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Yukawa unification and neutralino dark matter in $SU(4)_c \times SU(2)_L \times SU(2)_R$

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We consider a left-right symmetric $SU(4)_c \times SU(2)_L \times SU(2)_R$ (4-2-2) model with gravity mediated supersymmetry breaking. We find that with 4-2-2 compatible nonuniversal gaugino masses, $t - b - \tau$ Yukawa coupling unification is consistent with neutralino dark matter abundance and with constraints from collider experiments (except $(g-2)_\mu$). The gluino mass lies close to that of the lightest neutralino, so that the gluino coannihilation channel plays an important role in determining the neutralino relic abundance. By relaxing the Yukawa unification constraint we find stau and stop masses as low as 200–220 GeV. We highlight some benchmark points for these cases with $40 \leq \tan\beta \leq 58$.

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I. INTRODUCTION

As a maximal subgroup of Spin(10) (commonly known as $SO(10)$), the gauge symmetry $SU(4)_c \times SU(2)_L \times SU(2)_R$ (4-2-2) [1] captures many salient features exhibited by its covering group. Even as a stand alone symmetry group, 4-2-2 implements electric charge quantization, albeit in units of $\pm e/6$, rather than $\pm e/3$ [2,3]. It explains the standard model quantum numbers of the quark and lepton families by assigning them in bifundamental representations and it also predicts the existence of right-handed neutrinos [1]. However, there are some important differences between models based on $SO(10)$ and 4-2-2 which, in principle, can be experimentally tested. For instance, in 4-2-2 the lightest magnetic monopole carries two quanta of Dirac magnetic charge [4]. (In $SO(10)$ the lightest monopole carries one quantum of Dirac magnetic charge, unless $SO(10)$ breaks via 4-2-2.) By the same token, 4-2-2 predicts the existence of $SU(3)$ color singlet states carrying electric charges $\pm e/2$ [3,5]. Finally, gauge boson mediated proton decay is a characteristic feature of $SO(10)$ which is absent in the 4-2-2 framework.

While these different experimental signatures can help distinguish $SO(10)$ from 4-2-2, they mostly rely on physics operating at superheavy scales. A major motivation for this paper is to highlight some important differences in the low energy predictions of supersymmetric $SO(10)$ and 4-2-2 models, stemming from the Higgs and sparticle sectors of these models. An exciting new feature is that these predictions can be sufficiently different so that they can be compared at the LHC.

Supplementing 4-2-2 with a discrete left-right (LR) symmetry [1,6] (more precisely C parity) [7] reduces from three to two the number of independent gauge couplings in supersymmetric 4-2-2. In combination with Yukawa unification [8], this has important implications for low energy Higgs and sparticle spectroscopy which we will explore in this paper and compare with the corresponding predictions from an $SO(10)$ model.

In 4-2-2 the matter fields are unified into three generations of ψ (4, 2, 1), and the antimatter fields are in three generations of ψ_c (4, 1, 2). If the minimal supersymmetric standard model (MSSM) electroweak doublets come from the bidoublet H(1, 2, 2), the third family Yukawa coupling H $\psi_c \psi$ yields the following relation valid at the GUT scale ($M_{\rm GUT}$), namely

$$Y_t = Y_b = Y_\tau = Y_{\rm Dirac}. \tag{1}$$

We will assume that due to C-parity the soft mass^2 terms, induced at $M_{\rm GUT}$ through gravity mediated supersymmetry breaking [9], are equal in magnitude for the scalar squarks and sleptons of the three families. The asymptotic MSSM gaugino masses, on the other hand, can be nonuniversal from the following consideration. From C parity, we can expect that the gaugino masses at $M_{\rm GUT}$ associated with $SU(2)_L$ and $SU(2)_R$ are the same. However, the asymptotic $SU(4)_c$ and consequently $SU(3)_c$ gaugino masses can be different. With the hypercharge generator in 4-2-2 given by $Y = \sqrt{\frac{2}{5}}(B-L) + \sqrt{\frac{3}{5}} I_{3R}$, where $B-L$ and I_{3R} are the diagonal generators of $SU(4)_c$ and $SU(2)_R$, we have the following asymptotic relation between the three MSSM gaugino masses:

$$M_1 = \tfrac{3}{5}M_2 + \tfrac{2}{5}M_3. \tag{2}$$

The supersymmetric 4-2-2 model with C parity thus has two independent parameters (M_2, M_3) in the gaugino sector.

In this paper we wish to explore whether Yukawa coupling unification in 4-2-2 is compatible with recent observations of the dark matter relic abundance and other

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collider-based experimental constraints. A similar analysis for $SO(10)$, which we closely follow, has been carried out by Baer *et al.* [10]. Solutions consistent with $SO(10)$ Yukawa unification have been obtained in [10] only for very special values of the fundamental parameters. Furthermore, it turns out to be quite difficult in this model to reconcile the lightest neutralino primordial abundance with the observed dark matter densities.

