Mixed heavy quark hybrid mesons, decay puzzles, and RHIC

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(Received 16 March 2009; published 23 June 2009)

We estimate the energy of the lowest charmonium and upsilon states with hybrid admixtures using the method of QCD sum rules. Our results show that the $\Psi'(2S)$ and $\Upsilon(3S)$ states both have about a 50% admixture of hybrid and meson components. From this we find explanations of both the famous $\rho - \pi$ puzzle for charmonium and the unusual pattern of σ decays that have been found in Υ decays. Moreover, this picture can be used for predictions of heavy quark production with the octet model for RHIC.

DOI: 10.1103/PhysRevD.79.114026 PACS numbers: 14.40.Gx, 11.55.Hx, 12.38.Aw, 13.25.Gv

I. INTRODUCTION

There is great interest in studying states with active glue, such as hybrid mesons, a color singlet composed of a quark-antiquark in a color octet and a gluon, in order to better understand nonperturbative QCD. Recently we have used the method of QCD sum rules in an attempt to find the lowest hybrid charmonium state [1]. Our conclusion was that the physical states with active glue must be mixed states, with both charmonium and hybrid charmonium components. In the present work we use QCD sum rules for $J^{PC} = 1^{--}$ vector states to find the lowest mixed meson-hybrid meson states for both charmonium and upsilon systems.

In addition to the importance of finding states with active glue, we are motivated by several experimental considerations. First, the ratio of hadronic decays of the charmonium $\Psi'(2S)$ compared to the $J/\Psi(1S)$ state is more than an order of magnitude smaller than predicted by perturbative QCD (PQCD), the so-called $\rho - \pi$ puzzle, which was discussed at length in Ref. [1]. Second, the $\Upsilon(nS)$ states have an unusual pattern of decays into two pions, which also cannot be consistent with PQCD [2], which we call the Vogel $\Upsilon(\Delta n = 2)$ puzzle. Third, our theory of heavy quark states provides a basis for the color octet model predictions of RHIC heavy quark production [3,4].

In Sec. II we discuss the motivation for the present work on mixed heavy quark and hybrid mesons. In Sec. III we review the method of QCD sum rules, the work in Ref. [1] on hybrid charmonium, and apply the method of QCD sum rules for mixed meson-hybrid meson charmonium and upsilon states. In Sec. IV we discuss our solution to the $\rho - \pi$ puzzle, the Vogel $\Upsilon(\Delta n = 2)$ puzzle, and applications of our mixed hybrid states for the RHIC search for the QCD phase transition via heavy quark state production. In Sec. V we review our conclusions.

II. HEAVY QUARK PUZZLES AND RHIC EXPERIMENTS

First let us look at the lowest charmonium and upsilon (nS) states (Fig. 1):

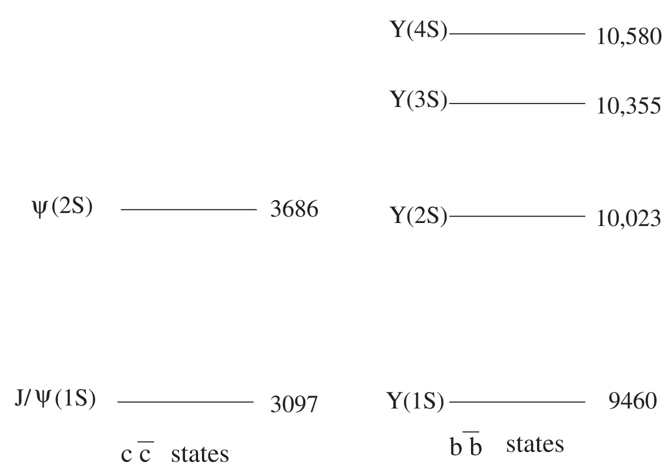

FIG. 1. Lowest nS charmonium and upsilon states.

Note that the separation in energy between the $\psi'(2S)$ and $J/\psi(1S)$ states is nearly the same as the separation energy of the $\Upsilon(2S)$ and the $\Upsilon(1S)$ states. This will be important for our studies of heavy quark hybrids, but turns out to be misleading.

