Light scalar mesons in QCD sum rules with inclusion of instantons

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The light scalar meson nonet above 1 GeV (i.e., the a_0, K_0^* and f_0) are studied within the framework of QCD sum rules. In conventional QCD sum rules, the calculated masses of this nonet are degenerate, and the mass of K_0^* is always larger than the a_0 in contradiction with the observed spectrum. After improving the correlation function by including instanton effects, the masses are well separated from each other. In particular, our results show glueball content plays an important role in the underlying structure of $f_0(1500)$. The sensitivity of the results on the instanton density is also discussed.

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I. INTRODUCTION

The SU(3) classification of strongly interacting particles originally proposed by Gell-Mann [1] and Zweig [2][1] has been a very successful paradigm in particle physics. It is observed from the hadronic spectrum that the results following from this naive quark model agree better with the heavy meson systems than the light ones. This is understandable because the heavy quarks inside a heavy meson are nonrelativistic, and hence we can deal with their kinematics in the framework of nonrelativistic quantum mechanics as a good approximation. However, for light mesons where the light components are relativistic, it is hard to say whether the nonrelativistic approximation is applicable. We can see that there is more complexity in light mesons than the heavy ones from the observed spectrum. The situation is even worse in combination with the proliferation of light scalars and their production in charmless B decays. In order to accommodate these light mesons in theories consistent with QCD, models beyond the naive quark model have been developed, including glueballs [4], multiquark states [5], and hybrid states [6]. One hopes that these models can supply some reasonable, or at least qualitative, explanation of the observed light mesons.

The underlying structure of mesons with mass near 1 GeV attract much attention. It has widely been suggested that the light scalars below or near 1 GeV [the isoscalars $f_0(600)$, $f_0(980)$, the isodoublet $K_0^*(800)$(or κ) and the isovector $a_0(980)$] form a SU(3) flavor nonet, while scalar mesons above 1 GeV [$f_0(1370)$, $a_0(1450)$, $K_0^*(1430)$, and $f_0(1500)/f_0(1710)$] form another nonet [3,7–9]. Refs. [5,10] suggest that the light scalar nonet above 1 GeV can be accommodated in the conventional $\bar{q}q$ model with some gluonic component, while the light scalars around 1 GeV are dominated by $\bar{q}q\bar{q}q$ states with some 0^+ $\bar{q}q$ and glueball states. But this interpretation is still far from deciphering the puzzle presented by the light scalars.

It is obvious that the starting point of all the models mentioned above concentrates on the kinematic aspect of the component inside the scalars, i.e., in order to reproduce the spectrum in theories consistent with QCD, the complexity of the light scalars is attributed to their constituents. Maybe one can refer to this as a kinematics-dependent approach. There is another viewpoint we can adopt. We should recognize that in the hadronic region perturbative QCD breaks down, and the nonperturbative aspects of QCD are dominant. It is well known the nonperturbative aspect of QCD is difficult to analyze. The nonzero quark condensate signals that the QCD vacuum is nontrivial and has a complex structure. In other words, the dynamics in QCD vacuum is very different from the trivial one. It is possible that the complexity of the light scalar mesons can be attributed to the enigmatic QCD vacuum, in which the particle treated as the excitation of the QCD vacuum from the viewpoint of quantum field theory. The nonzero value of QCD vacuum expectation values is one of the main ingredients of QCD sum rules [11–13], which deals with the low-energy nonperturbative aspects of QCD. In conventional QCD rules the physical quantities are expressed by a dominant perturbative part and corrections associated with vacuum expectation values of various operators. This method works well in many cases, but when we apply this method to the pions, it is difficult to obtain reasonable results. This difficulty was solved by introducing instanton contribution into the QCD sum rules [14]. Instantons—the nontrivial solution to the Yang-Mills field equation [15]—play an important role in solving the puzzle.[2] Recent work involving QCD sum rules with instanton effects include the electromagnetic pion form factor [17] and glueballs [18–21]. Furthermore, instanton effects within QCD the nonstrange sum rules for scalar currents have previously been shown to split the degeneracy between the a_0 and f_0 [22].[3]

[1]Following Ref. [3] we refer to this model as the naive quark model.

[2]For details on instantons in QCD, see the excellent review by Schäfer and Shuryak [16].

[3]We note that these works did not consider the structure of the entire nonet.

All these works pave a new way to resolving the controversy concerning the nature of the light scalars.

Comparing with the kinematics-dependent approach, we refer to the instanton effects as a dynamics-dependent approach, because here one attempts to solve the problem by further investigating low-energy QCD itself. Keeping these motivations in mind, in this paper we investigate the masses of the scalar nonet above 1 GeV [i.e, $f_0(1370)$, $a_0(1450)$, $K_0^*(1430)$, and $f_0(1500)/f_0(1710)$] from QCD sum rules based on scalar interpolating fields including the corresponding instanton contribution. Because there is no mixing between a_0, K_0^* meson and the glueball, these two members are ideally suited to investigate the role of instantons in QCD sum rules, and we will analyze them in the naive quark model. The situation is more complicated for the f_0 because it is widely accepted that there is mixing with the isoscalar 0^{++} glueball ground state [3] around 1500 MeV[4] Because of this mixing, a more consistent analysis should consider the mixing of quark and gluonic content in analyzing f_0 meson. So we will employ a mixed quark-glueball current to discuss f_0 meson if necessary. Specifically, we assign $f_0(1500)$ and $f_0(1710)$ to be a mixed current of quark and gluonic content, while $f_0(1370)$ is still assumed to be purely of quark content. As will be demonstrated below, the validity of these assignments is upheld by the results of the QCD sum-rule analysis. The instanton contributions to the sum rules are calculated using the semiclassical approximation with quark zero modes.

In Sec. II, we derive the QCD sum rules with scalar interpolating fields in the absence of instantons and note the shortcomings associated with the results of this analysis. In Sec. III, we present the sum rule including instanton contribution based on scalar current or its mixing with gluonic current, and the masses of the nonet are calculated. Section IV is devoted to our conclusions.

