# Anisotropic cosmology and (super)stiff matter in Hořava's gravity theory

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We study anisotropic cosmology in Hořava's gravity theory and obtain Kasner-type solutions, valid for any number, $d$, of spatial dimensions. The corresponding exponents satisfy two relations, one involving the marginal coupling $\lambda$. Also, Hořava's (super)renormalizable theory predicts (super)stiff matter whose equation of state is $p = w\rho$, with $w \geq 1$. We discuss briefly the implications of these results for the nature of cosmological collapse.

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## I. INTRODUCTION

Recently, Hořava has proposed [1] a candidate theory for gravity based on anisotropic scaling of space and time coordinates:

$$x^i \longrightarrow l x^i, \qquad t \longrightarrow l^z t \tag{1}$$

where $z$ is the scaling exponent. He has constructed an action for the metric fields invariant under the above scaling and also under foliation preserving diffeomorphic transformations. The action is required to have no more than two time derivatives. The kinetic part of the action is then universal and is characterized by a marginal coupling $\lambda$. The potential part has numerous terms containing various powers and spatial derivatives of curvatures of the spatial metric. Hořava has invoked "the principle of detailed balance" to constrain such terms, but this seems unnecessary and may even be problematic. The action reduces to the Einstein action in the IR if $\lambda \longrightarrow 1$ and $z \longrightarrow 1$. The theory then has full space-time diffeomorphism symmetry and may, therefore, be a candidate for a renormalizable Einstein theory of gravity. Hořava's theory may also acquire an anisotropic Weyl symmetry at $\lambda = \frac{1}{d}$, where $d$ is the number of spatial dimensions.

Such a theory has many appealing properties. For example, it is ghost-free since there are no more than two time derivatives. By construction, it is power-counting renormalizable in the UV if $z = d$ and is superrenormalizable if $z > d$, so it is believed to be UV complete. It contains many higher powers and derivatives of curvature, hence it may be able to resolve singularities. It singles out time, so the causal structure in the UV is likely to be modified, which may have nontrivial implications for black hole physics. In such a theory, the speed of light generically diverges in the UV, so the horizon problem may perhaps be solved without requiring inflation. See [1,2] for more details.

Various aspects of such a theory are being actively studied. See, for example, [2–20]. In this paper, we focus on implications of such a theory in early universe cosmology where differences from Einstein's theory are likely to be manifest. Such implications have also been studied in [5,6,8,9,13,14] for a $d = 3$ homogeneous isotropic Friedmann-Robertson-Walker universe. It is found that scale invariant cosmological perturbations can be generated without requiring inflation if the scale factor $a(t)$ evolves as $\sim t^n$ with $n > \frac{1}{3}$; it is also found that there can be a bounce in the early universe if the spatial curvature is nonzero. The scale invariance of perturbations is due to the modifications of dispersion relations arising from anisotropic scaling symmetry and, hence, is likely to be a generic feature of Hořava's theory. The bounce is due to the nonzero spatial curvature of the Friedmann-Robertson-Walker universe and due to higher powers of the curvature in the action. The bounce is thus a possible but nongeneric feature of Hořava's theory. For example, it is absent for a spatially flat universe.

Our main motivation here is to find the implications of Hořava's theory which differ from those of Einstein's theory and which are not crucially dependent on the spatial curvature. We find that the generic equation of state for matter in the UV is $\frac{p}{\rho} = w = \frac{z}{d}$. This is independent of whether the spatial curvature is zero or nonzero. Thus, in the early universe, $w = 1$ for renormalizable theory whereas $w > 1$ for superrenormalizable theory. The corresponding matter is sometimes referred to as (super)stiff.

We consider the general action given in [6] and study the evolution of a homogeneous anisotropic universe with $d$ spatial dimensions. We obtain an anisotropic Kasner-type solution in the limit where the universe is collapsing to zero size. The exponents in the corresponding scale factors satisfy two relations, one of them involving $\lambda$. The marginal coupling $\lambda$ may be different from 1 in the UV, and may even be close to $\frac{1}{d}$, where the theory may acquire a Weyl symmetry. Such a behavior of $\lambda$ and the presence of (super)stiff matter have interesting implications for the nature of collapse, which we explain briefly.

