PHYSICAL REVIEW D **79,** 125002 (2009)

High-energy supersymmetry at finite temperature

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(Received 26 January 2009; published 1 June 2009)

We study the leading thermal corrections to various observables in the high-energy limit in supersymmetric theories and observe that they preserve supersymmetry. Our findings generalize previous observations on the equality of asymptotic thermal masses in weakly coupled plasmas. We observe supersymmetry in the leading thermal effects for both the real and imaginary parts of self-energies, on the light cone and away from it, in both weakly and strongly interacting theories. All observed supersymmetry violations are found to be suppressed by more than two powers of the (large) energy.

DOI: 10.1103/PhysRevD.79.125002 PACS numbers: 11.10.Wx, 11.15.Bt, 11.25.Tq, 11.30.Pb

I. INTRODUCTION

Supersymmetry is usually considered as being broken at finite temperature (see e.g. [1]). A simple reason is the different statistics and population functions assumed by Bose and Fermi fields, making it hard to see how a Bose-Fermi symmetry could be preserved. In Euclidean space, Bose-Fermi symmetry is explicitly broken by boundary conditions along the periodic time direction (which has period $1/T$), which are, respectively, periodic for bosonic fields and antiperiodic for fermionic fields. Nevertheless, one can still ask whether the supersymmetry of the underlying equations of motion leaves any trace in physical observables.

One example along these lines was described in [2]: due to the existence of a conserved supercurrent, the effective hydrodynamics theory which describes the long-wavelength modes of the plasma must contain fermionic degrees of freedom. In this paper we will consider the opposite end of the energy spectrum, high-energy observables.

In the strict high-energy limit one expects the plasma to decouple, and supersymmetry to be recovered, provided it is present in the vacuum theory. More interestingly, one may look to the leading thermal corrections received by high-energy observables. We see no obvious reason why these should preserve supersymmetry. Nevertheless, the aim of this paper is to report the intriguing fact that, for a wide class of high-energy observables, the leading corrections indeed do.

This work was motivated by the well-known observation of supersymmetry preservation for the asymptotic thermal masses in weakly coupled plasmas. We will find that it also applies to other quantities, in fact to all high-energy correlators we could study. More precisely, parametrizing supersymmetry violations by the relative power of the energy E^{-n} by which they are suppressed, in all cases we find $n > 2$ with strict inequality. Since the leading thermal corrections have $n \leq 2$ in all cases this is a nontrivial statement.

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We will discuss in turn the relevant effective theories for the various observables we have considered. These include the effective particle masses at weak coupling, in Sec. II, where previously unknown next-to-leading order (NLO) results will also be reported; the imaginary part of self-energies at weak coupling (including collinear bremsstrahlung processes and $2 \to 2$ collisions), in Sec. III; the self-energies of neutral particles in strongly interacting plasmas having a gravity dual, in Sec. IV; and finally the operator product expansion (OPE) for deeply virtual correlators, in Sec. V.

By use of the phrase "effective theory" we mean to emphasize that the details of the plasma are always probed only through a restricted set of low-energy operators, whose expectation values provide the parameters of medium-independent high-energy effective theories. We thus understand the phenomenon of supersymmetry preservation as an intrinsic property of these effective theories: the thermal or equilibrium nature of the underlying medium probably plays no significant role.

II. THERMAL MASSES AT WEAK COUPLING

At the leading order in perturbation theory, thermal dispersion relations (of massless particles) are known to approach the form $E^2 = p^2 + m_\infty^2$ for any energy $E \gg gT$ [3], with $g = \sqrt{4\pi\alpha_s}$ a coupling strength. In applications to supersymmetric theories, it has been repeatedly observed that the asymptotic masses $m_\infty \sim gT$ are the same among particles within a supersymmetry multiplet [4]. Compiling results from the literature [6], or by direct evaluation of one-loop diagrams such as those shown in Fig. 1, they can be summarized by the formulas:

$$m_{\infty,g}^2 = m_{\infty,\lambda}^2 = g^2 C_A (Z_g + Z_f^\lambda) + g^2 N_{\text{matter}} T_M (Z_f^\psi + Z_\phi), \tag{1a}$$

$$m_{\infty,\psi}^2 = m_{\infty,\phi}^2 = g^2 C_M (Z_g + Z_f^\lambda + Z_f^\psi + Z_\phi) + y^2 (Z_f^\psi + Z_\phi), \tag{1b}$$

where the Z_i are tree-level condensates that we give

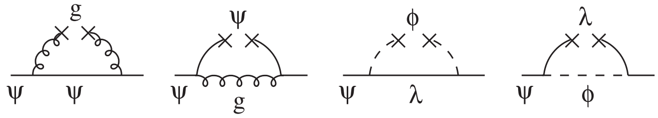

FIG. 1. One-loop fermion self-energy of a fermion ψ due to the gauge interaction, at large energy E. At leading order the asymptotic thermal mass is the sum of four condensates, which are extracted by letting each of the propagator become soft in turn (e.g., with all components $\sim T$ in Minkowski space-time) and expanding the rest of the diagram in powers of T/E.

shortly; g, λ, ϕ, and ψ stand for gluon, gluino, scalar, and fermionic matter fields, respectively, $N_{\rm matter}$ is the number of chiral superfields, and $C_{A,M}$, $T_{A,M}$, and $d_{A,M}$ are the quadratic Casimirs, Dynkin indices and dimensions of the adjoint and matter representations, respectively. For simplicity, the Yukawa contribution in Eq. (1) is normalized to correspond to a term $\sim \frac{y}{\sqrt{2}}\phi\psi\psi + \text{c.c.}$ in the Lagrangian of a single-field Wess-Zumino model. We expect supersymmetry to be preserved for more general (e.g., nonrenormalizable) superpotentials, though we have not checked this explicitly.

