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Semileptonic B_c to P-wave charmonia transitions within QCD sum rules

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The form factors of the semileptonic $B_c \to S(AV)\ell\nu$ ($\ell = \tau, \mu, e$) transitions, where S and AV denote the scalar X_{c0} and axial-vector (X_{c1}, h_c) mesons, are calculated within the framework of the three-point QCD sum rules. The heavy quark effective theory limit of the form factors is also obtained and compared with the values of the original transition form factors. The results of form factors are used to estimate the total decay widths and branching ratios of these transitions. A comparison of our results on branching ratios with the predictions of other approaches is also presented.

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I. INTRODUCTION

The B_c meson is the only known meson composed of two heavy quarks of different flavor and charge; a charm quark and a bottom antiquark. It was discovered by the collider detector at Fermilab (CDF Collaboration) in $p\bar{p}$ collision via the decay mode $B_c \to J/\psi l^{\pm}\nu$ at $\sqrt{s} = 1$, 8 TeV [1]. Discovery of the B_c meson has demonstrated the possibility of the experimental study of the charm-beauty system and has created considerable interest in its spectroscopy [2–6]. When the Large Hadron Collider (LHC) runs, an abundant number of B_c mesons, which are expected to be about $10^8 \sim 10^{10}$ per year with the luminosity values of $L = 10^{34}\ \mathrm{cm}^{-2}\,\mathrm{s}^{-1}$ and $\sqrt{s} = 14$ TeV, will be produced [7,8]. Therefore, not only experimental but also theoretical study on B_c mesons will be of great interest in many respects.

Among the B mesons, the B_c carries a distinctive signature and has attracted great interest recently for the following reasons: First, the B_c meson decay channels are expected to be very rich in comparison with other B mesons, so investigation of such types of decays can be used in the calculation of the Cabibbo-Kabayashi-Maskawa (CKM) matrix elements, leptonic decay constant, as well as the origin of the CP and T violation. Second, the B_c meson, because it contains the heavy quarks, provides more accuracy and confidence in the understanding of the QCD dynamics.

The B_c meson can decay via the $b \to u, d, s, c$ and also the $c \to u, d, s$ transitions. Among those transitions at quark level, the tree-level $b \to c$ transition, governing the B_c to P-wave charmonia, plays a significant role, because this is the most dominant transition. In the literature, there are several studies on the B_c mesons in different models. Some possible B_c meson decays such as $B_c \to l\bar{\nu}\gamma$, $B_c \to \rho^+\gamma$, $B_c \to K^{*+}\gamma$ and $B_c \to B_u^* l^+ l^-$, $B_c \to B_u^*\gamma$, $B_c^- \to D^{*0}\ell\nu$, $B_c \to P(D, D_s) l^+ l^- / \nu\bar{\nu}$, $B_c \to D_{s,d}^* l^+ l^-$, $B_c \to X\nu\bar{\nu}$ and $B_c \to D^*{}_s\gamma$ have been studied in the frame of light-cone QCD and three-point QCD sum rules [9–17]. The weak productions of new charmonium in semileptonic decays of B_c were also studied in the framework of light-cone QCD sum rules in [18]. In [19], a larger set of exclusive nonleptonic and semileptonic decays of the B_c meson were investigated in the relativistic constituent quark model. Weak decays of the B_c meson to charmonium and D mesons in the relativistic quark model have been discussed in [20,21]. Moreover, the $B_c \to (D^*, D^*{}_s)\nu\bar{\nu}$ transitions were also studied within the relativistic constituent quark model in [22].

Present work is devoted to the study of the $B_c \to S(AV)\ell\nu$. The long-distance dynamics of such transitions can be parametrized in terms of some form factors, which play a fundamental role in analyzing such transitions. For evaluation of the form factors, the QCD sum rules as a nonperturbative approach based on the fundamental QCD Lagrangian is used. The obtained results for the form factors are used to estimate the total decay rate and branching fractions for the related transitions. The heavy quark effective theory (HQET) limit of the form factors is also calculated and compared with their values. In these transitions, the main contribution comes from the perturbative part since the heavy quark condensates are suppressed by the inverse of the heavy quark masses and can be safely omitted and two-gluon condensate contributions are very small and we will ignore them. Note that the B_c to P-wave charmonia transitions have also been investigated in the framework of the covariant light-front quark model (CLQM), the renormalization group method (RGM), relativistic constituent quark model (RCQM), and nonrelativistic constituent quark model (NRCQM) in [23–27]. For more about those transitions see also [28–35].

The outline of the paper is as follows: In Sec. II, the same rules for the transition form factors relevant to the $B_c \to S(AV)\ell\nu$ decays are obtained. Section III encompasses the calculation of the HQET limit of the form factors and, Sec. IV is devoted to the numerical analysis

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of the form factors and their HQET limits, decay rates, branching ratios, conclusion and comparison of our results with the other approaches.

II. SUM RULES FOR THE $B_c \to S(AV)\ell\nu$ TRANSITION FORM FACTORS

The $B_c \to X_{c0}(X_{c1}, h_c)\ell\nu$ decays proceed via the $b \to c$ transition at the quark level. The effective Hamiltonian responsible for these transitions can be written as

$$H_{\rm eff} = \frac{G_F}{\sqrt{2}} V_{cb} \bar{\nu}\gamma_\mu(1-\gamma_5) l \bar{c}\gamma_\mu(1-\gamma_5) b. \qquad (1)$$

We need to sandwich Eq. (1) between initial and final meson states in order to obtain the matrix elements of $B_c \to S(AV)\ell\nu$. Hence, the amplitude of this decay is written as follows:

$$M = \frac{G_F}{\sqrt{2}} V_{cb} \bar{\nu}\gamma_\mu(1-\gamma_5) l \langle S(AV)(p') \times |\bar{c}\gamma_\mu(1-\gamma_5) b| B_c(p)\rangle. \qquad (2)$$

