Hyperon bulk viscosity in strong magnetic fields

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We study the bulk viscosity of neutron star matter including Λ hyperons in the presence of quantizing magnetic fields. Relaxation time and bulk viscosity due to both the nonleptonic weak process involving Λ hyperons and direct Urca processes are calculated here. In the presence of a strong magnetic field of 10^{17} G, the hyperon bulk viscosity coefficient is reduced, whereas bulk viscosity coefficients due to direct Urca processes are enhanced compared with their field free cases when many Landau levels are populated by protons, electrons, and muons.

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I. INTRODUCTION

R-mode instability plays an important role in regulating spins of newly born neutron stars as well as old and accreting neutron stars in low mass x-ray binaries [1]. Gravitational radiation drives the r mode unstable due to the Chandrasekhar-Friedman-Schutz mechanism [2–10]. R-mode instability could be a promising source of gravitational radiation. It would be possible to probe neutron star interior if it is detected by gravity wave detectors.

Like gravitational radiation, electromagnetic radiation also drives the r mode unstable through the Chandrasekhar-Friedman-Schutz mechanism. There exists a class of neutron stars called magnetars [11] with strong surface magnetic fields 10^{14}–10^{15} G as predicted by observations on soft gamma-ray repeaters and anomalous x-ray pulsars [12,13]. The effects of magnetic fields on the spin evolution and r modes in protomagnetars were investigated by different groups [14–16]. On the one hand, it was shown that the growth of the r mode due to electromagnetic and Alfvén wave emission for a strong magnetic field and slow rotation could compete with that of gravitational radiation [15]. On the other hand, it was argued that the distortion of magnetic fields in neutron stars due to r modes might damp the mode when the field is $\sim 10^{16}$ G or more [14,16].

The evolution of r modes proceeds through three steps [17]. In the first phase, the mode amplitude grows exponentially with time. In the next stage, the mode saturates due to nonlinear effects. In this case viscosity becomes important. Finally, viscous forces dominate over gravitational radiation driven instability and damp the r mode. This shows that viscosity plays an important role on the evolution of r mode. Bulk and shear viscosities were extensively investigated in connection with the damping of the r-mode instability [1,18–36]. In particular, it was shown that the hyperon bulk viscosity might effectively damp r-mode instability [25]. However, all these calculations of viscosity were performed in the absence of magnetic fields. The only calculation of bulk viscosity due to the Urca process in magnetized neutron star matter was presented in Ref. [37]. This motivates us to investigate bulk viscosity due to nonleptonic process involving hyperons in the presence of strong magnetic fields. It is to be noted that the magnetic field in neutron star's interior might be higher by several orders of magnitude than the surface magnetic field [38]. Further it was shown that neutron stars could sustain a strong interior magnetic field $\sim 10^{18}$ G [39,40].

The paper is organized in the following way. In Sec. II, we describe hyperon matter in strong magnetic fields. We calculate bulk viscosity due to the nonleptonic process involving Λ hyperons and due to leptonic processes in Sec. III. We discuss results in Sec. IV and a summary is given in Sec. V.

II. HYPERON MATTER IN MAGNETIC FIELD

We describe β equilibrated and charge neutral neutron star matter made of neutrons, protons, Λ hyperons, electrons, and muons within a relativistic mean field approach [41,42]. The baryon-baryon interaction is mediated by σ, ω, and ρ mesons. In the absence of a magnetic field, the baryon-baryon interaction is given by the Lagrangian density [43,44]

$$\begin{aligned}\mathcal{L}_B = \sum_{B=n,p,\Lambda} &\bar{\psi}_B(i\gamma_\mu\partial^\mu - m_B + g_{\sigma B}\sigma - g_{\omega B}\gamma_\mu\omega^\mu \\ &- g_{\rho B}\gamma_\mu \mathrm{t}_B\cdot\boldsymbol{\rho}^\mu)\psi_B + \frac{1}{2}(\partial_\mu\sigma\partial^\mu\sigma - m_\sigma^2\sigma^2) \\ &- U(\sigma) - \frac{1}{4}\omega_{\mu\nu}\omega^{\mu\nu} + \frac{1}{2}m_\omega^2\omega_\mu\omega^\mu \\ &- \frac{1}{4}\boldsymbol{\rho}_{\mu\nu}\cdot\boldsymbol{\rho}^{\mu\nu} + \frac{1}{2}m_\rho^2\boldsymbol{\rho}_\mu\cdot\boldsymbol{\rho}^\mu. \end{aligned} \quad (1)$$

The scalar self-interaction term [43–45] is

$$U(\sigma) = \tfrac{1}{3}g_1 m_N(g_{\sigma N}\sigma)^3 + \tfrac{1}{4}g_2(g_{\sigma N}\sigma)^4, \quad (2)$$

and

$$\omega_{\mu\nu} = \partial_\nu\omega_\mu - \partial_\mu\omega_\nu, \quad (3)$$

$$\boldsymbol{\rho}_{\mu\nu} = \partial_\nu\boldsymbol{\rho}_\mu - \partial_\mu\boldsymbol{\rho}_\nu. \quad (4)$$

In mean field approximation, the effective mass of baryons

B is

$$m_B^* = m_B - g_{\sigma B}\sigma, \tag{5}$$

where σ is given by its ground state expectation value

$$\sigma = \frac{1}{m_\sigma^2}\left(\sum_B g_{\sigma B} n_S^B - \frac{\partial U}{\partial \sigma}\right). \tag{6}$$

The scalar density is given by

$$n_S^B = \frac{2}{(2\pi)^3}\int_0^{k_{F_B}} \frac{m_B^*}{\sqrt{k_B^2 + m_B^{*2}}} d^3k_B. \tag{7}$$

The chemical potential for baryons B is

$$\mu_B = \sqrt{k_{F_B}^2 + m_B^{*2}} + \omega^0 g_{\omega B} + \rho_3^0 g_{\rho B} I_{3B}, \tag{8}$$

where I_{3B} is the isospin projection and

$$\omega^0 = \frac{1}{m_\omega^2}\sum_B g_{\omega B} n_B, \tag{9}$$

$$\rho_3^0 = \frac{1}{m_\rho^2}\sum_B g_{\rho B} I_{3B} n_B. \tag{10}$$

The total baryon number density is $n_b = \sum_B n_B$.

Now we consider the effects of strong magnetic fields on hyperon matter. The motion of charged particles in a magnetic field is Landau quantized in the plane perpendicular to the direction of the field. We solve Dirac equations for charged particles using the gauge corresponding to the constant magnetic field B_m along the z axis as $A_0 = 0$, $\vec{A} = (0, xB_m, 0)$. In the presence of a constant magnetic field, the Lagrangian density for protons is taken from Ref. [46]. The positive energy solutions for protons are

$$\psi_\alpha = \frac{\left(\frac{\sqrt{b}}{2^\nu \nu! \sqrt{\pi}}\right)^{1/2}}{\sqrt{L_y L_z}} e^{-\xi^2/2} e^{-i(\epsilon t - k_y y - k_z z)} \mathcal{U}_{\alpha,\nu}(k, x), \tag{11}$$

with $\xi = \sqrt{b}\left(x - \frac{k_y}{qB_m}\right)$ and $b = qB_m$.

The positive energy spinors, $\mathcal{U}_\nu(k, x)$, [47–50] are given by

$$\mathcal{U}_{\uparrow,\nu}(k, x) = \sqrt{\epsilon' + m_p^*}\begin{pmatrix} H_\nu(\xi) \\ 0 \\ \frac{p_z}{\epsilon' + m_p^*} H_\nu(\xi) \\ \frac{-\sqrt{2\nu b}}{\epsilon' + m_p^*} H_{\nu+1}(\xi) \end{pmatrix}, \tag{12}$$

and

$$\mathcal{U}_{\downarrow,\nu}(k, x) = \sqrt{\epsilon' + m_p^*}\begin{pmatrix} 0 \\ H_\nu(\xi) \\ \frac{-\sqrt{2\nu b}}{\epsilon' + m_p^*} H_{\nu-1}(\xi) \\ \frac{-p_z}{\epsilon' + m_p^*} H_\nu(\xi) \end{pmatrix}, \tag{13}$$

where $\epsilon' = \sqrt{p_z^2 + m_p^{*2} + 2\nu q B_m}$.

The proton number density n_p and scalar density n_S^p are given by [46]

$$n_p = \frac{qB_m}{2\pi^2}\sum_{\nu=0}^{\nu_{\max}} g_\nu k_p(\nu), \tag{14}$$

$$n_S^p = \frac{qB_m}{2\pi^2} m_p^* \sum_{\nu=0}^{\nu_{\max}} g_\nu \ln\frac{k_p(\nu) + \mu_p^*}{\sqrt{(m_p^{*2} + 2\nu q B_m)}}, \tag{15}$$

where $\mu_B^* = \sqrt{k_{FB}^2 + m_B^{*2}}$ and $k_p(\nu) = \sqrt{k_{F_p}^2 - 2\nu q B_m}$. The maximum number of Landau levels populated is denoted by $\nu_{\max}$, and the Landau level degeneracy g_ν is 1 for $\nu = 0$ and 2 for $\nu > 0$. Similarly, we treat noninteracting electrons and muons in constant magnetic fields.