II. SUM RULES WITHOUT INSTANTONS

In this section we will discuss QCD sum rules without instantons and the results following it. The starting point is the following correlator defined in terms of scalar interpolating current:

$$\Pi(q^2) = i\int d^4x e^{iq\cdot x}\langle 0|j(x)j^\dagger(0)|0\rangle, \tag{1}$$

where $j(x)$ is a scalar composite operator defined as

$$j(x) = \bar{q}_1(x)q_2(x). \tag{2}$$

Compared with the definition in [14], we have suppressed the renormalization invariant factor $(\ln(\mu/\Lambda))^{-4/b}$, with μ as the normalization pint and $b = (11N_c - 2n_f)/3$. The correlator can be expressed in terms of operator product expansion, up to order-α_s perturbative correction and dimension six. The operator product expansion we get is [12,24]

$$\begin{aligned}\Pi^{\rm OPE}(q^2) = &-\frac{3}{8\pi^2}\left(1+\frac{11}{3}\frac{\alpha_s}{\pi}\right)q^2\ln\frac{-q^2}{\mu^2} + \frac{3}{4\pi^2}m_1m_2\ln\frac{-q^2}{\mu^2} - \frac{1}{8\pi}\frac{1}{q^2}\langle\alpha_s G^2\rangle - \frac{1}{q^2}\left(\frac{m_1}{2}+m_2\right)\langle\bar{q}_1q_1\rangle \\ &-\frac{1}{q^2}\left(\frac{m_2}{2}+m_1\right)\langle\bar{q}_2q_2\rangle - \frac{1}{2q^4}m_2\langle g_s\bar{q}_1\sigma Gq_1\rangle - \frac{1}{2q^4}m_1\langle g_s\bar{q}_2\sigma Gq_2\rangle - \frac{16\pi}{27}\frac{\alpha_s}{q^4}[\langle\bar{q}_1q_1\rangle^2 + \langle\bar{q}_2q_2\rangle^2] \\ &-\frac{48\alpha_s}{9}\frac{1}{q^4}\langle\bar{q}_1q_1\rangle\langle\bar{q}_2q_2\rangle.\end{aligned} \tag{3}$$

This is the theoretical side of the QCD sum rule from the quark-gluon dynamics point of view. On the other hand, the correlator can also be derived phenomenologically:

$$\Pi(q^2) = \frac{1}{\pi}\int_0^\infty ds\frac{{\rm Im}\Pi^{\rm ph}(s)}{s-q^2} + \text{subtraction constants.} \tag{4}$$

The quantity ${\rm Im}\Pi^{\rm ph}(s)$ obtained by inserting a complete set of quantum states $\Sigma|n\rangle\langle n|$ into Eq. (1), which reads

$${\rm Im}\Pi^{\rm ph}(q^2) = m_S^2f_S^2\pi\delta(q^2-m^2) + \left[\frac{3}{8\pi^2}\pi\left(1+\frac{11}{3}\frac{\alpha_s}{\pi}\right)q^2 - \frac{3}{4\pi^2}m_1m_2\pi\right]\theta(q^2-s_0), \tag{5}$$

where s_0 represents the onset of the QCD continuum. The decay constant in Eq. (5) is defined as

$$\langle S|\bar{q}_2q_1|0\rangle = m_Sf_S.$$

By equating both the theoretical and phenomenological sides, we obtain the total dispersion integral

$$\Pi^{\rm OPE}(q^2) = \frac{1}{\pi}\int_0^\infty ds\frac{{\rm Im}\Pi^{\rm ph}(s)}{s-q^2} + \text{substraction constants,} \tag{6}$$

After Borel transform and subtracting the perturbative continuum contributions, we obtain the following sum rule:

[4]Lattice gauge calculations predict a glueball mass of 1400 to 1800 MeV [23]

$$m_S^2 f_S^2 \exp\left[-\frac{m_S^2}{M^2}\right] = \int_0^{s_0} ds\left[\frac{3}{8\pi^2}\left(1+\frac{11}{3}\frac{\alpha_s}{\pi}\right)s - \frac{3}{4\pi^2}m_1 m_2\right]\exp\left[-\frac{s}{M^2}\right] + \frac{1}{8\pi}\langle\alpha_s G^2\rangle + \left(\frac{m_1}{2}+m_2\right)\langle\bar{q}_1 q_1\rangle + \left(m_1+\frac{m_2}{2}\right)\langle\bar{q}_2 q_2\rangle - \frac{1}{2M^2}m_2\langle g_s\bar{q}_1\sigma G q_1\rangle - \frac{1}{2M^2}m_1\langle g_s\bar{q}_2\sigma G q_2\rangle + \frac{16\pi}{27}\frac{\alpha_s}{M^2}[\langle\bar{q}_1 q_1\rangle^2 + \langle\bar{q}_2 q_2\rangle^2] - \frac{48}{9}\frac{\alpha_s}{M^2}\langle\bar{q}_1 q_1\rangle\langle\bar{q}_2 q_2\rangle. \tag{7}$$

The parameters in Eq. (7) are as follows: [25,26]:

$$\alpha_s = 0.517, \qquad \left\langle\frac{\alpha_s}{\pi}G^2\right\rangle = 0.012 \pm 0.006\ \text{GeV}^4, \qquad \langle\bar{u}u\rangle = \langle\bar{d}d\rangle = -(0.24\pm 0.1)^3\ \text{GeV}^3,$$
$$\langle\bar{s}s\rangle = (0.8\pm 0.2)\langle\bar{u}u\rangle, \qquad \frac{m_u+m_d}{2} = 5\ \text{MeV}, \qquad m_s = 120\ \text{MeV},$$
$$\langle g_s\bar{u}\sigma G u\rangle = \langle g_s\bar{d}\sigma G d\rangle = 0.8\ \text{GeV}^2\langle\bar{u}u\rangle, \qquad \langle g_s\bar{s}\sigma G s\rangle = 0.8\langle g_s\bar{u}\sigma G u\rangle.$$

All the values adopted here are given at the scale $\mu = 1$ GeV. By taking the logarithm of both sides of Eq. (7) and applying the differential operator $M^4\partial/\partial M^2$ to them, we derive the desired mass formula, which is free of the decay constant.

The task now is to find ranges of parameters M^2 and the continuum threshold s_0 such that the resulting mass does not depend too much on the value of these parameters. In addition, the continuum contribution that is the part of dispersive integral from s_0 to ∞ subtracted from both sides of Eq. (7) should not be too large (less than 30% of the total dispersive integral), and the contribution of the dimension-six operators is less than 10%. One more requirement is the value of the continuum threshold s_0 should not stray too much away from the next known resonance in that channel [27].

