Primordial "f_{NL}" non-Gaussianity and perturbations beyond the present horizon

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We show a primordial nonlinear ("f_{NL}") term may produce unphysically large CMB anisotropy for a red-tilted primordial power spectrum ($n < 1$) because of coupling to primordial fluctuation on the largest scale. We consider a primordial power spectrum models of a running spectral index, and a transition at very low wave numbers. We find that only negative running spectral index models are allowed, provided that there is no transition at a low wave numbers (i.e. $k \ll 1$). For models of a constant spectral index, we find $\log(k_c/k_0) \gtrsim -184$, at 1σ level, on the transition scale of sharp cutoff models, using recent CMB and SDSS data.

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I. INTRODUCTION

Recently, there have been great successes in measurement of cosmic microwave background (CMB) anisotropy by ground and satellite observations [1–8]. The 5 yr data of the Wilkinson Microwave Anisotropy Probe (WMAP) [1–3] is released and the recent ground-based CMB observations such as the ACBAR [4,5] and QUaD [6–8] provide information complementary to the WMAP data. In the near future, PLANCK surveyor [9,10] is going to measure CMB temperature and polarization anisotropy with great accuracy over a wide range of angular scales. Using the observational data, we are able to impose strong constraints on cosmological models [11–13], and, in particular, on the class of inflation models of very large number of e-folds $N_e \gg 100$. They may provide a new window on physics beyond the Planck scale [14,15]. Another important feature of the recent CMB observations is the testing of the non-Gaussianity and statistical anisotropy of the CMB sky [16–27]. They provide a unique opportunity to test the modern theories of inflation through the observational data (see for review [28]).

The fluctuations of the gravitational potential $\Phi(\mathbf{x})$ (equivalent to Bardeen's gauge invariant variable Φ_H [29]) is related to primordial perturbation in complicated ways [30,31]. When considered up to second order, there exists a nonlinear term $f_{\mathrm{NL}}\Phi_{\mathrm{L}}^2(\mathbf{x})$, where f_{NL} is a coupling constant or some even function of the spatial coordinates. The nonlinear term $f_{\mathrm{NL}}\Phi_{\mathrm{L}}^2(\mathbf{x})$ leads to coupling between largest scales and scales relevant to observable Universe. The recent constraint of the WMAP data shows $f_{\mathrm{NL}} \sim 60 \pm 30$ (see [31,32,32–34] for the recent analysis).

Much of studies have been focused on the behavior of the primordial power spectrum on small scales. In this paper, we focus on the "infrared" asymptotic behavior of a red-tilted ($n < 1$) primordial power spectrum. Given the red-tilted primordial power spectra [31,35], coupling to the fluctuation on largest scales may produce unphysically large CMB anisotropy, which could be in disagreement with CMB observational data. There have been attempts to remove the singularity of a primordial power spectrum by renormalization [36], and the author notes that the residual k-dependent term, which is not removed by renormalization, is negligible for observable scales of galaxy surveys. However, we note the residual k-dependent term may produce very large excess power on the low multipole CMB anisotropy ($l \leq 10$), which are associated with scales much larger than galaxy surveys. Not to produce unphysical excess power for CMB anisotropy, we require a primordial power spectrum to satisfy 1) a spectral index of negative running or 2) a transition at very large scale (e.g. sharp cutoff in the power spectrum at a very low wave number). We find at least one of them should be satisfied to make agreement with the recent CMB observational data. As will be discussed in this paper, the imprints of the largest scales due to the $f_{\mathrm{NL}}\Phi_{\mathrm{L}}^2(\mathbf{x})$ term may improve our understanding on the properties of primordial perturbations on the scales larger than the present particle horizon.

The outline of this paper is as follows: In Sec. II, we discuss the primordial power spectrum associated with a primordial nonlinear ("f_{NL}") term. In Sec. III, we discuss the effect of a f_{NL} term on CMB power spectra. In Sec. IV, we show the primordial power spectrum should satisfy some requirement not to produce unphysically large CMB power spectra. In Sec. V, we summarize our investigation and discuss prospects.

II. THE EFFECT OF THE f_{NL} TERM ON A PRIMORDIAL POWER SPECTRUM

Up to second order, primordial perturbation is given by [31,37–39]:

$$\Phi(\mathbf{x}) = \Phi_{\mathrm{L}}(\mathbf{x}) + f_{\mathrm{NL}}[\Phi_{\mathrm{L}}^2(\mathbf{x}) - \langle\Phi_{\mathrm{L}}^2(\mathbf{x})\rangle], \tag{1}$$

where $\Phi_{\mathrm{L}}(\mathbf{x})$ is a linear part of primordial perturbation, and f_{NL} is the nonlinear coupling parameter. The last term on the right-hand side ensures $\langle\Phi(\mathbf{x})\rangle = 0$, and is given by

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$$\langle\Phi_{\rm L}^2(\mathbf{x})\rangle = \int P_\Phi(k)\frac{d^3\mathbf{k}}{(2\pi)^3},$$

where

$$P_\Phi(k) = \frac{\Delta^2_{\mathcal{R}}(k)}{k^3}.$$

$\Delta^2_{\mathcal{R}}(k)$ is the variance of curvature perturbation per logarithmic interval $d\ln k$ [12,31]. Using Eq. (1), we find primordial perturbation in Fourier space

$$\Phi(\mathbf{k}) = \Phi_{\rm L}(\mathbf{k}) + \Phi_{\rm NL}(\mathbf{k}), \tag{2}$$

where

$$\Phi_{\rm NL}(\mathbf{k}) = f_{\rm NL}\Big(\int \Phi_{\rm L}(\mathbf{k}+\mathbf{p})\Phi^*_{\rm L}(\mathbf{p})\frac{d^3\mathbf{p}}{(2\pi)^3} - (2\pi)^3\delta(\mathbf{k})\langle\Phi_{\rm L}^2(\mathbf{x})\rangle\Big). \tag{3}$$

