Possible $J^{PC} = 0^{--}$ exotic state

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In order to explore the possible existence of the exotic 0^{--} state, we have constructed the tetraquark interpolating operators systematically. As a byproduct, we notice the 0^{+-} tetraquark operators without derivatives do not exist. The special Lorentz structure of the 0^{--} currents forbids the four-quark type of corrections to the spectral density. Now the gluon condensates are the dominant power corrections. None of the seven interpolating currents support a resonant signal. Therefore we conclude that the exotic 0^{--} state does not exist below 2 GeV, which is consistent with the current experimental observations.

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I. INTRODUCTION

Most of the experimentally observed hadrons can be interpreted as $q\bar{q}/qqq$ states and accommodated in the quark model [1,2]. Up to now there has accumulated some evidence of the exotic state with $J^{PC} = 1^{-+}$ [3–5]. Such a quantum number is not accessible for a pair of quark and antiquark. It is sometimes labeled as an exotic hybrid meson with the particle contents $\bar{q}g_s G_{\mu\nu}\gamma^\nu q$. Recently we have investigated the 1^{-+} state using the tetraquark currents [6]. The extracted mass and characteristic decay pattern are quite similar to those expected for the exotic hybrid meson. Such a result is expected. Since the gluon field creates a pair of $q\bar{q}$ easily, the hybrid operator $\bar{q}g_s G_{\mu\nu}\gamma^\nu q$ transforms into a tetraquark interpolating operator with the same exotic quantum number. In quantum field theory different operators with the same quantum number mix and tend to couple to the same physical state.

Using the same tetraquark formalism developed in the study of the low-lying scalar mesons [7] and the exotic 1^{-+} mesons [6], we study the possible $J^{PC} = 0^{--}$ states composed of light quarks. For a neutral quark model state $q\bar{q}$, we know that $J = 0$ ensures $L = S$ hence $C = (-)^{L+S} = +1$. In other words, states with $J^{PC} = 0^{--}$, 0^{+-}, are strictly forbidden. On the other hand, the gauge invariant scalar and pseudoscalar operators composed of a pair of the gluon field are $g_s^2 G^a_{\mu\nu}G^{a\mu\nu}$ and $\epsilon^{\mu\nu\alpha\beta}g_s^2 G^a_{\mu\nu}G^a_{\alpha\beta}$, both of which carry the even C parity.

We construct all the local tetraquark currents with $J^{PC} = 0^{--}$. There are two kinds of constructions: $(qq)(\bar{q}\bar{q})$ and $(\bar{q}q)(\bar{q}q)$. They can be related to each other by using the Fierz transformation. As usual, we use the first set [7]. Their flavor structure can be $\bar{\mathbf{3}}_f \otimes \mathbf{3}_f$, $\mathbf{6}_f \otimes \bar{\mathbf{6}}_f$, and $\bar{\mathbf{3}}$ $[(qq)(\bar{q}\bar{q})]$. With all these independent currents, we perform the QCD sum rule analysis. As a byproduct, we notice that there does not exist any tetraquark interpolating operator without derivative for the $J^{PC} = 0^{+-}$ case.

This paper is organized as follows. In Sec. II, we construct the tetraquark currents with $J^{PC} = 0^{--}$ using the diquark (qq) and antidiquark $(\bar{q}\bar{q})$ fields. The tetraquark currents constructed with the quark-antiquark $(\bar{q}q)$ pairs are shown in Appendix. A. We present the spectral density in Sec. III and perform the numerical analysis in Sec. IV. For comparison, we present the finite energy sum rule analysis in the Appendix. B. The last section is a short summary.

II. TETRAQUARK INTERPOLATING CURRENTS

A. The $J^{PC} = 0^{--}$ tetraquark interpolating currents

In this section, we construct the tetraquark interpolating currents with $J^{PC} = 0^{--}$ using diquark and antidiquark fields. Such a quantum number can not be accessed by a $q\bar{q}$ pair. The currents can be similarly constructed by using the quark-antiquark pairs. However, as shown in Appendix. A, these two constructions are equivalent as we have shown several times in our previous studies [6,7].

The pseudoscalar tetraquark currents can be constructed using five independent diquark fields, which are constructed by five independent γ matrices

$$\begin{aligned}
S_{abcd} &= (q_{1a}^T C q_{2b})(\bar{q}_{3c}\gamma_5 C\bar{q}_{4d}^T),\\
V_{abcd} &= (q_{1a}^T C\gamma_5 q_{2b})(\bar{q}_{3c} C\bar{q}_{4d}^T),\\
T_{abcd} &= (q_{1a}^T C\sigma_{\mu\nu} q_{2b})(\bar{q}_{3c}\sigma^{\mu\nu}\gamma_5 C\bar{q}_{4d}^T),\\
A_{abcd} &= (q_{1a}^T C\gamma_\mu q_{2b})(\bar{q}_{3c}\gamma^\mu\gamma_5 C\bar{q}_{4d}^T),\\
P_{abcd} &= (q_{1a}^T C\gamma_\mu\gamma_5 q_{2b})(\bar{q}_{3c}\gamma^\mu C\bar{q}_{4d}^T),
\end{aligned} \tag{1}$$

where $q_{1\text{–}4}$ represents the *up*, *down*, and *strange* quarks, and $a - d$ are the color indices.

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To compose a color singlet pseudoscalar tetraquark current, the diquark and antidiquark should have the same color and spin symmetries. So the color structure of the tetraquark is either $\mathbf{6}\otimes\bar{\mathbf{6}}$ or $\bar{\mathbf{3}}\otimes\mathbf{3}$, which is denoted by labels **6** and **3** respectively. Therefore, considering both the color and Lorentz structures, there are altogether ten terms of products

$$\{S \oplus V \oplus T \oplus A \oplus P\}_{\text{Lorentz}} \otimes \{3 \oplus 6\}_{\text{Color}}. \tag{2}$$

We list them as follows

$$\begin{aligned}
6_F\otimes\bar{6}_F(S)&\begin{cases} S_6 = q_{1a}^T C q_{2b}(\bar{q}_{3a}\gamma_5 C\bar{q}_{4b}^T + \bar{q}_{3b}\gamma_5 C\bar{q}_{4a}^T),\\ V_6 = q_{1a}^T C\gamma_5 q_{2b}(\bar{q}_{3a} C\bar{q}_{4b}^T + \bar{q}_{3b} C\bar{q}_{4a}^T),\\ T_3 = q_{1a}^T C\sigma_{\mu\nu} q_{2b}(\bar{q}_{3a}\sigma^{\mu\nu}\gamma_5 C\bar{q}_{4b}^T - \bar{q}_{3b}\sigma^{\mu\nu}\gamma_5 C\bar{q}_{4a}^T),\end{cases}\\
\bar{3}_F\otimes 3_F(A)&\begin{cases} S_3 = q_{1a}^T C q_{2b}(\bar{q}_{3a}\gamma_5 C\bar{q}_{4b}^T - \bar{q}_{3b}\gamma_5 C\bar{q}_{4a}^T),\\ V_3 = q_{1a}^T C\gamma_5 q_{2b}(\bar{q}_{3a} C\bar{q}_{4b}^T - \bar{q}_{3b} C\bar{q}_{4a}^T),\\ T_6 = q_{1a}^T C\sigma_{\mu\nu} q_{2b}(\bar{q}_{3a}\sigma^{\mu\nu}\gamma_5 C\bar{q}_{4b}^T + \bar{q}_{3b}\sigma^{\mu\nu}\gamma_5 C\bar{q}_{4a}^T),\end{cases}\\
\bar{3}_F\otimes\bar{6}_F(M)&\begin{cases} A_6 = q_{1a}^T C\gamma_\mu q_{2b}(\bar{q}_{3a}\gamma^\mu\gamma_5 C\bar{q}_{4b}^T + \bar{q}_{3b}\gamma^\mu\gamma_5 C\bar{q}_{4a}^T),\\ P_3 = q_{1a}^T C\gamma_\mu\gamma_5 q_{2b}(\bar{q}_{3a}\gamma^\mu C\bar{q}_{4b}^T - \bar{q}_{3b}\gamma^\mu C\bar{q}_{4a}^T).\end{cases}\\
6_F\otimes 3_F(M)&\begin{cases} P_6 = q_{1a}^T C\gamma_\mu\gamma_5 q_{2b}(\bar{q}_{3a}\gamma^\mu C\bar{q}_{4b}^T + \bar{q}_{3b}\gamma^\mu C\bar{q}_{4a}^T),\\ A_3 = q_{1a}^T C\gamma_\mu q_{2b}(\bar{q}_{3a}\gamma^\mu\gamma_5 C\bar{q}_{4b}^T - \bar{q}_{3b}\gamma^\mu\gamma_5 C\bar{q}_{4a}^T).\end{cases}
\end{aligned} \tag{3}$$

