_\beta(y)|0\rangle = \int \frac{d^4k}{(2\pi)^4} e^{-ik(x-y)}\left(\frac{i}{\not k - m}\right)_{\alpha\beta}, \tag{B16}$$

$$\langle 0|T\bar{\psi}_\alpha(x)\psi_\beta(y)|0\rangle = \int \frac{d^4k}{(2\pi)^4} e^{-ik(x-y)}\left[\left(\frac{i}{\not k + m}\right)^T\right]_{\alpha\beta}. \tag{B17}$$

We then need to use a line with arrow to represent the Dirac propagator. The arrow reflects the flow of charge which flows out of $\bar{\psi}$ and into ψ. If the direction of charge flow coincides with the direction of the fermion momentum flow, we should use $\langle\psi\bar{\psi}\rangle = \frac{i}{\not k - m}$; otherwise we should use $\langle\bar{\psi}\psi\rangle = (\frac{i}{\not k + m})^T$.

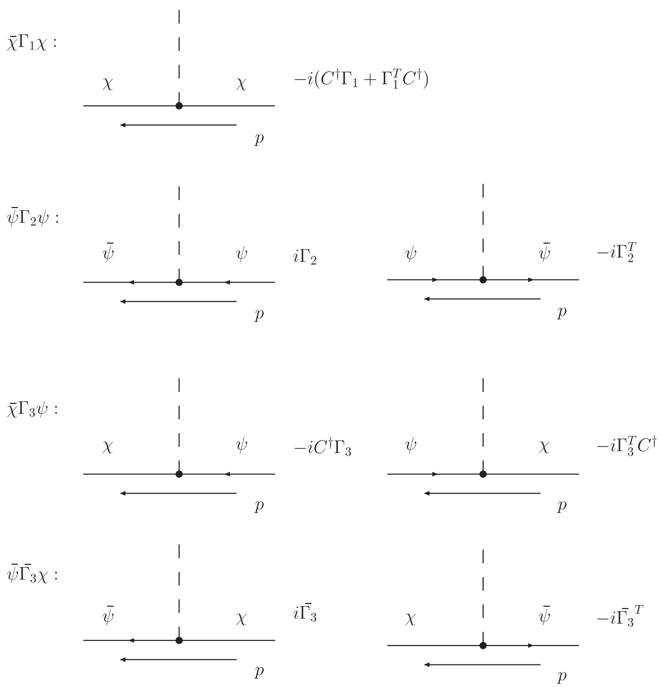

FIG. 11. Feynman rules for vertices.

The Feynman rules for vertices are also doubled. For the Dirac-Dirac interaction, one has

$$\bar{\psi}_\alpha\Gamma_{\alpha\beta}\psi_\beta = \psi_\alpha(-\Gamma^T)_{\alpha\beta}\bar{\psi}_\beta. \tag{B18}$$

If the direction of the charge flow coincides with the direction of the momentum flow, we should use $i\Gamma$; otherwise we should use $-i\Gamma^T$. The vertex rules of the Majorana-Dirac interaction can be derived similarly from the identities

$$\bar{\chi}_\alpha\Gamma_{\alpha\beta}\psi_\beta = \chi_\alpha(-C^\dagger\Gamma)_{\alpha\beta}\psi_\beta = \psi_\alpha(-\Gamma^T C^\dagger)_{\alpha\beta}\chi_\beta, \tag{B19}$$

$$\bar{\psi}_\alpha\bar{\Gamma}_{\alpha\beta}\chi_\beta = \chi_\alpha(-\bar{\Gamma}^T)_{\alpha\beta}\bar{\psi}_\beta. \tag{B20}$$

The RSIF can be determined by using the same method as we mentioned above.

With the above considerations, we can summarize our Feynman rules. The solid lines are still used to denote the fermions. Dirac fermions lines carry arrows which reflect the direction of charge flow, and Majorana lines do not carry arrows. We may write down Feynman amplitudes according to the following steps:

(1) Draw all topologically distinctive, connected Feynman diagrams for a given process.
(2) Fix an arbitrary direction for each fermion chain. This is the direction of the fermion momentum flow, which means that the momentum of every internal fermion line should follow this direction. We should write down the Dirac matrices proceeding opposite to the chosen direction through the chain.
(3) For the external fermion lines, the rules are shown in Fig. 9.
If the spinors are located at the beginning (end) of the contraction, we should add a superscript T appropriately to write them as row (column) vectors.
(4) For the fermion propagators, the rules are shown in Fig. 10.

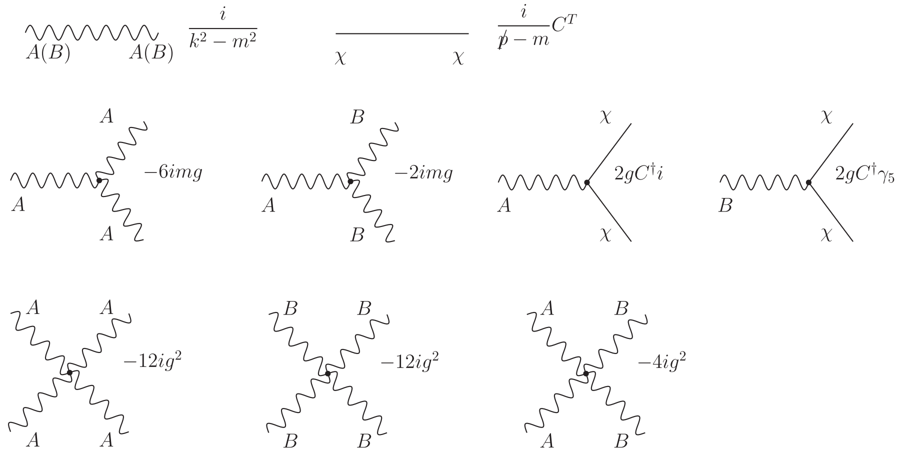

FIG. 12. Feynman rules of the massive Wess-Zumino model.

(5) For the general fermion interactions $\bar{\chi}\Gamma_1\chi$, $\bar{\psi}\Gamma_2\psi$, $\bar{\chi}\Gamma_3\psi + \bar{\psi}\bar{\Gamma}_3\chi$, where $\bar{\Gamma}_3 = \gamma^0\Gamma_3^\dagger\gamma^0$, the Feynman rules are shown in Fig. 11, respectively.

(6) To determine the RSIF. For each diagram, multiply by a factor (-1) for each closed fermion loop and multiply by the permutation parity of the spinors in the obtained analytical expression with respect to some reference order.

(7) Multiplying a symmetry factor S^{-1} for each diagram. The Majorana fermions may be treated just as real scalar fields to obtain the symmetry factor.

$$S = g \prod_{n=2,3,\ldots} 2^\beta (n!)^{\alpha_n}, \tag{B21}$$

where α_n is the number of pairs of vertices connected by n identical self-conjugate lines, β is the number of lines connecting a vertex with itself, and g is the number of permutations of vertices which leave the diagram unchanged with fixed external lines.

APPENDIX C: FEYNMAN RULES OF THE MASSIVE WESS-ZUMINO MODEL

We present all the Feynman rules of this model in Fig. 12.

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