In most of inflationary models, $\Phi_{\rm L}(\mathbf{k})$ follows a Gaussian distribution [11–13,31,38,39], and hence have the following statistical properties:

$$\langle\Phi_{\rm L}(\mathbf{k})\rangle = 0, \tag{4}$$

$$\langle\Phi_{\rm L}(\mathbf{k})\Phi^*_{\rm L}(\mathbf{k}')\rangle = (2\pi)^3 P_\Phi(k)\delta(\mathbf{k}-\mathbf{k}'), \tag{5}$$

$$\langle\Phi_{\rm L}(\mathbf{k}+\mathbf{p})\Phi^*_{\rm L}(\mathbf{p})\rangle = (2\pi)^3\delta(\mathbf{k})P_\Phi(p), \tag{6}$$

$$\begin{aligned}&\langle\Phi_{\rm L}(\mathbf{k}+\mathbf{p})\Phi^*_{\rm L}(\mathbf{p})\Phi^*_{\rm L}(\mathbf{k}'+\mathbf{p}')\Phi_{\rm L}(\mathbf{p}')\rangle\\ &= (2\pi)^6\times[P_\Phi(k+p)P_\Phi(p)\delta(\mathbf{k}-\mathbf{k}')(\delta(\mathbf{p}-\mathbf{p}')\\ &\quad+\delta(\mathbf{k}+\mathbf{p}+\mathbf{p}')) + \delta(\mathbf{k})\delta(\mathbf{k}')P_\Phi(p)P_\Phi(p')].\end{aligned} \tag{7}$$

Using Eqs. (3)–(7), we may easily show

$$\langle\Phi_{\rm L}(\mathbf{k})\Phi^*_{\rm NL}(\mathbf{k}')\rangle = 0, \tag{8}$$

$$\langle\Phi_{\rm NL}(\mathbf{k})\Phi^*_{\rm NL}(\mathbf{k}')\rangle = 8\pi(f_{\rm NL})^2\delta(\mathbf{k}-\mathbf{k}')\times\Big[\int P_\Phi(k+p)P_\Phi(p)p^2dp\Big]. \tag{9}$$

Finally, by using Eq. (2), (5), (8), and (9) we find

$$\langle\Phi^*(\mathbf{k})\Phi(\mathbf{k}')\rangle = (2\pi)^3[P_\Phi(k) + P_{\Phi,\rm NL}(k)]\delta(\mathbf{k}-\mathbf{k}'),$$

where

$$P_{\Phi,\rm NL}(k) = \frac{(f_{\rm NL})^2}{\pi^2}\int P_\Phi(k+k')P_\Phi(k')k'^2dk'. \tag{10}$$

III. THE EFFECT OF THE $f_{\rm NL}$ TERM ON CMB POWER SPECTRA

The Stokes parameters of CMB anisotropy are conveniently decomposed in terms of spin 0 and spin ± 2 spherical harmonics

$$T(\hat{\mathbf{n}}) = \sum_{lm} a_{T,lm}Y_{lm}(\hat{\mathbf{n}}),$$

$$Q(\hat{\mathbf{n}}) \pm iU(\hat{\mathbf{n}}) = \sum_{l,m} -(a_{E,lm}\pm i a_{B,lm})_{\pm 2}Y_{lm}(\hat{\mathbf{n}}),$$

where $a_{T,lm}$, $a_{E,lm}$, and $a_{B,lm}$ are decomposition coefficients. The decomposition coefficients are related to primordial perturbations as

$$a_{T,lm} = 4\pi(-\imath)^l\int\frac{d^3\mathbf{k}}{(2\pi)^3}\Phi(\mathbf{k})g_{Tl}(k)Y^*_{lm}(\hat{\mathbf{k}}), \tag{11}$$

$$a_{E,lm} = 4\pi(-\imath)^l\int\frac{d^3\mathbf{k}}{(2\pi)^3}\Phi(\mathbf{k})g_{El}(k)Y^*_{lm}(\hat{\mathbf{k}}), \tag{12}$$

$$a_{B,lm} = 4\pi(-\imath)^l\int\frac{d^3\mathbf{k}}{(2\pi)^3}\Phi(\mathbf{k})g_{Bl}(k)Y^*_{lm}(\hat{\mathbf{k}}), \tag{13}$$

where $g_{Tl}(k)$, $g_{El}(k)$, and $g_{Bl}(k)$ are the radiation transfer functions and can be numerically computed by a computer software CAMB [40]. In the absence of tensor perturbation, CMB power spectra are given by

$$C_l^{TT} = \frac{2}{\pi}\int k^2dk[P_\Phi(k) + P_{\Phi,\rm NL}(k)]g^2_{Tl}(k), \tag{14}$$

$$C_l^{EE} = \frac{2}{\pi}\int k^2dk[P_\Phi(k) + P_{\Phi,\rm NL}(k)]g^2_{El}(k), \tag{15}$$

$$C_l^{TE} = \frac{2}{\pi}\int k^2dk[P_\Phi(k) + P_{\Phi,\rm NL}(k)]g_{Tl}(k)g_{El}(k), \tag{16}$$

where $P_{\Phi,\rm NL}(k)$ is the primordial power spectrum associated with the $f_{\rm NL}$ term and given by Eq. (10). Note that CMB anisotropy, excluding the dipole, is sensitive to primordial perturbation of wave numbers $k \gtrsim 2/\eta_0$, where η_0 is the present conformal time.