Before proceeding with our analysis we note that experimentally, except for the $f_0(1710)$, there is a small mass difference between other members, and the mass difference between a_0 and K_0^* is even smaller. Thus, it is reasonable to deal with them using the same threshold and Borel window; we think this criterion also holds true for other multiplets with small mass differences considered by other QCD practitioners. Of course one can analyze each member of a multiplet with a separate threshold and Borel window, but it is too artificial to be adopted because the sum rules are sensitive to the threshold. In considering this, we will analyze this nonet within same threshold and Borel window below.

If one uses the same threshold and Borel window, it is easy to see the mass spectrum of the nonet following from Eq. (7) will be similar to the naive quark model; there is a mass degeneracy broken by a tiny difference resulting from the SU(3) flavor symmetry breaking.[5] Even worse, the mass of K_0^* with underlying structure $s\bar{d}$ is always larger than the a_0 with underlying structure $\frac{1}{\sqrt{2}}(u\bar{u} - d\bar{d})$. For definiteness, when we select $s_0 = 4.1\ \text{GeV}^2$ and M^2 within the range [1.3, 1.6] GeV^2, the calculated mass of K_0^* and a_0 is shown in Fig. 1. One can see the mass of the K_0^* is above the a_0, which is inverted compared to the experimental results. All the results following from Eq. (7) are unsatisfactory.

To summarize Sec. II, we conclude that in the conventional QCD sum-rule analysis based on the naive quark model, one can not separate the this nonet with the same threshold and Borel window, the masses following from Eq. (7) are degenerate, and the results for the K_0^* and a_0, are in contradiction with experiment. This suggests that important effects have been neglected in Eq. (7).

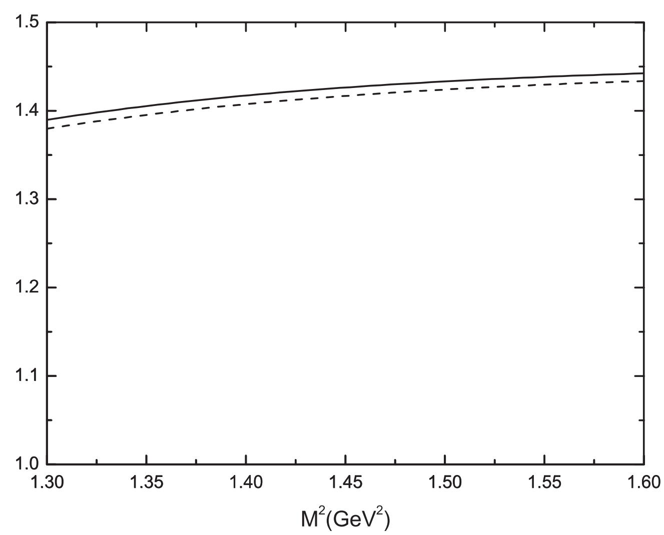

FIG. 1. Mass of K_0^* (solid line) and a_0 meson (dashed line) from sum-rule Eq. (7) based on naive quark model as a function of the Borel parameter M^2 without instanton.

[5] In addition, the degeneracy could also be broken when mass corrections proportional to $\alpha_s^2 m_{\mu(s)}^2$ are taken into account. These corrections are negligible compared with instanton effects, which we will consider in the next section.

III. SUM RULE WITH INCLUSION OF INSTANTON CONTRIBUTIONS

A. Basic formula

It has been known for a long time that the instanton plays an important role in nonperturbative QCD. The starting point on this subject is the solution of classical field equations in four-dimension Euclidean gauge-field theories given by Belavin *et al.* [15]. Subsequently, t' Hooft derived the instanton with topological quantum number $n = 1$ in Euclidean space [28]:

$$A^a_\mu(x) = \frac{2}{g}\eta_{a\mu\nu}\frac{(x-x_0)_\nu}{(x-x_0)^2+\rho^2},$$
$$G^a_{\mu\nu}(x) = -\frac{4}{g}\eta_{a\mu\nu}\frac{\rho^2}{[(x-x_0)^2+\rho^2]^2}, \tag{8}$$

where ρ is instanton size, $\eta_{a\mu\nu}$ is the t' Hooft η symbol, and x_0 is an any point in Euclidean space. The density $n(\rho)$ of instanton with size ρ in the vacuum can be parameterized as [14,29]

$$n(\rho) = n_c\delta(\rho-\rho_c), \tag{9}$$

with two parameters n_c and ρ_c, called the average instanton density and size. The original values

$$n_c = \frac{1}{2}\ \text{fm}^{-4} = 8\times 10^{-4}\ \text{GeV}^4,$$
$$\rho_c = \frac{1}{3}\ \text{fm} = \frac{1}{0.6}\ \text{GeV}^{-1}, \tag{10}$$

are adopted by some instanton practitioners [14,17,30,31]. Up to now, the value of the instanton size ρ_c is agreed upon both from phenomenological and other estimates, but it seems there is no consensus on the value of the instanton density n_c. In order to reproduce the values of vacuum quark and gluon condensates in lattice calculations, a value of the instanton density of order 1 fm^{-4} is needed [32], and recently the work of M. Cristoforetti *et al.* based on the interacting instanton liquid model has shown even a larger one is needed [33], i.e., $n_c = 3\ \text{fm}^{-4}$, to reproduce the nucleon mass and the low-energy constants in chiral perturbation theory. So it is instructive to investigate the sensitivity of our results on the choice of the value of instanton density. We hope the results presented in this paper will also shed some light on the choice of instanton density.

When we include the instanton contribution in the correlator (1), there is a new term [14]:

$$\Pi^{\bar{q}q,\text{inst}}(q^2) = \left|\int d^4x e^{iq\cdot x}\bar{q}_{10}(x)q_{20}(x)\right|^2\frac{n_c}{m_1^* m_2^*}, \tag{11}$$

where q_{10} and q_{20} is the t' Hooft quark zero mode, respectively, m_1^* and m_2^* is the effective mass, correspondingly. Similarly, applying dispersive relation to Eq. (11) we can rewrite Eq. (11) as

$$\Pi^{\bar{q}q,\text{inst}}(q^2) = \frac{1}{\pi}\int_0^\infty ds\frac{\text{Im}\Pi^{\bar{q}q,\text{inst}}(s)}{s-q^2}. \tag{12}$$

After the Borel transformation we get the desired form of the instanton contributions of the current with isospin I:

$$\Pi^{\bar{q}q,\text{inst}}(M^2) = (-1)^I\frac{n_c\rho^4M^6}{2m_1^*m_2^*}\exp\left[-\frac{M^2\rho^2}{2}\right]\times\left[K_0\left(\frac{M^2\rho^2}{2}\right)+K_1\left(\frac{M^2\rho^2}{2}\right)\right], \tag{13}$$

and the instanton continuum contribution is

$$\Pi^{\bar{q}q,\text{inst,cont}}(s_0, M^2) = (-1)^I\frac{\pi n_c\rho^2}{m_1^*m_2^*}\times\int_{s_0}^\infty ds s J_1(\rho\sqrt{s})Y_1(\rho\sqrt{s})e^{-s/M^2}, \tag{14}$$

where K_0, K_1 are the McDonald functions, and J_1, Y_1 are the Bessel functions.