It is necessary to calculate the matrix elements $\langle S(AV)\times(p') \mid \bar{c}\gamma_\mu(1-\gamma_5)b \mid B_c(p)\rangle$ appearing in Eq. (2). In the S case in the final state, the only axial-vector part of the transition current, $\bar{c}\gamma_\mu(1-\gamma_5)b$, can contribute to the matrix element stated above. However, in the AV case, both vector and axial-vector parts have contributions. Considering the parity and Lorentz invariances, the aforementioned matrix element can be parametrized in terms of the form factors in the following way:

$$\langle S(p')|\bar{c}\gamma_\mu\gamma_5 b|B_c(p)\rangle = f_1(q^2)P_\mu + f_2(q^2)q_\mu, \qquad (3)$$

$$\langle AV(p',\varepsilon)|\bar{c}\gamma_\mu\gamma_5 b|B_c(p)\rangle = i\frac{f_V(q^2)}{(m_{B_c}+m_{X_{c1}})}\varepsilon_{\mu\nu\alpha\beta}\varepsilon^{*\nu}p^\alpha p'^\beta, \qquad (4)$$

$$\langle AV(p',\varepsilon)|\bar{c}\gamma_\mu b|B_c(p)\rangle = i\Big[f_0(q^2)(m_{B_c}+m_{X_{c1}})\varepsilon^*_\mu - \frac{f_+(q^2)}{(m_{B_c}+m_{X_{c1}})}(\varepsilon^* p)P_\mu - \frac{f_-(q^2)}{(m_{B_c}+m_{X_{c1}})}(\varepsilon^* p)q_\mu\Big], \qquad (5)$$

where $f_1(q^2)$, $f_2(q^2)$, $f_V(q^2)$, $f_0(q^2)$, $f_+(q^2)$, and $f_-(q^2)$ are transition form factors and $P_\mu = (p+p')_\mu$, $q_\mu = (p-p')_\mu$.

From the general philosophy of the QCD sum rules, we see a hadron from two different windows. First, we see it from the outside, so we have a hadron with hadronic parameters such as its mass and leptonic decay constant. Second, we see the internal structure of the hadron, namely, quarks and gluons and their interactions in a QCD vacuum. In technical language, we start with the main object in QCD sum rules called the correlation function. The correlation function is calculated in two different ways: From one side, it is saturated by a tower of hadrons called the phenomenological or physical side. On the other hand, in the QCD or theoretical side, it is calculated in terms of quark and gluons interacting in a QCD vacuum with the help of the operator product expansion (OPE), where the short- and long-distance effects are separated. The former is calculated using the perturbation theory (perturbative contribution); however, the latter is parametrized in terms of vacuum condensates with different mass dimensions. In the present work there are no light quarks, and the heavy quark condensate contributions are suppressed by the inverse of the heavy quark mass and can be safely removed. The two-gluon contributions are also very small, and here we will ignore those contributions. Hence, the only contribution comes from the perturbative part. Equating two representations of the correlation function and applying double Borel transformation with respect to the momentum of the initial and final states to suppress the contribution of the higher states and continuum, sum rules for the physical quantities, form factors, are obtained. To proceed, we consider the following correlation functions:

$$\Pi_\mu(p^2,p'^2) = i^2\int d^4x d^4y e^{-ipx}e^{ip'y}\langle 0|T \times [J_S(y)J_\mu^{V;A}(0)J_{B_c}(x)]|0\rangle, \qquad (6)$$

$$\Pi_{\mu\nu}(p^2,p'^2) = i^2\int d^4x d^4y e^{-ipx}e^{ip'y}\langle 0|T \times [J_{\nu AV}(y)J_\mu^{V;A}(0)J_{B_c}(x)]|0\rangle, \qquad (7)$$

where $J_S(y) = \bar{c}Uc$, $J_{\nu AV}(y) = \bar{c}\gamma_\nu\gamma_5 c$, $J_{B_c}(x) = \bar{b}\gamma_5 c$ are the interpolating currents of the S, AV, and B_c mesons, respectively, and $J_\mu^V(0) = \bar{c}\gamma_\mu b$, $J_\mu^A = \bar{c}\gamma_\mu\gamma_5 b$ are the vector and axial-vector parts of the transition current. In order to calculate the phenomenological or physical part of the correlator given in Eq. (6), two complete sets of intermediate states with the same quantum numbers as the interpolating currents $J_{S(AV)}$ and J_{B_c} are inserted. As a result, the following representations of the abovementioned correlators are obtained:

$$\Pi_\mu(p^2,p'^2) = \frac{\langle 0|J_S(0)|S(p')\rangle\langle S(p')|J_\mu^{V;A}(0)|B_c(p)\rangle\langle B_c(p)|J_{B_c}(0)|0\rangle}{(p'^2-m_S^2)(p^2-m_{B_c}^2)} + \cdots, \qquad (8)$$

$$\Pi_{\mu\nu}(p^2, p'^2) = \frac{\langle 0|J^{\nu}_{AV}(0)|AV(p', \varepsilon)\rangle\langle AV(p', \varepsilon)|J^{V;A}_{\mu}(0)|B_c(p)\rangle\langle B_c(p)|J_{B_c}(0)|0\rangle}{(p'^2 - m^2_{AV})(p^2 - m^2_{B_c})} + \cdots, \tag{9}$$

where $\cdots$ represents the contributions coming from higher states and the continuum. The vacuum to the hadronic state matrix elements in Eq. (8) can be parametrized in terms of the leptonic decay constants as

$$\langle 0|J_S|S(p')\rangle = -if_S, \qquad \langle B_c(p)|J_{B_c}|0\rangle = -i\frac{f_{B_c}m^2_{B_c}}{m_b + m_c}, \qquad \langle 0|J^{\nu}_{AV}|AV(p', \varepsilon)\rangle = f_{AV}m_{AV}\varepsilon^{\nu}. \tag{10}$$