Nonzero expectation values for the D or F auxiliary fields, not considered in Eq. (1), could break the supersymmetry by lifting bosonic masses. Since this is not a specifically thermal source of supersymmetry breaking [7] this will not be considered in this paper.

The (nonlocal) dimension-2 condensates in Eq. (1), each normalized to give the contribution from 2 degrees of freedom, admit the following gauge-invariant definitions and tree-level thermal expectation values:

$$Z_g \equiv \frac{1}{d_A}\left\langle v_\sigma F^{\sigma\mu}\frac{-1}{(v\cdot D)^2}v_{\sigma'}F^{\sigma'}_{\mu}\right\rangle = 2\int_q \frac{n_B(q)}{q} = \frac{T^2}{6}, \tag{2a}$$

$$Z_S \equiv \frac{2}{d_M}\langle\phi^*\phi\rangle = 2\int_q\frac{n_B(q)}{q} = \frac{T^2}{6}, \tag{2b}$$

$$Z_f^\psi \equiv \frac{1}{2d_M}\left\langle\bar\psi\frac{\not v}{v\cdot D}\psi\right\rangle = 2\int_q\frac{n_F(q)}{q} = \frac{T^2}{12}. \tag{2c}$$

Here $v^\mu = (1,\mathbf{v})$ is the four-velocity of the hard particle, $\int_q = \int d^3q/(2\pi)^3$ and $n_{B,F}$ are the standard Bose-Einstein and Fermi-Dirac distribution functions. All condensates are time-ordered products, as is appropriate due to the high energy of the probe [8]. Useful examples include the thermal masses in $\mathcal{N}=4$ super Yang-Mills (SYM) theory, which are all equal to $m_\infty^2 = g^2N_cT^2$, and the gluon and gluino masses in pure glue supersymmetric QCD, $m^2_{\infty,g(\lambda)} = \frac{1}{4}g^2N_cT^2$.

The structures in Eq. (2) are identical to those entering the hard thermal loop (HTL) effective action [10]. This is not a coincidence: these are the unique dimension-2 gauge-invariant operators that can be built out of a lightlike four-vector v^μ.

Although our derivation of Eq. (1) was only carried out at the leading order in the coupling, we claim that it correctly describes next-to-leading order corrections, which are $\mathcal{O}(g)$. The point is that, $\mathcal{O}(g)$ corrections arise only from gT scale HTL physics [10] but not from the hard scale $\sim E$ (from which only $\sim g^2$ quantum corrections arise). But Eq. (1) is precisely designed to separate high-energy physics from low-energy physics, in the spirit of a (real-time) operator product expansion, so we conclude that at $\mathcal{O}(g)$ only the matrix elements in Eq. (2) receive corrections but not the coefficients, which contain only hard scale physics. In particular, the $\mathcal{O}(g)$ corrections also preserve supersymmetry. The evaluation of the condensates Eq. (2) at $\mathcal{O}(g)$, which requires HTL resummation, has not previously appeared in the literature and is performed in the Appendix. For completeness we record the results here (with $Z_f^{\rm NLO} = Z_f^{\rm LO} + \mathcal{O}(g^2T^2)$):

$$Z_g^{\rm NLO} = \frac{T^2}{6} - \frac{Tm_{\infty,g}}{\pi\sqrt{2}} + \mathcal{O}(g^2T^2), \tag{3a}$$

$$Z_S^{\rm NLO} = \frac{T^2}{6} - \frac{Tm_{\infty,S}}{2\pi} + \mathcal{O}(g^2T^2). \tag{3b}$$

Since the energy scale from which the corrections originate is gT, the NLO mass shifts obtained by substituting Eq. (3) into Eq. (1) should be valid, up to $\mathcal{O}(g^2)$ effects, for any energy $E \gg gT$.

Next-to-leading order (momentum-averaged) thermal masses were also obtained in [11] by means of an indirect thermodynamic argument, by relating them to the well-known $\sim g^3T^3$ corrections to the QCD entropy. The results are in agreement with Eqs. (3) [12].

We have no idea about how one should make sense of Eqs. (1) and (2) beyond NLO order (at order g^2), when genuine quantum corrections and renormalization group effects will first appear; at present we view the factorized form Eq. (1) as simply a convenient way to summarize the known leading-order results (and NLO results, we have argued). In particular, we have no idea as to whether supersymmetry will survive at higher orders in perturbation theory, should it be possible at all to define asymptotic masses.

Following our derivation, we interpret the supersymmetry of the thermal masses as a statement about the couplings of soft particles (of all spin) to hard propagators: these turn out to be the same among hard superpartners.

III. IMAGINARY PARTS OF SELF-ENERGIES AT WEAK COUPLING

The imaginary parts of self-energies at weak coupling, or scattering rates, are due to $2\to 2$ scattering against plasma particles as well as to induced collinear radiative processes (bremsstrahlung or pair production). For charged particles in gauge theories, the dominant contribution to $\mathrm{Im}\Pi$ is $\sim g^2TE$ due to small-angle elastic Coulomb scattering, though the dominant inelastic contribution $\sim g^4T^{3/2}E^{1/2}$ (barring logarithms) is due to induced collinear processes which we will discuss first. These processes also dominate the self-energies of neutral particles in gauge theories, provided these particles are allowed to pair produce charged ones. In nongauge theories, self-energies begin at $\sim g^4T^2E^0$ due to ordinary $2\to 2$ scattering, which we will discuss in Sec. III B.