By introducing nonuniversality in the gaugino sector, we can allow the neutralinos in 4-2-2 to be closely degenerate in mass with the gluino, which is not possible in $SO(10)$. This opens up, in particular, the bino-gluino coannihilation channel [11], which turns out to be an essential difference between the 4-2-2 and $SO(10)$ models. In order to make Yukawa coupling unification compatible with radiative electroweak symmetry breaking (REWSB), one needs to implement some splitting in the Higgs sector, with $m_{Hu}^2 < m_{Hd}^2$. Such a splitting may be introduced via a D-term contribution to all scalar masses [12], or it can be generated via GUT-scale threshold corrections related to a large Dirac neutrino Yukawa coupling [13]. It has been noted [14] that a splitting just in the Higgs soft terms, as opposed to splitting in all scalar masses, yields better Yukawa unification, and so we focus on this approach. Since one of our goals is a comparison of 4-2-2 and $SO(10)$ models, we follow the same notation as in [10]. We parametrize the Higgs soft mass splitting by $m_{H_{u,d}}^2 = m_{10}^2 \mp 2M_D^2$, where m_{10}^2 is the MSSM universal Higgs soft mass^2 term. The supersymmetric 4-2-2 model we are discussing thus has the following fundamental parameters:

$$m_{16}, m_{10}, M_D, M_2, M_3, A_0, \tan\beta, \mathrm{sign}\mu. \tag{3}$$

Thus, compared to the $SO(10)$ model of [10], we have one additional parameter in 4-2-2 which plays a crucial role in realizing Yukawa unification consistent with the desired neutralino relic density.

The outline for the rest of the paper is as follows. In Sec. II we summarize the scanning procedure and the experimental constraints that we have employed. We present the results from our scan in Sec. III, where we compare the 4-2-2 and $SO(10)$ models and then proceed to highlight some of the predictions of the 4-2-2 model. Our conclusions are summarized in Sec. IV.

II. PHENOMENOLOGICAL CONSTRAINTS AND SCANNING PROCEDURE

We employ ISAJET 7.78 package [15] to perform random scans over the parameter space. In this package, the weak scale values of gauge and third generation Yukawa couplings are evolved to $M_{\rm GUT}$ via the MSSM renormalization group equations (RGEs) in the $\overline{\rm DR}$ regularization scheme, where $M_{\rm GUT}$ is defined to be the scale at which $g_1 = g_2$. We do not enforce an exact unification of the strong coupling $g_3 = g_1 = g_2$ at $M_{\rm GUT}$, since a few percent deviation from unification can be assigned to unknown GUT-scale threshold corrections [16]. At $M_{\rm GUT}$, the boundary conditions are imposed and all the soft supersymmetry breaking (SSB) parameters, along with the gauge and Yukawa couplings, are evolved back to the weak scale M_Z. The effect of the neutrino Dirac Yukawa coupling in the running of the RGEs has been shown in [17] to be significant for coupling values $\sim$2. In the 4-2-2 model with $t - b - \tau$ unification, the asymptotic neutrino Dirac Yukawa coupling has the same value as $y_t(M_{\rm GUT})$ which is relatively small (~ 0.5). Thus, in the following discussion we will ignore it.

In the evaluation of Yukawa couplings the SUSY threshold corrections [18] are taken into account at the common scale $M_{\rm SUSY} = \sqrt{m_{\tilde{t}_L} m_{\tilde{t}_R}}$. The entire parameter set is iteratively run between M_Z and $M_{\rm GUT}$ using the full two-loop RGEs until a stable solution is obtained. To better account for leading-log corrections, one-loop step-beta functions are adopted for gauge and Yukawa couplings, and the SSB parameters m_i are extracted from RGEs at multiple scales $m_i = m_i(m_i)$. The RGE-improved one-loop effective potential is minimized at an optimized scale $M_{\rm SUSY}$, which effectively accounts for the leading two-loop corrections. Full one-loop radiative corrections are incorporated for all sparticle masses.

The requirement of radiative electroweak symmetry breaking (REWSB) [19] puts an important theoretical constraint on the parameter space. Another important constraint comes from limits on the cosmological abundance of stable charged particles [20]. This excludes regions in the parameter space where charged SUSY particles, such as $\tilde{\tau}_1$ or $\tilde{t}_1$, become the lightest supersymmetric particle (LSP). We accept only those solutions for which one of the neutralinos is the LSP.

We have performed random scans for the following parameter range:

$$\begin{gathered} 0 \le m_{16} \le 20\ \text{TeV}, \qquad 0 \le M_2 \le 1\ \text{TeV}, \\ 0 \le M_3 \le 1\ \text{TeV}, \qquad -3 \le A_0/m_{16} \le 0, \\ 0 \le M_D/m_{16} \le 0.95, \qquad 0 \le m_{10}/m_{16} \le 1.5, \\ 40 \le \tan\beta \le 58, \end{gathered} \tag{4}$$

with $\mu > 0$, and $m_t = 172.6$ GeV [21].

We first collected 150 000 points for both the $SO(10)$ and 4-2-2 models. All of these points satisfy the requirement of REWSB with the neutralino being the LSP in each case. Furthermore, all of these points satisfy the constraint $\Omega_{\rm CDM}h^2 \le 10$. This is done so as to collect more points with a WMAP compatible value of cold dark matter relic abundance. Once we identify good regions in parameter space, we perform a random scan focused around those regions for the 4-2-2 case. After collecting the data, we use the ISATOOLS package [22] to implement the following phenomenological constraints:

$$\begin{aligned}
m_{\tilde{\chi}_1^{\pm}}(\text{chargino mass}) &\geq 103.5\ \text{GeV} \quad [20];\\
m_h(\text{lightest Higgs mass}) &\geq 114.4\ \text{GeV} \quad [23],\\
m_{\tilde{\tau}}(\text{stau mass}) &\geq 86\ \text{GeV} \quad [20],\\
m_{\tilde{g}}(\text{gluino mass}) &\geq 220\ \text{GeV} \quad [20],\\
\text{BR}(B_s \rightarrow \mu^+\mu^-) &< 5.8\times 10^{-8} \quad [24],\\
2.85\times 10^{-4} \leq \text{BR}(b\rightarrow s\gamma) &\leq 4.24\times 10^{-4}(2\sigma) \quad [25],\\
\Omega_{\text{CDM}}h^2 &= 0.111^{+0.028}_{-0.037}(5\sigma) \quad [26],\\
3.4\times 10^{-10} \leq \Delta a_\mu &\leq 55.6\times 10^{-10}(3\sigma) \quad [27].
\end{aligned} \tag{5}$$

We apply the experimental constraints successively on the data that we acquire from ISAJET. As a first step we apply the constraints from $\text{BR}(B_s \rightarrow \mu^+\mu^-)$, $\text{BR}(b \rightarrow s\gamma)$, the WMAP upper bound on the relic density of cold dark matter, and the (s)particle mass bounds. We then apply the WMAP lower bound on the relic density of dark matter, followed by the constraint on the muon anomalous magnetic moment $a_\mu = (g-2)_\mu/2$ at the 3σ allowed region. The data is then plotted showing the successive application of these constraints.