Using Eqs. (3)–(10), the final expressions of the phenomenological side of the correlation functions are obtained as

$$\begin{aligned}\Pi_{\mu}(p^2, p'^2) &= -\frac{f_S}{(p'^2 - m^2_S)(p^2 - m^2_{B_c})}\frac{f_{B_c}m^2_{B_c}}{m_b + m_c}[f_1(q^2)P_{\mu} + f_2(q^2)q_{\mu}] + \text{excited states},\\ \Pi_{\mu\nu}(p^2, p'^2) &= \frac{f_{B_c}m^2_{B_c}}{(m_b + m_c)}\frac{f_{AV}m_{AV}}{(p'^2 - m^2_{AV})(p^2 - m^2_{B_c})}\bigg[f_0(q^2)g_{\mu\nu}(m_{B_c} + m_{AV}) - \frac{f_+(q^2)P_{\mu}p_{\nu}}{(m_{B_c} + m_{AV})} - \frac{f_-(q^2)q_{\mu}p_{\nu}}{(m_{B_c} + m_{AV})}\\ &\quad + \varepsilon_{\alpha\beta\mu\nu}p^{\alpha}p'^{\beta}\frac{f_V(q^2)}{(m_{B_c} + m_{AV})}\bigg] + \text{excited states},\end{aligned} \tag{11}$$

where we will choose the structures P_{μ}, q_{μ}, $\varepsilon_{\mu\nu\alpha\beta}p'^{\alpha}p^{\beta}$, $g_{\mu\nu}$ and $\frac{1}{2}(p_{\mu}p_{\nu} \pm p'_{\mu}p_{\nu})$ to evaluate the form factors f_1, f_2, f_V, f_0 and $f_{\pm}$, respectively.

On the QCD side, the aforementioned correlation functions can be calculated with the help of the OPE in the deep spacelike region where $p^2 \ll (m_b + m_c)^2$ and $p'^2 \ll (2m_c)^2$. As we mentioned before, the main contributions to the theoretical part of the correlation functions come from bare-loop (perturbative) diagrams. To calculate those contributions, the correlation functions are written in terms of the selected structures as follows:

$$\begin{aligned}\Pi_{\mu} &= \Pi^{\text{per}}_1 P_{\mu} + \Pi^{\text{per}}_2 q_{\mu},\\ \Pi_{\mu\nu} &= \Pi^{\text{per}}_V \varepsilon_{\mu\nu\alpha\beta}p'^{\alpha}p^{\beta} + \Pi^{\text{per}}_0 g_{\mu\nu}\\ &\quad + \frac{1}{2}\Pi^{\text{per}}_+(p_{\mu}p_{\nu} + p'_{\mu}p_{\nu})\\ &\quad + \frac{1}{2}\Pi^{\text{per}}_-(p_{\mu}p_{\nu} - p'_{\mu}p_{\nu}),\end{aligned} \tag{12}$$

where each Π^{per}_i function is written in terms of the double dispersion representation in the following way:

$$\Pi^{\text{per}}_i = -\frac{1}{(2\pi)^2}\int ds\int ds'\frac{\rho_i(s, s', q^2)}{(s - p^2)(s' - p'^2)} + \text{subtraction terms}, \tag{13}$$

where the functions $\rho_i(s, s', q^2)$ are called the spectral densities. Using the usual Feynman integral for the bare-loop diagram, the spectral densities can be calculated with the help of Cutkosky rules, i.e., by replacing the quark propagators with Dirac delta functions: $\frac{1}{p^2 - m^2} \to -2\pi\delta(p^2 - m^2)$, which implies that all quarks are real. After some straightforward calculations, the spectral densities are obtained as follows:

$$\begin{aligned}\rho_1(s, s', q^2) &= N_cI_0(s, s', q^2)[2(m_b - 3m_c)m_c + 2A\{2(m_b - m_c)m_c - s\} + 2B\{2(m_b - m_c)m_c - s'\}],\\ \rho_2(s, s', q^2) &= N_cI_0(s, s', q^2)[-2(m_b + m_c)m_c + 2A\{2(m_b - m_c)m_c + s\} - 2B\{2(m_b - m_c)m_c + s'\}],\\ \rho_V(s, s', q^2) &= 4N_cI_0(s, s', q^2)[(m_c - m_b)A + 2m_cB + m_c],\\ \rho_0(s, s', q^2) &= 2N_cI_0(s, s', q^2)[4(m^2_c - C)(m_c - m_b) + m_cu + \{m_c(4s + u) - m_bu\}A + 2[-m_bs' + m_c(s' + u)]B],\\ \rho_+(s, s', q^2) &= 2N_cI_0(s, s', q^2)[-m_c + (m_b - 3m_c)A - 2m_cB + 2(m_b - m_c)D + 2(m_b - m_c)E],\\ \rho_-(s, s', q^2) &= 2N_cI_0(s, s', q^2)[m_c - (m_c + m_b)A + 2m_cB + 2(m_b - m_c)D + 2(m_c - m_b)E],\end{aligned} \tag{14}$$

where

$$
\begin{aligned}
I_0(s,s',q^2) &= \frac{1}{4\lambda^{1/2}(s,s',q^2)},\\
\lambda(a,b,c) &= a^2+b^2+c^2-2ac-2bc-2ab,\\
\Delta &= m_b^2-m_c^2-s,\\
\Delta' &= -s',\\
u &= s+s'-q^2,\\
A &= \frac{1}{\lambda(s,s',q^2)}(\Delta' u-2\Delta s'),\\
B &= \frac{1}{\lambda(s,s',q^2)}(\Delta u-2\Delta' s),\\
C &= \frac{1}{2\lambda(s,s',q^2)}[\Delta'^2 s+\Delta^2 s'-\Delta\Delta' u\\
&\quad + m_c^2(-4ss'+u^2)],\\
D &= \frac{1}{\lambda(s,s',q^2)^2}[-6\Delta\Delta' s' u+\Delta'^2(2ss'+u^2)\\
&\quad + 2s'(3\Delta^2 s'+m_c^2(-4ss'+u^2))],\\
E &= \frac{1}{\lambda(s,s',q^2)^2}[-3\Delta^2 s' u+2\Delta\Delta'(2ss'+u^2)\\
&\quad - u(3\Delta'^2 s+m_c^2(-4ss'+u^2))], \qquad (15)
\end{aligned}
$$

and $N_c=3$ is the number of colors. The integration region for the perturbative contribution in Eq. (13) is determined requiring that the arguments of the three δ functions vanish simultaneously. Therefore, the physical region in the s and s' plane is described by the following nonequality:

$$
-1 \le f(s,s') = \frac{2ss'+(s+s'-q^2)(m_b^2-s-m_c^2)}{\lambda^{1/2}(m_b^2,s,m_c^2)\lambda^{1/2}(s,s',q^2)} \le +1. \qquad (16)
$$