In the above expressions, the flavor structure is fixed at the same time due to the Pauli principle. The currents S_6, V_6, T_3 belong to the symmetric flavor representation $\mathbf{6_F}\otimes\bar{\mathbf{6}}_\mathbf{F}(S)$ where both diquark and antidiquark fields have the symmetric flavor structure. The currents S_3, V_3, T_6 belong to the antisymmetric flavor representation $\bar{\mathbf{3}}_\mathbf{F}\otimes\mathbf{3_F}(A)$, where both diquark and antidiquark fields have the antisymmetric flavor structure. A_6, P_3 for $\bar{\mathbf{3}}_\mathbf{F}\otimes\bar{\mathbf{6}}_\mathbf{F}(M)$ and A_3, P_6 for $\mathbf{6_F}\otimes\mathbf{3_F}(M)$, where M represents the mixed flavor symmetry. The isovector states with charges can be observed in the experiments more easily, therefore in this paper we concentrate on the isovector currents which was shown in the $SU(3)$ tetraquark weight diagram in Fig. 1 [6]. We have

$$\begin{aligned} qq\bar{q}\,\bar{q}(S),\, qs\bar{q}\,\bar{s}(S) &\in 6_F\otimes\bar{6}_F(S),\\ qs\bar{q}\,\bar{s}(A) &\in \bar{3}_F\otimes 3_F(A),\\ qq\bar{q}\,\bar{q}(M),\, qs\bar{q}\,\bar{s}(M) &\in (\bar{3}_F\otimes\bar{6}_F)\oplus(6_F\otimes 3_F)(M).\end{aligned} \tag{4}$$

We do not differentiate *up* and *down* quarks and denote them by q. We are only interested in those neutral components. The other states do not carry definite C parity. It turns out that the neutral isovector and isoscalar states have the same QCD sum rules. Our following discussions are valid for both of them. Taking the charge-conjugation transformation, we get

$$\begin{aligned} &\mathbb{C}S_6\mathbb{C}^{-1} = V_6, \quad \mathbb{C}A_6\mathbb{C}^{-1} = P_6, \quad \mathbb{C}A_3\mathbb{C}^{-1} = P_3,\\ &\mathbb{C}S_3\mathbb{C}^{-1} = V_3, \quad \mathbb{C}T_6\mathbb{C}^{-1} = T_6, \quad \mathbb{C}T_3\mathbb{C}^{-1} = T_3.\end{aligned} \tag{5}$$

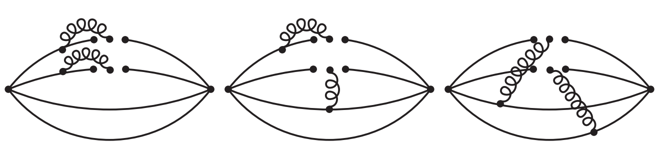

FIG. 1. Feynman diagrams for the quark gluon mixed condensate.

T_6 and T_3 have even charge-conjugation parity. We conclude that the currents with $J^{PC} = 0^{--}$ are

$$\begin{aligned}
\eta^{(S)} &= S_6 - V_6\\
&= q_{1a}^T C q_{2b}(\bar{q}_{3a}\gamma_5 C\bar{q}_{4b}^T + \bar{q}_{3b}\gamma_5 C\bar{q}_{4a}^T)\\
&\quad - q_{1a}^T C\gamma_5 q_{2b}(\bar{q}_{3a} C\bar{q}_{4b}^T + \bar{q}_{3b} C\bar{q}_{4a}^T),\\
\eta_1^{(M)} &= A_6 - P_6\\
&= q_{1a}^T C\gamma_\mu q_{2b}(\bar{q}_{3a}\gamma^\mu\gamma_5 C\bar{q}_{4b}^T + \bar{q}_{3b}\gamma^\mu\gamma_5 C\bar{q}_{4a}^T)\\
&\quad - q_{1a}^T C\gamma_\mu\gamma_5 q_{2b}(\bar{q}_{3a}\gamma^\mu C\bar{q}_{4b}^T + \bar{q}_{3b}\gamma^\mu C\bar{q}_{4a}^T),\\
\eta_2^{(M)} &= A_3 - P_3\\
&= q_{1a}^T C\gamma_\mu q_{2b}(\bar{q}_{3a}\gamma^\mu\gamma_5 C\bar{q}_{4b}^T - \bar{q}_{3b}\gamma^\mu\gamma_5 C\bar{q}_{4a}^T)\\
&\quad - q_{1a}^T C\gamma_\mu\gamma_5 q_{2b}(\bar{q}_{3a}\gamma^\mu C\bar{q}_{4b}^T - \bar{q}_{3b}\gamma^\mu C\bar{q}_{4a}^T),\\
\eta^{(A)} &= S_3 - V_3\\
&= q_{1a}^T C q_{2b}(\bar{q}_{3a}\gamma_5 C\bar{q}_{4b}^T - \bar{q}_{3b}\gamma_5 C\bar{q}_{4a}^T)\\
&\quad - q_{1a}^T C\gamma_5 q_{2b}(\bar{q}_{3a} C\bar{q}_{4b}^T - \bar{q}_{3b} C\bar{q}_{4a}^T).
\end{aligned} \tag{6}$$

Adding different quauk contents as shown in Eq. (4), there are altogether seven independent currents as shown:

(1) For $6_F \otimes \bar{6}_F(S)$:

$$\begin{aligned}\eta_1 &= S_6(qq\bar{q}\,\bar{q}) - V_6(qq\bar{q}\,\bar{q})\\ &= u_a^T C d_b(\bar{u}_a\gamma_5 C\bar{d}_b^T + \bar{u}_b\gamma_5 C\bar{d}_a^T)\\ &\quad - u_a^T C\gamma_5 d_b(\bar{u}_a C\bar{d}_b^T + \bar{u}_b C\bar{d}_a^T),\\ \eta_2 &= S_6(qs\bar{q}\,\bar{s}) - V_6(qs\bar{q}\,\bar{s})\\ &= u_a^T C s_b(\bar{u}_a\gamma_5 C\bar{s}_b^T + \bar{u}_b\gamma_5 C\bar{s}_a^T)\\ &\quad - u_a^T C\gamma_5 s_b(\bar{u}_a C\bar{s}_b^T + \bar{u}_b C\bar{s}_a^T),\end{aligned} \tag{7}$$

where η_1 belongs to the $\mathbf{27_F}$ representation and contains up and down quarks only while η_2 belongs to the $\mathbf{8_F}$ representation and contains one $s\bar{s}$ quark pair.