A. Collinear radiation

At the risk of oversimplifying matters, the key aspects of collinear radiative processes may be briefly summarized as follows. These processes are only relevant in gauge theories, where they are initiated by the very frequent small-angle (Coulomb) scatterings suffered by either the parent or the daughter particles. At high energies $E \gg gT$, their long formation times (associated with the collinearity) allow multiple soft scatterings to occur during them and these must be summed coherently. This causes a parametrically significant destructive interference, the so-called LPM effect [14], that is responsible for the nonanalytic power $\Pi \propto E^{1/2}$ (neglecting logarithms). For relativistic plasmas, a complete leading-order treatment was given (for photons) in [15] (see also [16,17], in which different approximations are made). Somewhat schematically, the result may be written in the form ($-2\,\mathrm{Im}\Pi = 2E\Gamma$):

$$-2\,\mathrm{Im}\Pi_a(E) = \sum_{bc}\int_0^1 dz P_{a\to bc}(z)F_{a\to bc}(E,z), \qquad (4)$$

where $P_{a\to bc}$ are ordinary Dokshitzer-Gribov-Lipatov-Altarelli-Parisi (DGLAP) kernels [18], governing collinear physics, b and c denote the final states, and $z = E_b/E_a$ is the longitudinal momentum fraction. We have omitted final state Bose-enhancement or Pauli-blocking factors, which are not needed unless z or $(1-z)$ is very small, $\sim T/E$. The functions $F(E,z)$ depend in a complicated way on E and z and are to be obtained by solving an effective inhomogeneous Schrödinger equation for the wave function of the pair in the transverse plane [15]. This equation depends on the details of the plasma through a collision kernel $d\Gamma/d^2q_\perp$, which is a function of the transverse momentum transfer.

Its only property that we need, however, is that it involves only eikonal physics: it cares not about the spins of the particles. For our purposes, $F(E,z)$ in Eq. (4) is thus just some universal function that is the same for all final states among a given supersymmetry multiplet. In the leading logarithmic approximation [15],

$$F(E,z) \sim g^4N_c^2T^{3/2}E^{1/2}z^{-1/2}(1-z)^{-(1/2)} \times \left(\log\left(\frac{ET}{g^2T^2z(1-z)}\right)\right)^{1/2}.$$

The only ingredients in Eq. (4) which could break supersymmetry are thus the DGLAP splitting kernels $P_{a\to bc}(z)$. Such kernels are listed in Table I, for various supermultiplets of initial and final states. As shown in the table, when complete supermultiplets of final states are summed over [thereby enforcing the symmetry under $z\to(1-z)$], supersymmetry with respect to the initial particle is restored. Not shown in the table (it is related to the first three entries by a crossing symmetry [18]), but which also preserves supersymmetry, is the process of bremsstrahlung of a gauge multiplet off a matter particle. Thus, all in-medium splitting rates preserve supersymmetry.

Observations of supersymmetry in DGLAP kernels were made long (for instance, in the last reference of [18]), and subsequently given an explanation in [19]. Here we are simply reporting on their implications in a medium.

We expect coupling constant corrections to Eq. (4) to first arise at $\mathcal{O}(g)$. In thermal perturbation theory, $\sim g$ factors arise from ordinary loop factors g^2 multiplied by large bosonic occupation numbers $n_B \sim T/p^0 \sim T/gT$, and come strictly from gT scale physics. Such soft physics can only interfere with scatterings that have a sufficiently long duration, such as the soft scatterings contributing to collision rate $d\Gamma/d^2q_\perp$ with $q_\perp \sim gT$, so we believe that this is the only ingredient in Eq. (4) (through the function F) which receives $\mathcal{O}(g)$ corrections. These soft collisions have a purely diffusive effect, so equivalently we are claiming that all $\mathcal{O}(g)$ corrections at high energies should

TABLE I. DGLAP splitting kernels for various branching processes. Supersymmetry is restored when complete supermultiplets of final states are summed over.

Process	DGLAP kernel $P(z)$	Sum
$\gamma\to\psi^\dagger\psi$	$e^2[z^2+(1-z)^2]$	e^2
$\gamma\to\phi^\dagger\phi$	$e^2[2z(1-z)]$	
$\tilde\gamma\to\phi^\dagger\psi$	$e^2[2z]$	e^2
$g\to gg$	$2g^2C_A[\frac{(1-z)}{z}+\frac{z}{1-z}+z(1-z)]$	$g^2C_A[\frac{2}{z}+\frac{2}{1-z}-3]$
$g\to\lambda^\dagger\lambda$	$g^2C_A[z^2+(1-z)^2]$	
$\lambda\to g\lambda$	$g^2C_A[\frac{4z}{1-z}+2(1-z)]$	$g^2C_A[\frac{2}{z}+\frac{2}{1-z}-3]$
$\phi\to\psi^\dagger\psi^\dagger$	$y^2[1]$	y^2
$\psi\to\phi^\dagger\psi^\dagger$	$y^2[2z]$	y^2

be restricted to the so-called transverse momentum diffusion coefficient "$\hat{q}$." Since only eikonal physics should enter their calculation, we thus expect that $\mathcal{O}(g)$ corrections will trivially preserve supersymmetry.

The $\mathcal{O}(g^2)$ corrections to Eq. (4) are expected to possess a much more interesting and richer structure. For instance, they will most certainly require dealing with the scale dependence of the partonic constituents of the plasma, which could ultimately lead to "saturation" effects [20] at very high energies, upon summation of large logarithms $\alpha_s \log(E/T)$ and $\alpha_s \log(q_\perp^2/T^2)$ with $q_\perp^2 \sim E^{1/2}T^{3/2}$. The scale evolution of the constituents of the probe, which has to be treated in the presence of the LPM effect, should also enter at this order. Other interesting (though manifestly supersymmetry-preserving) effects may include sensitivity to nonperturbative g^2T-scale magnetic physics, which we believe contributes to $\hat{q}$ at $\mathcal{O}(g^2)$. We leave to future work a detailed analysis of these effects and of the question of whether they preserve supersymmetry.