Equating the coefficient of the selected structures from the phenomenological and the OPE expressions and applying double Borel transformations with respect to the variables p^2 and p'^2 ($p^2\to M_1^2$, $p'^2\to M_2^2$) in order to suppress the contributions of the higher states and continuum, the QCD sum rules for the form factors $f_1(q^2)$ and $f_2(q^2)$ for the $B_c\to X_{c0}\ell\nu$ decay can be acquired:

$$
\begin{aligned}
f_{1,2}(q^2) &= \frac{(m_b+m_c)}{f_{B_c}m_{B_c}^2}\frac{1}{f_{X_{c0}}}e^{m_{B_c}^2/M_1^2}e^{m_{X_{c0}}^2/M_2^2}\\
&\times\left\{\frac{1}{(2\pi)^2}\int_{(m_b+m_c)^2}^{s_0}ds\int_{(2m_c)^2}^{s_0'}ds'\rho_{1,2}(s,s',q^2)\right.\\
&\left.\times\,\theta[1-f^2(s,s')]e^{-s/M_1^2}e^{-s'/M_2^2}\right\}. \qquad (17)
\end{aligned}
$$

The form factors f_V, f_0, f_+, and f_- for $B_c\to AV\ell\nu$ decays are also obtained as

$$
\begin{aligned}
f_i(q^2) &= \kappa\frac{(m_b+m_c)}{f_{B_c}m_{B_c}^2}\frac{\eta}{f_{AV}m_{AV}}e^{m_{B_c}^2/M_1^2+m_{AV}^2/M_2^2}\\
&\times\left[\frac{1}{(2\pi)^2}\int_{4m_c^2}^{s_0'}ds'\int_{(m_b+m_c)^2}^{s_0}ds\rho_i(s,s',q^2)\right.\\
&\left.\times\,\theta[1-f^2(s,s')]e^{-s/M_1^2-s'/M_2^2}\right], \qquad (18)
\end{aligned}
$$

where $i=V,0,\pm$, and $\eta=m_{B_c}+m_{AV}$ for $i=V,\pm$ and $\eta=\frac{1}{m_{B_c}+m_{AV}}$ for $i=0$ are considered. Here $\kappa=+1$ for $i=\pm$ and $\kappa=-1$ for $i=0$ and V. In the above equations, the s_0 and s_0' are continuum thresholds in s and s' channels, respectively.

In order to subtract the contributions of the higher states and continuum, the quark-hadron duality assumption is used, i.e., it is assumed that

$$
\rho^{\text{higher states}}(s,s') = \rho^{\text{OPE}}(s,s')\theta(s-s_0)\theta(s'-s_0'). \qquad (19)
$$

Note that the double Borel transformation used in calculations is written as

$$
\begin{aligned}
&\hat{B}\frac{1}{(p^2-m_1^2)^m}\frac{1}{(p'^2-m_2^2)^n}\\
&\to(-1)^{m+n}\frac{1}{\Gamma(m)}\frac{1}{\Gamma(n)}e^{-m_1^2/M_1^2}e^{-m_2^2/M_2^2}\frac{1}{(M_1^2)^{m-1}(M_2^2)^{n-1}}. \qquad (20)
\end{aligned}
$$

Now, we would like to explain our reason for ignoring the contributions of the gluon condensates to the QCD side of the correlation function. These contributions for the related form factors are obtained as the following orders:

$$
\begin{aligned}
f_{1,2}^{\langle G^2\rangle} &\sim \left\langle\frac{\alpha_s}{\pi}G^2\right\rangle\frac{m_b^{n_1}m_c^{m_1}}{M_1^{2k_1}M_2^{2l_1}}, \qquad n_1+m_1=2k_1+2l_1,\\
f_{V,+,-}^{\langle G^2\rangle} &\sim \left\langle\frac{\alpha_s}{\pi}G^2\right\rangle\frac{m_b^{n_2}m_c^{m_2}}{M_1^{2k_2}M_2^{2l_2}}, \qquad n_2+m_2=2k_2+2l_2-1,\\
f_0^{\langle G^2\rangle} &\sim \left\langle\frac{\alpha_s}{\pi}G^2\right\rangle\frac{m_b^{n_3}m_c^{m_3}}{M_1^{2k_3}M_2^{2l_3}}, \qquad n_3+m_3=2k_3+2l_3+1,
\end{aligned}
\qquad (21)
$$

where α_s is the strong coupling constant and M_1^2 and M_2^2 are Borel mass parameters. Recalling the magnitude of the $\langle\frac{\alpha_s}{\pi}G^2\rangle=0.012\ \text{GeV}^4$ [36] and considering the working region of the Borel parameters (see numerical analysis section), the gluon condensate contributions become very small, and here we ignore those small contributions (maximum contribution is obtained for $f_0^{\langle G^2\rangle}$, which is not more than few percent).