(2) For $(\bar{3}_F \otimes \bar{6}_F) \oplus (6_F \otimes 3_F)(M)$:

$$\begin{aligned}\eta_3 &= A_6(qq\bar{q}\,\bar{q}) - P_6(qq\bar{q}\,\bar{q})\\ &= u_a^T C\gamma_\mu d_b(\bar{u}_a\gamma^\mu\gamma_5 C\bar{d}_b^T + \bar{u}_b\gamma^\mu\gamma_5 C\bar{d}_a^T)\\ &\quad - u_a^T C\gamma_\mu\gamma_5 d_b(\bar{u}_a\gamma^\mu C\bar{d}_b^T + \bar{u}_b\gamma^\mu C\bar{d}_a^T),\\ \eta_4 &= A_6(qs\bar{q}\,\bar{s}) - P_6(qs\bar{q}\,\bar{s})\\ &= u_a^T C\gamma_\mu s_b(\bar{u}_a\gamma^\mu\gamma_5 C\bar{s}_b^T + \bar{u}_b\gamma^\mu\gamma_5 C\bar{s}_a^T)\\ &\quad - u_a^T C\gamma_\mu\gamma_5 s_b(\bar{u}_a\gamma^\mu C\bar{s}_b^T + \bar{u}_b\gamma^\mu C\bar{s}_a^T),\\ \eta_5 &= A_3(qq\bar{q}\,\bar{q}) - P_3(qq\bar{q}\,\bar{q})\\ &= u_a^T C\gamma_\mu d_b(\bar{u}_a\gamma^\mu C\bar{d}_b^T - \bar{u}_b\gamma^\mu C\bar{d}_a^T)\\ &\quad - u_a^T C\gamma_\mu\gamma_5 d_b(\bar{u}_a\gamma^\mu C\bar{d}_b^T - \bar{u}_b\gamma^\mu C\bar{d}_a^T),\\ \eta_6 &= A_3(qs\bar{q}\,\bar{s}) - P_3(qs\bar{q}\,\bar{s})\\ &= u_a^T C\gamma_\mu s_b(\bar{u}_a\gamma^\mu C\bar{s}_b^T - \bar{u}_b\gamma^\mu C\bar{s}_a^T)\\ &\quad - u_a^T C\gamma_\mu\gamma_5 s_b(\bar{u}_a\gamma^\mu C\bar{s}_b^T - \bar{u}_b\gamma^\mu C\bar{s}_a^T),\end{aligned} \tag{8}$$

where η_3 and η_5 belong to the $\overline{\mathbf{10}}_\mathbf{F}$ representation and contain only u, d quarks while η_4 and η_6 belong to the $\mathbf{8_F}$ representation and contain one $s\bar{s}$ quark pair.

(3) For $\bar{3}_F \otimes 3_F(A)$:

$$\begin{aligned}\eta_7 &= S_3(qs\bar{q}\,\bar{s}) - V_3(qs\bar{q}\,\bar{s})\\ &= u_a^T C s_b(\bar{u}_a\gamma_5 C\bar{s}_b^T - \bar{u}_b\gamma_5 C\bar{s}_a^T)\\ &\quad - u_a^T C\gamma_5 s_b(\bar{u}_a C\bar{s}_b^T - \bar{u}_b C\bar{s}_a^T).\end{aligned} \tag{9}$$

where η_7 belongs to the $\mathbf{8_F}$ and contains one $s\bar{s}$ quark pair.

It is understood that there exists the other part $\pm[u \leftrightarrow d]$ in the expressions of $\eta_{2,4,6,7}$.

B. The $J^{PC}=0^{+-}$ tetraquark currents

Now we move on to the $J^{PC}=0^{+-}$ case. There are also ten independent scalar tetraquark currents without derivative:

$$\begin{aligned}S_6' &= q_{1a}^T C q_{2b}(\bar{q}_{3a} C\bar{q}_{4b}^T + \bar{q}_{3b} C\bar{q}_{4a}^T),\\ V_6' &= q_{1a}^T\gamma_\mu C q_{2b}(\bar{q}_{3a} C\gamma^\mu\bar{q}_{4b}^T + \bar{q}_{3b} C\gamma^\mu\bar{q}_{4a}^T),\\ T_6' &= q_{1a}^T\sigma_{\mu\nu} C q_{2b}(\bar{q}_{3a} C\sigma^{\mu\nu}\bar{q}_{4b}^T + \bar{q}_{3b} C\sigma^{\mu\nu}\bar{q}_{4a}^T),\\ A_6' &= q_{1a}^T\gamma_\mu\gamma_5 C q_{2b}(\bar{q}_{3a} C\gamma^\mu\gamma_5\bar{q}_{4b}^T + \bar{q}_{3b} C\gamma^\mu\gamma_5\bar{q}_{4a}^T),\\ P_6' &= q_{1a}^T\gamma_5 C q_{2b}(\bar{q}_{3a} C\gamma_5\bar{q}_{4b}^T + \bar{q}_{3b} C\gamma_5\bar{q}_{4a}^T),\\ S_3' &= q_{1a}^T C q_{2b}(\bar{q}_{3a} C\bar{q}_{4b}^T - \bar{q}_{3b} C\bar{q}_{4a}^T),\\ V_3' &= q_{1a}^T\gamma_\mu C q_{2b}(\bar{q}_{3a} C\gamma^\mu\bar{q}_{4b}^T - \bar{q}_{3b} C\gamma^\mu\bar{q}_{4a}^T),\\ T_3' &= q_{1a}^T\sigma_{\mu\nu} C q_{2b}(\bar{q}_{3a} C\sigma^{\mu\nu}\bar{q}_{4b}^T - \bar{q}_{3b} C\sigma^{\mu\nu}\bar{q}_{4a}^T),\\ A_3' &= q_{1a}^T\gamma_\mu\gamma_5 C q_{2b}(\bar{q}_{3a} C\gamma^\mu\gamma_5\bar{q}_{4b}^T - \bar{q}_{3b} C\gamma^\mu\gamma_5\bar{q}_{4a}^T),\\ P_3' &= q_{1a}^T\gamma_5 C q_{2b}(\bar{q}_{3a} C\gamma_5\bar{q}_{4b}^T - \bar{q}_{3b} C\gamma_5\bar{q}_{4a}^T).\end{aligned} \tag{10}$$

The flavor structure is again fixed due to the Pauli principle. To have a charge-conjugation parity, we fix the quark contents to be $q_1 = q_3$ and $q_2 = q_4$ (or $q_1 = q_4$ and $q_2 = q_3$). After performing the charge-conjugation transformation, we find that they all have an even charge-conjugation parity, for example:

$$\mathbb{C}\, S_6' \mathbb{C}^{-1} = +S_6'. \tag{11}$$

Therefore, the $J^{PC}=0^{+-}$ tetraquark interpolating currents without derivatives do not exist.

III. THE SPECTRAL DENSITY

We consider the two-point correlation function in the framework of QCD sum rule [8,9]:

$$\Pi(q^2) \equiv \int d^4x e^{iqx}\langle 0|T\eta(x)\eta^\dagger(0)|0\rangle, \tag{12}$$

where η is an interpolating current. We can calculate $\Pi(q^2)$ at the quark gluon level using the propagator:

$$\begin{aligned}iS_q^{ab} &\equiv \langle 0|T[q^a(x)\bar{q}^b(0)]|0\rangle\\ &= \frac{i\delta^{ab}}{2\pi^2x^4}\hat{x} + \frac{i}{32\pi^2}\frac{\lambda_{ab}^n}{2}gG_{\mu\nu}^n\frac{1}{x^2}(\sigma^{\mu\nu}\hat{x} + \hat{x}\sigma^{\mu\nu})\\ &\quad - \frac{\delta^{ab}}{12}\langle\bar{q}q\rangle + \frac{\delta^{ab}x^2}{192}\langle g_s\bar{q}\sigma Gq\rangle - \frac{m_q\delta^{ab}}{4\pi^2x^2}\\ &\quad + \frac{i\delta^{ab}m_q\langle\bar{q}q\rangle}{48}\hat{x} + \frac{i\delta^{ab}m_q^2\langle\bar{q}q\rangle}{8\pi^2x^2}\hat{x},\end{aligned} \tag{13}$$

where $\hat{x} \equiv \gamma_\mu x^\mu$. With the dispersion relation $\Pi(q^2)$ is related to the observable at the hadron level

$$\Pi(p^2) = \int_0^\infty \frac{\rho(s)}{s - p^2 - i\varepsilon}ds, \tag{14}$$

where

$$\begin{aligned}\rho(s) &\equiv \sum_n \delta(s - M_n^2)\langle 0|\eta|n\rangle\langle n|\eta^\dagger|0\rangle\\ &= f_X^2\delta(s - M_X^2) + \text{continuum}.\end{aligned} \tag{15}$$