IV. THE SHAPE OF A PRIMORDIAL POWER SPECTRUM

Inflation models predict the power spectrum of primordial perturbation nearly follow a power law [3,11–13,31,35,41,42]. Since fluctuations, which were once on sub-Planckian scales, are stretched to the observable scales by inflation, we need to consider trans-Planckian effects on a primordial power spectrum [14,43–49]. Since trans-Planckian corrections are highly model dependent [50,51], we consider general forms of trans-Planckian correction. Following the approach of the WMAP team, we model the variance of curvature perturbation $\Delta^2_{\mathcal{R}}(k)$ by two general forms [35]:

$$\Delta^2_{\mathcal{R}}(k) = A_0\Big(\frac{k}{k_0}\Big)^{n-1}\Big[1 + \epsilon_{\rm TP}\cos\Big(\upsilon\frac{k}{k_0} + \phi\Big)\Big], \tag{17}$$

$$\Delta^2_{\mathcal{R}}(k) = A_0\left(\frac{k}{k_0}\right)^{n-1}\left[1 + \epsilon_{\rm TP}\cos\left(\upsilon \ln\frac{k}{k_0} + \phi\right)\right], \quad (18)$$

where the pivot scale k_0 is set to the WMAP team's pivot scale 0.002/Mpc [35], and the spectral index n is given by

$$n = n(k_0) + \frac{1}{2}\frac{dn}{d\ln k}\ln\left(\frac{k}{k_0}\right).$$

$\epsilon_{\rm TP}$, υ, and ϕ are the amplitude, the frequency, and the phase of trans-Planckian effect. We denote the spectrum in Eq. (17) and (18) respectively as "model I" and "model II," which differ in the parametrized form of trans-Planckian corrections. Using Eqs. (10), (17), and (18), we find

$$P_{\Phi,\rm NL}(k) = \frac{(f_{\rm NL})^2}{\pi^2}A_0^2\int_0^\infty \frac{dk'}{k_0}\left(\frac{k+k'}{k_0}\right)^{n-4} \times [1 + \epsilon_{\rm TP}\cos\theta(k+k')]\left(\frac{k'}{k_0}\right)^{n-2} \times [1 + \epsilon_{\rm TP}\cos\theta(k')], \quad (19)$$

where $\theta(k) = \upsilon\frac{k}{k_0} + \phi$ for model I and $\theta = \upsilon\ln\frac{k}{k_0} + \phi$ for model II.

Most of inflationary models predict that a primordial spectrum is slightly red-tilted (i.e. $n(k_0) < 1$) [12,13], which is in good agreement with observations [31]. Given a slightly red-tilted spectral index $n < 1$, $P_{\Phi,\rm NL}(k)$ shown in Eq. (19) may approach an infinity, therefore producing unphysically large CMB power spectra [see Eqs. (14)–(16)]. CMB power spectra, which are finite physical observables, are well measured by recent satellite and ground observations [1,2,4,6–8,52]. Not to produce unphysical excess power, a primordial power spectrum should satisfy some condition, which will be discussed in the following subsections.

A. Running spectral index

We consider a running spectral index (i.e. $dn/d\ln k \neq 0$). Since significant contribution to the integral comes from $k'/k_0 \ll 1$, we find $P_{\Phi,\rm NL}(k)$ for $k \gtrsim 2/\eta_0$ and model I:

$$P_{\Phi,\rm NL}(k) \approx \frac{(f_{\rm NL})^2}{\pi^2}A_0^2\left(\frac{k}{k_0}\right)^{n-4}\left[1 + \epsilon_{\rm TP}\cos\left(\upsilon\frac{k}{k_0} + \phi\right)\right] \times \int_0^\infty\left(1 + \epsilon_{\rm TP}\cos\phi - \epsilon_{\rm TP}\upsilon\sin\phi\frac{k'}{k_0}\right) \times \left(\frac{k'}{k_0}\right)^{n-2}\frac{dk'}{k_0}. \quad (20)$$

Note that we have set $k_{\rm max}$ to ∞, because the integrand converges to zero for $k'/k_0 \gg 1$. If $dn/d\ln k < 0$, Eq. (20) is given by

$$P_{\Phi,\rm NL}(k) \approx \frac{(f_{\rm NL})^2}{\pi^2}A_0^2\left(\frac{k}{k_0}\right)^{n-4}\left[1 + \epsilon_{\rm TP}\cos\left(\upsilon\frac{k}{k_0} + \phi\right)\right] \times\sqrt{\frac{2\pi}{-\alpha}}\left[(1 + \epsilon_{\rm TP}\cos\phi)\exp\left(-\frac{(n-1)^2}{2\alpha}\right) - \epsilon_{\rm TP}\upsilon\sin\phi\exp\left(-\frac{n^2}{2\alpha}\right)\right],$$

where $\alpha = dn/d\ln k$. On the other hand, if $dn/d\ln k \geq 0$ and the lower integration bound $k_{\rm min} \to 0$, Eq. (20) approaches an infinity $P_{\Phi,\rm NL}(k) \to \infty$. Hence, we see that $dn/d\ln k < 0$ is required to keep CMB power spectra finite.