As for effects subleading in T/E at leading order on g, we expect supersymmetry-breaking effects in Π not to be larger than $\sim T^{5/2}E^{-1/2}$ (relative to the $\sim E^2$ natural size); these could arise from various $\sim T/E$ or $\sim q_\perp^2/E^2 \sim (T/E)^{3/2}$ corrections to ingredients entering $F(E, z)$, such as the eikonal vertices.

B. 2 → 2 scattering at weak coupling

Ordinary $2 \to 2$ collisions dominate self-energies in nongauge models, which we will now discuss; their total rate will also be found to preserve supersymmetry. We first recall the general formula for the total collision rate:

$$-2\,\mathrm{Im}\Pi(p_1) = \int \frac{d^3p_2 d^3p_3 d^3p_4}{(2\pi)^5 2E_2 2E_3 2E_4} \times \delta^4(p_1 + p_2 - p_3 - p_4) \times \sum_{s_2 s_3 s_4} |\mathcal{M}_{1s_2\to s_3 s_4}|^2 n_b(E_2)(1 \pm n_c(E_3)) \times (1 \pm n_d(E_4)). \tag{5}$$

Here the particle labels are as defined in Fig. 2, the s_i labels are the corresponding particle species and n_i are the corresponding distribution functions.

Let us first assume, for a moment, that the distribution functions can be omitted in the final state ("Bose-enhancement" and "Pauli-blocking") factors $(1 \pm n_i)$, which is justified for generic final state energies $E_3 \sim E_4 \sim \sqrt{E_1E_2} \sim \sqrt{ET}$. The integrand then depends only on the sum $\sum_{s_3s_4} |\mathcal{M}|^2_{1s_2\to s_3s_4}$. Such matrix elements summed over final states turn out to obey supersymmetry identities, with respect to the particle 1, for fixed identities of particle 2. This is exemplified in Table II for a single-field Wess-Zumino model with cubic superpotential and a general proof will be given shortly. Therefore, contributions

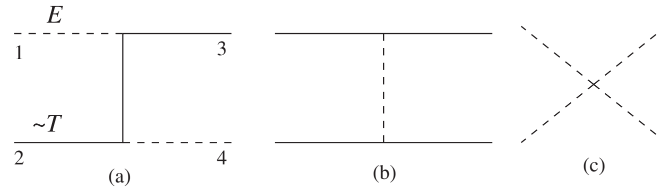

FIG. 2. $2 \to 2$ scattering processes in the Wess-Zumino model; solid lines are fermions and dashed lines are scalars.

to Eq. (5) from the region $E_3, E_4 \gg T$ preserve supersymmetry.

It is easy to convince oneself that for bounded amplitudes $|\mathcal{M}|^2$, the regions $E_3 \sim T$ or $E_4 \sim T$ suffer from $\sim T/E$ phase-space suppressions, justifying the neglect of the final state distributions in Eq. (5). However, $s/t \sim ET/T^2$ singularities in squared matrix elements when $t \lesssim T^2$ can overcome this suppression and a separate discussion is required for the singular terms [21] ($u \to 0$ singularities can be treated similarly). To establish the supersymmetry of these contributions, for which the distribution function $n(E_4)$ must be kept, we need another ingredient: the universality of the $1/t$ singularities. Indeed, the coefficient of $1/t$ at $t \to 0$, which is due to soft fermion exchange, is left unchanged when the hard particle 1 is replaced by its superpartner [e.g. if particles 1 and 3 are exchanged in Fig. 2(a)]. This shows that the complete $\sim T^2E^0$ self-energies in the Wess-Zumino model preserve supersymmetry, up to $\sim T^3E^{-1}$ corrections.

This universality of soft couplings is reminiscent of that which played a role for thermal masses in Sec. II, and can in fact be analyzed using the same tools. Indeed, the region $E_4 \sim T$, $t \sim T^2$ in Fig. 2 is characterized by soft fields coupled to a hard line and is thus governed by the gradient expansion of Fig. 1. This means that the $\sim T^2E^0$ contribution to Eq. (5) from soft fermion exchange is equivalently captured by an imaginary part of the dimension-2 fermion condensates in Eq. (2) at one loop in thermal perturbation theory [22].

We now prove, as claimed, that the supersymmetry of scattering amplitudes summed over final states holds in any supersymmetric theory as a property of its vacuum S matrix. Introducing the notation $P_{i_1\cdots i_n}$ for projection operators which perform the sum over complete super-

TABLE II. Left panel: scattering amplitudes $|\mathcal{M}|^2$ in the Wess-Zumino model, with amplitudes related by crossing symmetry not shown. Right panel: amplitudes summed over final states, for which supersymmetry is restored as a function of particle 1 with particle 2 held fixed.

Process	$\|\mathcal{M}\|^2/4y^2$	Processes	$\|\bar{\mathcal{M}}\|^2/4y^2$
$\psi\psi \to \psi\psi$	1	$\psi\psi \to X,\ \phi\psi \to X$	1
$\phi\phi \to \phi\phi$	1	$\psi\bar{\psi} \to X,\ \phi\bar{\psi} \to X$	$[2 + \frac{u}{t} + \frac{t}{u}]$
$\phi\psi \to \phi\psi$	$-u/s$	$\psi\phi \to X,\ \phi\phi \to X$	1
		$\psi\bar{\phi} \to X,\ \phi\bar{\phi} \to X$	$[2 + \frac{u}{t} + \frac{t}{u}]$

multiplets of scattering states with n particles (at fixed momenta), this follows from considering the following trace (over scattering states):

$$\mathrm{Tr}[S^\dagger P_{34} S(|2\rangle\langle 2| \otimes [Q, |1\rangle\langle\tilde{1}|])], \tag{6}$$

with S the S matrix and $\tilde{1}$ denotes the superpartner of particle 1. For any supersymmetry generator Q which does not annihilate particle 1, the commutator $[Q, |1\rangle \times \langle\tilde{1}|] \propto (|1\rangle\langle 1| - |\tilde{1}\rangle\langle\tilde{1}|)$ so Eq. (6) computes the difference:

$$\sum_{s_3,s_4} (|M_{12\to s_3 s_4}|^2 - |M_{\tilde{1}2\to s_3 s_4}|^2). \tag{7}$$

For a massless particle 2 it is always possible to choose Q so as to annihilate particle 2; such a Q commutes with $|2\rangle\langle 2|$, with the S matrix as well as with the projectors $P_{i_1\cdots i_n}$ (by construction), showing that Eq. (6) [and thus Eq. (7)] vanishes, being the trace of a commutator. Thus the contributions to Eq. (5) from $E_3, E_4 \gg T$ preserve supersymmetry in any theory.