Here, the usual pole plus continuum parametrization of the hadronic spectral density is adopted. Up to dimension 12, the spectral density $\rho_i(s)$ at the quark and gluon level reads

$$\rho_1(s) = \frac{s^4}{15360\pi^6} - \frac{m_q^2}{192\pi^6}s^3 - \left(\frac{\langle g_s^2 GG\rangle}{3072\pi^6} - \frac{m_q\langle\bar{q}q\rangle}{24\pi^4}\right)s^2 + \left[\frac{\langle g_s^2 GG\rangle m_q^2}{256\pi^6} + \frac{\langle g_s^3 fGGG\rangle}{768\pi^6}\left(3\ln\left(\frac{s}{\tilde{\mu}^2}\right) - 5\right)\right]s - \left(\frac{3m_q^2\langle\bar{q}q\rangle^2}{2\pi^2} + \frac{\langle g_s^2 GG\rangle m_q\langle\bar{q}q\rangle}{192\pi^4}\right) + \left(\frac{16}{9}m_q\langle\bar{q}q\rangle^3 - \frac{1}{\pi^2}m_q^2\langle\bar{q}q\rangle\langle g_s\bar{q}\sigma Gq\rangle\right)\delta(s), \tag{16}$$

$$\rho_2(s) = \frac{s^4}{15360\pi^6} - \frac{m_s^2}{384\pi^6}s^3 + \left(\frac{m_s^4}{64\pi^6} + \frac{m_s\langle\bar{s}s\rangle}{48\pi^4} - \frac{\langle g_s^2 GG\rangle}{3072\pi^6}\right)s^2 + \left[\frac{\langle g_s^3 fGGG\rangle}{768\pi^6}\left(3\ln\left(\frac{s}{\tilde{\mu}^2}\right) - 5\right) - \left(\frac{m_s^3\langle\bar{s}s\rangle}{8\pi^4} - \frac{m_s^2\langle g_s^2 GG\rangle}{512\pi^6}\right)\right]s + \left(\frac{m_s^2\langle\bar{s}s\rangle^2}{12\pi^2} - \frac{m_s^2\langle\bar{q}q\rangle^2}{3\pi^2} - \frac{m_s\langle\bar{s}s\rangle\langle g_s^2 GG\rangle}{384\pi^4}\right) - \left(\frac{m_s^2\langle\bar{u}u\rangle\langle g_s\bar{q}\sigma Gq\rangle}{6\pi^2} - \frac{8}{9}m_s\langle\bar{s}s\rangle\langle\bar{q}q\rangle^2\right)\delta(s), \tag{17}$$

$$\rho_3(s) = \frac{s^4}{3840\pi^6} - \frac{m_q^2}{48\pi^6}s^3 + \left(\frac{5\langle g_s^2 GG\rangle}{1536\pi^6} + \frac{m_q\langle\bar{q}q\rangle}{6\pi^4}\right)s^2 + \left[\frac{\langle g_s^3 fGGG\rangle}{192\pi^6}\left(3\ln\left(\frac{s}{\tilde{\mu}^2}\right) - 5\right) - \frac{5\langle g_s^2 GG\rangle m_q^2}{128\pi^6}\right]s - \left(\frac{6m_q^2\langle\bar{q}q\rangle^2}{\pi^2} - \frac{5\langle g_s^2 GG\rangle m_q\langle\bar{q}q\rangle}{96\pi^4}\right) + \left(\frac{64}{9}m_q\langle\bar{q}q\rangle^3 - \frac{4}{\pi^2}m_q^2\langle\bar{q}q\rangle\langle g_s\bar{q}\sigma Gq\rangle\right)\delta(s), \tag{18}$$

$$\rho_4(s) = \frac{s^4}{3840\pi^6} - \frac{m_s^2}{96\pi^6}s^3 + \left(\frac{m_s^4}{16\pi^6} + \frac{m_s\langle\bar{s}s\rangle}{12\pi^4} + \frac{5\langle g_s^2 GG\rangle}{1536\pi^6}\right)s^2 + \left[\frac{\langle g_s^3 fGGG\rangle}{192\pi^6}\left(3\ln\left(\frac{s}{\tilde{\mu}^2}\right) - 5\right) - \left(\frac{m_s^3\langle\bar{s}s\rangle}{2\pi^4} + \frac{5m_s^2\langle g_s^2 GG\rangle}{256\pi^6}\right)\right]s + \left(\frac{m_s^2\langle\bar{s}s\rangle^2}{3\pi^2} - \frac{4m_s^2\langle\bar{q}q\rangle^2}{3\pi^2} + \frac{5m_s\langle\bar{s}s\rangle\langle g_s^2 GG\rangle}{192\pi^4}\right) - \left(\frac{2m_s^2\langle\bar{u}u\rangle\langle g_s\bar{q}\sigma Gq\rangle}{3\pi^2} - \frac{32}{9}m_s\langle\bar{s}s\rangle\langle\bar{q}q\rangle^2\right)\delta(s), \tag{19}$$

$$\rho_5(s) = \frac{s^4}{7680\pi^6} - \frac{m_q^2}{96\pi^6}s^3 + \left(\frac{\langle g_s^2 GG\rangle}{1536\pi^6} + \frac{m_q\langle\bar{q}q\rangle}{12\pi^4}\right)s^2 + \left[\frac{\langle g_s^3 fGGG\rangle}{384\pi^6}\left(3\ln\left(\frac{s}{\tilde{\mu}^2}\right) - 5\right) - \frac{\langle g_s^2 GG\rangle m_q^2}{128\pi^6}\right]s - \left(\frac{3m_q^2\langle\bar{q}q\rangle^2}{\pi^2} - \frac{\langle g_s^2 GG\rangle m_q\langle\bar{q}q\rangle}{96\pi^4}\right) + \left(\frac{32}{9}m_q\langle\bar{q}q\rangle^3 - \frac{2}{\pi^2}m_q^2\langle\bar{q}q\rangle\langle g_s\bar{q}\sigma Gq\rangle\right)\delta(s), \tag{20}$$

$$\rho_6(s) = \frac{s^4}{7680\pi^6} - \frac{m_s^2}{192\pi^6}s^3 + \left(\frac{m_s^4}{32\pi^6} + \frac{m_s\langle\bar{s}s\rangle}{24\pi^4} + \frac{\langle g_s^2 GG\rangle}{1536\pi^6}\right)s^2 + \left[\frac{\langle g_s^3 fGGG\rangle}{384\pi^6}\left(3\ln\left(\frac{s}{\tilde{\mu}^2}\right) - 5\right) - \left(\frac{m_s^3\langle\bar{s}s\rangle}{4\pi^4} + \frac{m_s^2\langle g_s^2 GG\rangle}{256\pi^6}\right)\right]s + \left(\frac{m_s^2\langle\bar{s}s\rangle^2}{6\pi^2} - \frac{2m_s^2\langle\bar{q}q\rangle^2}{3\pi^2} + \frac{m_s\langle\bar{s}s\rangle\langle g_s^2 GG\rangle}{192\pi^4}\right) - \left(\frac{m_s^2\langle\bar{u}u\rangle\langle g_s\bar{q}\sigma Gq\rangle}{3\pi^2} - \frac{16}{9}m_s\langle\bar{s}s\rangle\langle\bar{q}q\rangle^2\right)\delta(s), \tag{21}$$

$$\rho_7(s) = \frac{s^4}{30720\pi^6} - \frac{m_s^2}{768\pi^6}s^3 + \left(\frac{m_s^4}{128\pi^6} + \frac{m_s\langle\bar{s}s\rangle}{96\pi^4} + \frac{\langle g_s^2 GG\rangle}{3072\pi^6}\right)s^2 + \left[\frac{\langle g_s^3 fGGG\rangle}{1536\pi^6}\left(3\ln\left(\frac{s}{\tilde{\mu}^2}\right) - 5\right) - \left(\frac{m_s^3\langle\bar{s}s\rangle}{16\pi^4} + \frac{m_s^2\langle g_s^2 GG\rangle}{512\pi^6}\right)\right]s + \left(\frac{m_s^2\langle\bar{s}s\rangle^2}{24\pi^2} - \frac{m_s^2\langle\bar{q}q\rangle^2}{6\pi^2} + \frac{m_s\langle\bar{s}s\rangle\langle g_s^2 GG\rangle}{384\pi^4}\right) - \left(\frac{m_s^2\langle\bar{u}u\rangle\langle g_s\bar{q}\sigma Gq\rangle}{12\pi^2} - \frac{4}{9}m_s\langle\bar{s}s\rangle\langle\bar{q}q\rangle^2\right)\delta(s). \tag{22}$$

It is interesting to note several important features of the above spectral densities:

(i) First the special Lorentz structure of the $J^{PC} = 0^{--}$ interpolating currents forbids the appearance of the four-quark type of condensates $\langle\bar{q}q\rangle^2$, $\langle\bar{q}q\rangle\langle g_s\bar{q}\sigma Gq\rangle$, and $\langle g_s\bar{q}\sigma Gq\rangle^2$. Usually these terms play an important role in the multiquark sum rules. The Feynman diagrams for the dimension 10 condensate $\langle g_s\bar{q}\sigma Gq\rangle^2$ are shown in Fig. 1.