This paper is organized as follows. In Sec. II we present the setup. In Sec. III we discuss the dispersion relation and the consequent equation of state. In Sec. IV we present the equations of motion and some solutions, and briefly dis-

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cuss their implications. In Sec. V we conclude with a brief summary and a few comments.

## II. ANSATZ FOR ACTION AND METRIC

In Hořava's theory, the fields are the lapse function $N$, the shift vector $N^i$, and the spatial metric $g_{ij}$. The scaling dimensions of various quantities in momentum units are

$$[x^i] = -1, \qquad [t] = -z,$$
$$[N] = [g_{ij}] = 0, \qquad [N^i] = z - 1.$$

The action $S = S_K + S_V$ is required to contain no more than two time derivatives, and to be invariant under the scaling in Eq. (1) and foliation preserving diffeomorphism. The kinetic part $S_K$ of the action is then universal and may be written as

$$S_K = \frac{1}{2\kappa^2} \int dt d^d x N \sqrt{g} (K_{ij} K^{ij} - \lambda K^2) \qquad (2)$$

where $\kappa^2$ is a parameter with dimension $[\kappa^2] = z - d$, $\lambda$ is a dimensionless parameter, the spatial indices $i, j, \cdots = 1, 2, \cdots, d$ are to be lowered or raised using $g_{ij}$ or its inverse $g^{ij}$,

$$K_{ij} = \frac{1}{2N} (\dot{g}_{ij} - \nabla_i N_j - \nabla_j N_i), \qquad K = g^{ij} K_{ij},$$

and the covariant derivatives, as well as curvature tensors below, are all with respect to $g_{ij}$. For $z = d$, the parameter $\kappa$ becomes dimensionless and the theory is power-counting renormalizable; for $z > d$ it is superrenormalizable [1]. Our interest is in the $d = 3$ case, but most of the expressions below are valid for any value of $d$.

The potential part $S_V$ of the action contains various powers and spatial derivatives of the Riemann tensor $R_{ijkl}$, or equivalently of the Ricci tensor $R_{ij}$ in the $d = 3$ case. It suffices our purposes to write $S_V$ symbolically as

$$S_V = \int dt d^d x N \sqrt{g} \bigg( \sigma + \xi R + \sum_{n=2}^{n_*} \zeta_n R^n + \sum_{p,q=1}^{p_*, q_*} \beta_{pq} R \nabla^p R^q \bigg), \qquad (3)$$

where the first sum denotes various powers of the curvature tensor and the second denotes various derivatives acting on various powers of the curvature tensor. The upper limits of $(n, p, q)$ depend on the value of $z$. For the renormalizable case, for example, $z = d$ and $n_* = z$, $p_* + 2q_* + 2 = 2z$.

In [1], Hořava invokes the principle of detailed balance which will constrain the above form for $S_V$. For example, in the $d = 3$ case, $S_V$ will not contain $R^3$ terms and the coefficients of various terms in $S_V$ depend only on three new parameters. By construction, the resulting action is not the most general one. However, quantum corrections may not obey the principle of detailed balance and may induce other possible terms. This principle may even be problematic since the corresponding static spherically symmetric solutions reduce to the IR ones only on scales beyond the cosmological horizon, and not on smaller scales where Einstein's theory has been well tested. Rectifying this problem requires going beyond the detailed balance. See [5–7] and, in particular, [10] for detailed discussions of these issues.

For these reasons, we will not invoke detailed balance in this paper, and consider the general form for the action $S_V$. The most general form of $S_V$ for $z = d = 3$ is given, for example, in [6], where the corresponding equations of motion are also obtained. These equations are very long and, hence, are not presented here but will be used for the present cosmological study. Note that, for such a study, one can set $N = 1$ and $N^i = 0$ in the equations of motion with no loss of generality, and, also, that these equations are applicable for any value of $d$, as we later explain.