At the end of this section, we would like to present the differential decay rates of the $B_c\to S(AV)\ell\nu$ in terms of the transition form factors. The differential decay width for $B_c\to S\ell\nu$ is obtained as follows:

$$\frac{d\Gamma}{dq^2} = \frac{1}{192\pi^3 m_{B_c}^3} G_F^2 |V_{cb}|^2 \lambda^{1/2}(m_{B_c}^2, m_S^2, q^2)\left(\frac{q^2 - m_\ell^2}{q^2}\right)^2 \left\{ -\frac{1}{2}(2q^2 + m_\ell^2)[|f_1(q^2)|^2(2m_{B_c}^2 + 2m_S^2 - q^2) + 2(m_{B_c}^2 - m_S^2)\,\mathrm{Re}[f_1(q^2)f_2^*(q^2)] + |f_2(q^2)|^2 q^2] + \frac{(q^2 + m_\ell^2)}{q^2}[|f_1(q^2)|^2(m_{B_c}^2 - m_S^2)^2 + 2(m_{B_c}^2 - m_S^2)q^2 \times \mathrm{Re}[f_1(q^2)f_2^*(q^2)] + |f_2(q^2)|^2 q^4] \right\}, \tag{22}$$

and also the differential decay width corresponding to $B_c \to AV\ell\nu$ decays are acquired as

$$\frac{d\Gamma}{dq^2} = \frac{1}{16\pi^4 m_{B_c}^2} |\vec{p}'| G_F^2 |V_{cb}|^2 \Bigg(4\{(2A_1 + A_2 q^2)[|f_V|^2(4m_{B_c}^2|\vec{p}'|^2) + |f_0|^2]\} + \Bigg\{ (2A_1 + A_2 q^2)\Bigg[|f_V|^2\left(4m_{B_c}^2|\vec{p}'|^2 + m_{B_c}^2\frac{|\vec{p}'|^2}{m_{AV}^2}(m_{B_c}^2 - m_{AV}^2 - q^2)\right) + |f_0|^2 - |f_+|^2\frac{m_{B_c}^2|\vec{p}'|^2}{m_{AV}^2}(2m_{B_c}^2 + 2m_{AV}^2 - q^2) - |f_-|^2\frac{m_{B_c}^2|\vec{p}'|^2}{m_{AV}^2}q^2 - 2\frac{m_{B_c}^2|\vec{p}'|^2}{m_{AV}^2}(\mathrm{Re}(f_0'f_+ + f_0'f_- + (m_{B_c}^2 - m_{AV}^2)f_+f_-))\Bigg] - 2A_2\frac{m_{B_c}^2|\vec{p}'|^2}{m_{AV}^2}\Bigg[|f_0|^2 + (m_{B_c}^2 - m_{AV}^2)^2|f_+|^2 + q^4|f_-|^2 + 2(m_{B_c}^2 - m_{AV}^2)\,\mathrm{Re}(f_0 f_+) + 2q^2 f_0 f_- + 2q^2(m_{B_c}^2 - m_{AV}^2)\,\mathrm{Re}(f_+f_-)\Bigg]\Bigg\}\Bigg), \tag{23}$$

where

$$|p'| = \frac{\lambda^{1/2}(m_{B_c}^2, m_{AV}^2, q^2)}{2m_{B_c}}, \qquad A_1 = \frac{1}{12q^2}(q^2 - m_l^2)^2 I_0',$$
$$A_2 = \frac{1}{6q^4}(q^2 - m_l^2)(q^2 + 2m_l^2)I_0', \qquad I_0' = \frac{\pi}{2}\left(1 - \frac{m_l^2}{q^2}\right). \tag{24}$$

III. HEAVY QUARK EFFECTIVE THEORY LIMIT OF THE FORM FACTORS

In this section, we calculate the heavy quark effective theory (HQET) limits of the transition form factors for $B_c \to S(AV)\ell\nu$. For this aim, following Refs. [37–40], we use the parametrization

$$y = \nu\nu' = \frac{m_{B_c}^2 + m_{S(AV)}^2 - q^2}{2m_{B_c}m_{S(AV)}}, \tag{25}$$

where ν and ν' are the four-velocities of the initial and final meson states, respectively. Next, we try to find the y dependent expressions of the form factors by taking $m_b \to \infty$, $m_c = \frac{m_b}{\sqrt{z}}$, where z is given by $\sqrt{z} = y + \sqrt{y^2 - 1}$. In this limit, the new Borel parameters $T_1 = M_1^2/2(m_b + m_c)$ and $T_2 = M_2^2/4m_c$ are defined. The new continuum thresholds ν_0, and ν_0' are also parametrized as

$$\nu_0 = \frac{s_0 - (m_b + m_c)^2}{m_b + m_c}, \qquad \nu_0' = \frac{s_0' - 4m_c^2}{2m_c}, \tag{26}$$

and the new integration variables take the following form:

$$\nu = \frac{s - (m_b + m_c)^2}{m_b + m_c}, \qquad \nu' = \frac{s' - 4m_c^2}{2m_c}. \tag{27}$$

The leptonic decay constants are rescaled:

$$\hat{f}_{B_c} = \sqrt{m_b + m_c} f_{B_c}, \qquad \hat{f}_{S(AV)} = \sqrt{2m_c} f_{S(AV)}. \tag{28}$$

To evaluate the form factors in HQET, we also need to redefine the form factors in the following form:

$$f_{1,2}' = \frac{f_{1,2}}{(m_{B_c} + m_S)^2} \qquad f_{V,0,+,-}' = \frac{f_{V,0,+,-}}{m_{B_c} + m_{AV}}. \tag{29}$$