For $k \gtrsim 2/\eta_0$ and model II, we find

$$\begin{aligned} P_{\Phi,\rm NL}(k) &\approx \frac{(f_{\rm NL})^2}{\pi^2}A_0^2\left(\frac{k}{k_0}\right)^{n-4}\left[1 + \epsilon_{\rm TP}\cos\left(\upsilon\log\frac{k}{k_0} + \phi\right)\right] \\ &\quad\times\int_0^\infty\left[1 + \epsilon_{\rm TP}\cos\left(\upsilon\log\frac{k}{k_0} + \phi\right)\right]\left(\frac{k'}{k_0}\right)^{n-2}\frac{dk'}{k_0} \\ &= \frac{(f_{\rm NL})^2}{\pi^2}A_0^2\left(\frac{k}{k_0}\right)^{n-4}\left[1 + \epsilon_{\rm TP}\cos\left(\upsilon\log\frac{k}{k_0} + \phi\right)\right] \\ &\quad\times\int_{-\infty}^\infty[1 + \epsilon_{\rm TP}\cos(\upsilon x + \phi)]e^{x(n-1)}dx, \end{aligned}$$

where $x = \log(k'/k_0)$. We have also set $k_{\rm max}$ to ∞, because the integrand converges to zero for $k'/k_0 \gg 1$.

If $dn/d\ln k < 0$, we get

$$P_{\Phi,\rm NL}(k) \approx \frac{(f_{\rm NL})^2}{\pi^2}A_0^2\left(\frac{k}{k_0}\right)^{n-4}\left[1 + \epsilon_{\rm TP}\cos\left(\upsilon\frac{k}{k_0} + \phi\right)\right] \times\sqrt{\frac{2\pi}{-\alpha}}\exp\left(-\frac{(n-1)^2}{2\alpha}\right) \times\left[1 + \epsilon_{\rm TP}\cos\left(\phi - \frac{(n-1)\upsilon}{\alpha}\right)\exp\left(\frac{\upsilon^2}{2\alpha}\right)\right],$$

where $\alpha = dn/d\ln k$. On the other hand, if $dn/d\ln k \geq 0$ and the lower integration bound $k_{\rm min} \to 0$, Eq. (20) also approach an infinity $P_{\Phi,\rm NL} \to \infty$. Therefore, we require $dn/d\ln k < 0$ in running spectral index models. Many inflationary models predict a negative $dn/d\ln k$, and hence meet the requirement, while a few inflationary models fail to satisfy the requirement. For instance, the model of a mass term potential $V(\phi)/V_0 = 1 \pm 4\pi G|c|\phi^2$ predicts $dn/d\ln k = 0$, and the model of the potential $V(\phi)/V_0 = 1 - 4\pi G|c|\phi^2\ln\phi/|Q|$, which belongs to the class of a softly broken supersymmetry models, predicts $dn/d\ln k > 0$. Therefore, these models are in disagreement with observations, provided the primordial power spectrum in Eqs. (17) and (18) are valid up to the largest spatial scale.

B. Sharp cutoff in the primordial power spectrum

We consider a model of a constant spectral index (i.e. $dn/d\ln k = 0$), and discuss some requirements to avoid unphysically large $P_{\rm NL}$. We may consider a transition in the shape of the primordial power spectrum at a very low wave number, below which Eq. (17) or (18) are no longer valid. For instance, the WMAP team have considered a model of a sharp cutoff, and found that the cutoff at $k_c \sim 3 \times 10^{-4}/\mathrm{Mpc}$ makes a slightly better fit [35]. For $k > k_c$, we may write Eq. (19) as follows:

$$\begin{aligned} P_{\Phi,\mathrm{NL}}(k) = \frac{(f_{\mathrm{NL}})^2 A_0}{\pi^2}\bigg(&\int_0^{k_c} \frac{dk'}{k_0}\left(\frac{k+k'}{k_0}\right)^{n-4} \\ &\times [1+\epsilon_{\mathrm{TP}}\cos\theta(k+k')]P_\Phi(k')\left(\frac{k'}{k_0}\right)^2 \\ &+ A_0\int_{k_c}^{k_{\max}} \frac{dk'}{k_0}\left(\frac{k+k'}{k_0}\right)^{n-4} \\ &\times [1+\epsilon_{\mathrm{TP}}\cos\theta(k+k')]\left(\frac{k'}{k_0}\right)^{n-2} \\ &\times [1+\epsilon_{\mathrm{TP}}\cos\theta(k')]\bigg), \end{aligned} \tag{21}$$

where $\theta(k) = \upsilon \frac{k}{k_0} + \phi$ for model I and $\theta = \upsilon \ln\frac{k}{k_0} + \phi$ for model II.

Just as the WMAP team did, we consider a sharp cutoff at k_c, and set $P_\Phi(k') = 0$ for $k' < k_c$. However, $P_\Phi(k')$ of $k' < k_c$ may take on some nonzero value, though they are assumed to differ significantly from Eqs. (17) and (18). Therefore, our estimate on a transition scale should be interpreted as a lower bound, since a true $P_{\Phi,\mathrm{NL}}(k)$ is more likely to be higher than that of our sharp cutoff model, and a higher k_c is needed to make a true $P_{\Phi,\mathrm{NL}}(k)$ equal to that of our sharp cutoff model. In our sharp cutoff model, Eq. (21) is given by

$$\begin{aligned} P_{\Phi,\mathrm{NL}}(k) = \frac{(f_{\mathrm{NL}})^2}{\pi^2}A_0^2 &\int_{k_c}^{k_{\max}} \frac{dk'}{k_0}\left(\frac{k+k'}{k_0}\right)^{n-4} \\ &\times [1+\epsilon_{\mathrm{TP}}\cos\theta(k+k')]\left(\frac{k'}{k_0}\right)^{n-2} \\ &\times [1+\epsilon_{\mathrm{TP}}\cos\theta(k')]. \end{aligned} \tag{22}$$

We have found that $P_{\Phi,\mathrm{NL}}(k)$ of $k \gtrsim 2/\eta_0$ is barely affected by the value of $k_{\max}$, as long as $\log(k_{\max}/k_0) \gtrsim 5$. Hence, we have fixed $k_{\max}$ to $\log(k_{\max}/k_0) = 10$, and numerically computed Eq. (22) by the Romberg integration method [53].