The total energy density of neutron star matter is

$$\begin{aligned}\varepsilon = {} & \frac{1}{2}m_\sigma^2\sigma^2 + U(\sigma) + \frac{1}{2}m_\omega^2\omega^{0^2} + \frac{1}{2}m_\rho^2\rho_3^{0^2} + \sum_{B=n,\Lambda}\frac{1}{8\pi^2}\left(2k_{F_B}\mu_B^{*^3} - k_{F_B} m_B^{*^2}\mu_B^* - m_B^{*^4}\ln\frac{k_{F_B} + \mu_B^*}{m_B^*}\right) \\ & + \frac{qB_m}{(2\pi)^2}\sum_{\nu=0}^{\nu_{\max}} g_\nu\left(k_p(\nu)\mu_p^* + (m_p^{*^2} + 2\nu q B_m)\ln\frac{k_p(\nu) + \mu_p^*}{\sqrt{(m_p^{*^2} + 2\nu q B_m)}}\right) \\ & + \frac{qB_m}{(2\pi)^2}\sum_{l=e,\mu}\sum_{\nu=0}^{\nu_{\max}}\left(k_l(\nu)\mu_l + (m_l^2 + 2\nu q B_m)\ln\frac{k_l(\nu) + \mu_l}{\sqrt{(m_l^2 + 2\nu q B_m)}}\right) + \frac{B_m^2}{8\pi}.\end{aligned} \tag{16}$$

Similarly, the total pressure of the system is given by

$$\begin{aligned}P = {} & -\frac{1}{2}m_\sigma^2\sigma^2 - U(\sigma) + \frac{1}{2}m_\omega^2\omega^{0^2} + \frac{1}{2}m_\rho^2\rho_3^{0^2} + \frac{1}{3}\sum_{B=n,\Lambda}\frac{2J_B + 1}{2\pi^2}\int_0^{k_{F_B}}\frac{k^4 dk}{(k^2 + m_B^{*2})^{1/2}} + \frac{qB_m}{(2\pi)^2}\sum_{\nu=0}^{\nu_{\max}}\Bigg\{k_p(\nu)\mu_p^* \\ & - (m_p^{*^2} + 2\nu q B_m)\ln\frac{k_p(\nu) + \mu_p^*}{\sqrt{(m_p^{*^2} + 2\nu q B_m)}}\Bigg\} + \frac{qB_m}{(2\pi)^2}\sum_{l=e,\mu}\sum_{\nu=0}^{\nu_{\max}}\left\{k_l(\nu)\mu_l - (m_l^2 + 2\nu q B_m)\ln\frac{k_l(\nu) + \mu_l}{\sqrt{(m_l^2 + 2\nu q B_m)}}\right\} + \frac{B_m^2}{8\pi},\end{aligned} \tag{17}$$

where $k_l(\nu) = \sqrt{k_{F_l}^2 - 2\nu q B_m}$. The relation between pressure and energy density defines the equation of state (EoS).

III. BULK VISCOSITY

The macroscopic compression (or expansion) of a fluid element leads to departure from chemical equilibrium. Nonequilibrium processes cause dissipation of energy, which is the origin of bulk viscosity in neutron stars. Weak interaction processes bring the system back to equilibrium. In this calculation, we consider the nonleptonic reaction

$$n + p \rightarrow p + \Lambda, \tag{18}$$

as well as direct Urca (dUrca) processes, which are represented by

$$n \rightarrow p + l^- + \bar{\nu}_l, \tag{19}$$

where l stands for e or μ. When the chemical equilibrium is achieved, chemical potentials involved in the above reactions satisfy $\mu_n - \mu_\Lambda = 0$ and $\mu_n - \mu_p - \mu_l = 0$, respectively. In this case the forward and reverse reaction rates Γ_f and Γ_r are same. The departure from chemical equilibrium due to macroscopic perturbation gives rise to the difference between forward and reverse reaction rates $\Gamma = \Gamma_f - \Gamma_r \neq 0$. For a rotating neutron star, the r-mode oscillation provides the macroscopic perturbation, which drives the system out of chemical equilibrium.

The real part of bulk viscosity coefficient can be written as [51]

$$\zeta = -\frac{n_b^2 \tau}{1 + (\omega\tau)^2}\left(\frac{\partial P}{\partial n_n}\right)\frac{d\bar{x}_n}{dn_b}, \tag{20}$$

where $\bar{x}_i = n_i/n_b$ is the equilibrium fraction of i-th species, ω is the angular velocity of (l, m) r mode, and τ is the microscopic relaxation time. For a neutron star rotating with angular velocity Ω, the angular velocity (ω) of (l, m) r mode is given by

$$\omega = \frac{2m}{l(l+1)}\Omega. \tag{21}$$

We are interested in $l = m = 2$ r mode in this calculation. The relaxation time is given by

$$\frac{1}{\tau} = \frac{\Gamma}{\delta\mu}\frac{\delta\mu}{n_b \delta x_n}, \tag{22}$$

where $\delta\mu$ refers to the chemical imbalance. Here, Γ is the total reaction rate.

The partial derivative of pressure with respect to neutron number density can be evaluated from the EoS under consideration as

$$\frac{\partial P}{\partial n_n} = \frac{k_{F_n}^2}{3\mu_n^*} - \frac{\frac{g_{\sigma N}}{m_\sigma}\frac{m_n^*}{\mu_n^*}}{D}\sum_B n_B \frac{g_{\sigma B}}{m_\sigma}\frac{m_B^*}{\mu_B^*} + g_{\omega N}\omega^0 + g_{\rho N} I_{3n}\rho_3^0, \tag{23}$$

$$D = 1 + \sum_B\left(\frac{g_{\sigma B}}{m_\sigma}\right)^2 \frac{\partial n_S^B}{\partial m_B^*} + \frac{1}{m_\sigma^2}\frac{\partial^2 U}{\partial \sigma^2}. \tag{24}$$

The total derivative dx_n/dn_b can be evaluated numerically.

Now, we calculate relaxation times for above mentioned processes in the presence of a magnetic field B_m using the EoS as described in Sec. II.

A. Nonleptonic process

Here, we consider the nonleptonic process given by Eq. (18). In this case, only protons are affected by magnetic fields. The reaction rate is given by

$$\Gamma = \int\frac{Vd^3k_n}{(2\pi)^3}\int\frac{L_z dk_{p_{iz}}}{2\pi}\int_{-(bL_x/2)}^{bL_x/2}\frac{L_y dk_{p_{iy}}}{2\pi}\int\frac{L_z dk_{p_{fz}}}{2\pi} \times \int_{-(bL_x/2)}^{bL_x/2}\frac{L_y dk_{p_{fy}}}{2\pi}\int\frac{Vd^3k_\Lambda}{(2\pi)^3} W_{fi} F(\epsilon_n, \epsilon_{p_i}, \epsilon_{p_f}, \epsilon_\Lambda), \tag{25}$$

$k_{p_{iz}}$ and $k_{p_{fz}}$ being the z component of momenta of initial and final protons, respectively, and k_n and k_Λ denote momenta of neutrons and Λ hyperons. The Pauli blocking factor is given by

$$F(\epsilon_n, \epsilon_{p_i}, \epsilon_{p_f}, \epsilon_\Lambda) = f(\epsilon_n)f(\epsilon_{p_i})\{1 - f(\epsilon_{p_f})\}\{1 - f(\epsilon_\Lambda)\} - f(\epsilon_\Lambda)f(\epsilon_{p_f})\{1 - f(\epsilon_{p_i})\} \times \{1 - f(\epsilon_n)\}, \tag{26}$$

with the Fermi distribution function at temperature T

$$f(\epsilon_i) = \frac{1}{1 + e^{(\epsilon_i - \mu)/kT}}. \tag{27}$$

The matrix element W_{fi} is given by

$$W_{fi} = \frac{1}{V^3(L_y L_z)}\frac{(2\pi)^3}{16\epsilon_n\epsilon_{p_i}\epsilon_{p_f}\epsilon_\Lambda}|\mathcal{M}|^2 e^{-Q^2}\delta(\epsilon)\delta(k_y)\delta(k_z), \tag{28}$$

where

$$Q^2 = \frac{(k_{nx} - k_{\Lambda x})^2 + (k_{p_{iy}} - k_{p_{fy}})^2}{2b} \quad \text{and}$$
$$\delta(k) \equiv \delta(k_n + k_{p_i} - k_{p_f} - k_\Lambda). \tag{29}$$