Here, we consider only a spatially curved, homogeneous, isotropic universe, or a spatially flat, homogeneous, anisotropic universe. The line element of a spatially curved, homogeneous, isotropic universe may be written as

$$ds^2 = -dt^2 + a^2 d\Sigma^2_{d,\hat{k}} \qquad (4)$$

where $a(t)$ is the scale factor, $d\Sigma_{d,\hat{k}}$ is the line element of a $d$-dimensional space of constant curvature, and $\hat{k} = +1$, $-1$, $0$ for positive, negative, or zero curvature. The Hubble parameter $H$ is defined by $H = \frac{\dot{a}}{a}$, where an overdot denotes the time derivative.

The line element of a spatially flat, homogeneous, anisotropic universe may be written as

$$ds^2 = -dt^2 + \sum_{i=1}^{d} a_i^2 (dx^i)^2, \qquad (5)$$

where $a_i(t)$ are the scale factors. The corresponding Hubble parameters $h_i$ are defined by $h_i = \frac{\dot{a}_i}{a_i}$. Also, define the geometric mean $a$ of the scale factors by $a^d = \prod_i a_i$. Then $H = \frac{\dot{a}}{a} = \frac{1}{d} \sum_i h_i$ is the average of the Hubble parameters $h_i$.

With the above definitions, the conservation equation for a matter source with pressure $p$ and density $\rho$ is given in both of the above cases by

$$\dot{\rho} + dH(\rho + p) = 0. \qquad (6)$$

If the equation of state is given by $p = w\rho$, then we have $\rho = \rho_0 a^{-d(1+w)}$, where $\rho_0$ is an initial value.

## III. (SUPER)STIFF MATTER

Consider now matter sources, e.g. radiation, and their equations of state. The matter action which is invariant under the scaling in Eq. (1) will lead to a modified dispersion relation in the UV, typically of the form $\omega^2 \sim k^{2z}$ [1,2]. From the standard statistical mechanical methods using such a dispersion relation, it follows that the dependence of free energy $F$ on temperature $T$ is of the form $F \sim T^{1+(d/z)}$ [2,17]. For renormalizable theories, $z = 1$ in the IR and $z = d$ in the UV. It then follows that $F \sim T^{1+d}$ at low temperatures and $F \sim T^{1+1}$ at high temperatures. As noted in [2], similar free-energy behavior at high temperatures appears also in string theory. We further note here that similar free-energy behavior, at both low and high temperatures, appears also in the context of a particular version of the generalized uncertainty principle [21].

With free energy $F \sim T^{1+(d/z)}$, it follows upon using thermodynamical relations that the corresponding equation of state is given by $p = w\rho$, where $w = \frac{z}{d}$. Thus for radiation in $d = 3$, we have $z = 1$ and $w = \frac{1}{3}$ in the IR. We have $z = d$ in the UV for renormalizable theories, which then implies that $w = 1$,[1] the corresponding matter sometimes referred to as stiff matter. Also, $z > d$ for super-renormalizable theories, which then implies that $w$ can be $>1$, the corresponding matter sometimes referred to as superstiff matter.

Such an UV dispersion relation, namely $\omega^2 \sim k^{2z}$, is ubiquitous in Hořava's theory and arises from an underlying principle: it is a consequence of invariance under the anisotropic scaling in Eq. (1). Also, it is independent of whether spatial curvature is zero or nonzero. Thus, Hořava's theory can be taken to predict that the early universe, and more generally the UV regime, is dominated by matter whose equation of state is given by $p = w\rho$, where $w = \frac{z}{d} = 1$ for renormalizable theories and $>1$ for superrenormalizable theories.[2]

## IV. EQUATIONS OF MOTION, SOLUTIONS, AND THEIR IMPLICATIONS

Consider now the equations of motion. They are given in [6] for $d = 3$. Consider, for any $d$, the general form of the contributions of various terms in the action to the equations of motion. For the cases of interest here, namely, where the line element is given by Eq. (4) or (5), we observe the following:

(i) A matter source with an equation of state $p = w\rho$ will contribute terms $\propto a^{-d(1+w)}$ in the equations of motion; see Eq. (6).