By making a small modification to CAMB, we have computed theoretical CMB and matter power spectrum,

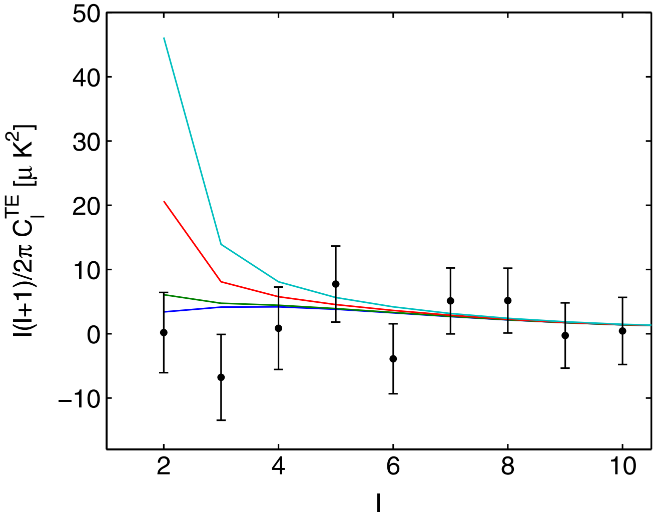

FIG. 2 (color online). CMB TE correlation of $\log(k_c/k_0) = -120, -100, -60, -5$ (from the highest curve to the lowest), dots denote the WMAP data.

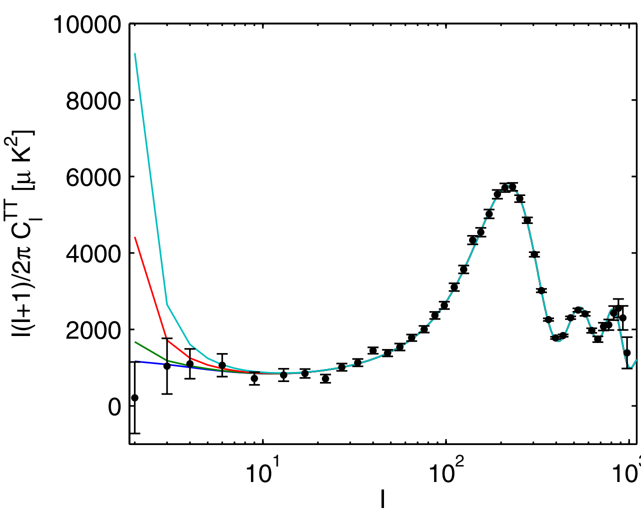

FIG. 1 (color online). CMB temperature power spectra of $\log(k_c/k_0) = -120, -100, -60, -5$ (from the highest curve to the lowest), dots denote the WMAP and the ACBAR data.

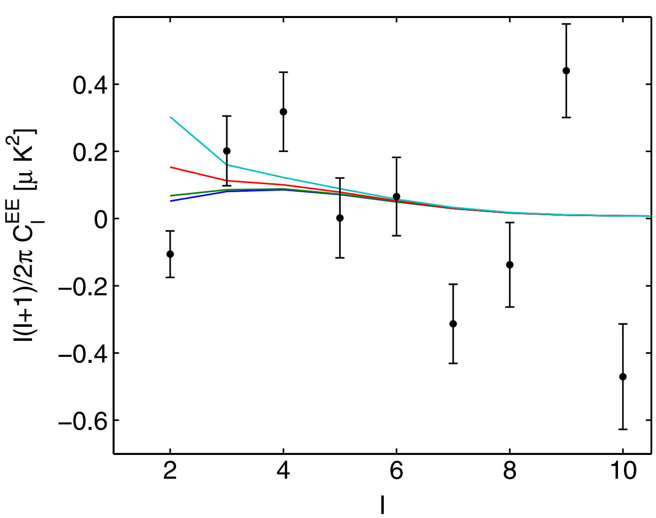

FIG. 3 (color online). E mode power spectrum of $\log(k_c/k_0) = -120, -100, -60, -5$ (from the highest curve to the lowest), dots denote the WMAP data.

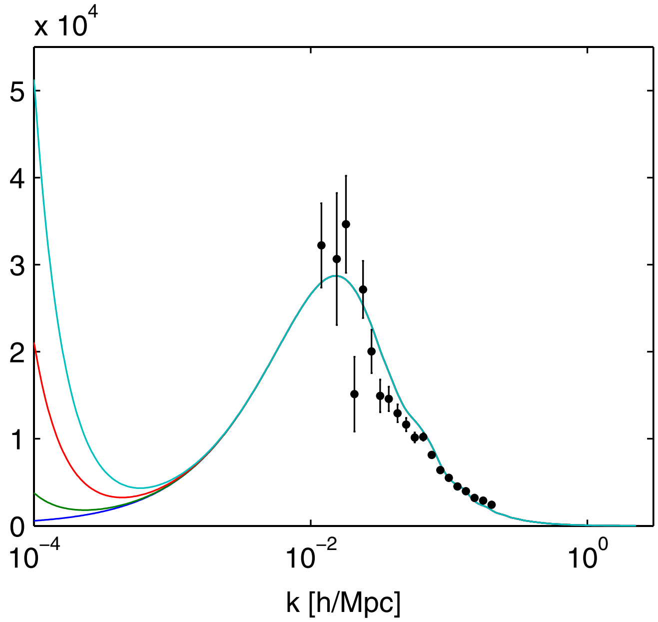

FIG. 4 (color online). matter power spectrum of $\log(k_c/k_0) = -120, -100, -60, -5$ (from the highest to the lowest), dots denote SDSS data.