When the smoke clears, we get the final result

$$m_S^2f_S^2\exp[-m_S^2/M^2] = \Pi^{\text{OPE}}(M^2) - \Pi^{\text{OPE,cont}}(s_0, M^2) + \Pi^{\bar{q}q,\text{inst}}(M^2) - \Pi^{\bar{q}q,\text{inst,cont}}(s_0, M^2). \tag{15}$$

This is the sum rule we obtained including instanton effects in the correlation function. Similarly, we can obtain the mass from Eq. (15) with the same manipulation as the previous section.

As an important parameter in sum-rule Eq. (15) it is necessary to discuss the value of the effective mass m^*'s. In the mean-field approximation [16]

$$m_u^* = m_d^* = \pi\rho(\tfrac{2}{3})^{1/2}(N/V)^{1/2} = 170\ \text{MeV}. \tag{16}$$

The value of N/V is $N/V = 1\ \text{fm}^{-4}$, phenomenologically. However, this value has been updated in Ref. [34], which suggested a significantly lower value

$$m_u^* = m_d^* = 86\ \text{MeV}, \tag{17}$$

which will be adopted in our numerical calculation. We also need the corresponding value for the strange quark m_s^*. Using

$$\langle\bar{q}q\rangle = -\frac{N/V}{m_q^*}, \tag{18}$$

combined with the relation between $\langle\bar{u}u\rangle$ and $\langle\bar{s}s\rangle$

$$\langle\bar{s}s\rangle = (0.8\pm 0.2)\langle\bar{u}u\rangle, \tag{19}$$

we obtain the effective mass of the strange

$$m_s^* = 114^{+28}_{-28}\ \text{MeV}. \tag{20}$$

Hereafter, we take the central value of m_s^*, i.e., $m_s^* = 114$ MeV in our numerical calculations.

B. Mass of K_0^*, a_0, f_0 meson and analysis

All the parameters needed in numerical calculation have now been fixed. Firstly, we take $j = \bar{d}s$, which corresponds to the K_0^* meson as our "sample," since in this case there is no mixing with glueball, allowing us to examine the effects of instanton without the complications presented by mixing.

Using standard QCD sum-rule methodologies, we obtain the threshold $s_0 = 3.5\ \mathrm{GeV}^2$, which is just below the next excited state $K^*(1950)$, and the Borel window is within the range $[2.2, 2.4]\ \mathrm{GeV}^2$. The calculated masses of K_0^* and a_0 from assigning the current $j = \frac{1}{\sqrt{2}}(\bar{u}u - \bar{d}d)$, with the same threshold and Borel window, are shown in Fig. 2 for different instanton densities. (For clarity we show the results separately for each different instanton density, otherwise, there are irrelevant crossovers entangling with the desired point.)

We can see from Fig. 2 that there is a crossover at some point for K_0^* and a_0 for different instanton densities. For example, for instanton density $n_c = 1/2\ \mathrm{fm}^{-4}$ the crossing point is $M^2 \approx 2.30$ MeV corresponding to a mass $m = 1445$ MeV, which is very close to the experimental value of K_0^* and a_0. Similarly, crossovers occur for other instanton densities involved in sum-rule Eq. (15). We refer to this crossover value of M^2 as the "reference point." There are four "reference points" we can get, each one corresponding to one choice of instanton density. These points will play an important role in our analysis because they represent the Borel scale where the mass hierarchy between the a_0 and K_0^* reverses. The more important aspect observed from Fig. 2 is that the calculated mass for K_0^* and a_0 present the right picture: there is a range where the mass of the a_0 is larger than the K_0^*, a result that cannot be obtained in the sum rule without the instanton contribu-