The invariant amplitude squared for the process is

$$|\mathcal{M}|^2 = 4G_F^2\sin^2 2\theta_c[2m_n^* m_p^{*2} m_\Lambda^*(1-g_{np}^2)(1-g_{p\Lambda}^2) - m_n^* m_p^*(k_{p_i}\cdot k_\Lambda)(1-g_{np}^2)(1+g_{p\Lambda}^2) - m_p^* m_\Lambda^*(k_n\cdot k_{p_f})(1+g_{np}^2)(1-g_{p\Lambda}^2) + (k_n\cdot k_{p_i})(k_{p_f}\cdot k_\Lambda)\{(1+g_{np}^2)(1+g_{p\Lambda}^2) + 4g_{np}g_{p\Lambda}\} + (k_n\cdot k_\Lambda)(k_{p_i}\cdot k_{p_f}) \times\{(1+g_{np}^2)(1+g_{p\Lambda}^2) - 4g_{np}g_{p\Lambda}\}]. \quad (30)$$

In calculating the matrix element given by Eq. (28) we use the solutions of Dirac equation for protons in the magnetic field given by Eqs. (12) and (13). We also assume that the magnetic field is so strong that only zeroth Landau level is populated. Now we integrate over $k_{p_{iy}}$ and $k_{p_{fy}}$ using $\delta(k_y)$ and obtain

$$\Gamma = \frac{L_y L_z}{(2\pi)^7 V16} bL_x \int d^3k_n \int dk_{p_{iz}} \int dk_{p_{fz}} \times \int d^3k_\Lambda \left(\frac{|\mathcal{M}|^2}{\epsilon_n\epsilon_{p_i}\epsilon_{p_f}\epsilon_\Lambda}\right)_{\delta(k_y)} e^{-[(k_{nx}-k_{\Lambda x})^2+(k_{ny}-k_{\Lambda y})^2]/2b} \times F(\epsilon_n,\epsilon_{p_i},\epsilon_{p_f},\epsilon_\Lambda)\delta(\epsilon_n+\epsilon_{pi}-\epsilon_{pf}-\epsilon_\Lambda)\delta(k_z). \quad (31)$$

Here, the subscript $\delta(k_y)$ denotes that this condition has been imposed on the quantity within the parenthesis. Next we perform the integration over $\boldsymbol{k}_n$ and $\boldsymbol{k}_\Lambda$ and write $d^3k = k^2dkd(\cos\theta)d\phi$. The delta function of z components of momenta implies $k_{nz}+k_{p_{iz}} = k_{p_{fz}}+k_{\Lambda z}$. Here, we note that when protons occupy only the zeroth Landau level, they have momenta along the direction of the magnetic field, i.e. in z direction. Hence, we have $k_{pz} = k_{F_p}$. Then depending upon whether the initial and final protons are moving in the same or opposite direction we have $k_{\Lambda z} - k_{nz} = 0$ or $k_{\Lambda z} - k_{nz} = 2k_{F_p}$. Next we perform the angle integrations using $\delta(k_z)$ and change variable k to ϵ to get

$$\Gamma = \frac{b}{(2\pi)^5 8}\int d\epsilon_n d\epsilon_{pi} d\epsilon_{pf} d\epsilon_\Lambda \frac{k_{F_\Lambda}}{k_{F_p}k_{F_p}} \times ((|\mathcal{M}|^2)_{\theta_{\rm int}})_{\delta(k_y),\delta(k_z)}[\Theta\{(k_{F_n}-k_{F_\Lambda})^2\}e^{-[(k_{F_n}-k_{F_\Lambda})^2]/2b} + \Theta\{(k_{F_n}-k_{F_\Lambda})^2 - 4k_{F_p}^2\}e^{-[(k_{F_n}-k_{F_\Lambda})^2-4k_{F_p}^2]/2b}] \times F(\epsilon_n,\epsilon_{p_i},\epsilon_{p_f},\epsilon_\Lambda)\delta(\epsilon_n+\epsilon_{p_i}-\epsilon_{p_f}-\epsilon_\Lambda). \quad (32)$$

Here, the subscript $\theta_{\rm int}$ denotes the angle integrated value. As particles reside near their Fermi surfaces in a degenerate matter we replace momenta and energies under integration by their respective values at their Fermi surfaces.

The matrix element squared is rewritten as

$$((|\mathcal{M}|^2)_{\theta_{\rm int}})_{\delta(k_y),\delta(k_z)} = 4G_F^2\sin^2 2\theta_c\Big[2m_n^* m_p^{*2} m_\Lambda^*(1-g_{np}^2)(1-g_{p\Lambda}^2) - m_n^* m_p^*\mu_p\mu_\Lambda(1-g_{np}^2)(1+g_{p\Lambda}^2) - m_p^* m_\Lambda^*\mu_n\mu_p(1+g_{np}^2)(1-g_{p\Lambda}^2) + \mu_n\mu_p^2\mu_\Lambda\{(1+g_{np}^2)(1+g_{p\Lambda}^2)+4g_{np}g_{p\Lambda}\} + \mu_n\mu_p^2\mu_\Lambda\left(1-\frac{k_{F_p}^2}{\mu_p^2}\right)\{(1+g_{np}^2)(1+g_{p\Lambda}^2) - 4g_{np}g_{p\Lambda}\}\Big]. \quad (33)$$

As $\delta\mu \ll kT$, the energy integration of Eq. (32) can be written as [51]

$$\int d\epsilon_n d\epsilon_{p_i} d\epsilon_{p_f} d\epsilon_\Lambda F(\epsilon_n,\epsilon_{p_i},\epsilon_{p_f},\epsilon_\Lambda) \times\delta(\epsilon_n+\epsilon_{p_i}-\epsilon_{p_f}-\epsilon_\Lambda) = (kT)^2\frac{2\pi^2}{3}\delta\mu. \quad (34)$$

Finally, we get

$$\Gamma = \frac{1}{384\pi^3}\frac{qB_m k_{F_\Lambda}}{k_{F_p}^2}((|\mathcal{M}|^2)_{\theta_{\rm int}})_{\delta(k_y),\delta(k_z)}[\Theta\{(k_{F_n}-k_{F_\Lambda})^2\} \times e^{-[(k_{F_n}-k_{F_\Lambda})^2]/2b} + \Theta\{(k_{F_n}-k_{F_\Lambda})^2-4k_{F_p}^2\} \times e^{-[(k_{F_n}-k_{F_\Lambda})^2-4k_{F_p}^2]/2b}](kT)^2\delta\mu. \quad (35)$$

The expression of the reaction rate for a zero magnetic field is given by [51]

$$\Gamma = \frac{1}{192\pi^3}\langle|\mathcal{M}|^2\rangle k_{F_\Lambda}(kT)^2\delta\mu, \quad (36)$$

where the angle averaged matrix element squared is the same as that given by [51].

Now the quantity $\delta\mu/\delta x_n$ in Eq. (22) is to be evaluated under the condition of total baryon number conservation [51]

$$\delta n_n + \delta n_\Lambda = 0, \quad (37)$$

which leads to

$$\frac{\delta\mu}{\delta x_n} = \alpha_{nn} - \alpha_{n\Lambda} - \alpha_{\Lambda n} + \alpha_{\Lambda\Lambda}, \quad \text{with} \quad \alpha_{ij} = \frac{\partial\mu_i}{\partial n_j}. \quad (38)$$

Further, we have

$$\alpha_{ij} = \frac{\pi^2}{k_{F_i}\mu_i^*}\delta_{ij} - \frac{m_i^*}{\mu_i^*}\frac{(\frac{g_{\sigma i}}{m_\sigma})(\frac{g_{\sigma j}}{m_\sigma})\frac{m_j^*}{\mu_j^*}}{D} + \frac{1}{m_\omega^2}g_{\omega i}g_{\omega j} + \frac{1}{m_\rho^2}g_{\rho i}I_{3i}g_{\rho j}I_{3j}. \quad (39)$$

Here, D is the same as given by Eq. (24). Next we evaluate the relaxation time of the nonleptonic reaction at a given baryon density using Eq. (22) along with Eqs. (35), (38), and (39).

As soon as we know the relaxation time, we can calculate the bulk viscosity coefficient ζ due to the nonleptonic interaction at a given baryon density from Eq. (20).