III. RESULTS

Following Baer *et al.* [10] we introduce a parameter R to quantify Yukawa unification. Namely, R is the ratio,

$$R = \frac{\max(y_t, y_b, y_\tau)}{\min(y_t, y_b, y_\tau)}, \tag{6}$$

so that $R = 1$ corresponds to perfect unification and a higher value of R signifies a larger deviation from unification.

We next present the results of the random scan. We first compare the $SO(10)$ model with the 4-2-2 model in Figs. 1 and 2 following the treatment in [10]. In Fig. 1 we plot the results in the (R, m_{16}), $(R, \tan\beta)$, and $(\Omega h^2, R)$ planes for $SO(10)$ (left panel) and 4-2-2 (right panel). All of these points satisfy the theoretical requirement of REWSB and correspond to a neutralino LSP. In addition, these points satisfy the various experimental constraints listed earlier. The light blue points satisfy the constraints from $\text{BR}(B_s \rightarrow \mu^+\mu^-)$, $\text{BR}(b \rightarrow s\gamma)$, the Higgs, chargino, gluino, and stau mass bounds, and the upper bound on the relic density of dark matter from WMAP. Shown in dark blue are points that also satisfy the lower bound on $\tilde{\chi}_1^0$ dark matter abundance. In Fig. 2 we similarly present results in the $(m_{10}/m_{16}, R)$, $(M_D/m_{16}, R)$, and $(A_0/m_{16}, R)$ planes for $SO(10)$ (left) and 4-2-2 (right). It is quite obvious from the results that, as expected, using just a random scan it is quite difficult to realize acceptable Yukawa unification in $SO(10)$ consistent with the experimental constraints. Reference [10] employs a modified scanning algorithm based on Markov Chain Monte Carlo (MCMC) to search the parameter space more efficiently. It is shown there that they show that only the h-resonance (light Higgs) channel is available to bring the neutralino dark matter density in the right (WMAP) ball park. While this channel does yield acceptable Yukawa unification consistent with WMAP, it is more or less ruled out by the lower bound of 114.4 GeV on the the SM Higgs mass.

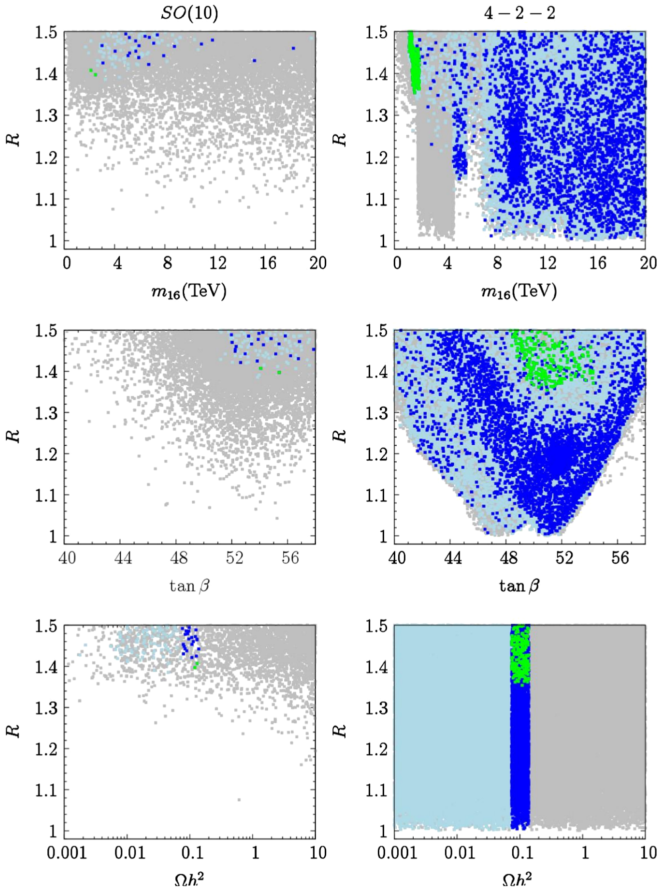

FIG. 1 (color online). Plots in the (m_{16}, R), $(\tan\beta, R)$, and $(\Omega h^2, R)$ planes for $SO(10)$ (left panels) and 4-2-2 (right panels). Gray points are consistent with REWSB and $\tilde{\chi}_1^0$ LSP. Light blue points satisfy the WMAP upper bound on $\tilde{\chi}_1^0$ abundance and various constraints from colliders ($\text{BR}(B_s \rightarrow \mu^+\mu^-)$, $\text{BR}(b \rightarrow s\gamma)$, and (s)particle mass bounds). Dark blue points also satisfy the lower bound on $\tilde{\chi}_1^0$ density. Green points, additionally, satisfy the constraint from $(g-2)_\mu$.