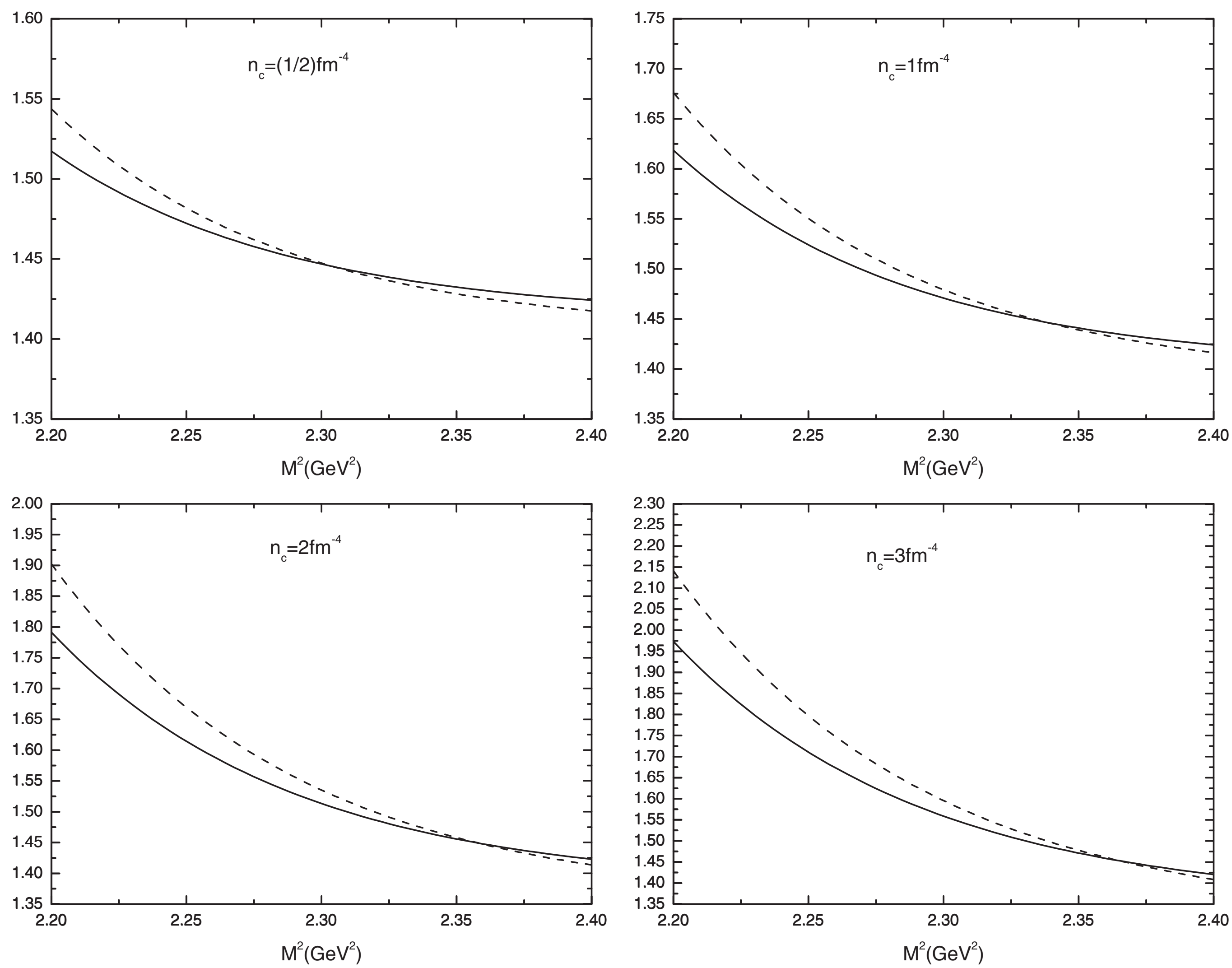

FIG. 2. Masses of K_0^* (solid line) and a_0 (dashed line) from sum-rule Eq. (15) as a function of the Borel parameter M^2 include instanton for different instanton density n_c. Mass of both K_0^* and a_0 become larger as n_c increase on the lower energy side, but the position of M_R^2 is nearly invariant.

tions shown in Fig. 1. In other words, the sum rule including instanton contributions can reproduce realistic results, which are in agreement with the light meson spectrum.

The first impression provided by Fig. 2 is that the results from sum-rule Eq. (15) are very sensitive to the instanton density since there is significant deviation from the experimental value on the low-energy side when we change the instanton density. This impression is false because the relevant quantities are the values at and around the "reference point" (hereafter referred to as M_R^2), not the entire Borel window. As one can see from Fig. 2 the calculated masses do not change much in a Borel parameter range around M_R^2. The results are shown in Table I.

Then, if we set

$$j = \frac{1}{\sqrt{2}}(\bar{u}u + \bar{d}d), \tag{21}$$

with isospin $I = 0$, we immediately get the mass from sum-rule Eq. (15), which is shown in Fig. 3. Similarly, in line with the analysis for K_0^* and a_0, we present the results in Table II. The results agree well with the experimental one, and $f_0(1370)$ is separated from other members. In fact, we can write the current with small $s\bar{s}$ content

$$j = \frac{1}{\sqrt{2 + \delta^2}}[(\bar{u}u + \bar{d}d) + \delta\bar{s}s].$$

We find there will be little effects with the parameter δ, no more than 10%, so we conclude $f_0(1370)$ dominantly by $(u\bar{u} + d\bar{d})/\sqrt{2}$.

Because there are more controversies for the f_0 meson than the two members discussed above, it is useful to mention the model beyond the pure quark viewpoint. We also notice the glueball can mix with scalar mesons nearby, so it is possible that there is mixing between the $f_0(1370)$ glueball, or in other words, $f_0(1370)$ is not in a pure quark state, and could have some glue content; an idea that was first introduced in Ref. [35] and then generalized in Ref. [36]. The work in Ref. [36] suggested the $f_0(1710)$ is dominated by $s\bar{s}$ content, while $f_0(1500)$ and $f_0(1370)$ share roughly equal amounts of glueball ($\simeq 40\%$) [$f_0(1500)$ and $f_0(1710)$ will be detailed in the coming subsection]. Here, we see when including the instanton effects, $f_0(1370)$ can be accommodated naturally in pure

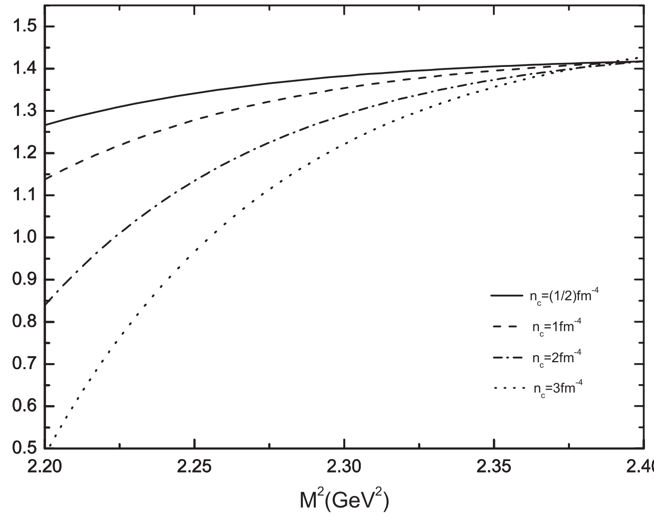

FIG. 3. Mass of f_0 with quark structure $(u\bar{u} + d\bar{d})/\sqrt{2}$ as a function of the Borel parameter M^2 includes instanton for different instanton density.

quark model. So we conclude that $f_0(1370)$ may be a pure quark state based when instanton effects are considered.

This separation is understandable from the viewpoint of instanton. Firstly, recalling the QCD sum-rule approach depends on the operator product expansion (OPE) of relevant normalized current, so if we assume an ideal SU(3) flavor symmetry, the degeneration is inevitable under the same threshold and Borel window in the conventional QCD sum rule since there is a common OPE for $K_0^*(1430)$, $a_0(1450)$, and $f_0(1370)$. While when instanton effects included, a new correction joins in, which is given by Eq. (13) after the continuum contribution is subtracted. This new contribution is very different from the ones characterized by QCD vacuum condensates of various operators constructed from quark and gluon fields with appropriate quantum numbers and a power suppress factor of $1/M^2$, where M^2 is the Borel parameter. These condensates are common quantities for various currents as we take the vacuum expectation value of their OPE as long as we select a fixed normalization point and scale in the QCD sum rule. As we can see from Eq. (13), the contribution from instanton is different for currents with different quark content and quantum numbers, which is reflected by a

TABLE I. Masses of K_0^* and a_0 (in units of MeV), assigning quark content $s\bar{d}$ and $\frac{1}{\sqrt{2}}(u\bar{u} - d\bar{d})$, respectively, from sum-rule Eq. (15) for different instanton density n_c. The third column $m_{M_R^2}$ denotes the value at M_R^2.