After standard calculations, we obtain the y-dependent expressions of the form factors for $B_c \to S\ell\nu$ transition as follows:

$$f_1' = \frac{3(-1+y^2)[3+z+y(-3-2\sqrt{z}+z)]}{8\sqrt{2}\pi^2\hat{f}_S\hat{f}_{B_c}z^{13/4}(1+y)\sqrt{1+\frac{1}{\sqrt{z}}\Big[\frac{(-1+y^2)(1+\sqrt{z})^2}{z^2}\Big]^{3/2}}} \times e^{((\Lambda/T_1)+(\bar{\Lambda}/T_2))}\bigg\{\int_0^{\nu_0}d\nu\int_0^{\nu_0'}d\nu' e^{-(\nu/(2T_1))-(\nu'/(2T_2))}\theta[1-\lim_{m_b\to\infty}f^2(v,v')]\bigg\}, \tag{30}$$

$$f_2' = \frac{-3(-1+y^2)[-1+y(1+\sqrt{z})^2+4\sqrt{z}+z)]}{8\sqrt{2}\pi^2\hat{f}_S\hat{f}_{B_c}z^{13/4}(1+y)\sqrt{1+\frac{1}{\sqrt{z}}[\frac{(-1+y^2)(1+\sqrt{z})^2}{z^2}]^{3/2}}} \times e^{((\Lambda/T_1)+(\bar{\Lambda}/T_2))}\bigg\{\int_0^{\nu_0}d\nu\int_0^{\nu_0'}d\nu' e^{-(\nu/(2T_1))-(\nu'/(2T_2))}\theta[1-\lim_{m_b\to\infty}f^2(v,v')]\bigg\}, \tag{31}$$

and for $B_c \to AV\ell\nu$ decay, the y-dependent expressions of the form factors are acquired as

$$f_V' = \frac{3(3+\sqrt{z})[-1+y+\sqrt{z}+y\sqrt{z}]}{8\sqrt{2}\pi^2\hat{f}_{AV}\hat{f}_{B_c}z^{5/4}(1+y)(1+\sqrt{z})\sqrt{1+\frac{1}{\sqrt{z}}\sqrt{\frac{(-1+y^2)(1+\sqrt{z})^2}{z^2}}}} \times e^{((\Lambda/T_1)+(\bar{\Lambda}/T_2))}\bigg\{\int_0^{\nu_0}d\nu\int_0^{\nu_0'}d\nu' e^{-(\nu/(2T_1))-(\nu'/(2T_2))}\theta[1-\lim_{m_b\to\infty}f^2(v,v')]\bigg\}, \tag{32}$$

$$f_0' = \frac{3(-1+y)[1+2y(1+\sqrt{z})+3\sqrt{z}]}{8\sqrt{2}\pi^2\hat{f}_{AV}\hat{f}_{B_c}z^{5/4}(1+y)(3+\sqrt{z})\sqrt{1+\frac{1}{\sqrt{z}}\sqrt{\frac{(-1+y^2)(1+\sqrt{z})^2}{z^2}}}} \times e^{((\Lambda/T_1)+(\bar{\Lambda}/T_2))}\bigg\{\int_0^{\nu_0}d\nu\int_0^{\nu_0'}d\nu' e^{-(\nu/(2T_1))-(\nu'/(2T_2))}\theta[1-\lim_{m_b\to\infty}f^2(v,v')]\bigg\}, \tag{33}$$

$$f_+' = \frac{3(-1+y^2)(3+\sqrt{z})[2+2y^2(1+\sqrt{z})^2+5y(-1+z)-10\sqrt{z}]}{32\sqrt{2}\pi^2\hat{f}_{AV}\hat{f}_{B_c}z^{13/4}(1+y)^2\sqrt{1+\frac{1}{\sqrt{z}}[\frac{(-1+y^2)(1+\sqrt{z})^2}{z^2}]^{3/2}}} \times e^{((\Lambda/T_1)+(\bar{\Lambda}/T_2))}\bigg\{\int_0^{\nu_0}d\nu\int_0^{\nu_0'}d\nu' e^{-(\nu/(2T_1))-(\nu'/(2T_2))}\theta[1-\lim_{m_b\to\infty}f^2(v,v')]\bigg\}, \tag{34}$$

$$f_-' = \frac{-3(-1+y^2)(3+\sqrt{z})[-2+2y^2(1+\sqrt{z})^2+y(3+8\sqrt{z}+5z)+10\sqrt{z}]}{32\sqrt{2}\pi^2\hat{f}_{AV}\hat{f}_{B_c}z^{13/4}(1+y)^2\sqrt{1+\frac{1}{\sqrt{z}}[\frac{(-1+y^2)(1+\sqrt{z})^2}{z^2}]^{3/2}}} \times e^{((\Lambda/T_1)+(\bar{\Lambda}/T_2))}\bigg\{\int_0^{\nu_0}d\nu\int_0^{\nu_0'}d\nu' e^{-(\nu/(2T_1))-(\nu'/(2T_2))}\theta[1-\lim_{m_b\to\infty}f^2(v,v')]\bigg\}, \tag{35}$$

where $\Lambda = m_{B_c} - (m_b + m_c)$ and $\bar{\Lambda} = m_{S(AV)} - 2m_c$.

IV. NUMERICAL ANALYSIS

This section is devoted to the numerical analysis of the form factors, their HQET limit, and branching ratios. The sum rules expressions for the form factors depict that they mainly depend on the leptonic decay constants, continuum thresholds s_0 and s_0', and Borel parameters M_1^2 and M_2^2. In calculations, the quark masses are taken to be $m_c(\mu = m_c) = 1.275 \pm 0.015$ GeV, $m_b = (4.7 \pm 0.1)$ GeV [41], and the meson masses are chosen as $m_{B_c} = 6.286$ GeV, $m_{h_c} = 3.52528$ GeV, $m_{X_{c0}} = 3.41476$ GeV, $m_{X_{c1}} = 3.51066$ GeV [42]. For the values of the leptonic decay constants, we use $f_{B_c} = (400 \pm 40)$ MeV and $f_{X_{c0}} = f_{X_{c1}} = f_{h_c} = (340^{+119}_{-101})$ MeV[23]. The two-point QCD sum rules are used to determine the continuum thresholds s_0 and s_0'. These thresholds are not completely arbitrary and they are related to the energy of the exited states. The result of the physical quantities, form factors, should be stable with respect to the small variation of these parameters. Generally, the s_0 are obtained to be $(m_{\rm hadron} + 0.5)^2$ [36]. Here, we use $s_0 = (45 \pm 5)$ GeV^2 and $s_0' = (16 \pm 2)$ GeV^2. Since the Borel parameters M_1^2 and M_2^2 are not physical quantities, the form factors should not depend on them. The reliable regions for the Borel parameters M_1^2 and M_2^2 can be determined by requiring that not only the contributions of the higher states and continuum are effectively suppressed, but the contribution of the operator with the highest dimension be small. As a result of the above-mentioned requirements, the working regions are determined to be 15 GeV$^2 \leq M_1^2 \leq 35$ GeV^2 and

TABLE I. The values of the form factors for the $B_c \to X_{c0}\ell\nu$ decay at $M_1^2 = 25$ GeV^2, $M_2^2 = 15$ GeV^2, and $q^2 = 0$.