In the initial sweep of the $SO(10)$ model around 150 000 points were identified, consistent with REWSB and the requirement that is a LSP neutralino. Yukawa unification consistent with the experimental data was found to be no better than 40%, even if we ignore the constraint from Δa_μ. The 4-2-2 model yields "good" solutions with Yukawa unification to better than 10%. More concentrated searches around such good points have yielded "near perfect" unification. Such concentrated searches were not performed for the $SO(10)$ model as they have already been reported in [10] with the conclusion that a narrow, almost excluded, light Higgs funnel region is the only one that is viable from the point of view of Yukawa unification and dark matter relic density.

We now focus on the 4-2-2 model, which does much better than the $SO(10)$ model in terms of Yukawa unification and most of the experimental constraints, including the WMAP bounds on dark matter abundance. The constraint from $(g-2)_\mu$ is found to be largely incompatible with Yukawa unification (Yukawa unification is worse than 35% if one insists on $(g-2)_\mu$). From Figs. 1 and 2 we find that the following parameter values are preferred:

$$\begin{aligned}
&m_{16} \gtrsim 7\ \text{TeV},\\
&46 \lesssim \tan\beta \lesssim 48 \quad \text{and} \quad 50 \lesssim \tan\beta \lesssim 52,\\
&0.6 \lesssim m_{10}/m_{16} \lesssim 0.8 \quad \text{and} \quad m_{10}/m_{16} \approx 1.1,\\
&0.3 \lesssim M_D/m_{16} \lesssim 0.5,\\
&A_0 \approx -2m_{16} \quad \text{and} \quad A_0 \approx -2.5m_{16}.
\end{aligned} \tag{7}$$

In Table I we show a few benchmark points that are consistent with Yukawa unification. Point 1 displays the spectrum corresponding to essentially perfect unification ($R = 1.00$). Point 2 gives a light gluino (~ 265 GeV) consistent with good unification ($\sim 9\%$). Point 3 has the lightest stop (1911 GeV), again consistent with respectable Yukawa unification ($\sim 7\%$). Note that most of the sparticles are rather heavy as a consequence of requiring Yukawa unification. Note that for all three benchmark points the lightest neutralino (LSP) relic abundance is compatible with the WMAP dark matter bounds. This comes about because of the relatively small mass splitting between the neutralino (essentially binolike) and gluino which leads to efficient coannihilation [11].

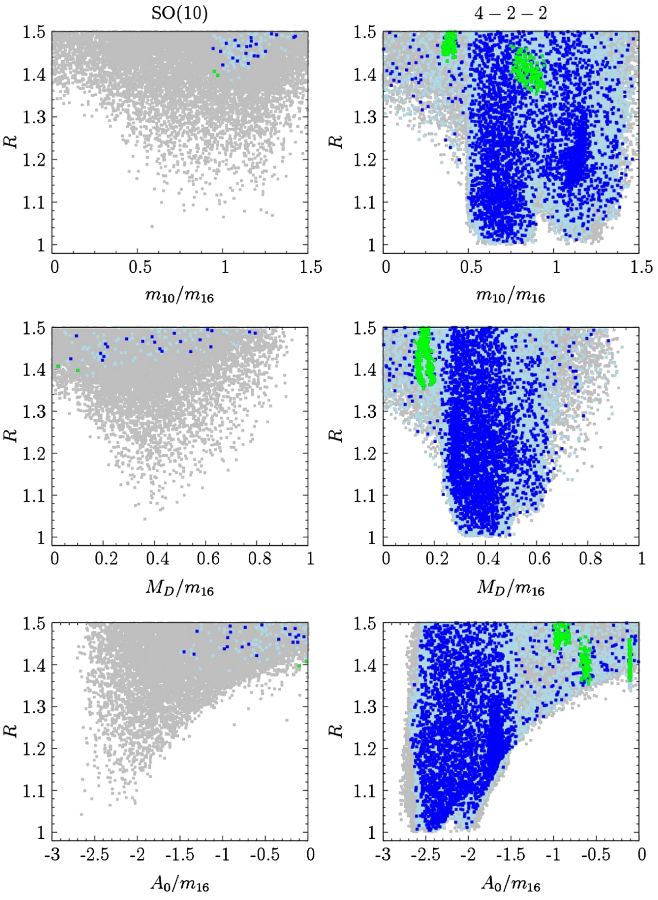

FIG. 2 (color online). Plots in the $(m_{10}/m_{16}, R)$, $(M_D/m_{16}, R)$, and $(A_0/m_{16}, R)$ planes for $SO(10)$ (left) and 4-2-2 (right). Color coding is the same as in Fig. 1.

Figure 3 shows plots in the (M_3, m_{16}), $(R, M_2/M_3)$, $(M_3, m_{10}/m_{16})$, $(M_3, M_D/m_{16})$, $(M_3, \tan\beta)$, and $(M_3, A_0/m_{16})$ planes for the 4-2-2 model. Color coding is essentially the same as in Fig. 1, except that we now also show red points that are consistent with all experimental constraints (except $(g-2)_\mu$) and have Yukawa unification better than 10%. It appears that the points with Yukawa

TABLE I. Sparticle and Higgs masses in 4-2-2 model (in units of GeV), with $m_t = 172.6$ GeV and $\mu > 0$. Point 1 corresponds to exact Yukawa unification ($R = 1.00$) while point 2 (3) shows the spectrum corresponding to the lightest stop (gluino) with Yukawa unification of 10% or better. Note that in each case gluino coannihilation plays as essential role.