(ii) Consider terms of the form $R^n$ in $S_V$. For the isotropic case, it is easy to see that they contribute a term $\propto \hat{k}^n a^{-2n}$ in the equations of motion. It thus follows that such terms act as sources with equations of state $p = w\rho$, where $w = \frac{2n}{d} - 1$ and $\rho = C_n \hat{k}^n a^{-2n}$. The constant $C_n$ depends on the index structure of the $R^n$ terms and their coefficients in $S_V$. For the spatially flat case, $\hat{k} = 0$ and the corresponding contributions all vanish.
Note that $n = 1$ for the $R$ term and $w = \frac{2}{d} - 1 = -\frac{1}{3}$ for $d = 3$; $n = 2$ for the $R^2$ term and $w = \frac{4}{d} - 1 = \frac{1}{3}$ for $d = 3$; and, formally, $n = 0$ for the cosmological constant term and $w = -1$ for any value of $d$. For the term $R^{n_*}$ with the highest power of curvature, we have $n_* = z = d$ for the renormalizable case and $w = \frac{2n_*}{d} - 1 = 1$ for any value of $d$.

(iii) $R_{ijkl}$ for a constant curvature space is given in terms of $g_{ij}$, and the scale factor which depends on $t$ only. Hence covariant derivatives acting on curvature tensors will all vanish. Therefore the terms in the second sum in Eq. (3) do not contribute to the equations of motion.

(iv) The kinetic part $S_K$ of the action is universal for any values of $d$ and $z$. Hence, the corresponding terms in the equations of motion are just those given in [6].

Using observations (i)–(iv) above and the expressions given in [6], we can now write the equations of motion.

### A. Isotropic case

For the isotropic case, the metric is given in Eq. (4), and the equations of motion may be written as

$$d(\lambda d - 1)H^2 = 2\kappa^2 \sum \rho, \tag{7}$$

$$(\lambda d - 1)(\dot{H} + dH^2) = \kappa^2 \sum (\rho - p), \tag{8}$$

where $H = \frac{\dot{a}}{a}$ and $\dot{H} = \frac{\ddot{a}}{a} - (\frac{\dot{a}}{a})^2$. The sum $\sum$ in the equations above denotes contributions from the matter source, and also those from $R^n$ terms in $S_V$ for which $p = (\frac{2n}{d} - 1)\,\rho$ and $\rho = C_n \hat{k}^n a^{-2n}$, where the constant $C_n$ depends on the index structure of the $R^n$ terms and their coefficients in $S_V$. See [6] for explicit expressions for the $d = 3$ case.

We will assume in this paper that $\lambda d > 1$, since this is the case in Einstein's theory for which $\lambda = 1$, and also that $\lambda$ is not arbitrarily close to $\frac{1}{d}$. Replacing $\kappa^2$ by $(\frac{\lambda d - 1}{d - 1})\kappa^2$ then renders Eqs. (7) and (8) identical to those in Einstein's theory.

The evolution of the scale factor is then straightforward to understand. Let $a \to 0(\infty)$ in the limit $t \to 0(\infty)$. The evolution is then dictated by those sources for which $w$ is largest (smallest). If the total coefficient of the dominant sources is positive, then it follows that $a(t) \sim t^{2/d(1+w)}$.

If the spatial curvature is nonzero, then sources arising from curvature terms can have negative coefficients. Then the total coefficient of the dominant sources can be negative, and this will generically lead to a bounce in the evolution of $a(t)$. The details, and even the presence itself, of the bounce depend on the nature and strength of other

[1] That $w = 1$ for radiation in the UV is also pointed out in [18] which appeared while this paper was being written.

[2] The idea that the early universe must be dominated by $w \geq 1$ matter also appears in different contexts. For example, see [24,25] for the $w = 1$ case; see [26] and references therein for the $w > 1$ case.

sources present and can only be obtained by further analysis incorporating these data.

Let $z = d = 3$. Hořava's theory then predicts the existence of stiff matter in the UV for which $w = 1$. Assume the total coefficient of the sources with $w = 1$, which are the dominant ones in the limit $a \rightarrow 0$, to be positive. This will be the case for a spatially flat universe for which there are no contributions from curvature terms. It then follows that there is no bounce and that $a(t) \sim t^{1/3}$ in the limit $t \rightarrow 0$.