	$f_1(0)$	$f_2(0)$
$B_c \to X_{c0}\ell\nu$	0.673 ± 0.195	-1.458 ± 0.437

TABLE II. The values of the form factors for the $B_c \to AV\ell\nu$ decays at $M_1^2 = 25$ GeV^2, $M_2^2 = 15$ GeV^2, and $q^2 = 0$.

	$f_0(0)$	$f_V(0)$	$f_+(0)$	$f_-(0)$
$B_c \to X_{c1}\ell\nu$	0.084 ± 0.025	0.949 ± 0.261	0.211 ± 0.061	-0.586 ± 0.179
$B_c \to h_c\ell\nu$	0.084 ± 0.025	0.954 ± 0.282	0.211 ± 0.061	-0.588 ± 0.181

$10\text{ GeV}^2 \leq M_2^2 \leq 20\text{ GeV}^2$. The numerical values of the form factors at $q^2 = 0$ for $B_c \to X_{c0}\ell\nu$ and $B_c \to AV\ell\nu$ transitions are given in the Tables I and II, respectively.

In order to estimate the decay width of the $B_c \to S(AV)\ell\nu$ transitions, we need to know the q^2 dependent form factors in the whole physical region, $m_l^2 \leq q^2 \leq (m_{B_c} - m_{S(AV)})^2$. Our form factors are truncated at about $q^2 = 4$ GeV^2. To extend our results to the full physical region, we search for parametrization of the form factors in such a way that in the region $0 \leq q^2 \leq 4$ GeV^2, this parametrization coincides with the sum rules predictions. The following fit parametrization is chosen for the form factors with respect to q^2:

$$f_i(q^2) = \frac{a}{(1 - \frac{q^2}{m_{\rm fit}^2})} + \frac{b}{(1 - \frac{q^2}{m_{\rm fit}^2})^2}, \qquad (36)$$

where the values of the parameters a, b, and $m_{\rm fit}$ for the $B_c \to X_{c0}\ell\nu$ and $B_c \to (X_{c1}, h_c)\ell\nu$ are given in Tables III and IV, respectively.

TABLE III. Parameters appearing in the form factors of the $B_c \to X_{c0}\ell\nu$ decay at $M_1^2 = 25$ GeV^2 and $M_2^2 = 15$ GeV^2.

	a	b	$m_{\rm fit}$
$f_1(B_c \to X_{c0}\ell\nu)$	0.218	0.455	5.043
$f_2(B_c \to X_{c0}\ell\nu)$	−0.721	−0.738	4.492

TABLE IV. Parameters appearing in the form factors of the $B_c \to X_{c1}\ell\nu$ and $B_c \to h_c\ell\nu$ decays at $M_1^2 = 25$ GeV^2 and $M_2^2 = 15$ GeV^2.

	a	b	$m_{\rm fit}$
$f_0(B_c \to X_{c1}\ell\nu)$	0.211	−0.126	5.241
$f_V(B_c \to X_{c1}\ell\nu)$	0.512	0.438	4.711
$f_+(B_c \to X_{c1}\ell\nu)$	0.279	−0.068	3.872
$f_-(B_c \to X_{c1}\ell\nu)$	−0.594	0.008	3.735
$f_0(B_c \to h_c\ell\nu)$	0.211	−0.127	5.256
$f_V(B_c \to h_c\ell\nu)$	0.498	0.456	4.735
$f_+(B_c \to h_c\ell\nu)$	0.282	−0.702	3.839
$f_-(B_c \to h_c\ell\nu)$	−0.620	0.031	3.686

To calculate the numerical values of the form factors at HQET limit, the values of $\Lambda = 0.31$ GeV and $\bar{\Lambda} = 0.86$ GeV (0.96 GeV) are used for $B_c \to S\ell\nu$ ($B_c \to AV\ell\nu$) transitions, respectively (see [43,44]). In Tables, V, VI, and VII, we compare the values of the form factors and their HQET limits for considered transitions in the interval $0 \leq q^2 \leq 7$ and corresponding values of the y. Comparing the form factors and their HQET values in those tables, we see that all form factors and their HQET limits have the same behavior with respect to the q^2, i.e., they both grow or fail by increasing the values of q^2. The HQET limit of the form factors is comparable with their original values, and in large q^2 those form factors and their HQET values become very close to each other. The results presented in Tables, VI and VII also indicate that the form factors and their HQET limits for $B_c \to X_{c1}\ell\nu$ and $B_c \to h_c\ell\nu$ have values very close to each other since

TABLE V. Values of the form factors and their HQET limits for the $B_c \to X_{c0}\ell\nu$ at $M_1^2 = 25$ GeV^2, $M_2^2 = 15$ GeV^2, $T_1 = 2.09$ GeV, and $T_2 = 2.94$ GeV.

q^2 (GeV^2)	0	1	2	3	4	5	6	7
y	1.1920	1.1687	1.1454	1.1221	1.0988	1.0755	1.0522	1.0289
f_1	0.6735	0.7204	0.7732	0.8326	0.9001	0.9771	1.0656	1.1680
f_1(HQET)	0.3423	0.3614	0.4087	0.4637	0.5483	0.6523	0.7833	0.9432
f_2	−1.4594	−1.5760	−1.7102	−1.8658	−2.0480	−2.2636	−2.5218	−2.8354
f_2(HQET)	−0.8921	−0.9432	−1.0824	−1.1841	−1.3682	−1.6112	−2.0571	−2.4633