Combining the results of the preceding sections, we have reached a simple conclusion: the full thermal self-energies of gauge-neutral particles preserve supersymmetry, at leading order in the coupling, up to corrections suppressed by at least $T^{5/2}E^{-1/2}$. Although it seems conceivable that the analysis of this section could be generalized to charged particles (for which it is made more complicated by the stronger singularities $\mathcal{M} \sim 1/t$ associated with gluon exchange [23] and by various sources of infrared divergences which make these self-energies less cleanly defined), here we will refrain from doing so: we are content with a robust result for gauge-invariant self-energies.

IV. SELF-ENERGIES AT STRONG COUPLING

Maldacena's conjectured gauge/gravity correspondence [25] renders possible, among other things, the calculation of correlators of currents in certain strongly coupled large N_c gauge theories. In theories which have a continuous R symmetry, such as the SU(4) of $\mathcal{N} = 4$ super Yang-Mills theory, "photons" and "photinos" can be introduced by weakly gauging a U(1) subgroup of the R symmetry, whose self-energies are then given by two two-point functions of currents and of their superpartners.

In the case of the on-shell photon self-energy in $\mathcal{N} = 4$ SYM, it was argued by means of a WKB approximation [26] (in the Appendix) that at high energy the calculation localizes itself near the boundary of the anti–de Sitter (AdS) space. Here we generalize this phenomenon to other backgrounds, which leads to a (simplistic) effective theory for high-energy photon/photino propagation in these theories, of which we can state two of its properties. First, it only probes the underlying low-energy medium through the expectation value of the energy-momentum tensor (actually, only through one component $\propto p_\mu p_\nu T^{\mu\nu}$), which determines the leading corrections to the metric at large radii. Second, it preserves supersymmetry: the absorption rates and dispersion relations of a photon and of a photino are identical.

We will be considering five-dimensional metrics of the general form

$$ds^2 = R^2 \frac{g(z)dz^2 + h_{\mu\nu}(z)dx^\mu dx^\nu}{z^2}, \tag{8}$$

for which, near the boundary $z = 0$, the metric approaches that of AdS_5 with radius R [for which $g(z) = 1$ and $h_{\mu\nu}(z) = \eta_{\mu\nu}$]. The metric Eq. (8) should be sufficiently general to cover any system invariant under space-time translation that admits a gravity dual. For the AdS_5 black hole, relevant for $\mathcal{N} = 4$ SYM at finite temperature T, $-h_{00} = 1 - (\pi T z)^4$, $h_{ij} = \delta_{ij}$, $h_{i0} = 0$, and $g(z) = (-h_{00})^{-1}$. At certain steps below, rotational invariance will be assumed; these steps will be highlighted.

A. Bulk equations

The bulk dual of the spin-1 current which couples to the photon is a five-dimensional gauge field, whose field strength tensor obeys Maxwell's equations:

$$0 = \frac{z}{G(z)} \partial_z \left(\frac{h^{\nu\sigma} G(z)}{z g(z)} F_{z\sigma} \right) + h^{\nu\sigma} h^{\mu\rho} \partial_\mu F_{\rho\sigma}, \tag{9}$$

$$\partial_\alpha F_{\mu\nu} = \partial_\mu F_{\alpha\nu} - \partial_\nu F_{\alpha\mu}, \tag{10}$$

with $G(z) = \sqrt{g(z)\det(-h(z))}$. Here μ, ν, σ, and ρ are space-time indices and α may cover all five coordinates. We will restrict our attention to space-time momentum eigenstates $\partial_\mu = i p_\mu$. A closed equation for the transverse electric field $F_{0\perp}$, for $\nu = \perp$ a component perpendicular to p_μ, may be obtained by acting on the first equation with a partial time derivative ∂_0, and using the second equation. Specifically, one uses relations such as $\partial_0 F_{z\perp} = \partial_z F_{0\perp}$, which follow from dropping perpendicular derivatives $\partial_\perp$ in the latter. To fully exploit such simplifications, rotational invariance must be assumed, so that upstairs derivatives $h^{\perp\sigma}\partial_\sigma$ also vanish. This yields the closed equation:

$$\frac{z h_{\perp\perp}}{G(z)} \partial_z \left(\frac{h^{\perp\perp} G(z)}{z g(z)} \partial_z F_{0\perp} \right) = h^{\mu\nu} p_\mu p_\nu F_{0\perp}, \tag{11}$$

in which no summation over $\perp$ indices is implied.

The bulk dual of the spin-$\frac{1}{2}$ operator coupling to the photino is a five-dimensional Dirac fermion with bulk mass $m = \frac{1}{2}$ [27] (in units with $R = 1$). It possesses as many components as two four-dimensional Weyl spinors but it is dual to only one such spinor, the symmetry between the two Weyl components being broken by the sign of m. The bulk Dirac equation reads

$$[\not{D} + m]\psi = 0 \equiv [\gamma^a e^\alpha_a (\partial_\alpha + \tfrac{1}{4}\omega^{ab}_\alpha \gamma_a \gamma_b) + m]\psi, \tag{12}$$

with $\alpha, a \in 0, 1, 2, 3, 4,$ and e^α_a the orthogonal basis.