We now make a remark. It has been shown in [6,8,13,14] that, in Hořava's theory, the scale invariant primordial perturbation spectrum can be generated in the UV with an additional scalar field and *without requiring inflation*. Scale invariance arises, essentially, from the dispersion relation for the scalar field in the UV which is of the form $\omega^2 \sim k^6$. For the desired dynamics of the perturbations thus generated, it is also required that $H^2 a^6$ be an increasing function of $t$, or equivalently that $\int^\infty \frac{dt}{a^3}$ converge, which is taken to imply that the scale factor $a$ evolves as $\sim t^n$ with $n > \frac{1}{3}$. See [6,8,13,14] for details.

However, as described above, it is likely that $a(t) \sim t^{1/3}$ in the UV. Although this violates the requirement $n > \frac{1}{3}$, there may be no adverse effect on scale invariance of the spectrum since $H^2 a^6$ may still be an increasing function of $t$ because of the presence of other sources in Eq. (7) which will become important as $t$ increases. This is plausible but, nevertheless, it is desirable to study in detail the effects of $a(t) \sim t^{1/3}$ in the UV on the scale invariance of the spectrum obtained in [6,8,13,14] in Hořava's theory without requiring inflation.

### B. Anisotropic case

For the spatially flat anisotropic case, the metric is given in Eq. (5). There are no contributions from the $R^n$ terms, and the equations of motion may be written as

$$\lambda d^2 H^2 - \sum_i (h_i)^2 = 2\kappa^2 \rho, \tag{9}$$

$$(\lambda d - 1)(\dot{h}_i + dHh_i) = \kappa^2(\rho - p), \tag{10}$$

where $h_i = \frac{\dot{a}_i}{a_i}$, $H = \frac{1}{d}\sum_i h_i$, and $\dot{h}_i = \frac{\ddot{a}_i}{a_i} - (\frac{\dot{a}_i}{a_i})^2$. Note that summing Eq. (10) over $i$ gives

$$(\lambda d - 1)(\dot{H} + dH^2) = \kappa^2(\rho - p). \tag{11}$$

We have $H = \frac{\dot{a}}{a}$ from the definition $a^d = \prod_i a_i$. It then follows that[3]

$$h_i - H = A_i a^{-d}, \tag{12}$$

$$d(\lambda d - 1)H^2 = 2\kappa^2\rho + A^2 a^{-2d}, \tag{13}$$

where $A_i$ are initial values satisfying $\sum_i A_i = 0$ and $A^2 = \sum_i (A_i)^2$. Once the equation of state $p(\rho)$ is given, then, in principle, $\rho(a)$ can be obtained from Eq. (6), $a(t)$ from Eq. (13), and $a_i(t)$ from Eq. (12). Note that the initial values $A_i$ encode anisotropic initial conditions, e.g. during a collapse, and also that $A_i$ may be thought of as a source with equation of state $p = w\rho$, where $w = 1$ and $\rho_0 = \frac{A^2}{2\kappa^2}$; see Eqs. (6) and (13).

Consider now the dynamics of the evolution, assuming the equation of state to be $p = w\rho$. Equation (6) then implies that $\rho = \rho_0 a^{-d(1+w)}$, where $\rho_0 > 0$ is an initial value. The evolution in the limit $a \rightarrow \infty$, namely, the large universe limit, is similar to the standard one where $a \sim t^{2/d(1+w)}$. The effect of $\lambda$ is unimportant unless $\lambda$ is arbitrarily close to $\frac{1}{d}$.

Consider a universe collapsing to zero size, i.e. $a \rightarrow 0$, as $t \rightarrow 0$. Let the scale factors $a_i \sim t^{\alpha^i}$ in this limit. We study the following cases:

$$w > 1.$$

In the limit $a \rightarrow 0$, $2\kappa^2\rho \sim a^{-d(1+w)} \gg A^2 a^{-2d}$ in Eq. (13) since $w > 1$. It is then straightforward to show that

$$a \sim t^{2/d(1+w)}, \qquad a_i = c_i e^{ct^{(w-1)/(w+1)}} t^{2/d(1+w)} \tag{14}$$