	Point 1	Point 2	Point 3
m_{16}	14 110	8429	13 124
M_2	832.03	1020.2	689.4
M_3	0.7945	60.542	9.6261
$\tan\beta$	50.82	46.41	51.17
M_D/m_{16}	0.4543	0.5595	0.3323
m_{10}/m_{16}	0.7741	1.1584	1.3048
A_0/m_{16}	−2.4487	−2.1527	−1.8226
m_h	123	126	127
m_H	7569	2163	9882
m_A	7520	2150	9818
$m_{H^\pm}$	7571	2175	9883
$m_{\tilde{\chi}^\pm_{1,2}}$	887 13869	975 4047	712 3750
$m_{\tilde{\chi}^0_{1,2}}$	283 885	319 974	228 712
$m_{\tilde{\chi}^0_{3,4}}$	13 879,13 879	4049 4049	3784 3785
$m_{\tilde{g}}$	325	365	265
$m_{\tilde{u}_{L,R}}$	14 126,13 916	8435 8361	13 140,12 841
$m_{\tilde{t}_{1,2}}$	5337 5726	1911 2640	4931 5310
$m_{\tilde{d}_{L,R}}$	14 126,14 203	8435 8455	13 141,13 249
$m_{\tilde{b}_{1,2}}$	5237 5653	2521 2767	4115 5146
$m_{\tilde{\nu}_1}$	13988	8409	12926
$m_{\tilde{\nu}_3}$	10598	6577	9535
$m_{\tilde{e}_{L,R}}$	13 988,14 376	8408 8514	12 926,13 500
$m_{\tilde{\tau}_{1,2}}$	6412 10581	4270 6573	5580 9559
μ	14100	4110	3840
$\Omega_{\text{LSP}}h^2$	0.09	0.112	0.116
R	1.00	1.07	1.09

unification seem to favor a nonuniversal gaugino sector, with $M_2 \gtrsim 10M_3$. This ratio is higher still if we also require these solutions to satisfy constraints from experiments. This, of course, does not mean that solutions with $M_2 \approx M_3$ do not exist, as the latter have been reported in [10]. However, this does suggest a statistical preference for solutions with a significant splitting in the gaugino sector.

The PAMELA experiment has reported an excess in the observed positron flux with no corresponding antiproton excess [28]. It may be possible to explain this ''excess'' in the context of SUSY with the lightest neutralino as the dark matter candidate. One explanation invokes a neutralino of mass around 300 GeV decaying into positrons via ''tiny'' ($\sim 10^{-13}$) R-parity violating couplings [29]. This scenario is consistent with Yukawa unification as we can see in Fig. 4.

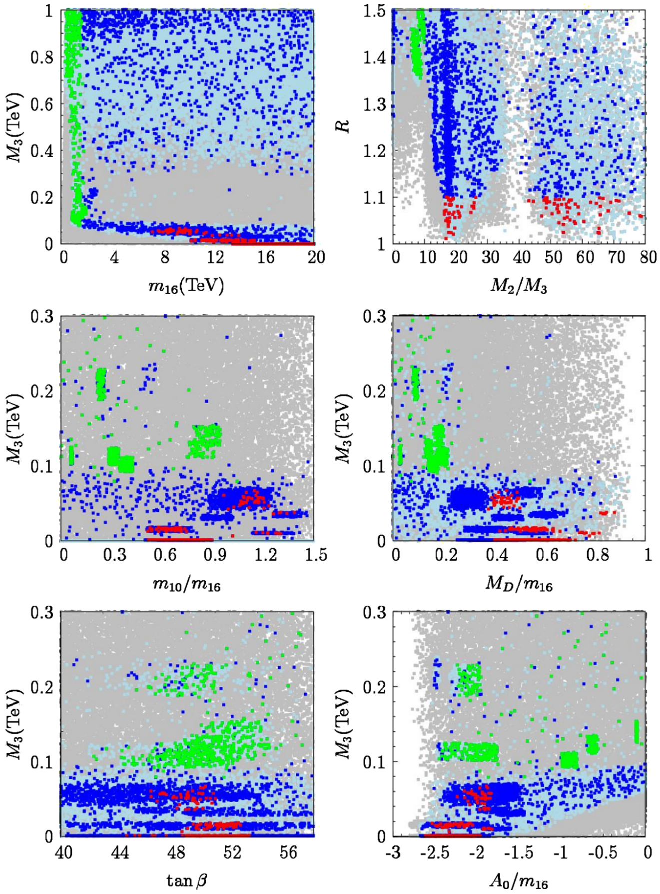

FIG. 3 (color online). Plots in the (M_3, m_{16}), $(R, M_2/M_3)$, $(M_3, m_{10}/m_{16})$, $(M_3, M_D/m_{16})$, $(M_3, \tan\beta)$, and $(M_3, A_0/m_{16})$ planes for 4-2-2. Gray points are consistent with REWSB and $\tilde{\chi}_1^0$ LSP. Light blue points satisfy the WMAP upper bound on $\tilde{\chi}_1^0$ abundance and various constraints from colliders (BR($B_s \rightarrow \mu^+\mu^-$), BR($b \rightarrow s\gamma$), and (s)particle mass bounds). Dark blue points also satisfy the lower bound on $\tilde{\chi}_1^0$ primordial abundance. Green points, additionally, satisfy the constraint from $(g-2)_\mu$. Points in red represent a subset of dark blue ones that is consistent with 10% or better Yukawa unification.