B. Leptonic processes

Here, we consider dUrca processes involving nucleons, electrons, and muons in a magnetic field. The forward reaction rate for this process is then given by [47–49]

$$\Gamma_f = \int \frac{V d^3 k_n}{(2\pi)^3} \int \frac{V d^3 k_\nu}{(2\pi)^3} \int \frac{L_z dk_{zp}}{2\pi} \int_{-(bL_x/2)}^{bL_x/2} \frac{L_y dk_{yp}}{2\pi} \times \int \frac{L_z dk_{zl}}{2\pi} \int_{-(bL_x/2)}^{bL_x/2} \frac{L_y dk_{yl}}{2\pi} W_{fi} F(\epsilon_n, \epsilon_p, \epsilon_l). \quad (40)$$

Here, $F(\epsilon_n, \epsilon_p, \epsilon_l)$ is given by

$$F(\epsilon_n, \epsilon_p, \epsilon_l) = f(\epsilon_n)\{1 - f(\epsilon_p)\}\{1 - f(\epsilon_l)\}. \quad (41)$$

Using the solutions of Dirac equations for protons and electrons in a magnetic field, we obtain the matrix element

$$W_{fi} = \frac{(2\pi)^3}{V^3(L_y L_z)} |\mathcal{M}|^2 \delta(\epsilon)\delta(k_y)\delta(k_z). \quad (42)$$

Firstly, we treat the case following the prescription of Baiko and Yakovlev [48] when protons and electrons populate large numbers of Landau levels. In this case, we have

$$\sum_{s_n, s_p} |\mathcal{M}|^2 = 2G_F^2 \cos^2\theta_c (1 + 3G_A^2) F^2, \quad (43)$$

where F is the Laguerre functions for both protons and electrons [48]. The forward reaction rate is given by

$$\Gamma_f = \frac{32\pi G_F^2 \cos^2\theta_c m_n^* m_p^* \mu_l}{(2\pi)^5} R_B^{qc} \int d\epsilon_\nu \epsilon_\nu^2 J(\epsilon_\nu), \quad (44)$$

where

$$R_B^{qc} = 2 \iint_{-1}^{1} d\cos\theta_p d\cos\theta_l \frac{K_{F_p} K_{F_l}}{4b} F^2_{N_p, N_l}(u) \times \Theta(k_{F_n} - |k_{F_p} \cos\theta_p + k_{F_l} \cos\theta_l|), \quad (45)$$

and

$$J(\epsilon_\nu) = \int d\epsilon_n d\epsilon_p d\epsilon_l F(\epsilon_n, \epsilon_p, \epsilon_l) \delta(\epsilon_n - \epsilon_p - \epsilon_l - \epsilon_\nu), = \frac{(kT)^2}{2} \frac{\pi^2 + (\epsilon_\nu/kT)^2}{1 + e^{\epsilon_\nu/kT}}. \quad (46)$$

As there is chemical imbalance due to the perturbation, the reverse reaction rate (Γ_r) differs from the forward reaction rate, and the net reaction rate is given by [26,48]

$$\Gamma_l = \frac{32\pi G_F^2 \cos^2\theta_c m_n^* m_p^* \mu_l}{(2\pi)^5} R_B^{qc} \int d\epsilon_\nu \epsilon_\nu^2 \{J(\epsilon_\nu - \delta\mu) - J(\epsilon_\nu + \delta\mu)\}. \quad (47)$$

One important aspect of the dUrca process is the opening of this channel in the forbidden regime $K_{F_n} > K_{F_p} + K_{F_l}$, which was otherwise closed in the field free case [48]. The dUrca process also operates in the allowed domain $K_{F_p} + K_{F_l} > K_{F_n}$ in the presence of a magnetic field. We adopt fitting formulas for R_B^{qc} in both domains as given by Ref. [48].

Next we focus on the case when both protons and electrons populate zeroth Landau levels [47–49]. In this case, we write the matrix element as

$$W_{fi} = \frac{(2\pi)^3}{V^3(L_y L_z)} \frac{1}{16\epsilon_n \epsilon_\nu \epsilon_p \epsilon_e} |\mathcal{M}|^2 e^{-Q^2} \delta(\epsilon)\delta(k_y)\delta(k_z), \quad (48)$$

$$Q^2 = \frac{(k_{nx} - k_{\nu x})^2 + (k_{py} + k_{ly})^2}{2b}. \quad (49)$$

In a magnetic field neutrons will be polarized because of their anomalous magnetic moments. Hence, for two different spin states of neutrons, matrix elements should be evaluated separately. The invariant amplitude squared is then $|\mathcal{M}|^2 = |\mathcal{M}_+|^2 + |\mathcal{M}_-|^2$, where

$$|\mathcal{M}_\pm|^2 = \frac{G_F^2}{2} \sum_s \{\bar{\mathcal{V}}_{\nu s}(k_\nu)(1 + \gamma^5)\gamma_\nu \mathcal{U}_{l-}(k_l)\} \times \{\bar{\mathcal{U}}_{n\pm}(k_n)(1 - g_{np}\gamma^5)\gamma^\nu \mathcal{U}_{p+}(k_p)\} \times \{\bar{\mathcal{U}}_{p+}(k_p)\gamma^\mu(1 + g_{np}\gamma^5)\mathcal{U}_{n\pm}(k_n)\} \times \{\bar{\mathcal{U}}_{l-}(k_e)\gamma_\mu(1 - \gamma^5)\mathcal{V}_{\nu s}(k_\nu)\}, \quad (50)$$

and $\pm$ signs denote the up and down spins, respectively. The spinors for nonrelativistic neutrons are given by

$$\mathcal{U}_{n\pm} = \sqrt{\epsilon_n + m_n^*} \begin{pmatrix} \chi_\pm \\ 0 \end{pmatrix}, \quad (51)$$

where

$$\chi_+ = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \quad \text{and} \quad \chi_- = \begin{pmatrix} 0 \\ 1 \end{pmatrix}. \quad (52)$$

For nonrelativistic protons in the zeroth Landau level, the spinor has the same form as given by Eq. (51). For spin down relativistic leptons in the zeroth Landau level, the spinor is given by

$$\mathcal{U}_{l-} = \sqrt{\epsilon_l + m_l} \begin{pmatrix} 0 \\ 1 \\ 0 \\ \frac{-p_{lz}}{\epsilon_l + m_l} \end{pmatrix}. \quad (53)$$

For spin up and down neutrons, invariant amplitudes squared are

$$|\mathcal{M}_+|^2 = 8G_F^2\cos^2\theta_c m_n^* m_p^*(1+g_{np})^2(\epsilon_l + p_l)(\epsilon_\nu + p_{\nu z}), \tag{54}$$

and

$$|\mathcal{M}_-|^2 = 32G_F^2\cos^2\theta_c m_n^* m_p^* g_{np}^2(\epsilon_l + p_l)(\epsilon_\nu - p_{\nu z}). \tag{55}$$

Following the same procedure as described in Sec. III A and neglecting the neutrino momenta in momentum conserving delta functions, the final expression of forward reaction rate Γ_f is given by

$$\begin{aligned}\Gamma_f = &\frac{b}{(2\pi)^5 8}\frac{m_n^* m_p^* \mu_l}{k_{F_p}k_{F_l}}[(|\mathcal{M}_+|_d^2)_{\delta(k_y),\delta(k_z)} \\ &+ (|\mathcal{M}_-|_d^2)_{\delta(k_y),\delta(k_z)}][\Theta\{k_{F_n}^2 - (k_{F_p} - k_{F_l})^2\} \\ &\times e^{-[k_{F_n}^2 - (k_{F_p}-k_{F_l})^2]/2b} + \Theta\{k_{F_n}^2 - (k_{F_p} + k_{F_l})^2\} \\ &\times e^{-[k_{F_n}^2 - (k_{F_p}+k_{F_l})^2]/2b}]\int d\epsilon_\nu \epsilon_\nu^2 \int d\epsilon_n d\epsilon_p d\epsilon_l \\ &\times F(\epsilon_n, \epsilon_p, \epsilon_l)\delta(\epsilon_n - \epsilon_p - \epsilon_l - \epsilon_\nu),\end{aligned} \tag{56}$$

where

$$(|\mathcal{M}_+|_d^2)_{\delta(k_y),\delta(k_z)} = 8G_F^2\cos^2\theta_c(1+g_{np})^2 \times \left(1+\frac{p_l}{\epsilon_l}\right)\left(1+\frac{p_{\nu z}}{\epsilon_\nu}\right). \tag{57}$$

Similarly, we have

$$(|\mathcal{M}_-|_d^2)_{\delta(k_y),\delta(k_z)} = 32G_F^2\cos^2\theta_c g_{np}^2\left(1+\frac{p_l}{\epsilon_l}\right)\left(1-\frac{p_{\nu z}}{\epsilon_\nu}\right). \tag{58}$$

It is to be noted that z component of the neutrino momentum is smaller than its energy. We obtain

$$\begin{aligned}\Gamma_f = &\frac{b}{(2\pi)^5 8}\frac{m_n^* m_p^* \mu_l}{k_{F_p}k_{F_l}}[(|\mathcal{M}_+|_d^2)_{\delta(k_y),\delta(k_z)} \\ &+ (|\mathcal{M}_-|_d^2)_{\delta(k_y),\delta(k_z)}][\Theta\{k_{F_n}^2 - (k_{F_p} - k_{F_l})^2\} \\ &\times e^{-[k_{F_n}^2 - (k_{F_p}-k_{F_l})^2]/2b} + \Theta\{k_{F_n}^2 - (k_{F_p} + k_{F_l})^2\} \\ &\times e^{-[k_{F_n}^2 - (k_{F_p}+k_{F_l})^2]/2b}]\int d\epsilon_\nu \epsilon_\nu^2 J(\epsilon_\nu).\end{aligned} \tag{59}$$