$n_c(\times\mathrm{fm}^{-4})$	M_R^2 (GeV^2)	$m_{M_R^2}$	$m_{K_0^*}$	m_{a_0}
$\frac{1}{2}$	2.3	1447	1472 ~ 1432	1483 ~ 1429
1	2.34	1447	1493 ~ 1425	1511 ~ 1416
2	2.36	1449	1514 ~ 1425	1539 ~ 1413
3	2.37	1452	1559 ~ 1423	1597 ~ 1413

TABLE II. Masses of f_0 (in units of MeV), assigning quark content $\frac{1}{\sqrt{2}}(u\bar{u} + d\bar{d})$, from sum-rule Eq. (15) for different instanton density n_c. The third column $m_{M_R^2}$ denotes the value at M_R^2.

$n_c(\times\mathrm{fm}^{-4})$	M_R^2 (GeV^2)	$m_{M_R^2}$	m_{f_0}
1/2	2.3	1384	1381 ~ 1418
1	2.34	1387	1356 ~ 1418
2	2.36	1387	1295 ~ 1418
3	2.37	1387	1218 ~ 1424

factor of $(-1)^I$ depending on the isospin of the current considered and the effective mass m_q^* (one should not confuse it with the effective mass in the constituent quark model) in the denominator. On the other hand, the instanton contribution is opposite for pseudoscalar and for scalar mesons: in the pseudoscalar case the instanton contribution is positive [14], and opposite in sign to the scalar meson. Taking pseudoscalar mesons as an example, when the isospin dependent factor $(-1)^I$ is taken into account, the instanton effects induce a positive contribution for η' with isospin $I = 0$, while the effect is negative for the π with isospin $I = 1$, and therefore $m_{\eta'} > m_\pi$. With all this in mind, it is easy to understand the degeneracy lifting of the scalar mesons. In the realistic world with broken flavor symmetry, the OPE part of K_0^* is larger than a_0 for the same threshold and Borel window as indicated in the previous section, while the instanton-induced part just reverses when we include the factor $(-1)^I$ because in the denominator $m_s^* > m_u^*$, so the entire effects will be that a_0 obtains a larger instanton-induced contribution than K_0^*. Thus, when Borel parameters vary, there will be a point where correlation functions are equivalent, and then separate. A similar analysis can be applied to f_0, but in this case one should notice the contribution from the instanton is opposite to a_0 because of the isospin dependent factor $(-1)^I$, i.e., negative effects to f_0 with quark content $(u\bar{u} + d\bar{d})/\sqrt{2}$ by instanton, therefore $m_{f_0} < m_{a_0}$. Since the instanton contribution is suppressed exponentially by the energy, the splitting in the nonet of the scalar mesons is quite smaller than that in the pseudoscalar case. Here, we see the degeneracy between $K_0^*(1430)$, $a_0(1450)$, and $f_0(1370)$ lift naturally when the instanton effects are included.

C. Mass of $f_0(1500)$ meson of underlying structure $s\bar{s}$ mixing with glueball

The $f_0(1500)$ may be the most controversial object in this nonet. In order to present a thorough investigation on this object, it is reasonable to write the current with isospin $I = 0$ in a general form:

$$j = \frac{1}{\sqrt{2c_1^2 + c_2^2}}[c_1(\bar{u}u + \bar{d}d) + c_2\bar{s}s], \tag{22}$$

which includes two adjustable parameters c_1 and c_2. In the case of $c_1 = 0$, Eq. (22) reduces to a pure $s\bar{s}$ state, which is free of instanton [14] so we can deal with the conventional QCD sum rule as in Sec. II. The calculated mass is shown in Fig. 4 from which we can read the mass with pure $s\bar{s}$ structure

$$m_{f_0} = 1413 \sim 1430 \text{ MeV},$$

which is too low compared with $f_0(1500)$. A reasonable choice in pure quark structure of $f_0(1500)$ is

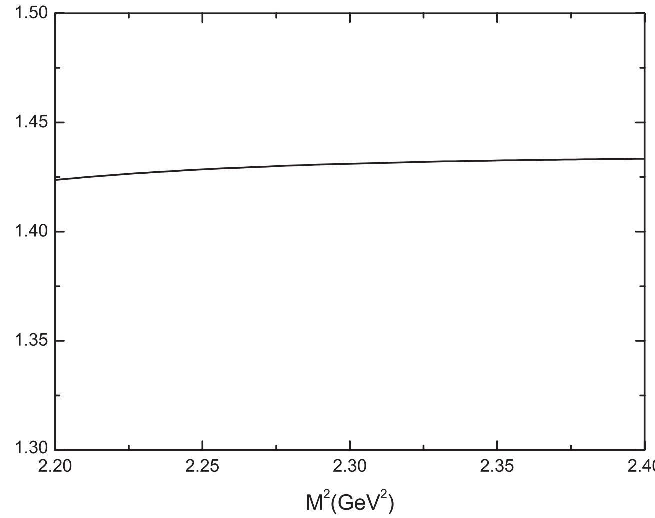

FIG. 4. Mass of f_0 with quark structure $s\bar{s}$ as a function of the Borel parameter M^2 without instanton.

$$j = \frac{1}{\sqrt{6}}[(\bar{u}u + \bar{d}d) - 2\bar{s}s], \tag{23}$$

while the masses following from this current are still unsatisfactory, only taking the largest instanton density can we obtain a mass close to $f_0(1500)$ within the selected Borel range around M_R^2, and all the values at M_R^2 are much lower (~ 1450 MeV) than the experimental one. Furthermore, based on Eq. (22), one can derive a sum rule depending on the two adjustable parameters c_1 and c_2. However, the results show a similar behavior as Eq. (23). In considering all of this, we conclude it is difficult to accommodate $f_0(1500)$ in pure quark picture. Next, we will resort to another solution of the problem, that is, mixing of quark and glueball currents.

As mentioned in Ref. [3], if we assume a $q\bar{q}$ structure, one concludes that $f_0(1500)$ is dominantly $s\bar{s}$, while this assignment cannot produce reasonable mass theoretically, as we can see from the previous paragraphs, it leads to contradictions experimentally [3]. There are some works [18] on this subject that take another extreme: they try to produce $f_0(1500)$ under the assumption of a pure glueball content. But what is the realistic structure of $f_0(1500)$ is still unknown.