(ii) The dominant nonperturbative correction arises from the gluon condensate, which is destructive for $\rho_{1-2}(s)$ and constructive for $\rho_{3-7}(s)$. Moreover there are corrections from the trigluon condensate $\langle g_s^3 f^{abc}G^aG^bG^c\rangle$ as shown in Fig. 2. In the above expressions we use the short-hand notation

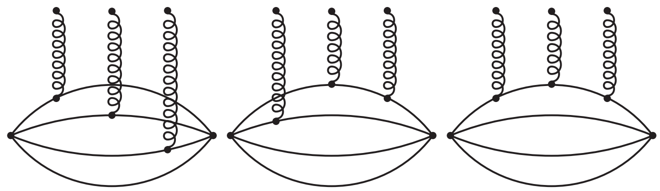

FIG. 2. Feynman diagrams for the trigluon condensate.

$\langle g_s^3 fGGG \rangle$ to denote the trigluon condensate. There are three types of Feynman diagrams. The first class of Feynman diagrams vanishes because of the product of the color matrices. The second class is proportional to m_q and could be omitted in the chiral limit. Only the third class leads to a nonvanishing trigluon correction. In fact the gluon condensates become the only power corrections in the chiral limit.

(iii) The second term in each $\rho_i(s)$ is destructive, which renders the spectral density negative when s is small. This $-m_q^2 s^3$ piece is an artifact of the expansion of the quark propagator $\frac{i}{\hat{p}-m_q}$ in terms of the quark mass m_q perturbatively. Without making such an expansion, the perturbative contribution to the spectral density is always positive definite. Such a destructive term will sometimes produce an artificial plateau and stability window in the sum rule analysis, which must be removed.

(iv) Although the tree-level four-quark condensate vanishes, one may wonder whether the four-quark condensate $g_s^2 \langle \bar{q} q \rangle^2$ plays a role since the latter is very important in the $q\bar{q}$ meson sum rules [8,9]. Two types of Feynman diagrams could produce such a correction. The first class of Feynman diagrams is very similar to that in the $q\bar{q}$ meson case where a gluon propagator is attached between two-quark condensates, as Fig. 3 shows. it is easy to check that they vanish due to the special Lorentz structure of the correlation function. One of the second class of diagrams is shown in Fig. 4. In this case, we use the mesonic type interpolating currents in Appendix A to simplify the derivation. After making a Wick contraction to the correlation function

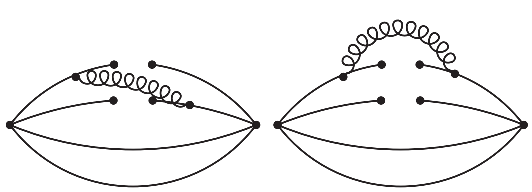

FIG. 3. One set of Feynman diagrams for the four-quark condensate.

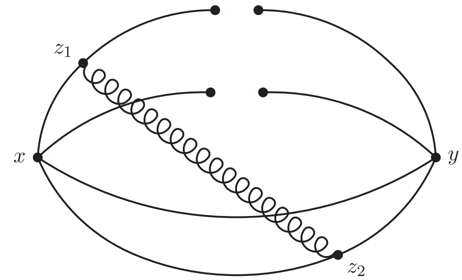

FIG. 4. Feynman diagrams for the four-quark condensate.

$$\bar{\psi}_3(x)\Gamma_1' \psi_4(x)\bar{\psi}_1(x)\Gamma_1 \psi_2(x)\bar{\psi}_1(z_1) g t^a \gamma^\mu \psi_1(z_1) \times A_\mu^a(z_1)\bar{\psi}_2(z_2) g t^b \gamma^\nu \psi_2(z_2) A_\nu^b(z_2)\bar{\psi}_2(y) \times \Gamma_2 \psi_1(y)\bar{\psi}_4(y)\Gamma_2' \psi_3(y),$$

we get

$$Tr[-\Gamma_1' S_Q(x-y)\Gamma_2' S_Q(y-x)] \times Tr[-S_Q(x-z_2)\gamma^\nu S_Q(z_2-y) \times \Gamma_2 S_Q(y-z_1)\gamma^\mu S_Q(z_1-x)\Gamma_1 \times g_{\mu\nu} \times S_G(z_2-z_1)],$$

where S_Q is the quark propagator and S_G is the gluon propagator. $\{\Gamma_1, \Gamma_2\}$ could be either $\{I, \gamma_5\}$ or $\{\gamma_\alpha, \gamma_5\gamma_\alpha\}$; $S_Q(y-z_1) \propto \langle \bar{q} q \rangle$. In fact, there would be three γ matrices or three γ matrices plus γ_5 left in the latter trace. Therefore this piece also vanishes.

IV. NUMERICAL ANALYSIS

In the chiral limit ($m_s = m_q = 0$) the spectral density reads

$$\begin{aligned}
\rho_{1-2}(s) &= \frac{s^4}{15360\pi^6} - \frac{\langle g_s^2 GG \rangle}{3072\pi^6} s^2 + \frac{\langle g_s^3 fGGG \rangle}{768\pi^6}\left(3\ln\left(\frac{s}{\tilde{\mu}^2}\right) - 5\right)s, \\
\rho_{3-4}(s) &= \frac{s^4}{3840\pi^6} - \frac{5\langle g_s^2 GG \rangle}{1536\pi^6} s^2 + \frac{\langle g_s^3 fGGG \rangle}{192\pi^6}\left(3\ln\left(\frac{s}{\tilde{\mu}^2}\right) - 5\right)s, \\
\rho_{5-6}(s) &= \frac{s^4}{7680\pi^6} - \frac{\langle g_s^2 GG \rangle}{1536\pi^6} s^2 + \frac{\langle g_s^3 fGGG \rangle}{384\pi^6}\left(3\ln\left(\frac{s}{\tilde{\mu}^2}\right) - 5\right)s, \\
\rho_7(s) &= \frac{s^4}{30720\pi^6} - \frac{\langle g_s^2 GG \rangle}{3072\pi^6} s^2 + \frac{\langle g_s^3 fGGG \rangle}{1536\pi^6}\left(3\ln\left(\frac{s}{\tilde{\mu}^2}\right) - 5\right)s,
\end{aligned} \tag{23}$$

where $\tilde{\mu} = 1$ GeV. Requiring the pole contribution is

TABLE I. The working region of M_B^2 with Ioffe's and SVZ's gluon condensates and $s_0 = 7\ \text{GeV}^2$.

\	$[M_{\min}^2, M_{\max}^2]$(SVZ)	$[M_{\min}^2, M_{\max}^2]$ (Ioffe)
ρ_{1-2}	$0.77 \sim 1.50$	$0.90 \sim 1.68$
ρ_{3-4}	$1.22 \sim 1.90$	$1.40 \sim 1.65$
ρ_{5-6}	$1.05 \sim 1.77$	$1.55 \sim 1.74$
ρ_7	$1.10 \sim 1.85$	$1.50 \sim 1.75$

larger than 40%, one gets the upper bound $M_{\max}^2$ of the Borel parameter M_B^2. The convergence of the operator expansion product leads to the lower bound $M_{\min}^2$ of the Borel parameter. In the present case, we require that the two-gluon condensate correction be less than one third of the perturbative term and the trigluon condensate correction less than one third of the gluon condensate correction. The working region of M_B^2 in the sum rule analysis is $[M_{\min}^2, M_{\max}^2]$, which is dependent on the threshold s_0.