in which $P_{\Phi,\rm NL}(k)$ is taken into account. We show the theoretical CMB power spectra, TE correlation in Fig. 1–3. The dots in the same plots denote the WMAP [2] and the ACBAR data [5]. We may see that anisotropy on largest scales ($l \lesssim 10$) is affected by $P_{\Phi,\rm NL}$ most. For an E-mode power spectrum and TE correlations, we show only low multipoles, since there is no visible effect on higher multipoles. We show a theoretical matter power spectrum and SDSS data in Fig. 4. It also shows that matter inhomogeneities on largest scales ($k \lesssim 10^{-3} h/{\rm Mpc}$) are affected by $P_{\rm NL}$ most. As also noted by [36], this excess power is, however, negligible on observable scales of the SDSS survey.

Using a modified CAMB and COSMOMC [40,54], we have estimated k_c, respectively, for model I and model II. For data constraints, we have used the SDSS data [55–57], the recent CMB observations (WMAP, ACBAR, and QUaD [1,2,4–8]), supernovae data [58–60] and big-bang nucleosynthesis [61]. We show the marginalized likelihood (solid lines) and mean likelihood (dotted lines) distribution of $\log(k_c/k_0)$ in Fig. 5. In Figs. 6 and 7, we show the marginalized likelihood distribution in the plane of $\log(k_c/k_0)$ versus other parameters. The k_c value of the best-fit cosmological model is $\log(k_c/k_0) = -1.98^{+0.35}_{-168.09}$ and $\log(k_c/k_0) = -1.98^{+0.35}_{-181.81}$ for model I and model II, respectively. Note that the confidence interval is marginalized over $\epsilon_{\rm TP}$, υ, ϕ, and $f_{\rm NL}$ besides the basic ΛCDM parameters. The best-fit values above do not coincide with the peak of likelihood distribution shown in Fig. 5. We attribute the discrepancy to the deviation of the multiparameter likelihood function from Gaussian distribution. The central values of our estimated k_c are very similar to the cutoff scale found by the WMAP team [35]. Since CMB power spectra are sensitive to $P_\Phi(k)$ of $k \gtrsim 2/\eta_0$, k_c higher than $2/\eta_0$ affects CMB power spectra through $P_\Phi(k)$ as well as $P_{\Phi,\rm NL}(k)$. This explains the similarity of the central values to the cutoff scale found by the WMAP team, even though $P_{\Phi,\rm NL}$ was not taken into account in their analysis. Note that the lower bounds above are associated mainly with $P_{\Phi,\rm NL}$, since $k_c \ll 2/\eta_0$.

As shown in Figs. 6 and 7, we find there is little degeneracy between $\log(k_c/k_0)$ and other parameters except for A_{sz}. As expected, the best-fit values of the basic ΛCDM parameters are similar to those of the WMAP concordance model.

C. Scale-dependent $f_{\rm NL}$

The nonlinear coupling parameter $f_{\rm NL}$ is a local parameter, and hence possesses some scale-dependence [62]. In a

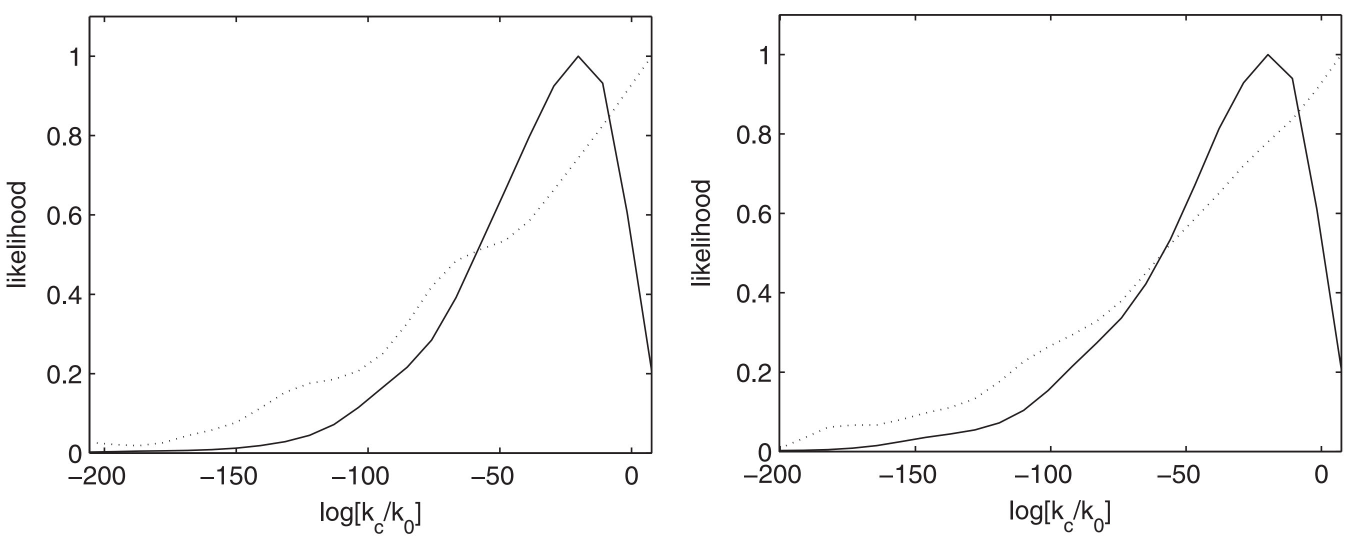

FIG. 5. Marginalized likelihood (solid lines) and mean likelihood of $\log(k_c/k_0)$ for model I (left) and model II (right).