There is another viewpoint that the light nonet above 1 GeV can be identified as conventional $\bar{q}q$ states with some possible gluonic content, that is, there is mixing of the pure glueball with the nearby two $N = n\bar{n}$ and $S = s\bar{s}$ scalar mesons as first introduced in Ref. [35], where $n\bar{n} = 1/2(u\bar{u} + d\bar{d})$. Based on this model, Ref. [36] obtained the results that $f_0(1710)$ is dominated by $s\bar{s}$ content, while there is roughly equal amounts of glue content in $f_0(1500)$. We have seen the key role of instanton in solving the puzzle on $K_0^*(1430)$ and $a_0(1450)$, $f_0(1370)$, and explore this possibility in the assumed mixing of scalar meson and pure glueball in $f_0(1500)$. With this motivation, we modified the current of $f_0(1500)$ as mixing of quark and gluonic current[6]:

$$j_{\rm mix} = A\bar{s}s + B\alpha_s G^a_{\mu\nu}G^{a\mu\nu}, \tag{24}$$

and in this case the decay constant is defined as

$$\langle S|j_{\rm mix}|0\rangle = m_S^2 f_S,$$

where A, B are both real, and one should notice that the parameter A has dimension one of mass, which insures the right dimension in the current. The parameters A and B accompany the Wilson coefficients of operators $\bar{s}s$ and $\alpha_s G^a_{\mu\nu}G^{a\mu\nu}$, respectively, and are therefore renormalization scale dependent. Here, we fix the renormalization scale of A and B so that they just are numbers in the following consideration. After this modification, there will be new contributions stemming from the glueball $\alpha_s G^a_{\mu\nu}G^{a\mu\nu}$ OPE, the glueball instanton, and the mixing instanton contribution, which will be presented below.

With the perturbative corrections and including nonperturbative terms up to dimension eight, the OPE of gluonic current is [37,38]

$$\begin{aligned}\Pi^{\rm GB,OPE}(q^2) = {}& q^4 \ln\frac{-q^2}{\mu^2}\left\{-2\left(\frac{\alpha_s}{\pi}\right)^2\left[1+\frac{659}{36}\frac{\alpha_s}{\pi}+247.48\left(\frac{\alpha_s}{\pi}\right)^2\right]+2\left(\frac{\alpha_s}{\pi}\right)^3\left(\frac{9}{4}+65.781\frac{\alpha_s}{\pi}\right)\ln\frac{-q^2}{\mu^2}\right.\\ &\left.-10.125\left(\frac{\alpha_s}{\pi}\right)^4\ln^2\frac{-q^2}{\mu^2}\right\}+\left[4\pi\frac{\alpha_s}{\pi}\left(1+\frac{175}{36}\frac{\alpha_s}{\pi}\right)-9\pi\left(\frac{\alpha_s}{\pi}\right)^2\ln\frac{-q^2}{\mu^2}\right]\langle\alpha_s G^2\rangle-8\pi^2\left(\frac{\alpha_s}{\pi}\right)^2\frac{1}{q^2}\langle\mathcal{O}_6\rangle\\ &+8\pi^2\frac{\alpha_s}{\pi}\frac{1}{q^4}\langle\mathcal{O}_8\rangle,\end{aligned} \tag{25}$$

where

$$\langle\mathcal{O}_6\rangle = \langle g_s f_{abc}G^a_{\mu\nu}G^b_{\nu\rho}G^c_{\rho\mu}\rangle = (0.27\ {\rm GeV}^2)\langle\alpha_s G^2\rangle,$$

and

$$\begin{aligned}\langle\mathcal{O}_8\rangle &= 14\langle(\alpha_s f_{abc}G^a_{\mu\nu}G^b_{\nu\rho})^2\rangle - \langle(\alpha_s f_{abc}G^a_{\mu\nu}G^b_{\rho\lambda})^2\rangle\\ &= \frac{9}{16}(\langle\alpha_s G^2\rangle)^2\end{aligned}$$

are the dimension-six and dimension-eight gluonic condensates, respectively. Because there is both quark and gluon current, we have to use the unsubtracted dispersive relation for the gluonic correlation function in order to be consistent with the entire correlation function. Applying the dispersion relation, and after subtracting the continuum contribution and taking the Borel transform, the glueball contribution is obtained [38,39]:

$$\begin{aligned}\Pi^{\rm GB,OPE}(s_0,M^2) = {}& \int_0^{s_0} ds s^2 e^{-s/M^2}\left\{2\left(\frac{\alpha_s}{\pi}\right)^2\left[1+\frac{659}{36}\frac{\alpha_s}{\pi}+247.48\left(\frac{\alpha_s}{\pi}\right)^2\right]-4\left(\frac{\alpha_s}{\pi}\right)^3\left(\frac{9}{4}+65.781\frac{\alpha_s}{\pi}\right)\ln\frac{s}{\mu^2}\right.\\ &\left.-10.125\left(\frac{\alpha_s}{\pi}\right)^4\left(\pi^2-3\ln^2\frac{s}{\mu^2}\right)\right\}+9\pi\left(\frac{\alpha_s}{\pi}\right)^2\langle\alpha_s G^2\rangle\int_0^{s_0} ds e^{-s/M^2}+8\pi^2\left(\frac{\alpha_s}{\pi}\right)^2\langle\mathcal{O}_6\rangle-8\pi^2\frac{\alpha_s}{\pi}\frac{1}{M^2}\langle\mathcal{O}_8\rangle.\end{aligned} \tag{26}$$

The contribution of glueball instanton after subtracting continuum is given by [30,38]

$$\begin{aligned}\Pi^{\rm GB,inst}(s_0,M^2) = {}& -2^4\pi^3 n\rho_c\int_0^{s_0} ds e^{-s/M^2}s^2 J_2(\rho\sqrt{s})\\ &\times Y_2(\rho\sqrt{s}),\end{aligned} \tag{27}$$

where J_2 and Y_2 are Bessel and Neumann functions, respectively.