TABLE VI. Values of the form factors and their HQET limits for the $B_c \to X_{c1}\ell\nu$ at $M_1^2 = 25\ \mathrm{GeV}^2$, $M_2^2 = 15\ \mathrm{GeV}^2$, $T_1 = 2.09$ GeV, and $T_2 = 2.94$ GeV.

q^2 (GeV^2)	0	1	2	3	4	5	6	7
y	1.1745	1.1518	1.1292	1.1065	1.0838	1.0612	1.0385	1.0159
f_0	0.0841	0.0823	0.0800	0.0770	0.0732	0.0684	0.0624	0.0548
f_0(HQET)	0.0683	0.0675	0.0664	0.0652	0.0641	0.0628	0.0604	0.0559
f_V	0.9506	1.0171	1.0925	1.1784	1.2771	1.3915	1.5252	1.6833
f_V(HQET)	0.4739	0.5421	0.6331	0.7566	0.9054	1.0872	1.1543	1.3421
f_-	−0.5862	−0.6309	−0.6828	−0.7442	−0.8176	−0.9071	−1.0185	−1.1612
f_-(HQET)	−0.2954	−0.3264	−0.3682	−0.4448	−0.5412	−0.6518	−0.7839	−1.0173
f_+	0.2108	0.2207	0.2312	0.2424	0.2539	0.2654	0.2761	0.2841
f_+(HQET)	0.1032	0.1157	0.1302	0.1545	0.1771	0.1998	0.2152	0.2305

TABLE VII. Values of the form factors and their HQET limits for the $B_c \to h_c\ell\nu$ at $M_1^2 = 25\ \mathrm{GeV}^2$, $M_2^2 = 15\ \mathrm{GeV}^2$, $T_1 = 2.09$ GeV, and $T_2 = 2.94$ GeV.

q^2 (GeV^2)	0	1	2	3	4	5	6	7
y	1.1745	1.1518	1.1292	1.1065	1.0838	1.0612	1.0385	1.0159
f_0	0.0842	0.0824	0.0801	0.0771	0.0733	0.0685	0.0625	0.0550
f_0(HQET)	0.0692	0.0683	0.0665	0.0653	0.0641	0.0629	0.0604	0.0561
f_V	0.9545	1.0213	1.0970	1.1833	1.2824	1.3970	1.5310	1.6890
f_V(HQET)	0.4781	0.5483	0.6383	0.7627	0.9061	1.0922	1.1633	1.3948
f_-	−0.5891	−0.6332	−0.6845	−0.7448	−0.8167	−0.9038	−1.0114	−1.1477
f_-(HQET)	−0.2983	−0.3291	−0.3704	−0.4457	−0.5404	−0.6487	−0.7671	−1.0102
f_+	0.2117	0.2217	0.2322	0.2433	0.2547	0.2659	0.2758	0.2823
f_+(HQET)	0.1043	0.1166	0.1314	0.1557	0.1783	0.2017	0.2163	0.2314

the X_{c1} and h_c mesons are both axial vectors, i.e., $J^P = 1^+$ and have nearly the same mass.

At the end of this section, we would like to calculate the values of the branching ratios for these decays. Taking into account the q^2 dependency of the form factors and performing integration over q^2 from the differential decay rates in Eqs. (22) and (23) in the interval $m_l^2 \le q^2 \le (m_{B_c} - m_{S(AV)})^2$ and also using the total lifetime of the B_c meson $\tau_{B_c} = 0.46 \pm 0.07 \times 10^{-12}$ s [42], we obtain the branching ratios of the related transitions as presented in Table VIII. This table also contains the predictions of the other approaches such as the covariant light-front quark

TABLE VIII. Branching ratios of the semileptonic $B_c \to (X_{c0}, X_{c1}, h_c)\ell\nu$ ($\ell = e,\ \mu,\ \tau$) transitions in different approaches.

	$B_c \to X_{c0}\ell\nu$	$B_c \to X_{c1}\ell\nu$	$B_c \to h_c\ell\nu$
Present work	0.182 ± 0.051	0.146 ± 0.042	0.142 ± 0.040
CLQM [23]	$0.21^{+0.02+0.01}_{-0.04-0.01}$	$0.14^{+0.00+0.01}_{-0.01-0.01}$	$0.31^{+0.05+0.01}_{-0.08-0.01}$
RGM [24]	0.12	0.15	0.18
RCQM [25]	0.17	0.092	0.27
RCQM [26]	0.18	0.098	0.31
NRCQM [27]	0.11	0.066	0.17

	$B_c \to X_{c0}\tau\nu$	$B_c \to X_{c1}\tau\nu$	$B_c \to h_c\tau\nu$
Present work	0.049 ± 0.016	0.0147 ± 0.0044	0.0137 ± 0.0038
CLQM [23]	$0.024^{+0.001+0.001}_{-0.003-0.001}$	$0.015^{+0.000+0.001}_{-0.001-0.002}$	$0.022^{+0.002+0.000}_{-0.004-0.000}$
RGM [24]	0.017	0.024	0.025
RCQM [25]	0.013	0.0089	0.017
RCQM [26]	0.018	0.012	0.027
NRCQM [27]	0.013	0.0072	0.015

model, renormalization group method, relativistic constituent quark model, and nonrelativistic constituent quark model [23–27]. These results can be tested in future experiments.

In conclusion, using the QCD sum rules approach, we investigated the semileptonic $B_c \to S(AV)\ell\nu$ decays. The q^2 dependencies of the transition form factors were calculated. The HQET limits of the form factors were also evaluated and compared with original form factors. The obtained results were used to estimate the total decay widths and branching ratios of these transitions. A comparison of the results for branching fractions was also presented.

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