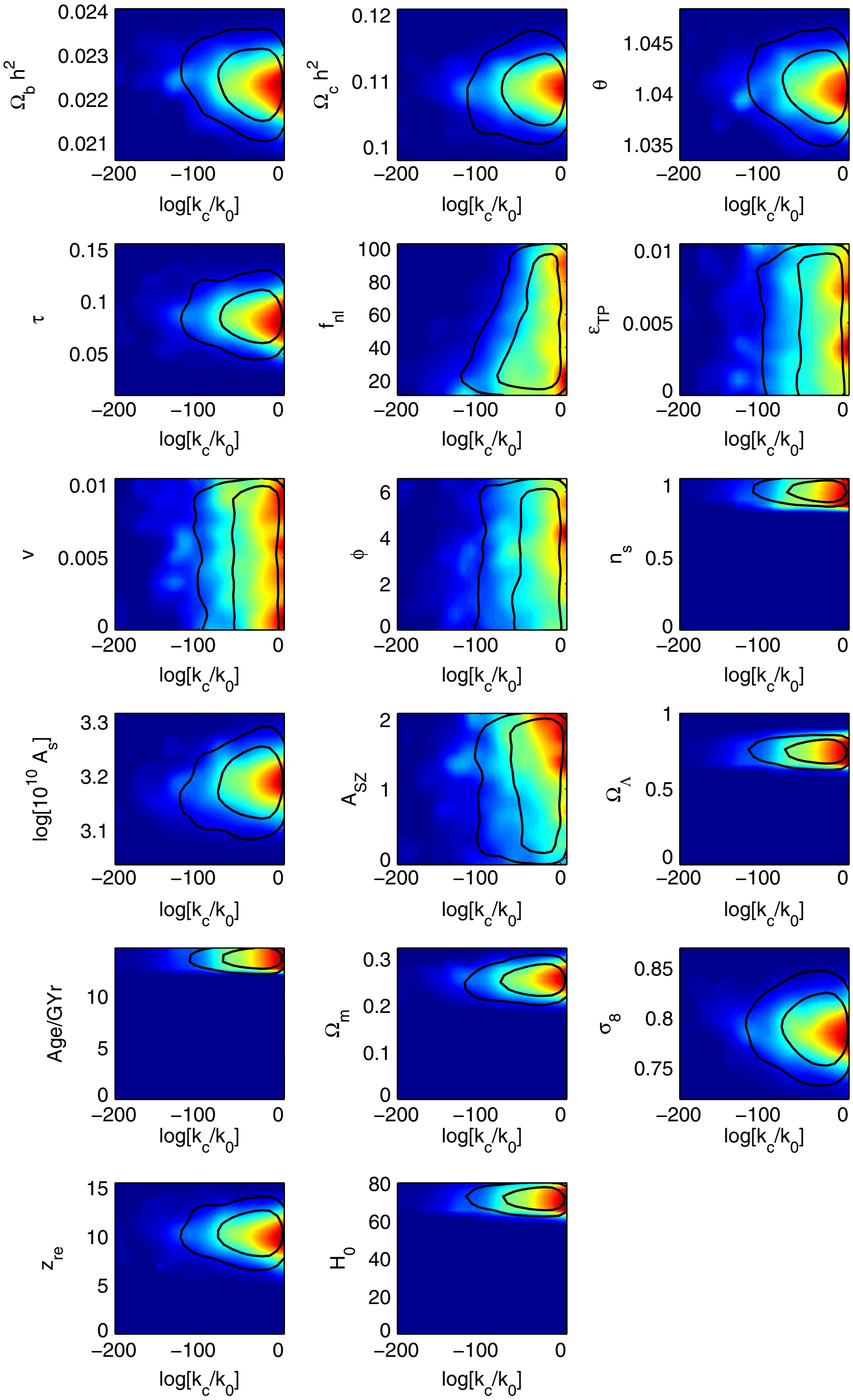

FIG. 6 (color online). Marginalized likelihood in the plane of $\log(k_c/k_0)$ versus others parameters for model I. Two contour lines correspond to 1σ and 2σ levels.

single-field inflation, for instance, $f_{\rm NL}$ in Eq. (3) is given by [62]

$$f_{\rm NL} = \frac{5}{6} - 3\frac{(\mathbf{k}\cdot\mathbf{p})^2}{k^4} - \frac{2\mathbf{k}\cdot\mathbf{p} - p^2}{k^2}. \qquad (23)$$

In the models of a constant spectral index $n \sim 0.962$, and no transition, all terms of $f_{\rm NL}$ should have k dependence $k^{\alpha \gtrsim 0.04}$ not to have unphysically large $P_{\rm NL}$. However, the $f_{\rm NL}$ predicted by most of inflationary models does not have such k dependence. Therefore, we find a scale-dependent $f_{\rm NL}$ alone does not provide a way to avoid unphysically large $P_{\rm NL}$.