Under the assumption of rotational invariance, the term involving the spin connection ω is proportional to γ_z and can be removed by a z-dependent field rescaling. We choose the rescaling

$$\psi = z^2(\det(-h))^{-1/4} e^{-m\int^z dz\sqrt{g(z)}/z}\tilde{\psi},$$

which leads to the following equations for the Weyl components of $\psi_{L,R}$ of $\tilde{\psi}$:

$$\partial_z \psi_L = \sqrt{g(z)}\not{p}_R \psi_R, \tag{13a}$$

$$\left[\frac{1}{\sqrt{g(z)}}\partial_z - \frac{2m}{z}\right]\psi_R = \not{p}_L \psi_L. \tag{13b}$$

Here $\not{p}_{L,R}$ are the Weyl operators associated with the four-dimensional metric $h_{\mu\nu}(z)$. With $m = +\frac{1}{2}$ the component relevant near the $z = 0$ boundary is ψ_L and we are calculating the self-energy of a left-handed photino. Equations (13) square to a closed equation for ψ_L,

$$\not{p}_R\left[\frac{1}{\sqrt{g(z)}}\partial_z - \frac{2m}{z}\right]\frac{1}{\not{p}_R\sqrt{g(z)}}\partial_z \psi_L = h^{\mu\nu}p_\mu p_\nu \psi_L. \tag{14}$$

B. WKB solution and supersymmetry

We are now in position to discuss the WKB approximation. By a change of variable $y \equiv y(z)$, Maxwell's equation (11) may be cast in a Schrödinger form with potential proportional to the squared energy p_0^2, provided

$$\frac{dy}{dz} = 2zh_{\perp\perp}\sqrt{\frac{g(z)}{\det(-h(z))}}. \tag{15}$$

For black holes (like the AdS_5 black hole metric given above) the function $g(z)$ has a pole at a finite value of z (the location of the horizon), while the function $\det(-h)$ vanishes there. In this limit y is mapped logarithmically to infinity. The rescaled potential remains finite there, though, and depends only on the energy $E = p^0$.

The qualitative features of the Schrödinger potential entering the equation $[\partial_y^2 - V(y)]F_{0\perp} = 0$ are sketched in Fig. 3. The shape of the potential depends on the geometry but not on the energy, which only determines its overall normalization. At large y the potential becomes constant, while for $y \to 0$ the leading term becomes, for on-shell and off-shell momenta, respectively,

$$V(y) \to \begin{cases} \frac{1}{4y}p^2, & p^2 \neq 0, \\ \frac{y}{4}p_\mu p_\nu \frac{dh^{\mu\nu}(z)}{d(z^4)} = -y\frac{\pi^2 T^{\mu\nu}p_\mu p_\nu}{2N_c^2}, & p^2 = 0. \end{cases} \tag{16}$$

Here we have used that the leading corrections to the metric near the boundary are proportional to z^4 and are related to the expectation value of the stress-energy tensor $T_{\mu\nu}$; its trace part, if nonzero, does not contribute when

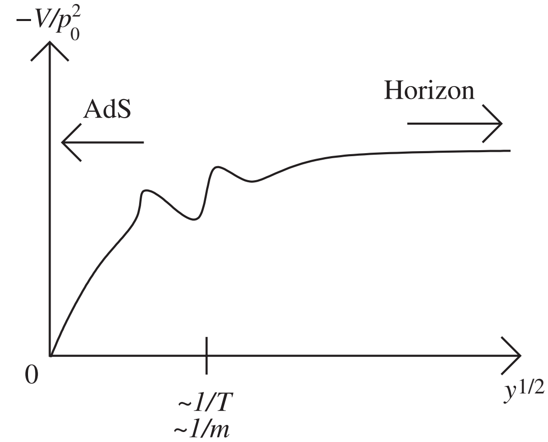

FIG. 3. Schematic features of the Schrödinger potential $V(y)/p_0^2$, when $p^2 = 0$. It approaches the universal linear behavior Eq. (16) near the boundary and tends to a constant at the horizon $y \to \infty$, with a transition regime that may depend on the details of the theory and on possible intrinsic mass scales m.

$p^2 = 0$. The normalization in Eq. (16) is appropriate to the $\mathcal{N} = 4$ SYM theory.

At the horizon $y \to \infty$ the solutions are oscillatory and in-falling boundary conditions $F_{0\perp} \propto e^{i\omega}$ must be imposed for calculating retarded correlators [28], with $\omega = p^0\pi T/2$ the natural frequency near the horizon. To obtain correlators of currents, as described shortly, this solution must be evolved to the AdS_5 boundary $z = 0$. For sufficiently large energies compared to all intrinsic scales in the metric a WKB approximation can be used. This is applicable for y down to $y \sim 1/p^2$ [respectively, $y \sim (T^4E^2)^{-1/3}$] for $p^2 \neq 0$ (respectively, $p^2 = 0$), at which it breaks down due to the redshift factors [29]. These scales are the intrinsic scales of the Schrödinger equations with approximate potentials Eq. (16). The problem is thus reduced to exactly solving those approximate equations, with large y behavior matching the WKB form $\propto V^{-1/4}e^{i\int^y dy\sqrt{V}}$.

The analysis is similar for the Dirac equation (14), with the change of variable Eq. (15) replaced with $\frac{dy}{dz} = 2z\sqrt{g(z)}\not{p}_R(z)/\not{p}_R(z=0)$. For the on-shell component ψ_L^- of a left-handed photino in a rotationally invariant background, the operator $\not{p}_R$ is nonsingular with eigenvalue $E(\sqrt{|h^{00}|} + \sqrt{h^{33}})$. Here h^{33} is the metric component along the longitudinal direction. Like for Eq. (15), near the boundary $y \sim z^2$ and the horizon is mapped logarithmically to $y = \infty$, and the same WKB approximation applies. More significantly, one readily sees comparing Eqs. (11) and (14) that the approximate potentials near the boundary will be *identical* to the photon case, Eqs. (16).