Now if the reverse reaction rate is Γ_r and there is slight departure from chemical equilibrium $\delta\mu$, then the net reaction rate is [26]

$$\begin{aligned}\Gamma_l &= \Gamma_r - \Gamma_f \\ &= \frac{b}{(2\pi)^5 8}\frac{m_n^* m_p^* \mu_l}{k_{F_p}k_{F_l}}[(|\mathcal{M}_+|_d^2)_{\delta(k_y),\delta(k_z)} \\ &\quad + (|\mathcal{M}_-|_d^2)_{\delta(k_y),\delta(k_z)}][\Theta\{k_{F_n}^2 - (k_{F_p} - k_{F_l})^2\} \\ &\quad \times e^{-[k_{F_n}^2 - (k_{F_p}-k_{F_l})^2]/2b} + \Theta\{k_{F_n}^2 - (k_{F_p} + k_{F_l})^2\} \\ &\quad \times e^{-[k_{F_n}^2 - (k_{F_p}+k_{F_l})^2]/2b}]\int d\epsilon_\nu \epsilon_\nu^2\{J(\epsilon_\nu - \delta\mu) \\ &\quad - J(\epsilon_\nu + \delta\mu)\}.\end{aligned} \tag{60}$$

Using the following result from Ref. [26]

$$\int d\epsilon_\nu \epsilon_\nu^2\{J(\epsilon_\nu - \delta\mu) - J(\epsilon_\nu + \delta\mu)\} = \frac{17(\pi kT)^4}{60}\delta\mu, \tag{61}$$

we get

$$\begin{aligned}\Gamma_l = &\frac{17qB_m}{480\pi}\frac{m_n^* m_p^* \mu_l}{k_{F_p}k_{F_l}}G_F^2\cos^2\theta_c\left(1+\frac{p_l}{\epsilon_l}\right) \\ &\times\left[\frac{1}{4}(1+g_{np})^2 + g_{np}^2\right][\Theta\{k_{F_n}^2 - (k_{F_p} - k_{F_l})^2\} \\ &\times e^{-[k_{F_n}^2 - (k_{F_p}-k_{F_l})^2]/2b} + \Theta\{k_{F_n}^2 - (k_{F_p} + k_{F_l})^2\} \\ &\times e^{-[k_{F_n}^2 - (k_{F_p}+k_{F_l})^2]/2b}](kT)^4\delta\mu.\end{aligned} \tag{62}$$

The zero magnetic field result is given by

$$\Gamma_l(B_m = 0) = \frac{17}{240\pi}m_n^* m_p^* \mu_l(|\mathcal{M}|_d^2)_{\theta_{\rm int}}(kT)^4\delta\mu, \tag{63}$$

where

$$\begin{aligned}(|\mathcal{M}|_d^2)_{\theta_{\rm int}} = &G_F^2\cos^2\theta_c\left\{(1+g_{np})^2\left(1-\frac{k_{F_n}}{m_n^*}\right)\right. \\ &\left. + (1-g_{np})^2\left(1-\frac{k_{F_p}}{m_p^*}\right) - (1-g_{np}^2)\right\}.\end{aligned} \tag{64}$$

IV. RESULTS AND DISCUSSION

Nucleon-meson coupling constants of the model are obtained by reproducing the properties of nuclear matter such as binding energy $E/B = -16.3$ MeV, saturation density $n_0 = 0.153$ fm^{-3}, asymmetry energy coefficient $a_{\rm asy} = 32.5$ MeV, and incompressibility $K = 240$ MeV and taken from Ref [52]. The coupling strength of Λ hyperons with ω mesons is determined from SU(6) symmetry of the quark model [53–55]. The coupling strength of Λ hyperons to σ mesons is determined from the potential depth of Λ hyperons in normal nuclear matter

$$U_\Lambda = -g_{\sigma\Lambda}\sigma + g_{\omega\Lambda}\omega_0. \tag{65}$$

We take the potential depth $U_\Lambda = -30$ MeV as obtained from the analysis of Λ hypernuclei [54,56].

We adopt a profile of a magnetic field given by [57],

$$B(n_b/n_0) = B_s + B_c\left(1 - e^{-\beta(n_b/n_0)^\gamma}\right). \qquad (66)$$

We consider different values for the central field $B_c = 10^{16}$ and 10^{17} G, whereas the surface field strength is taken as $B_s = 10^{14}$ G in this calculation. We chose $\beta = 0.01$ and $\gamma = 3$. The magnetic field strength depends on the baryon density in the above parameterization. Further, the magnetic field at each density point is constant and uniform. The effects of anomalous magnetic moments of nucleons and contributions of the magnetic field to energy density and pressure are negligible because the magnetic fields considered in this calculation are not too strong.

Numbers of Landau levels populated by electrons and protons, are sensitive to the magnetic field strength and baryon density. As the field strength increases, the population of Landau levels decreases. In a weak magnetic field, when many Landau levels are populated, we treat charged particles unaffected by the magnetic field. Further, the effects of magnetic fields are most pronounced when only zeroth Landau levels are populated. Protons, electrons and muons populate zeroth Landau levels if central field strength $B_c \sim 10^{19}$ G. Figure 1 shows fractions of various particle species with normalized baryon density. We find large numbers of Landau levels of charged particles even when the magnetic field reaches its central value 10^{17} G. Populations of charged particles are enhanced in the magnetic field due to Landau quantization than those of field free case (not shown in the figure). It is noted in Fig. 1 that the threshold density of Λ hyperons is shifted to $1.7n_0$ from its zero magnetic field value of $2.6n_0$ because of phase space modifications of charged particles in a magnetic field.

The variation of pressure with energy density in the presence of a magnetic field with central field strength $B_c = 10^{17}$ G (solid curve) is shown in Fig. 2. The dashed curve denotes the EoS without a magnetic field. The EoS in the presence of the magnetic field becomes stiffer when charged particles are Landau quantized. Here, magnetic field contributions to the energy density and pressure are insignificant.

Now we compute the relaxation time for both nonleptonic and leptonic reactions as given by Eq. (22). To calculate the matrix element we take $g_{np} = -1.27$ and $g_{p\Lambda} = -0.72$ [51], and the Cabibbo angle (θ_c) is given by $\sin\theta_c = 0.222$. As we have already noted, charged particles populate many Landau levels in a magnetic field having central value $B_c = 10^{17}$ G over the entire density range considered in our calculation. For the nonleptonic process, when protons populate large number of Landau levels, we use the field free expression of Γ as given by Eq. (36). For leptonic reactions we use the expression as given by Eq. (47) when leptons and protons populate finite numbers of Landau levels. Chemical potentials and Fermi momenta of constituent particles are obtained from the EoS. The partial derivative of chemical potentials with respect to baryon density can be calculated from the EoS. Using these inputs, we can compute relaxation times for both reactions as a function of baryon density at a particular temperature. Figure 3 displays relaxation time (τ) of the nonleptonic process involving Λ hyperons in a magnetic field having its central value $B_c = 10^{17}$ G and at different temperatures as a function of normalized baryon number density. Here, τ decreases with increasing baryon density. Further the relaxation time in a magnetic field increases with decreasing temperature as was earlier noted in the field free case [23].

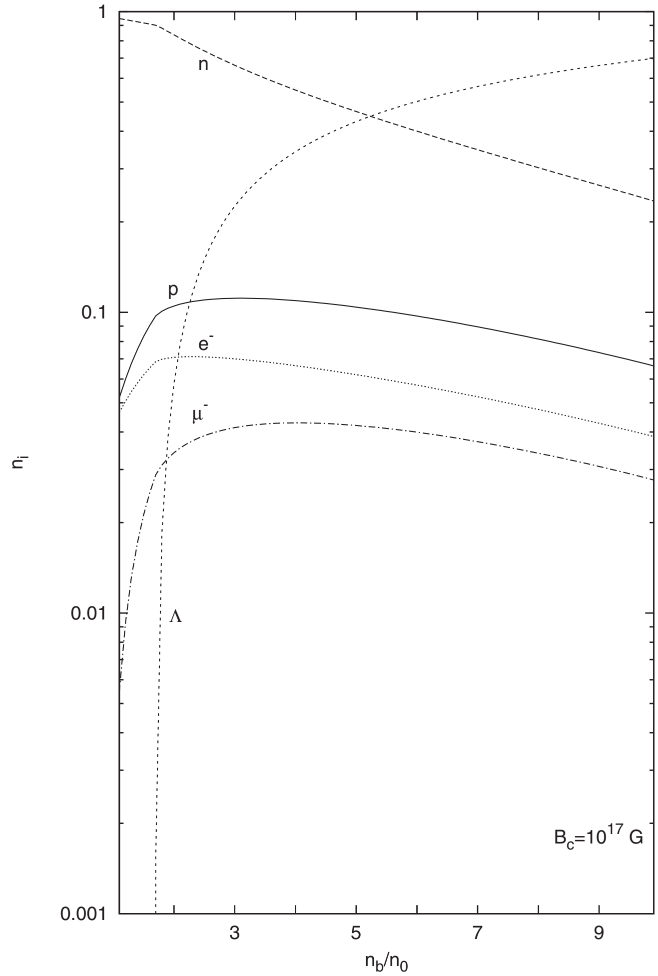

FIG. 1. Fractions of different particle species in Λ-hyperon matter in the presence of a magnetic field having central value $B_c = 10^{17}$ G as a function of normalized baryon density.