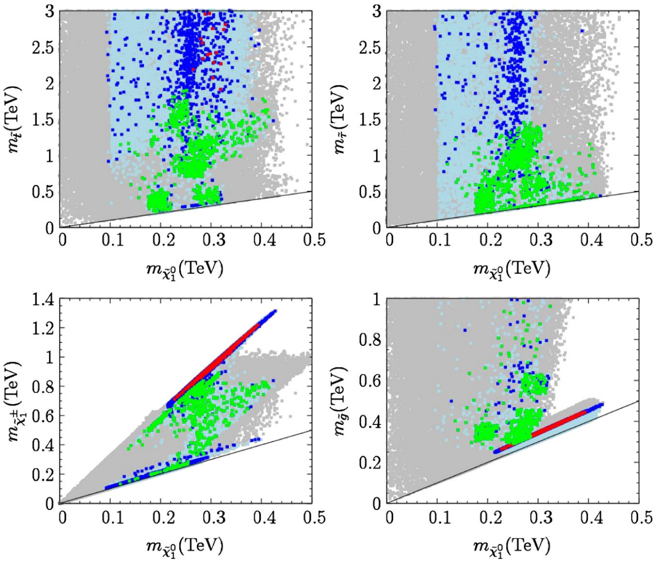

FIG. 4 (color online). Plots in the $(m_{\tilde{t}}, m_{\tilde{\chi}_1^0})$, $(m_{\tilde{\tau}}, m_{\tilde{\chi}_1^0})$, $(m_{\tilde{\chi}_1^\pm}, m_{\tilde{\chi}_1^0})$, and $(m_{\tilde{g}}, m_{\tilde{\chi}_1^0})$ planes for 4-2-2. Color coding is the same as in Fig. 3. Also shown is the unit slope line in each plane.

We have stressed that Yukawa unification seems incompatible with the current experimental bound on $(g-2)_\mu$. If we do not insist on Yukawa unification in 4-2-2, we can find a much lighter MSSM spectrum, which is consistent with all experimental constraints (including $(g-2)_\mu$). This can be seen from Figs. 4 and 5. In Fig. 4 we show plots in the $(m_{\tilde{t}}, m_{\tilde{\chi}_1^0})$, $(m_{\tilde{\tau}}, m_{\tilde{\chi}_1^0})$, $(m_{\tilde{\chi}_1^\pm}, m_{\tilde{\chi}_1^0})$, and $(m_{\tilde{g}}, m_{\tilde{\chi}_1^0})$ planes, with the same color coding as in Fig. 3. We also show the unit slope line in each plot, thus highlighting the

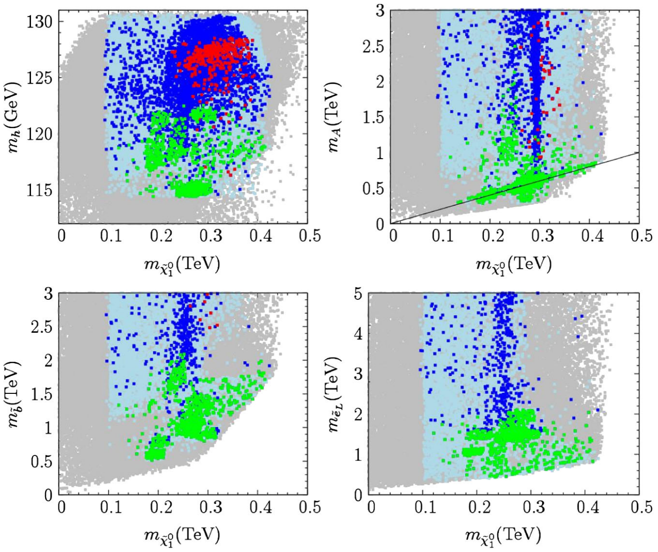

FIG. 5 (color online). Plots in the $(m_h, m_{\tilde{\chi}_1^0})$, $(m_A, m_{\tilde{\chi}_1^0})$, $(m_{\tilde{b}}, m_{\tilde{\chi}_1^0})$, and $(m_{\tilde{e}_L}, m_{\tilde{\chi}_1^0})$ planes for 4-2-2. Color coding is the same as in Fig. 3. In the $(m_A, m_{\tilde{\chi}_1^0})$ case we also show the line $m_A = 2m_{\tilde{\chi}_1^0}$.

TABLE II. Sparticle and Higgs masses (in units of GeV), with $m_t = 172.6$ GeV and $\mu > 0$. Points 1 through 5 correspond to the lightest chargino, *CP*-odd Higgs, gluino, stau, and stop for the 4-2-2 parameter space given in Eq. (4). Note that these points are not consistent with Yukawa unification ($R > 2.3$), but they satisfy all experimental constraints including the one from $(g-2)_\mu$.

	Point 1	Point 2	Point 3	Point 4	Point 5
m_{16}	1529.4	1038.5	1402.5	958.4	1469.8
M_2	158.0	630.1	736.5	607.6	630.1
M_3	467.9	122.0	79.43	117.8	103.6
$\tan\beta$	56.4	57.2	46.8	54.5	46.2
M_D/m_{16}	0.2185	0.2085	0.0721	0.1732	0.0276
m_{10}/m_{16}	0.459	0.339	0.317	0.291	0.059
A_{10}/m_{16}	−1.485	−1.976	−1.434	−2.063	−2.45
m_h	119	118	117	119	120
m_H	940	284	448	297	468
m_A	934	284	445	299	472
$m_{H^\pm}$	946	302	458	315	491
$m_{\tilde{\chi}^\pm_{1,2}}$	133 1545	526 1070	620 1238	505 1015	541 1676
$m_{\tilde{\chi}^0_{1,2}}$	121 132	186 526	208 620	178 504	186 539
$m_{\tilde{\chi}^0_{3,4}}$	1543 1543	1065 1068	1232 1236	1010 1014	1675 1676
$m_{\tilde{g}}$	1176	368	268	354	335
$m_{\tilde{u}_{L,R}}$	1784 1784	1135 1064	1475 1406	1056 986	1527 1481
$m_{\tilde{t}_{1,2}}$	1148 1392	409 764	777 1080	319 709	226 857
$m_{\tilde{d}_{L,R}}$	1785 1790	1138 1066	1477 1404	1059 987	1530 1479
$m_{\tilde{b}_{1,2}}$	1331 1497	613 799	972 1115	560 743	739 987
$m_{\tilde{\nu}_1}$	1526	1115	1482	1036	1526
$m_{\tilde{\nu}_3}$	1249	852	1293	797	1191
$m_{\tilde{e}_{L,R}}$	1528 1540	1118 1055	1483 1413	1039 974	1527 1478
$m_{\tilde{\tau}_{1,2}}$	878 1261	200 864	960 1299	198 808	586 1192
μ	1555	1077	1247	1020	1685
$\Omega_{\mathrm{LSP}}h^2$	0.079	0.076	0.074	0.114	0.127