A. The $\rho - \pi$ puzzle

The $\rho - \pi$ puzzle for $c\bar{c}$ 1^{--} states is based on the two diagrams for PQCD and electromagnetic decay of such states, shown in Fig. 2. By taking ratios the wave functions at the origin cancel, and this predicts the ratio of branching ratios for $c\bar{c}$ decays into hadrons (h)

$$R = \frac{B(\Psi'(2S) \to h)}{B(J/\Psi(1S) \to h)} = \frac{B(\Psi'(2S) \to e^+e^-)}{B(J/\Psi(1S) \to e^+e^-)} \simeq 0.12, \tag{1}$$

the famous 12% rule.

The $\rho - \pi$ puzzle: The $\Psi'(2S)$ to J/Ψ ratios for $\rho - \pi$ and other hadron decays are more than an order of magnitude smaller than predicted by Eq. (1). Many theorists have tried and failed to explain this puzzle. See Ref. [5] for a review. This and more recent attempts at a solution are also

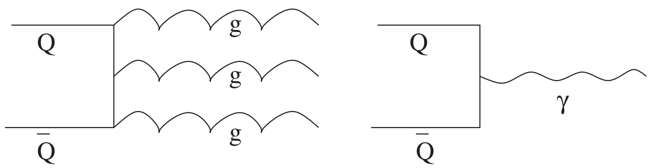

FIG. 2. Perturbative QCD and em diagrams for $Q\bar{Q}(1^{--})$ decays.

discussed in Ref. [1], and all agree that previous work has not produced a solution for this puzzle.

B. The sigma decays of $\Upsilon(nS)$ states puzzle

The puzzle of sigma decays of $b\bar{b}$ 1^{--} ($\Upsilon(nS)$) states is given by the following. The sigma is a low-energy broad two-pion scalar resonance. Experiments on $\Upsilon(nS)$ states find [2]

$\Upsilon(2S) \rightarrow \Upsilon(1S) + 2\pi$ has a large branching ratio, but no σ

$\Upsilon(3s) \rightarrow \Upsilon(1S) + 2\pi$ has a large branching ratio to σ

$\Delta n = 2$, emit σ

$\Delta n \neq 2$, no σ emitted.

This is the Vogel $\Delta n = 2$ puzzle, which cannot be understood using perturbative QCD, as expected for heavy bottomium states.

C. The octet model for RHIC and hybrids

The major goal of modern RHIC (Relativistic Heavy Ion Collision) experiments is to produce and study the quark-gluon plasma (QGP) which existed in the early universe before the QCD phase transition, about 10^{-5} seconds after the big bang. One important signal of this QGP is the production of heavy quark (charmonium and upsilon) states via $q\bar{q}$ interactions in the early universe. The most natural mechanism is $q\bar{q} \rightarrow g \rightarrow Q\bar{Q}$, in which an octet $q\bar{q}$ produces an octet $Q\bar{Q}$, which is just PQCD, followed by the nonperturbative (NPQCD) process in which the octet $Q\bar{Q}$ becomes a singlet $Q\bar{Q}$ with the emission of a gluon (or other color octet). This is depicted in Fig. 3:

The nonperturbative matrix elements for the transition from the color octet $\langle Q\bar{Q}(8)|$ state to a color singlet Ψ state, $\langle 0|\mathcal{O}_8^{\Psi}|0\rangle$ in the notation of Ref. [6], have been determined by fits to experiments using the octet model [6]. As we shall see, our determination of mixed heavy quark and heavy quark hybrid mesons will provide a mechanism for predicting these NPQCD matrix elements.

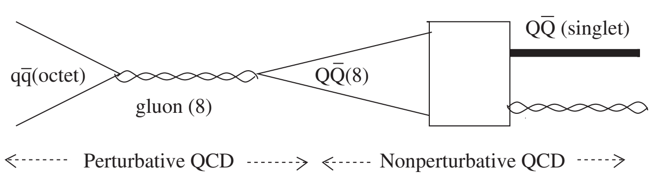

FIG. 3. Octet model for production of heavy quark mesons.