Relaxation times for dUrca reactions involving electrons and muons in a magnetic field with $B_c = 10^{17}$ G, and at different temperatures are plotted in Figs. 4 and 5, respectively. For leptonic processes, relaxation times are affected by the magnetic field. For the field free case, the dUrca process sets in at $1.4n_0$. In the magnetic field, relaxation times due to dUrca reactions drop sharply from large values in the forbidden domain $K_{F_n} > K_{F_p} + K_{F_e}$. This is

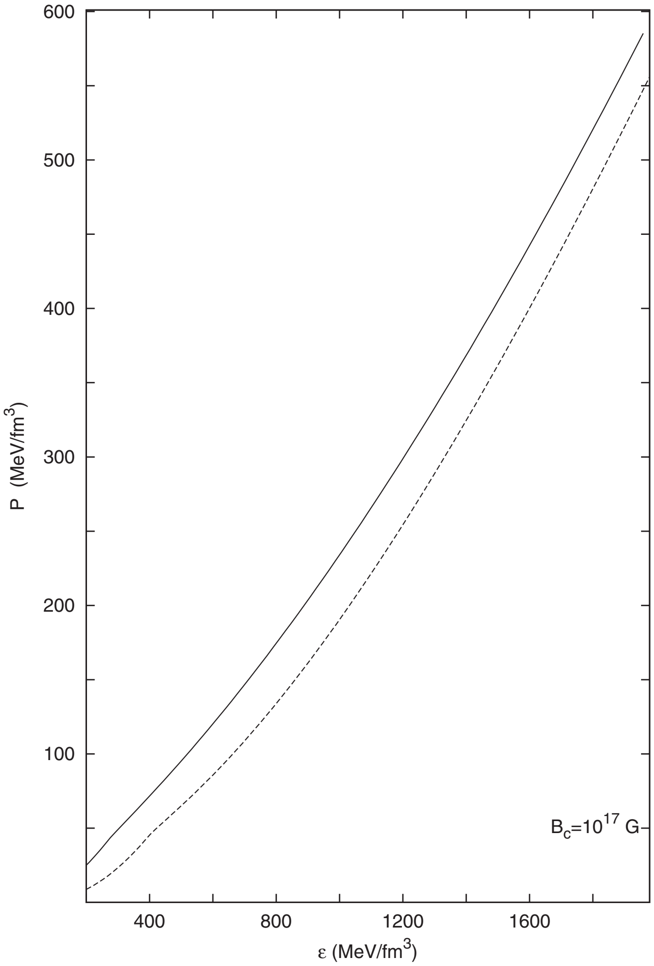

FIG. 2. Equation of state, pressure versus energy density, with a magnetic field having central value $B_c = 10^{17}$ G (solid line) and without a magnetic field (dashed curve).

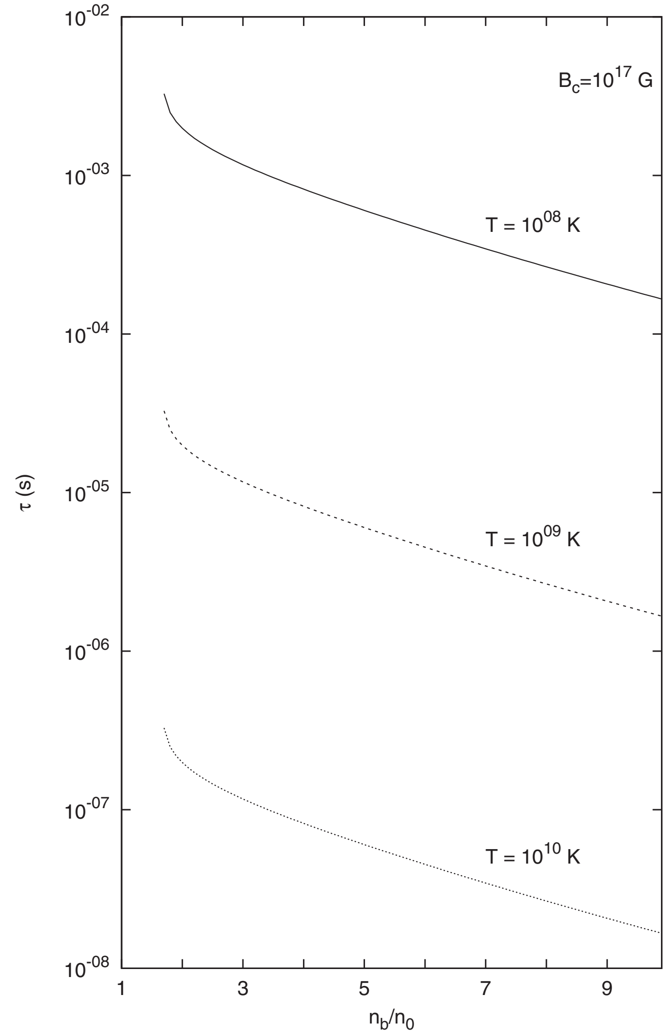

FIG. 3. Relaxation time for the nonleptonic reaction involving Λ hyperons in a magnetic field having central value $B_c = 10^{17}$ G and at different temperatures as a function of normalized baryon density.

attributed to the behavior of R_B^{qc}, which we discuss in detail in connection with bulk viscosity due to dUrca processes below. The forbidden domain joins with the allowed domain $K_{F_p} + K_{F_e} > K_{F_n}$ at a point from which relaxation times increase with baryon density. Like the nonleptonic case, relaxation times for dUrca processes also increase with decreasing temperature.

Now we focus on the calculation of bulk viscosity due to the nonleptonic and leptonic processes. As soon as we know relaxation times of nonleptonic and leptonic reactions, we compute bulk viscosity coefficients for the respective processes from Eq. (20). In this calculation we consider the $l = m = 2$ r mode and hence $\omega = 2/3\Omega$. Further, we take $\Omega = 3000\ \mathrm{s}^{-1}$. In the temperature regime considered here, we have always $\omega\tau \ll 1$ for the nonleptonic process involving Λ hyperons. Therefore, we neglect that term in the denominator of Eq. (20) to calculate the hyperon bulk viscosity. The partial derivative of pressure with respect to neutron number density is calculated from the EoS using Eq. (23) and the total derivative of neutron fraction with respect to baryon density is computed numerically from the EoS. As the relaxation time is a function of temperature, the bulk viscosity coefficient ζ also depends on temperature. The bulk viscosity coefficient for the nonleptonic process in a magnetic field with $B_c = 10^{17}$ G (dashed curve) and in the absence of a magnetic field (solid curve) are exhibited as a function of the normalized baryon number density in Fig. 6 at different temperatures. The nonleptonic reaction involves protons that populate many Landau levels in the magnetic field with $B_c = 10^{17}$ G. In this case, we adopt the field free expression of the reaction rate as given by Eq. (36) for the calculation of relaxation time and hyperon bulk viscosity coefficient in Eq. (20). Therefore, the effects of the magnetic field enter into hyperon bulk viscosity coefficient through the EoS, which is modified by Landau quantization of charged particles. In Fig. 6, we find hyperon bulk viscosity in the magnetic field is suppressed compared with the field free case.