In order to study the sensitivity of the sum rule to the condensate values, we adopt two sets of the gluon condensate values in our numerical analysis. One set is from Ioffe's recent review [10]: $\langle g_s^2 GG\rangle = (0.20 \pm 0.16)\ \text{GeV}^4$, $\langle g_s^3 fGGG\rangle = 0.12\ \text{GeV}^6$. We also use the original Shifman-Vainshtein-Zakharov's (SVZ) values [8]: $\langle g_s^2 GG\rangle = (0.48 \pm 0.14)\ \text{GeV}^4$, $\langle g_s^3 fGGG\rangle = 0.045\ \text{GeV}^6$. The working regions of the sum rules with the above two sets of gluon condensates and $s_0 = 7\ \text{GeV}^2$ are listed in Table I. The working region of the sum rule is very narrow even with $s_0 = 7\ \text{GeV}^2$. The variation of M_X with M_B^2 and s_0 is shown in Figs. 5–8 for the interpolating currents η_{1-2}, η_{3-4}, η_{5-6}, η_7 respectively using Ioffe's gluon condensate values. The variation of M_X with M_B^2 and s_0 and SVZ's gluon condensate values is presented in Figs. 9–12.

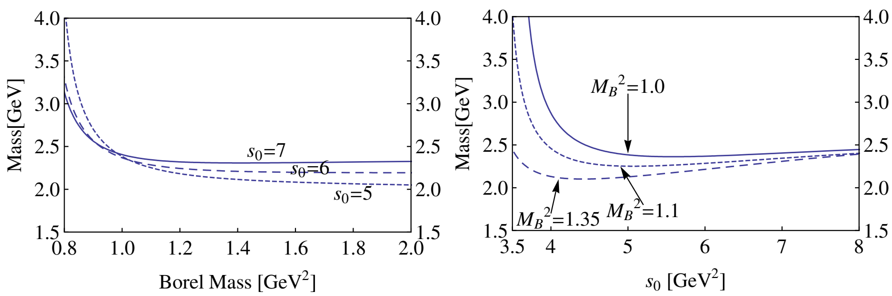

FIG. 5 (color online). The variation of M_X with M_B^2 (left) and s_0 (right) for the current η_{1-2} using Ioffe's gluon condensate values.

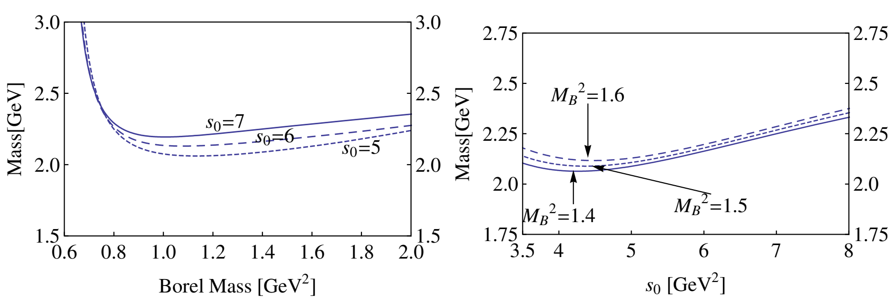

FIG. 6 (color online). The variation of M_X with M_B^2 (left) and s_0 (right) for the current η_{3-4} using Ioffe's gluon condensate values.

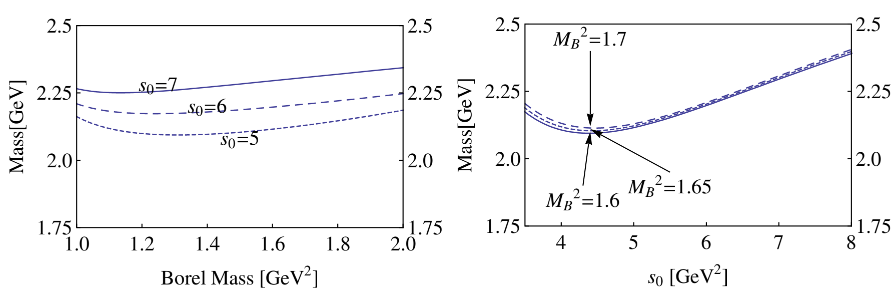

FIG. 7 (color online). The variation of M_X with M_B^2 (left) and s_0 (right) for the current η_{5-6} using Ioffe's gluon condensate values.

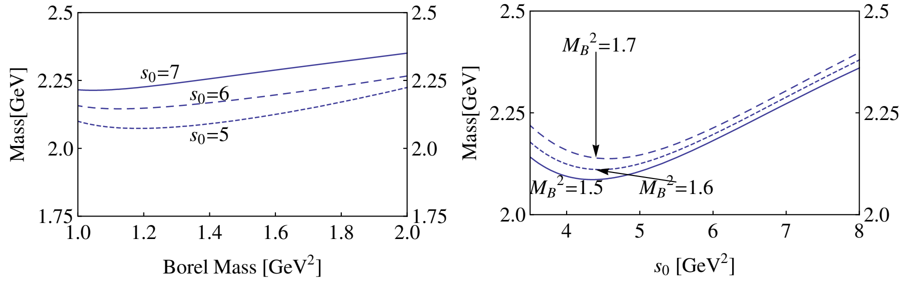

FIG. 8 (color online). The variation of M_X with M_B^2 (left) and s_0 (right) for the current η_7 using Ioffe's gluon condensate values.

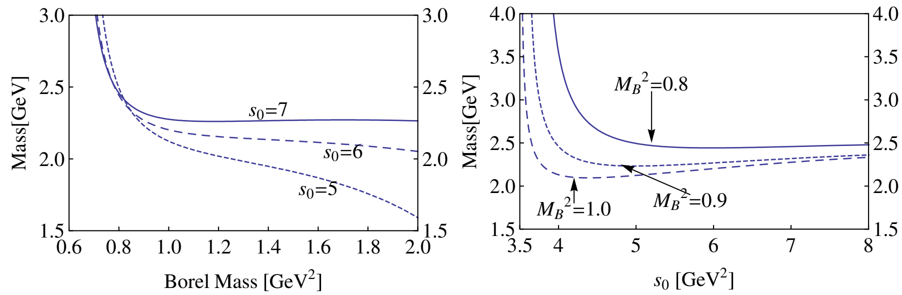

FIG. 9 (color online). The variation of M_X with M_B^2 (left) and s_0 (right) for the current η_{1-2} using SVZ's gluon condensate values.

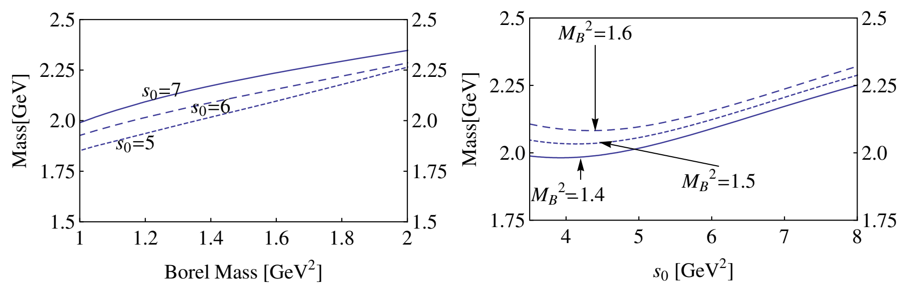

FIG. 10 (color online). The variation of M_X with M_B^2 (left) and s_0 (right) for the current η_{3-4} using SVZ's gluon condensate values.

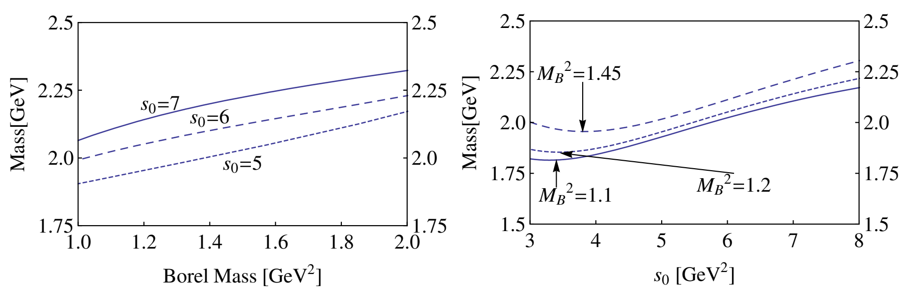

FIG. 11 (color online). The variation of M_X with M_B^2 (left) and s_0 (right) for the current η_{5-6} using SVZ's gluon condensate values.