Correlation functions are obtained by prescribing the limiting values of the fields $F_{0\perp}$ and ψ_L near the boundary and evaluating boundary terms $\propto \partial_y F_{0\perp}$ (see e.g. [26]), or proportional to $\bar{\psi}\psi \sim \psi_R/z \sim \frac{1}{\not{p}_R}\partial_y\psi_L$ [27]. In equations,

$$\Pi_\gamma = \frac{-N_c^2 T^2}{8\pi^2} \lim_{y\to 0} \frac{\partial_y F_{0\perp}(y)}{F_{0\perp}(y)},$$
$$\Pi_{\tilde{\gamma}} = \frac{-N_c^2 T^2}{8\pi^2} \lim_{y\to 0} \frac{\partial_y \psi_L^-(y)}{\psi_L^-(y)}. \quad (17)$$

Here $\Pi_{\tilde{\gamma}} \equiv \bar{u}\Sigma u$ is the photino self-energy sandwiched between on-shell polarization spinors u, whose real part yields the thermal mass squared. The normalization of Eq. (17) has been obtained by matching to the well-known supersymmetry-preserving vacuum result, $\Pi_\gamma = \Pi_{\tilde{\gamma}} = -N_c^2 p^2/32\pi^2 \log(p^2/\mu^2)$, $p^2 = \mathbf{p}^2 - p_0^2$. On the light cone, Schrödinger's equation with the approximate potential Eq. (16) is solved in terms of Bessel (Hankel) functions $F_{0\perp}(y) \sim \psi_L^- \sim y^{1/2} H_{1/3}(\frac{2}{3}\tilde{\omega} y^{3/2})$ with $\tilde{\omega}^2 = \pi^2 T_{\mu\nu} p^\mu p^\nu / 2N_c^2$, yielding with Eq. (17) the result:

$$\Pi_\gamma(p) = \Pi_{\tilde{\gamma}}(p) = \frac{N_c^2 \Gamma(\frac{2}{3})}{16\pi^2 \Gamma(\frac{1}{3})}(3^{1/3} - i3^{5/6})\tilde{\omega}^{2/3} \quad (18)$$

at large $p^0 = p$. The imaginary part of this result reproduces that given in [26] (see also [30]) in $\mathcal{N} = 4$ SYM (employing that $\tilde{\omega} = p^0 T^2 \pi^2/2$ then). Corrections in T/E to this result may be found by expanding the potential Eq. (16) to higher orders near the boundary; for the AdS_5 black hole this expansion proceeds in powers of $y^2 \sim \tilde{\omega}^{-4/3}$, so the first subleading corrections to Π are $\sim\tilde{\omega}^{-2/3}$.

We find it remarkable that photon self-energies at strong coupling and high energies depend on only *one* property of the plasma: its stress-energy tensor. On the gravity side this may be understood as due to the universal, spin-independent gravitational attraction towards the black hole at large distances. An heuristic field-theoretic picture of strongly coupled plasmas, based on the idea of parton saturation, has been proposed recently [30] in which such a universality also comes out naturally.

V. DEEPLY VIRTUAL CORRELATORS

Deeply virtual correlators, which for instance can be related to sum rules for spectral functions (e.g., dilepton production rates) or to their asymptotics, may be analyzed by means of the OPE [31]. The OPE is a systematic means of separating short-distance and long-distance physics, allowing the thermal corrections to deeply virtual (short-distance) correlators with $E \gg T$ to be expressed in terms of the expectation value of local operators. Thermal corrections are thus suppressed by powers $\sim E^{-\Delta}$ with the Δ's determined by the scaling dimensions of local operators [32].

The difference between a correlator of operators and of their superpartners is a supersymmetry variation (in agreement with the fact that it vanishes in supersymmetry-preserving vacua). For instance, for correlators of transverse currents $\epsilon_\mu J^\mu$ and of their superpartners λ_α, one schematically has

$$\epsilon_1 \cdot J(p)\epsilon_2 \cdot J - \tfrac{1}{2}\lambda^\dagger(p)\not{\epsilon}_1 \not{p} \not{\epsilon}_2 \lambda \propto \epsilon_1^{\alpha\dot{\alpha}} Q_\alpha(\lambda_{\dot{\alpha}}^\dagger(p)\epsilon_2 \cdot J), \quad (19)$$

with $p_\mu \epsilon_{1,2}^\mu = 0$, α, $\dot{\alpha}$ spinor indices, and Q_α a supersymmetry transformation. As an operator equation, the OPE must commute with the supersymmetries, so from the OPE of the right-hand side of Eq. (19) one concludes that the operators on its left-hand side must be *supersymmetry variations*. This has a simple consequence: supersymmetry violations of order E^{-2} or stronger, in the deeply virtual region, can only be seen if there exists *local* gauge-invariant fermionic operators of dimension $\frac{3}{2}$ or less.

In a wide class of theories there is an accidental symmetry: such operators do not exist. These theories certainly include all weakly coupled gauge theories containing no U(1) vector multiplets and no gauge-singlet chiral superfields. In these theories, the only gauge-invariant dimension-2 bosonic operators (such as $\mathrm{Tr}\phi\phi$ or $\mathrm{Tr}\phi^*\phi$) do not correspond to any supersymmetry variations, and thus cannot cause supersymmetry violations. The lowest dimensional fermionic operators are dimension-$\frac{5}{2}$ supercurrents, from which we conclude that thermal supersymmetry breaking can only be seen through dimension-3 operators, $\sim E^{-3}$.