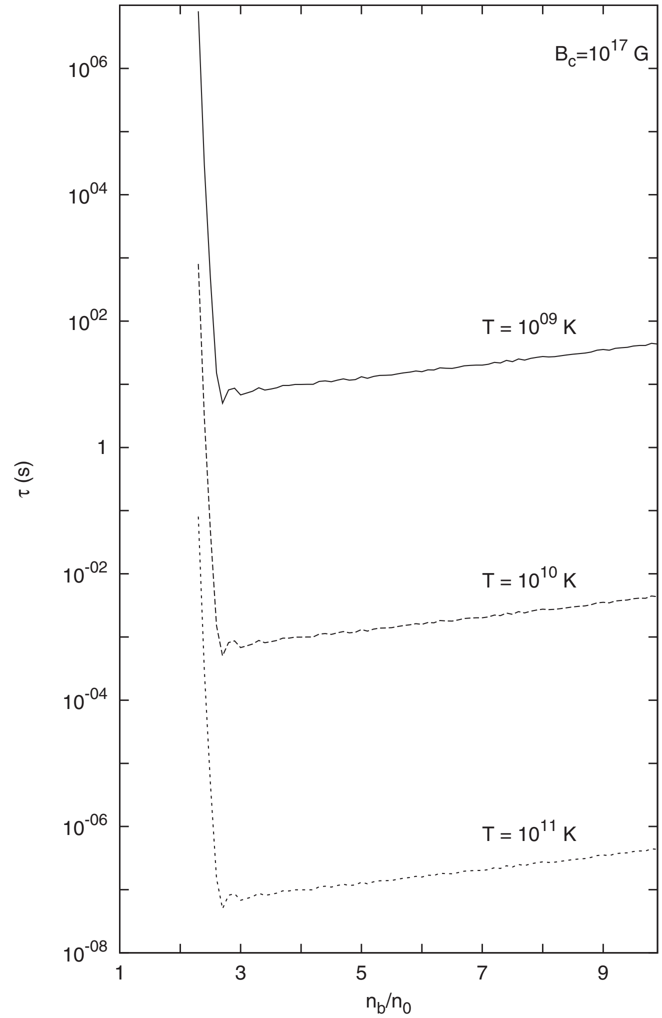

FIG. 4. Relaxation time of dUrca reaction involving electrons in a magnetic field having central value $B_c = 10^{17}$ G and at different temperatures as a function of normalized baryon density.

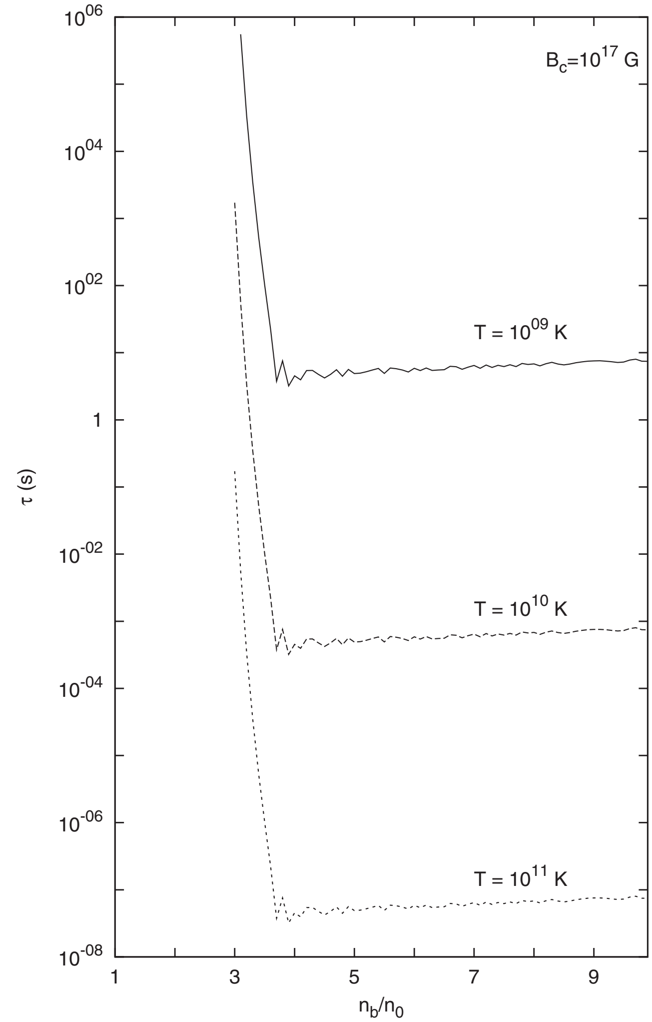

FIG. 5. same as Fig. 4 but for dUrca reaction including muons.

We display bulk viscosity coefficient for the dUrca process in a magnetic field with $B_c = 10^{16}$ G and at a temperature $T = 10^{11}$ K as a function of normalized baryon density in Fig. 7. In this case electrons and protons populate many Landau levels. The dotted line represents the dUrca contribution in the forbidden domain $K_{F_n} > K_{F_p} + K_{F_e}$. In this regime, reaction kinetics are characterized by two parameters $x = \frac{K_{F_n}^2 - (K_{F_p} + K_{F_e})^2}{K_{F_p}^2 N_{F_p}^{-2/3}}$ and $y = N_{F_p}^{2/3}$, where N_{F_p} is the number of proton Landau levels. The dUrca reaction in the forbidden domain is an efficient process as long as $x \leq 10$. This corresponds to baryon density $\leq 2.3 n_0$. The large enhancement of bulk viscosity coefficient in this domain is attributed to the behavior of R_B^{qc} [48]. It was noted that $R_B^{qc} = 1/3$ at $x = 0$, and it becomes very small when $x > 10$ [48]. At $x = 0$, the forbidden domain merges with the allowed domain $K_{F_p} + K_{F_e} > K_{F_n}$ of the dUrca process. The dUrca bulk viscosity in the allowed domain is shown by the dashed-dotted line. The result of zero field is shown by the solid line. The bulk coefficient increases with the magnetic field in the allowed domain at higher baryon densities.

Figures 8 and 9 show bulk viscosity coefficients for dUrca processes involving electrons and muons in the presence of the magnetic field with central value $B_c = 10^{17}$ G and at different temperatures as a function of normalized baryon density. In both cases contributions to bulk viscosity coefficients due to dUrca processes come from the forbidden as well as allowed domains. As discussed above, the forbidden domain merges with the allowed domain at $x = 0$. For temperatures $T = 10^9$ and 10^{10} K, bulk viscosity coefficients due to dUrca processes increase with baryon density. However, the bulk viscosity for $T = 10^{11}$ K initially decreases and later increases with baryon density. This behavior can be understood in the following way. For dUrca processes at 10^{11} K, we have $\omega\tau < 1$. On the other hand, we find $\omega\tau > 1$ for dUrca processes at 10^9 K and 10^{10} K. Consequently, bulk viscosity coefficients have a T^4 dependence when $\omega\tau > 1$,

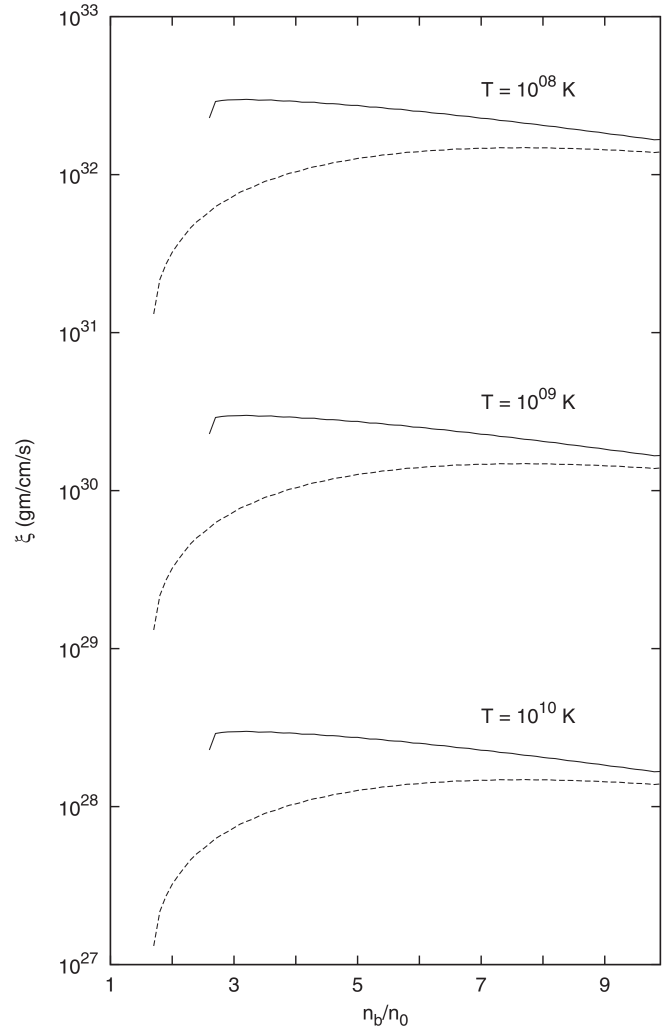

FIG. 6. Bulk viscosity coefficient (dashed line) for the nonleptonic processes involving Λ hyperons in a magnetic field having central value $B_c = 10^{17}$ G and at different temperatures as a function of normalized baryon density. Field free cases are shown by solid lines.