stop coannihilation region, the stau coannihilation region, and the mixed bino-wino dark matter region. In Fig. 5 we show similar plots in the $(m_h, m_{\tilde{\chi}^0_1})$, $(m_A, m_{\tilde{\chi}^0_1})$, $(m_{\tilde{b}}, m_{\tilde{\chi}^0_1})$, and $(m_{\tilde{e}_L}, m_{\tilde{\chi}^0_1})$ planes. We indicate the A-funnel region with the line $m_A = 2m_{\tilde{\chi}^0_1}$. In Table II we present points corresponding to the lightest spectrum found in our investigation (disregarding Yukawa unification, but consistent with all experimental constraints). Points 1 through 5, respectively, display the spectrum corresponding to the lightest chargino (133 GeV), *CP*-odd Higgs (284 GeV), gluino (268 GeV), stau (198 GeV), and stop (226 GeV).

IV. CONCLUSION

The 4-2-2 gauge symmetry, supplemented by left-right symmetry (*C* parity) captures many attractive features exhibited by the simplest $SO(10)$ models. One of these features happens, in some models, to be Yukawa unification. We have shown that by relaxing in 4-2-2 the assumption of universal gaugino masses, the resulting MSSM models have rather distinctive mass spectra which can be tested at the LHC. Moreover, the primordial abundance of the lightest neutralino in this case is consistent with the WMAP dark matter limits, something which is difficult to achieve in $SO(10)$ with $t - b - \tau$ Yukawa unification. We have also studied the implications of relaxing the Yukawa unification condition and identified several additional benchmark points which also can be explored at the LHC. Finally, we wish to note the recent observation that the little hierarchy problem can be largely resolved in the 4-2-2 framework [30]. The implication of this for sparticle spectroscopy will be discussed elsewhere.

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[1] J. C. Pati and A. Salam, Phys. Rev. D **10**, 275 (1974); **11**, 703(E) (1975).

[2] G. Lazarides and Q. Shafi, Nucl. Phys. **B189**, 393 (1981).

[3] S. F. King and Q. Shafi, Phys. Lett. B **422**, 135 (1998).

[4] G. Lazarides, M. Magg, and Q. Shafi, Phys. Lett. B **97**, 87 (1980).

[5] T. W. Kephart, C. A. Lee, and Q. Shafi, J. High Energy Phys. 01 (2007) 088; T. W. Kephart and Q. Shafi, Phys. Lett. B **520**, 313 (2001).

[6] R. N. Mohapatra and J. C. Pati, Phys. Rev. D **11**, 566 (1975); G. Senjanovic and R. N. Mohapatra, Phys. Rev. D **12**, 1502 (1975); G. Senjanovic, Nucl. Phys. **B153**, 334 (1979); M. Magg, Q. Shafi, and C. Wetterich, Phys. Lett. B **87**, 227 (1979); M. Cvetic, Nucl. Phys. **B233**, 387 (1984).

[7] T. W. B. Kibble, G. Lazarides, and Q. Shafi, Phys. Lett. B **113**, 237 (1982); T. W. B. Kibble, G. Lazarides, and Q. Shafi, Phys. Rev. D **26**, 435 (1982); R. N. Mohapatra and B. Sakita, Phys. Rev. D **21**, 1062 (1980).

[8] B. Ananthanarayan, G. Lazarides, and Q. Shafi, Phys. Rev. D **44**, 1613 (1991); Phys. Lett. B **300**, 245 (1993); V. Barger, M. Berger, and P. Ohmann, Phys. Rev. D **49**, 4908 (1994); M. Carena, M. Olechowski, S. Pokorski, and C. Wagner, Nucl. Phys. **B426**, 269 (1994); B. Ananthanarayan, Q. Shafi, and X. Wang, Phys. Rev. D **50**, 5980 (1994); R. Rattazzi and U. Sarid, Phys. Rev. D **53**, 1553 (1996); T. Blazek, M. Carena, S. Raby, and C. Wagner, Phys. Rev. D **56**, 6919 (1997); T. Blazek, S. Raby, and K. Tobe, Phys. Rev. D **62**, 055001 (2000); H. Baer, M. Diaz, J. Ferrandis, and X. Tata, Phys. Rev. D **61**, 111701 (2000); H. Baer, M. Brhlik, M. Diaz, J. Ferrandis, P. Mercadante, P. Quintana, and X. Tata, Phys. Rev. D **63**, 015007 (2000); S. Profumo, Phys. Rev. D **68**, 015006 (2003); C. Pallis, Nucl. Phys. **B678**, 398 (2004); M. Gomez, G. Lazarides, and C. Pallis, Phys. Rev. D **61**, 123512 (2000); Nucl. Phys. **B638**, 165 (2002); Phys. Rev. D **67**, 097701 (2003); I. Gogoladze, Y. Mimura, and S. Nandi, Phys. Lett. B **562**, 307 (2003); I. Gogoladze, Y. Mimura, S. Nandi, and K. Tobe, Phys. Lett. B **575**, 66 (2003); K. Tobe and J. D. Wells, Nucl. Phys. **B663**, 123 (2003); U. Chattopadhyay, A. Corsetti, and P. Nath, Phys. Rev. D **66**, 035003 (2002); M. Gomez, T. Ibrahim, P. Nath, and S. Skadhauge, Phys. Rev. D **72**, 095008 (2005).