When neutral chiral superfields or U(1) vector multiplets are present, nonzero expectation values for $D \sim \phi^*\phi$ or $F \sim \phi^*\phi^*$ auxiliary fields (which enter the supersymmetry transformations of gauginos and fermionic matter fields, respectively) could produce supersymmetry violations at dimension 2. A similar possibility was observed for thermal masses in Sec. II but, as we discussed there, we do not view it as being specifically related to thermal effects. Thus, we conclude that in weakly coupled theories, there generically cannot be supersymmetry breaking (in deeply virtual correlators) due to thermal effects below dimension 3.

It is not possible to analyze general theories at finite values of the coupling constants, because finite anomalous dimensions can alter the power counting. Nevertheless, for certain strongly coupled theories accessible to the AdS/CFT correspondence, it is easy to be more quantitative. For instance, it is known [34] that in $\mathcal{N} = 4$ SYM at large 't Hooft coupling $\lambda \gg 1$, only protected (chiral) operators have finite dimensions $\Delta \ll \lambda^{1/4}$ and that the lowest dimensional fermionic operator has dimension $2 + \frac{1}{2} = \frac{5}{2}$ (it is the supersymmetry variation of a dimension-2 primary field). Similarly, the $\mathcal{N} = 1$ theory dual to IIB string theory on $AdS_5 \times T^{11}$ [35] is known to contain no fermionic operator of dimension less than 2 [36]. Thus, in these theories, supersymmetry violations (in the deeply virtual regime) can only be seen at $\sim E^{-3}$ or $\sim E^{-(5/2)}$ levels, respectively. A discussion of more general strongly coupled theories will not be attempted here.

VI. CONCLUSIONS

In this paper we have shown that supersymmetry is a generic property of the effective theories which describe high-energy correlators in supersymmetric theories, even in the presence of an underlying medium. The correlators studied include self-energies at high energies on the light cone as well as far away from it (large virtuality).

For all correlators (except for the more tractable deeply virtual correlators treated in Sec. V) our analysis has been limited to the leading nontrivial order (and sometimes NLO) at both weak and strong coupling. Without an understanding of the structure of higher order corrections, which is presently lacking, it seems hard however to decide whether our findings highlight general structural properties of supersymmetric theories, or whether they are artifacts of these extreme limits. We find nevertheless the presented evidence to be suggestive.

We have found that thermal supersymmetry violations in all correlators are suppressed by a power of the energy E^{-n} relative to the vacuum correlators, with *n strictly greater than 2.* (Violations with $n = 2$ were observed in Secs. II and V due to nonvanishing D-term or F-term expectation values, but we do not regard these effects as being of a specifically thermal origin.) We find pleasing that such a simple and uniform bound holds: this makes one wonder whether it could be a consequence of some general principle which would be valid independently of a perturbation theory, though at present we have no concrete proposal to make along these lines.

ACKNOWLEDGMENTS

I would like to thank Guy D. Moore and K. Dasgupta for useful discussions and comments on a draft of this paper, and O. Saremi for helpful advice on fermions in AdS/CFT. This work was supported in part by the Natural Sciences and Engineering Research Council of Canada.

APPENDIX: NEXT-TO-LEADING ORDER CALCULATION OF THE CONDENSATES IN EQ. (3)

In this Appendix we evaluate the next-to-leading order $[\mathcal{O}(g)]$ corrections to the condensates Eq. (2), as given in Eq. (3). The corrections originate from the difference between using bare and HTL-resummed propagators [10].

For the scalar condensate we use the fact that the scalar HTL self-energy is simply a constant mass shift (see the third reference of [10]). The calculation of $Z_S = 2\Phi^*\Phi$ can be done in Euclidean space, where only the zero Matsubara mode contributes to the $\mathcal{O}(g)$ correction:

$$\delta Z_S = 2T \int_q \left[\frac{1}{q^2 + m_{\infty,S}^2} - \frac{1}{q^2} \right] = \frac{-Tm_{\infty,S}}{2\pi}. \tag{A1}$$

A quick way to evaluate the shift to the gluon condensate is to use the fact that $-m_D^2 d_A/4$ times the angular average of Z_g is precisely the HTL effective action [10], so $\langle Z_g \rangle = -4\langle \Gamma_{\mathrm{HTL}} \rangle / m_D^2 d_A$. Given the physical significance of this effective action, it should be possible to evaluate it in Euclidean space, where it reduces to a constant mass shift $\Gamma_{\mathrm{HTL}}^{\mathrm{Euclidean}} = -m_D^2 A_4 A_4/2$ for the zero Matsubara mode of the temporal gauge field (see, for instance, Chapter 5 of the review [37]), plus negligible corrections to the other modes. Thus,

$$\delta Z_g = \frac{2}{d_A} \delta\langle A_4 A_4 \rangle = 2T \int_q \left[\frac{1}{q^2 + m_D^2} - \frac{1}{q^2} \right] = \frac{-Tm_D}{2\pi}, \tag{A2}$$

which reproduces Eq. (3) upon using $m_D = m_{\infty,g}\sqrt{2}$.

The only seemingly weak point of the preceding paragraph is the appeal to Euclidean techniques. This can be rigorously justified using the sum rules developed by Aurenche, Gelis, and Zaraket [38]. In Appendix A of the second reference of [38] (which appeared after the initial submission of this paper), it is proved that such sum rules always reduce expectation values localized on the light cone in configuration space, such as Z_g, to Matsubara sums, which reduce in the classical approximation $[n_B(\omega) = T/\omega$, which is justified for the NLO correction] to the $\omega_E = 0$ contribution Eq. (A2). This proves the first equality of Eq. (A2). Of course, it is also possible to verify Eq. (A2) directly by numerically integrating the Minkowski-signature operator Z_g as defined in Eq. (2), evaluated with HTL-resummed propagators (and with bare propagators subtracted); we have also done this.

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