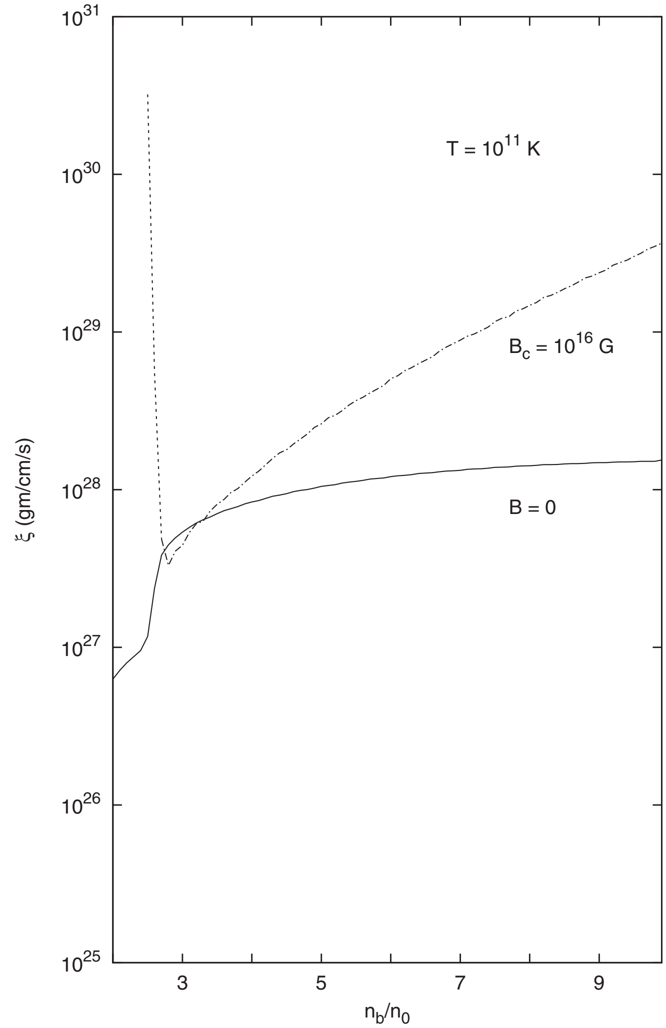

FIG. 7. Bulk viscosity coefficient for the dUrca process involving electrons in a magnetic field having central value $B_c = 10^{16}$ G and at a temperature 10^{11} K as a function of normalized baryon density. The field free case is shown by the solid line.

whereas it has a T^{-4} dependence when $\omega\tau < 1$. This inversion of temperature dependence of dUrca bulk viscosity coefficients is not found in the case of hyperon bulk viscosity.

Finally, we point out what happens in case of superstrong fields. We find that charged particles populate zeroth Landau levels when $B_c \sim 10^{19}$. Populations of charged particles are enhanced because of strong modification of their phase spaces. Further the EoS is modified due to magnetic field contributions to the energy density and pressure. The strong magnetic field enhances the hyperon bulk viscosity compared with the field free case. Similarly, we note significant modification in bulk viscosity coefficients due to dUrca processes when leptons and protons are in zeroth Landau levels. However, there is no observational evidence for the superstrong field $\sim 10^{19}$ G in the neutron star's interior so far.

V. SUMMARY

We have investigated bulk viscosity of nonleptonic process involving Λ hyperons and dUrca processes in the presence of strong magnetic fields. In this calculation we consider magnetic fields with different central values $B_c = 10^{16}$ and 10^{17} G. The equation of state has been constructed using the relativistic field theoretical model. Many Landau levels of charged particles are populated for the above values of central field. For a particular temperature, the hyperon bulk viscosity coefficient is reduced compared with that of the zero field case. Further, it is noted that the hyperon bulk viscosity decreases with increasing temperature as was earlier reported for the field free case. Bulk viscosity coefficients due to dUrca processes in a magnetic field have contributions from the forbidden as well as allowed domains. The bulk viscosity

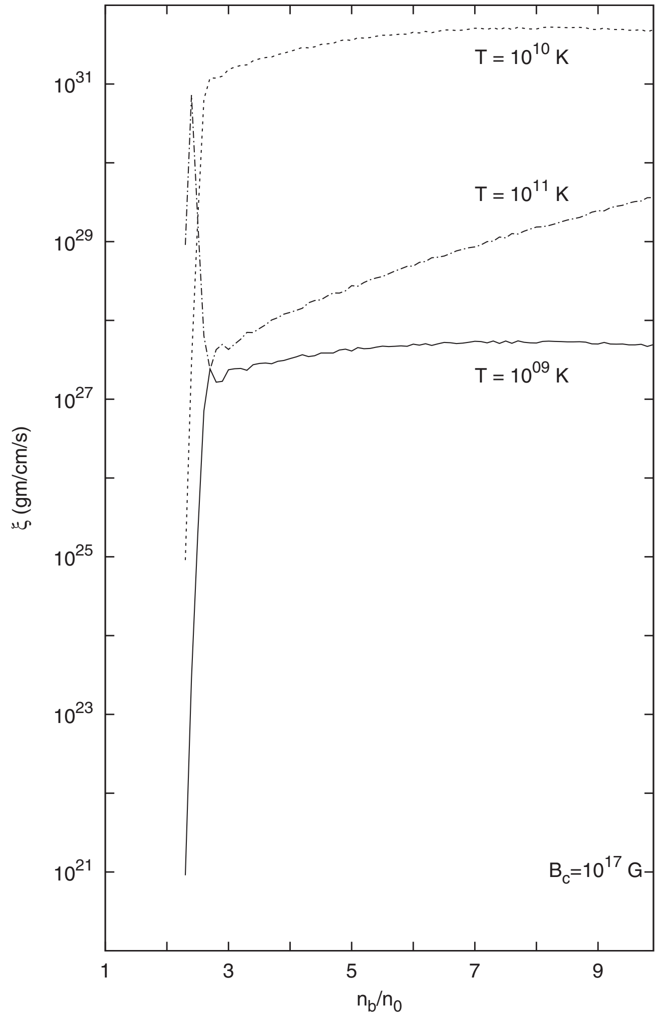

FIG. 8. Bulk viscosity coefficient for the dUrca process involving electrons in a magnetic field having central value $B_c = 10^{17}$ G and at different temperatures as a function of normalized baryon density.

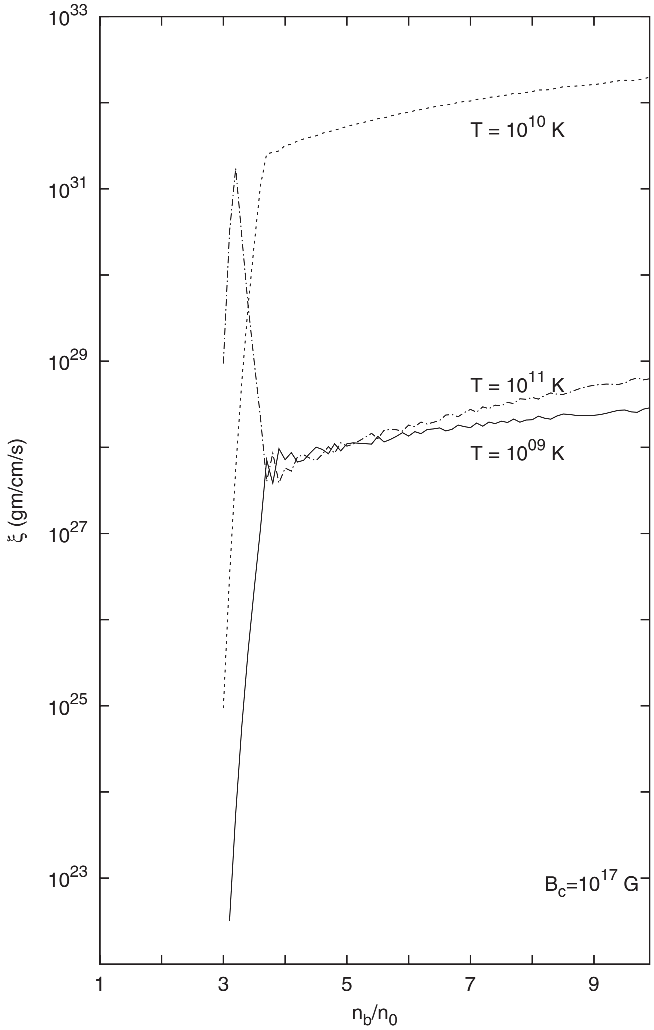

FIG. 9. Same as Fig. 8 but for the dUrca process involving muons.

coefficients in magnetic fields having central values $B_c = 10^{16}$ and 10^{17} G are enhanced in the allowed domain at higher baryon densities than those of field free cases. We find an inversion of the temperature dependence of dUrca bulk viscosity coefficients at 10^{11} K. We briefly discuss the effects of a superstrong magnetic field $\sim 10^{19}$ G on hyperon and dUrca bulk viscosities when zeroth Landau levels of charged particles are populated. However, such a superstrong magnetic field may not be a possibility in neutron stars.

In this calculation, we adopt the field free hyperon bulk viscosity relation when protons populate large number of Landau levels. This may be an approximate treatment of the actual case. However, the exact treatment of the effects of a magnetic field on the nonleptonic bulk viscosity would be worth studying when protons populate many Landau levels. Further, the investigation of bulk viscosity in magnetic fields has important implications for the r modes in magnetars. This will be reported in a future publication.

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