[9] A. H. Chamseddine, R. L. Arnowitt, and P. Nath, Phys. Rev. Lett. **49**, 970 (1982); R. Barbieri, S. Ferrara, and C. A. Savoy, Phys. Lett. B **119**, 343 (1982); L. J. Hall, J. D. Lykken, and S. Weinberg, Phys. Rev. D **27**, 2359 (1983); E. Cremmer, P. Fayet, and L. Girardello, Phys. Lett. B **122**, 41 (1983); N. Ohta, Prog. Theor. Phys. **70**, 542 (1983).

[10] H. Baer, S. Kraml, S. Sekmen, and H. Summy, J. High Energy Phys. 03 (2008) 056; H. Baer, M. Haider, S. Kraml, S. Sekmen, and H. Summy, J. Cosmol. Astropart. Phys. 02 (2009) 002.

[11] S. Profumo and C. E. Yaguna, Phys. Rev. D **69**, 115009 (2004); **70**, 095004 (2004), and references therein.

[12] M. Drees, Phys. Lett. B **181**, 279 (1986); J. S. Hagelin and S. Kelley, Nucl. Phys. **B342**, 95 (1990); A. E. Faraggi *et al.*, Phys. Rev. D **45**, 3272 (1992); Y. Kawamura and M. Tanaka, Prog. Theor. Phys. **91**, 949 (1994); Y. Kawamura *et al.*, Phys. Lett. B **324**, 52 (1994); N. Polonsky and A. Pomarol, Phys. Rev. D **51**, 6532 (1995); H.-C. Cheng and L. J. Hall, Phys. Rev. D **51**, 5289 (1995).

[13] T. Blazek, R. Dermisek, and S. Raby, Phys. Rev. Lett. **88**, 111804 (2002); Phys. Rev. D **65**, 115004 (2002).

[14] D. Auto, H. Baer, C. Balazs, A. Belyaev, J. Ferrandis, and X. Tata, J. High Energy Phys. 06 (2003) 023.

[15] H. Baer, F. E. Paige, S. D.Protopopescu, and X. Tata, arXiv:hep-ph/0001086.

[16] J. Hisano, H. Murayama, and T. Yanagida, Nucl. Phys. **B402**, 46 (1993); Y. Yamada, Z. Phys. C **60**, 83 (1993); J. L. Chkareuli and I. G. Gogoladze, Phys. Rev. D **58**, 055011 (1998).

[17] For recent discussions and additional references see V. Barger, D. Marfatia, and A. Mustafayev, Phys. Lett. B **665**, 242 (2008); M. E. Gomez, S. Lola, P. Naranjo, and J. Rodriguez-Quintero, J. High Energy Phys. 04 (2009) 043.

[18] D. M. Pierce, J. A. Bagger, K. T. Matchev, and R.-j. Zhang, Nucl. Phys. **B491**, 3 (1997).

[19] L. E. Ibanez and G. G. Ross, Phys. Lett. B **110**, 215 (1982); K. Inoue, A. Kakuto, H. Komatsu, and S. Takeshita, Prog. Theor. Phys. **68**, 927 (1982); **70**, 330(E) (1983); L. E. Ibanez, Phys. Lett. B **118**, 73 (1982); J. R. Ellis, D. V. Nanopoulos, and K. Tamvakis, Phys. Lett. B **121**, 123 (1983); L. Alvarez-Gaume, J. Polchinski, and M. B. Wise, Nucl. Phys. **B221**, 495 (1983).

[20] W. M. Yao *et al.*, J. Phys. G **33**, 1 (2006).

[21] Tevatron Electroweak Working Group, CDF Collaboration, and D0 Collaboration, arXiv:0803.1683.

[22] H. Baer, C. Balazs, and A. Belyaev, J. High Energy Phys. 03 (2002) 042; H. Baer, C. Balazs, J. Ferrandis, and X. Tata, Phys. Rev. D **64**, 035004 (2001).

[23] S. Schael *et al.*, Eur. Phys. J. C **47**, 547 (2006).

[24] T. Aaltonen *et al.* (CDF Collaboration), Phys. Rev. Lett. **100**, 101802 (2008).

[25] E. Barberio *et al.* (Heavy Flavor Averaging Group (HFAG) Collaboration), arXiv:0704.3575.

[26] For a recent discussion see E. Komatsu *et al.* (WMAP Collaboration), Astrophys. J. Suppl. Ser. **180**, 330 (2009).

[27] G. W. Bennett *et al.* (Muon G-2 Collaboration), Phys. Rev. D **73**, 072003 (2006).

[28] O. Adriani *et al.*, Nature (London) **458**, 607 (2009); O. Adriani *et al.*, Phys. Rev. Lett. **102**, 051101 (2009).

[29] I. Gogoladze, R. Khalid, Q. Shafi, and H. Yuksel, Phys. Rev. D **79**, 055019 (2009); P. f. Yin, Q. Yuan, J. Liu, J. Zhang, X. j. Bi, and S. h. Zhu, Phys. Rev. D **79**, 023512 (2009).

[30] I. Gogoladze, M. U. Rehman, and Q. Shafi (to be published).