where $c_i$ and $c$ are constants. Thus, since $t^{(w-1)/(w+1)} \rightarrow 0$ in the limit $t \rightarrow 0$, it follows that the exponents $\alpha^i$ in $a_i \sim t^{\alpha^i}$ are all equal, and are independent of the initial values $A^i$. Hence, the collapse is isotropic and stable under perturbations [26],

$$w \leq 1.$$

In the limit $a \rightarrow 0$, the right-hand side of Eq. (13) becomes $B^2 a^{-2d}$, where $B^2 = A^2$ if $w < 1$ and $B^2 = 2\kappa^2\rho_0 + A^2$ if $w = 1$. It is then straightforward to show that

$$a \sim t^{1/d}, \qquad a_i \sim t^{\alpha^i}, \qquad \alpha^i = \frac{1}{d} - \frac{A^i}{B}\left(\lambda - \frac{1}{d}\right)^{1/2}. \tag{15}$$

This is a Kasner-type solution. The exponents $\alpha^i$ depend on initial values $A^i$ and, since $\sum_i A^i = 0$, satisfy the relations

$$\sum_i \alpha^i = 1, \qquad X \equiv \sum_i (\alpha^i)^2 = \frac{1}{d} + \left(\lambda - \frac{1}{d}\right)\frac{A^2}{B^2}. \tag{16}$$

For $w < 1$ we have $B^2 = A^2$, hence $X = \lambda$ is the only possible value. For $w = 1$ we have $B^2 = 2\kappa^2\rho_0 + A^2$, hence $0 < \frac{A^2}{B^2} < 1$ and $\frac{1}{d} < X < \lambda$. Clearly $X \rightarrow \lambda$ if $2\kappa^2\rho_0 \ll A^2$. Also $X \rightarrow \frac{1}{d}$ if $2\kappa^2\rho_0 \gg A^2$, which is the only possibility in Einstein's theory, or if $\lambda \rightarrow \frac{1}{d}$, which is a new possibility in Hořava's theory and is valid for any values of $\rho_0$ and $A^2$.

[3] Equations (10) and (11) give Eq. (12). Using $\sum_i h_i = dH$ gives the constraint $\sum_i A_i = 0$. Substituting $h_i$ in Eq. (9) then gives Eq. (13).

Kasner-type solutions, in particular, the value of $X$ and the dependence of $\alpha^i$ on initial values, provide an insight into the stability of the cosmological collapse process under generic curvature and/or anisotropic perturbations: namely, an insight into whether the collapse will be isotropic or anisotropic, whether it will be smooth or will exhibit chaotic oscillatory behavior, etc.

Since $\alpha_i$ depend on the initial values $A_i$, the collapse will be generically anisotropic. Consider the $d = 3$ case. The exponents $\alpha_i$ must satisfy the constraints in Eq. (16). If $X = 1$, then one of the $\alpha_i$ must be negative. Then, under curvature perturbations, the collapse will not be smooth and will exhibit chaotic behavior. If $X$ is sufficiently close to $\frac{1}{3}$, then no $\alpha_i$ can be negative and the collapse will be stable and non-oscillatory under perturbations.

In Einstein's theory $\lambda = 1$ and, hence, smaller values of $X$ may result only through smaller values of $\frac{A^2}{2\kappa^2\rho_0}$, which necessarily requires stiff matter with $w = 1$. Stability also results if superstiff matter with $w > 1$ is present as follows from Eq. (14). See [26] and the references therein for a thorough discussion of these issues.

In Hořava's theory, on the other hand, $\lambda$ is typically different from 1 in the UV. This theory may acquire an anisotropic Weyl symmetry if $\lambda = \frac{1}{d}$, so it is possible that $\lambda \to \frac{1}{d}$ in the UV. If so, then $X$ may be naturally small even without stiff matter. But (super)stiff matter with $w \geq 1$ is also naturally present in this theory. This makes it more likely that the collapse process is stable and non-oscillatory.