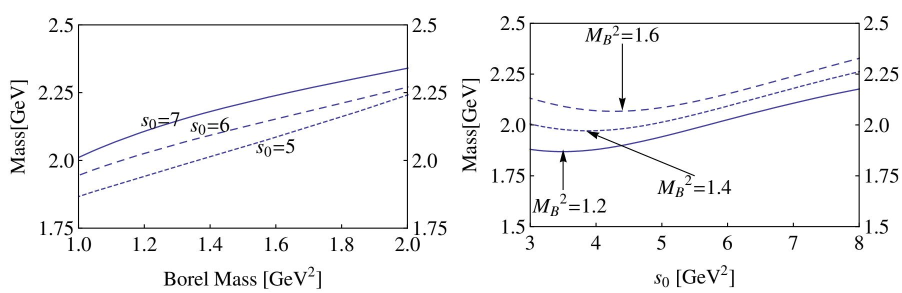

FIG. 12 (color online). The variation of M_X with M_B^2 (left) and s_0 (right) for the current η_7 using SVZ's gluon condensate values.

For a genuine hadron state, one expects that the extracted mass from the sum rule analysis is stable with the reasonable variation of the Borel parameter and the continuum threshold. In other words, there should exists dual stability in M_B^2 and s_0 in the working region of M_B^2. From all these figures we notice none of the mass curves satisfy the stability requirement. These interpolating currents do not support a low-lying resonant signal.

V. CONCLUSION

The exotic state with $J^{PC} = 0^{--}$ cannot be composed of a pair of gluons nor $q\bar{q}$. In order to explore the possible existence of these interesting states, we first construct the tetraquark type interpolating operators systematically. As a byproduct, we notice that the $J^{PC} = 0^{+-}$ tetraquark operators without derivatives do not exist. Then we make the operator product expansion (OPE) and extract the spectral density. The gluon condensate becomes the dominant power correction. Usually the four-quark type of condensates $\langle\bar{q}q\rangle^2$, $\langle\bar{q}q\rangle\langle g_s\bar{q}\sigma Gq\rangle$, and $\langle g_s\bar{q}\sigma Gq\rangle^2$ are the dominant nonperturbative corrections in the multiquark sum rules. However these terms vanish because of the special Lorentz structure imposed by the exotic 0^{--} quantum numbers.

Within the framework of the SVZ sum rule, we note that the absence of the $\langle\bar{q}q\rangle^2$, $\langle\bar{q}q\rangle\langle g_s\bar{q}\sigma Gq\rangle$, and $\langle g_s\bar{q}\sigma Gq\rangle^2$ terms destabilize the sum rule. There does not exist stability in either M_B^2 or s_0 in the working region of M_B^2. Therefore we conclude that none of these independent interpolating currents support a resonant signal below 2 GeV, which is consistent with the current experimental measurement [1].

ACKNOWLEDGMENTS

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APPENDIX A: INTERPOLATING CURRENTS IN $(\bar{q}q)(\bar{q}q)$ BASIS

For $6_F \otimes \bar{6}_F(S)$:

$$\begin{aligned}\eta_m^{(S)(1)} &= (\bar{q}_{1a}\gamma_\mu q_{1a})(\bar{q}_{2b}\gamma^\mu\gamma_5 q_{2b}) + (\bar{q}_{1a}\gamma_\mu\gamma_5 q_{1a})\\&\quad\times(\bar{q}_{2b}\gamma^\mu q_{2b}) + (\bar{q}_{1a}\gamma_\mu q_{2a})(\bar{q}_{2b}\gamma^\mu\gamma_5 q_{1b})\\&\quad+(\bar{q}_{1a}\gamma_\mu\gamma_5 q_{2a})(\bar{q}_{2b}\gamma^\mu q_{1b}),\\\eta_m^{(S)(8)} &= \lambda_{ab}\lambda_{cd}\{(\bar{q}_{1a}\gamma_\mu q_{1b})(\bar{q}_{2c}\gamma^\mu\gamma_5 q_{2d})\\&\quad+(\bar{q}_{1a}\gamma_\mu\gamma_5 q_{1b})(\bar{q}_{2d}\gamma^\mu q_{2d}) + (\bar{q}_{1a}\gamma_\mu q_{2b})\\&\quad\times(\bar{q}_{2c}\gamma^\mu\gamma_5 q_{1d}) + (\bar{q}_{1a}\gamma_\mu\gamma_5 q_{2b})(\bar{q}_{2c}\gamma^\mu q_{1d})\}.\end{aligned}$$

For $(\bar{3}_F \otimes \bar{6}_F) \oplus (6_F \otimes 3_F)(M)$:

$$\begin{aligned}\eta_{1m}^{(M)(1)} &= (\bar{q}_{1a}q_{1a})(\bar{q}_{2b}\gamma_5 q_{2b}) - (\bar{q}_{1a}\gamma_5 q_{1a})(\bar{q}_{2b}q_{2b}),\\\eta_{1m}^{(M)(8)} &= \lambda_{ab}\lambda_{cd}\{(\bar{q}_{1a}q_{1b})(\bar{q}_{2c}\gamma_5 q_{2d}) - (\bar{q}_{1a}\gamma_5 q_{1b})\\&\quad\times(\bar{q}_{2c}q_{2d})\},\\\eta_{2m}^{(M)(1)} &= (\bar{q}_{1a}\gamma_\mu q_{1a})(\bar{q}_{2b}\gamma^\mu\gamma_5 q_{2b}) - (\bar{q}_{1a}\gamma_\mu\gamma_5 q_{1a})\\&\quad\times(\bar{q}_{2b}\gamma^\mu q_{2b}),\\\eta_{2m}^{(M)(8)} &= \lambda_{ab}\lambda_{cd}\{(\bar{q}_{1a}\gamma_\mu q_{1b})(\bar{q}_{2c}\gamma^\mu\gamma_5 q_{2d})\\&\quad-(\bar{q}_{1a}\gamma_\mu\gamma_5 q_{1b})(\bar{q}_{2c}\gamma^\mu q_{2d})\}.\end{aligned}$$

For $\bar{3}_F \otimes 3_F(A)$:

$$\begin{aligned}\eta_m^{(A)(1)} &= (\bar{q}_{1a}\gamma_\mu q_{1a})(\bar{q}_{2b}\gamma^\mu\gamma_5 q_{2b}) + (\bar{q}_{1a}\gamma_\mu\gamma_5 q_{1a})\\&\quad\times(\bar{q}_{2b}\gamma^\mu q_{2b}) - (\bar{q}_{1a}\gamma_\mu q_{2a})(\bar{q}_{2b}\gamma^\mu\gamma_5 q_{1b})\\&\quad-(\bar{q}_{1a}\gamma_\mu\gamma_5 q_{2a})(\bar{q}_{2b}\gamma^\mu q_{1b}),\\\eta_m^{(A)(8)} &= \lambda_{ab}\lambda_{cd}\{(\bar{q}_{1a}\gamma_\mu q_{1b})(\bar{q}_{2c}\gamma^\mu\gamma_5 q_{2d})\\&\quad+(\bar{q}_{1a}\gamma_\mu\gamma_5 q_{1b})(\bar{q}_{2d}\gamma^\mu q_{2d}) - (\bar{q}_{1a}\gamma_\mu q_{2b})\\&\quad\times(\bar{q}_{2c}\gamma^\mu\gamma_5 q_{1d}) - (\bar{q}_{1a}\gamma_\mu\gamma_5 q_{2b})(\bar{q}_{2c}\gamma^\mu q_{1d})\},\end{aligned}$$

where the indices (1) and (8) represent the color singlet and octet. Now we get eight mesonic currents. Then we introduce the formula of the interchange of the color indices:

$$\begin{aligned}(q_{1a}q_{2b}\bar{q}_{3a}\bar{q}_{4b}) &= \tfrac{1}{3}(q_{1a}q_{2b}\bar{q}_{3b}\bar{q}_{4a})\\&\quad+\tfrac{1}{2}\lambda_{ab}\lambda_{cd}(q_{1a}q_{2c}\bar{q}_{3d}\bar{q}_{4b}),\\\lambda_{ab}\lambda_{cd}(q_{1a}q_{2c}\bar{q}_{3b}\bar{q}_{4d}) &= \tfrac{16}{9}(q_{1a}q_{2b}\bar{q}_{3b}\bar{q}_{4a})\\&\quad-\tfrac{1}{3}\lambda_{ab}\lambda_{cd}(q_{1a}q_{2c}\bar{q}_{3d}\bar{q}_{4b}).\end{aligned}\tag{A1}$$