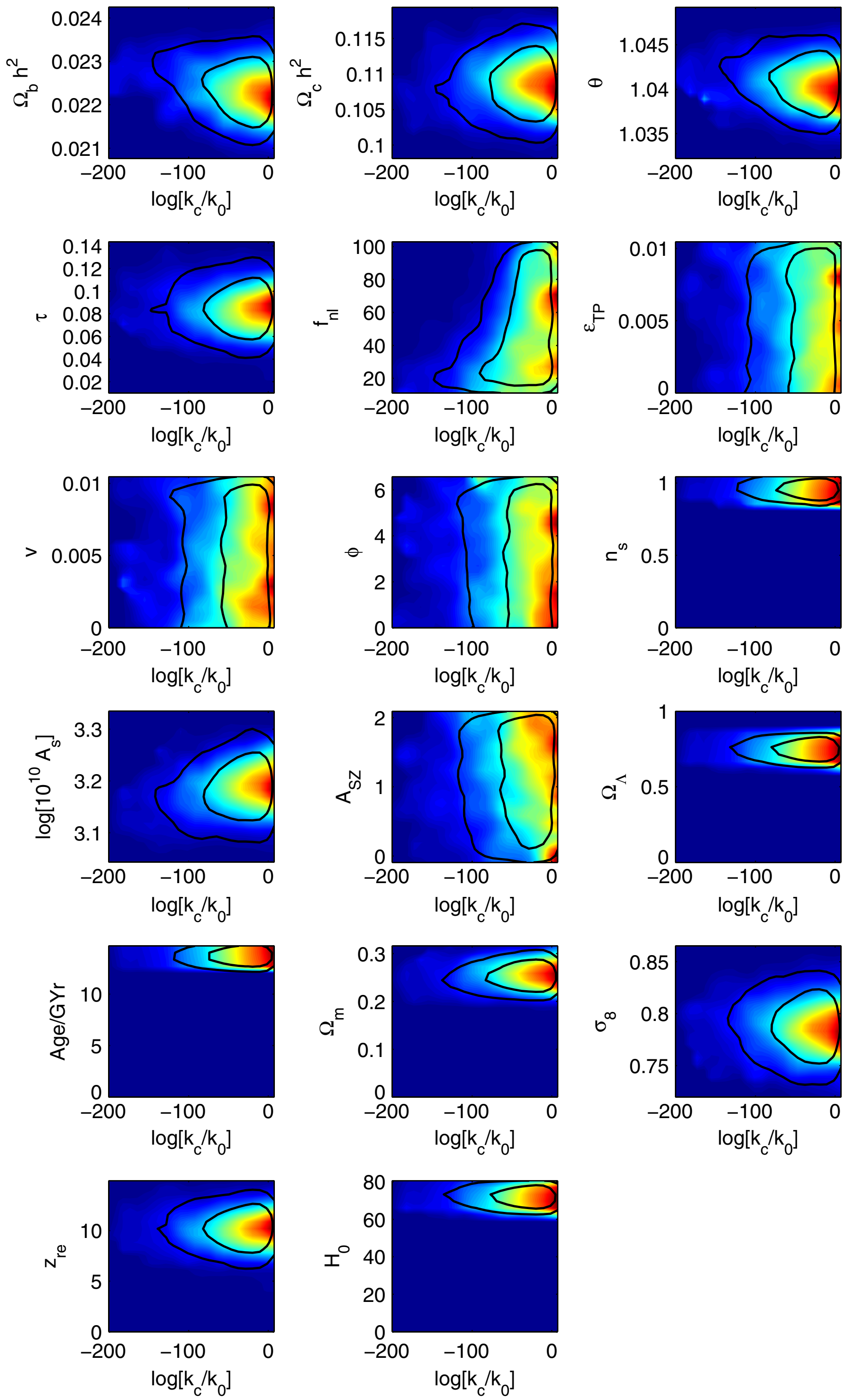

FIG. 7 (color online). Marginalized likelihood in the plane of $\log(k_c/k_0)$ versus others parameters for model II. Two contour lines corresponding to 1σ and 2σ levels.

V. DISCUSSION

We have shown a primordial nonlinear term ($f_{\rm NL}$ term) may produce unphysically large CMB anisotropy, because of coupling to primordial fluctuation on largest scales. Since such large excess power are not observed in CMB data, we have explored the following minimally extended power law models for a primordial power spectrum to explain the absence of the large excess power.

(i) A spectral index of a negative running: provided a power law model is valid up to the largest scale (i.e.

no transition at a very low wave number), running of the spectral index should be negative (i.e. $dn/d\ln k < 0$). We may rule out inflationary models of $dn/d\ln k \geq 0$ (e.g. a mass term potential and some models of softly broken supersymmetry models).

(ii) A transition at a very low wave number (e.g. cutoff): provided a spectral index is constant, there should exist some transition at a very low wave number, below which the power law is not valid. We have fitted a transition scale of a sharp cutoff model with the recent CMB and SDSS data, and obtained $\log(k_c/k_0) = -1.98^{+0.35}_{-168.09}$ and $\log(k_c/k_0) = -1.98^{+0.35}_{-181.81}$, respectively, for two models described by Eq. (17) and (18).

Though it is not clear which condition is true for the primordial power spectrum, it is certain that at least one of two conditions should be met to avoid unphysically large CMB anisotropy.

We shall be able to impose stronger constraints on inflationary models with the data from the upcoming PLANCK surveyor [9,10]. The improved constraints on a running spectral index of scalar perturbation $dn/d\ln k$, tensor-to-scalar ratio r, and the spectral index of tensor perturbation n_t improves our understanding of inflation, and improves our understanding of how unphysically large $P_{\rm NL}$ is avoided.

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