However, spatial curvature terms of high order are also allowed in Hořava's theory. As explained in remark (ii) in Sec. IV, this order is closely linked to the scaling exponent $z$, to which is also linked the presence of (super)stiff matter. Curvature terms typically lead to destabilizing effects under curvature perturbations, and our preliminary analysis indicates that they may be comparable to the stabilizing effects of (super)stiff matter. However, there is also a stabilizing effect that results if $\lambda$ is close to $\frac{1}{d}$ in the UV, but no comparable, $\lambda$-dependent, destabilizing curvature effects seem to be present. It is therefore possible that the sum total of all these effects in Hořava's theory results in a stable and non-oscillatory collapse. Clearly, further analysis is necessary but it is complicated and quite involved, and is beyond the scope of the present work.

## V. CONCLUSION

We now summarize briefly and make a few comments. Our main motivation for the present study is to find the implications of Hořava's theory which differ from those of Einstein's theory and which are not crucially dependent on the spatial curvature. The implications of Hořava's theory we find which differ from those of Einstein's theory are as follows: (1) The UV regime in the (super)renormalizable case is dominated by (super)stiff matter, namely, matter with $w \geq 1$. (2) $\sum_i (\alpha^i)^2$, where $\alpha^i$ are the Kasner exponents, can be different from and smaller than 1 even without stiff matter present. These two implications are generic. (3) Equations of motion contain curvature terms of the form $C_n \hat{k}^n a^{-2n}$, where $n \leq d$ for the renormalizable case. The constants $C_n$ can be positive or negative. The curvature terms may hence lead to a bounce in the evolution of the scale factor $a(t)$. The bounce is possible but nongeneric since it depends on the sign and the magnitude of $C_n \hat{k}^n$ as well as the nature and strength of other sources present.

We now make a few comments. Hořava's theory is believed to be UV complete. Also, it contains higher powers and derivatives of curvature. It is then reasonable to expect that this theory will also resolve the singularities. The big bang singularity is indeed absent if $a(t)$ bounces back. But the bounce is not a generic feature. If there is no bounce, then, as can be inferred from the present solutions, the big bang singularity is present. Note that a black hole singularity is also present in all the static spherically symmetric solutions to Hořava's theory studied so far (see, e.g., [7,10–12,15]), although these solutions do differ from those in Einstein's theory.

It is possible that, regarding the presence or absence singularities, this is the most one can see using classical action in Hořava's theory and that quantization of the action is necessary to see any further.

There is a similar situation in string/M theory. No classical string/M theory action so far has led to the generic absence of singularities. However, it is likely that when the temperatures become comparable to string theory scales, the classical description of the universe, and similarly of black holes, must be abandoned and a string theoretic description should be used; see [27,28]. But the details of such a description, or of how exactly the singularities get resolved, are not fully known at present. However, due to entropic reasons, the universe at this stage seems likely to be dominated by stiff matter for which $w = 1$ [24,25].[4]

In this context, note that Hořava's theory also predicts generically the presence of stiff matter in the early universe. The presence of such matter and/or the scaling arguments using Eq. (1) lead to the high temperature behavior of the free energy $F \sim T^2$ [2]. This behavior may be taken to signify that the spectral dimension of the spacetime in the UV is $1 + 1$. This is shown to be the case for Hořava's theory in [2]. Such an UV spectral dimension is also observed in many candidate theories for quantum

[4]The nature and quantum mechanical properties of stiff matter, and those of a universe dominated by stiff matter, as well as further evolution of such a universe are studied in a series of papers in [24]. A possible string/M theory scenario of how such a universe may arise is also given in [25]. Also, the properties of stars made up of stiff matter and their similarities to black holes are studied in [29].

gravity, e.g. causal dynamical triangulations, quantum Einstein gravity, spin foam theory, and string theory. This may perhaps be the case also for loop quantum gravity. It is indeed argued in [5] that two-dimensional effective gravitational theories in the UV may be a generic feature of UV complete theories of gravity. See [5] for more discussions and an extensive list of references.

If these similarities are more than just coincidences, then it may be that there are also similarities in the ways in which singularities get resolved in any of these theories and in Hořava's theory. It is therefore important to study if singularities can be resolved by quantizing the action in Hořava's theory. Such a resolution, besides being important on its own, may also provide insights into the theories mentioned above.

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