Next, we perform the Fierz rearrangement in the Lorrentz indices with the formula

$$\begin{aligned}(\bar{a}b)(\bar{b}a) &= \tfrac{1}{4}(\bar{a}a)(\bar{b}b) + \tfrac{1}{4}(\bar{a}\gamma_5 a)(\bar{b}\gamma_5 b)\\&\quad+\tfrac{1}{4}(\bar{a}\gamma_\mu a)(\bar{b}\gamma^\mu b) - \tfrac{1}{4}(\bar{a}\gamma_5\gamma_\mu a)(\bar{b}\gamma_5\gamma^\mu b)\\&\quad+\tfrac{1}{8}(\bar{a}\sigma_{\mu\nu}a)(\bar{b}\sigma^{\mu\nu}b).\end{aligned}\tag{A2}$$

For example, we have

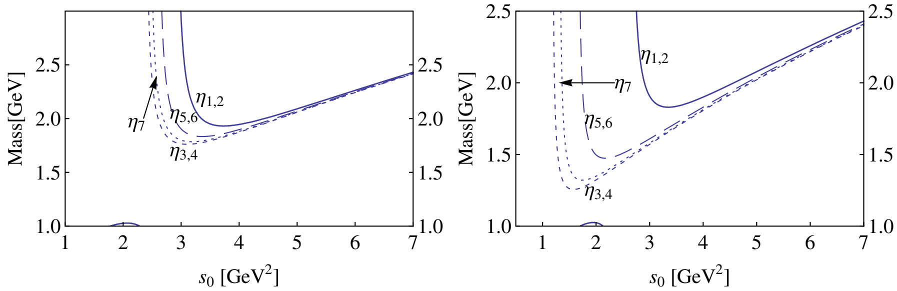

FIG. 13 (color online). The variation of M_X with s_0 and $n = 0$ from the finite energy sum rule. The left and right diagrams correspond to Ioffe's and SVZ's gluon condensate values, respectively.

$$\begin{aligned}(q_{1a}^T C q_{2b})(\bar{q}_{3a}\gamma_5 C\bar{q}_{4b}^T) &= -\tfrac{1}{4}(q_{1a}^T C\gamma_5 C\bar{q}_{4b}^T)(\bar{q}_{3a}q_{2b}) - \tfrac{1}{4}(q_{1a}^T C\gamma_\mu\gamma_5 C\bar{q}_{4b}^T)(\bar{q}_{3a}\gamma^\mu q_{2b}) - \tfrac{1}{8}(q_{1a}^T C\sigma_{\mu\nu}\gamma_5 C\bar{q}_{4b}^T)(\bar{q}_{3a}\sigma^{\mu\nu}q_{2b}) \\ &\quad + \tfrac{1}{4}(q_{1a}^T C\gamma_\mu\gamma_5\gamma_5 C\bar{q}_{4b}^T)(\bar{q}_{3a}\gamma^\mu\gamma_5 q_{2b}) - \tfrac{1}{4}(q_{1a}^T C\gamma_5\gamma_5 C\bar{q}_{4b}^T)(\bar{q}_{3a}\gamma_5 q_{2b}) \\ &= -\tfrac{1}{4}(\bar{q}_{4b}\gamma_5 q_{1a})(\bar{q}_{3a}q_{2b}) - \tfrac{1}{4}(\bar{q}_{4b}\gamma_\mu\gamma_5 q_{1a})(\bar{q}_{3a}\gamma^\mu q_{2b}) + \tfrac{1}{8}(\bar{q}_{4b}\sigma_{\mu\nu}\gamma_5 q_{1a})(\bar{q}_{3a}\sigma^{\mu\nu}q_{2b}) \\ &\quad - \tfrac{1}{4}(\bar{q}_{4b}\gamma_\mu q_{1a})(\bar{q}_{3a}\gamma^\mu\gamma_5 q_{2b}) - \tfrac{1}{4}(\bar{q}_{4b}q_{1a})(\bar{q}_{3a}\gamma_5 q_{2b}). \end{aligned} \tag{A3}$$

There are only four independent currents among those eight mesonic currents. Any four currents are independent and can be expressed by the other four

$$\eta_m^{(S)(8)} = \tfrac{4}{3}\eta_m^{(S)(1)}, \qquad \eta_{1m}^{(M)(8)} = -\tfrac{2}{3}\eta_{1m}^{(M)(1)} - \eta_{2m}^{(M)(1)},$$
$$\eta_{2m}^{(M)(8)} = -4\eta_{1m}^{(M)(1)} - \tfrac{2}{3}\eta_{2m}^{(M)(1)}, \qquad \eta_m^{(A)(8)} = -\tfrac{8}{3}\eta_m^{(A)(1)}.$$

We establish the relations between the diquark currents and the mesonic currents using the Fierz transformation. For instance, we can verify the relations

$$\eta_m^{(S)(1)} = -2\eta_d^S, \qquad \eta_{1m}^{(M)(1)} = \tfrac{1}{4}\eta_{1d}^M + \tfrac{1}{4}\eta_{2d}^M,$$
$$\eta_{2m}^{(M)(1)} = -\tfrac{1}{2}\eta_{1d}^M + \tfrac{1}{2}\eta_{2d}^M, \qquad \eta_m^{(A)(1)} = -2\eta_d^A.$$

APPENDIX B: FINITE ENERGY SUM RULE

Sometimes the finite energy sum rule is also employed in the numerical analysis. One first defines the nth moment using the spectral density

$$W(n, s_0) = \int_0^{s_0} \rho(s)s^n ds. \tag{B1}$$

With the quark-hadron duality, we have

$$W(n, s_0)|_{\rm Hadron} = W(n, s_0)|_{\rm OPE}. \tag{B2}$$

The mass of the ground state can be obtained as

$$M_X^2(n, s_0) = \frac{W(n+1, s_0)}{W(n, s_0)}. \tag{B3}$$

We have plotted the variation of M_X with s_0 for all the seven interpolating currents in Fig. 13. The left and right diagrams correspond to Ioffe's and SVZ's gluon condensate values, respectively. It seems that there exists a minimum of M_X for each current. However, a reasonable sum rule requires that the operator product expansion should converge well. In other words, we require that the two-gluon power correction be less than one third of the perturbative term and the trigluon power correction less than one third of two-gluon power correction in $W(0, s_0)$, which leads to the working window of this finite energy sum rule as:

\	s_0(SVZ)	s_0(Ioffe)
ρ_{1-2}	4.0	7.0
ρ_{3-4}	4.2	5.7
ρ_{5-6}	4.0	7.0
ρ_7	4.9	6.0

Clearly for each current the minimum of the mass curve lies outside of the working region in both of the figures and is not a real resonant signal. Starting from 4.0 GeV^2 each mass curve grows monotonically with s_0. Thus, there does not exist a resonant signal for every interpolating current.

[1] C. Amsler *et al.* (Particle Data Group), Phys. Lett. B **667**, 1 (2008).
[2] E. Klempt and A. Zaitsev, Phys. Rep. **454**, 1 (2007).
[3] G. S. Adams *et al.* (E862 Collaboration), Phys. Lett. B **657**, 27 (2007).
[4] A. Abele *et al.* (Crystal Barrel Collaboration), Phys. Lett.

B **446**, 349 (1999); **423**, 175 (1998).
[5] D.R. Thompson *et al.* (E852 Collaboration), Phys. Rev. Lett. **79**, 1630 (1997).
[6] H. X. Chen, A. Hosaka, and S. L. Zhu, Phys. Rev. D **78**, 054017 (2008); **78**, 117502 (2008).
[7] H. X. Chen, A. Hosaka, and S. L. Zhu, Phys. Rev. D **74**, 054001 (2006); **76**, 094025 (2007); Phys. Lett. B**650**, 369 (2007); H. X. Chen, X. Liu, A. Hosaka, and S. L. Zhu, Phys. Rev. D **78**, 034012 (2008).
[8] M. A. Shifman, A. I. Vainshtein, and V. I. Zakharov, Nucl. Phys. **B147**, 385 (1979).
[9] L. J. Reinders, H. Rubinstein, and S. Yazaki, Phys. Rep. **127**, 1 (1985).
[10] B. L. Ioffe, Prog. Part. Nucl. Phys. **56**, 232 (2006).