We have independently verified the following instanton contribution to the mixed correlator $\bar{s}s\alpha_s G^a_{\mu\nu}G^{a\mu\nu}$ [39]:

$$\begin{aligned}\Pi^{\rm mix,inst}(s_0,M^2) = {}& \frac{2\pi^2 n\rho^3}{m_s^*}\int_0^{s_0} ds e^{-s/M^2}s^{3/2}[J_1(\rho\sqrt{s})\\ &\times Y_2(\rho\sqrt{s})+Y_1(\rho\sqrt{s})J_2(\rho\sqrt{s})].\end{aligned} \tag{28}$$

Now we have determined all the terms induced by the current given by Eq. (24). It is convenient to write the entire result in a compact form as follows:

$$m_S^4 f_S^2\exp\left[-\frac{m_S^2}{M^2}\right] = \sum_X \Pi^X(s_0,M^2), \tag{29}$$

where X denotes

$$X = \{\{\bar{s}s,{\rm OPE}\},\{{\rm GB},{\rm OPE}\},\{{\rm GB},{\rm inst}\},\{{\rm mix},{\rm inst}\}\},$$

and we have absorbed the two parameters A and B in the Π^X's for convenience. Taking the same algorithm as the previous section one can obtain immediately the mass

[6]The renormalization-group invariant gluonic current has been used because the subleading perturbative effects will be included in the correlation function.

corresponding to the current given in Eq. (24). Assigning $A = 0.9$ GeV and $B = 2$ in Eq. (24), corresponding to a large glueball content (since the energy scale is ~1 GeV), we obtain the results shown in Fig. 5 and Table III, which are very stable within the selected Borel window when changing the instanton density.

Here, it is useful to mention some previous studies of $f_0(1500)$ such as the sum rule plus direct instanton calculation [19], the calculation based on the interacting instanton liquid model [20], and the single-instanton approximation calculation [21]. All of these calculations use a purely gluonic current, though some quark effects are also included. We should say that our result does not contradict these works, in part because quark-hadron duality does not establish an accurate equation between QCD-based OPE and the hadronic spectrum. In the QCD sum rules, the prediction is sensitive to the window of the Borel parameter M and the threshold S_0. Here, the "reference point" M_R and s_0 are determined by the sum rules for $a_0(1450)$ and the $K_0(1430)$. In addition, the results of the gluonic correlation function analyses of Refs. [19–21] do not exclude the possibility that $f_0(1500)$ is a mixed state; any state with a gluonic component will be present in a correlation function of purely gluonic currents.

Finally, we turn to the last state $f_0(1710)$. Generally it is assumed this state dominated by the $s\bar{s}$ content, so we can write the current as

$$j = A'\bar{s}s + B'\alpha_s G^a_{\mu\nu}G^{a\mu\nu}, \quad (30)$$

subject to the following orthogonality condition:

$$\langle 0|j|f_0(1500)\rangle = 0.$$

We find that the combination of $A' = 1.22$ GeV, $B' = 0.8$, and instanton density $n_c = 3\ \text{fm}^{-4}$ will produce a mass $1715 \sim 1720$ MeV. But we do not take it is as a reasonable

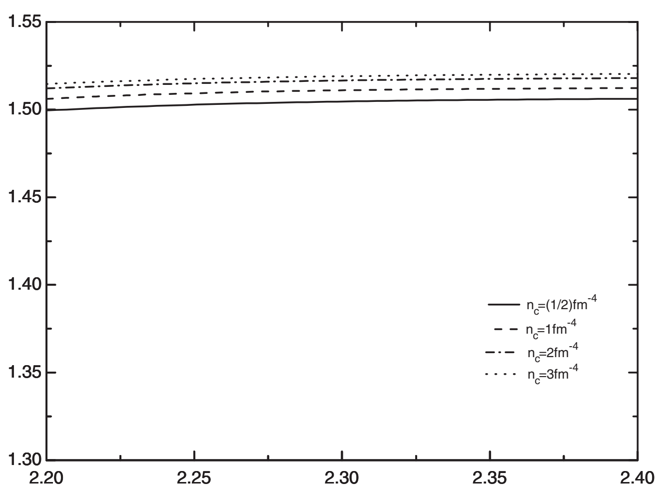

FIG. 5. Mass of f_0 with underlying structure 0.9 GeV $s\bar{s} + 2\alpha_s G^a_{\mu\nu}G^{a\mu\nu}$ as a function of the Borel parameter M^2 include glueball instanton and mixing instanton contributions for different instanton density.

TABLE III. Masses of f_0 (in units of MeV), assigning the underlying structure 0.9 GeV $s\bar{s} + 2\alpha_s G^a_{\mu\nu}G^{a\mu\nu}$, from sum-rule Eq. (29) for different instanton density n_c. The third column $m_{M_R^2}$ denotes the value at M_R^2.

$n_c(\times\text{fm}^{-4})$	M_R^2 (GeV^2)	$m_{M_R^2}$	m_{f_0}
1/2	2.3	1505	1505
1	2.34	1512	1512
2	2.36	1518	1518
3	2.37	1520	1520

interpretation for $f_0(1710)$, because other instanton densities with A, B fixed will produce a much lower mass within the entire Borel window, which cannot be accommodated by the QCD sum rule approach. Secondly, the combination of $A' = 1.22$ GeV, $B' = 0.8$ is not orthogonal to Eq. (24). So our method fails to predict the underlying structure of $f_0(1710)$. This can be understood intuitively because the threshold $s_0 = (1.9\ \text{GeV})^2$, which is adopted here only for the states with the masses around 1450 MeV, is too low to reproduce such a large stable mass.

IV. CONCLUSIONS

In this work we have studied the mass and decay constant of the light nonet a_0, K_0^*, and f_0 within the framework of the QCD sum rule with and without instanton contributions. Our main results are as follows:

(1) In the conventional QCD sum rule, the masses of this nonet are degenerate, the calculated mass of K_0^* is larger than the a_0 for the same threshold and the same Borel window.

(2) When we include instanton contributions in the sum rule, the masses of the nonet can be well separated, and the mass of K_0^* and a_0 agrees well with the observed results. The results suggest the underlying structure $K_0^*(1430)$ is $s\bar{d}$, $a_0(1450)$ is $\frac{1}{\sqrt{2}}(u\bar{u} - d\bar{d})$, and $f_0(1370)$ is $\frac{1}{\sqrt{2}}(u\bar{u} + d\bar{d})$. For $f_0(1500)$, our results suggest there is considerable glueball content in its underlying structure. Under the selected threshold and Borel window, the results are stable when there is a change in the instanton density.

(3) With a mixing current and the threshold and Borel window common to the multiplet, we cannot obtain the mass of $f_0(1710)$. One reason might be that the threshold suitable for $K^*(1430)$ is too low for $f_0(1710)$.

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