III. MIXED HEAVY QUARK HYBRID HEAVY QUARK 1^{--} STATES AND QCD SUM RULES

In this section we review the method of QCD sum rules, review our previous application of this method to attempt to find the lowest-energy hybrid charmonium 1^{--} state, and present our new application of the QCD sum rule method to find the lowest-energy mixed charmonium and upsilon states with hybrids.

A. Method of QCD sum rules

The starting point of the method of QCD sum rules [7] for finding the mass of a state A is the correlator,

$$\Pi^A(x) = \langle |T[J_A(x)J_A(0)]|\rangle, \tag{2}$$

with $|\rangle$ the vacuum state and the current $J_A(x)$ creating the states with quantum numbers A:

$$J_A(x)|\rangle = c_A|A\rangle + \sum_n c_n|n;A\rangle, \tag{3}$$

where $|A\rangle$ is the lowest-energy state with quantum numbers A, and the states $|n;A\rangle$ are higher energy states with the A quantum numbers, which we refer to as the continuum.

The QCD sum rule is obtained by evaluating Π^A in two ways. First, after a Fourier transform to momentum space, a dispersion relation gives the left-hand side (lhs) of the sum rule:

$$\Pi(q)^A_{\rm lhs} = \frac{\mathrm{Im}\,\Pi^A(M_A)}{\pi(M_A^2 - q^2)} + \int_{s_o}^{\infty} ds \frac{\mathrm{Im}\,\Pi^A(s)}{\pi(s - q^2)}, \tag{4}$$

where M_A is the mass of the state A (assuming zero width) and s_o is the start of the continuum—a parameter to be determined. The imaginary part of $\Pi^A(s)$, with the term for the state we are seeking shown as a pole (corresponding to a $\delta(s - M_A^2)$ term in $\mathrm{Im}\,\Pi$), and the higher-lying states produced by J_A shown as the continuum, is illustrated in Fig. 4:

Next $\Pi^A(q)$ is evaluated by an operator product expansion (OPE), giving the right-hand side (rhs) of the sum rule

$$\Pi(q)^A_{\rm rhs} = \sum_k c_k(q)\langle 0|\mathcal{O}_k|0\rangle, \tag{5}$$

where $c_k(q)$ are the Wilson coefficients and $\langle 0|\mathcal{O}_k|0\rangle$ are

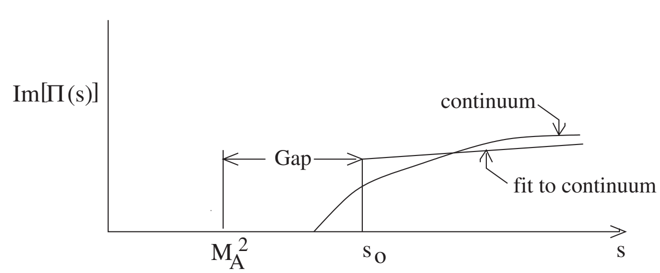

FIG. 4. QCD sum rule study of a state A with mass M_A (no width).

gauge invariant operators constructed from quark and gluon fields, with increasing k corresponding to increasing dimension of $\mathcal{O}_k$. It is important to note that the Wilson coefficients, $c_k(q)$ obey renormalization group equations [8].

After a Borel transform, $\mathcal{B}$, in which the q variable is replaced by the Borel mass, M_B,

$$\mathcal{B} = \lim_{q^2,n\to\infty} \frac{1}{(n-1)!}(q^2)^n\left(-\frac{d}{dq^2}\right)^n \Bigg|_{q^2/n=M_B^2}. \quad (6)$$

The final QCD sum rule, $\mathcal{B}\Pi_A(q)(\text{LHS}) = \mathcal{B}\Pi_A(q)\times$ (RHS), has the form

$$\frac{1}{\pi}e^{-M_A^2/M_B^2} + \mathcal{B}\int_{s_o}^{\infty}\frac{\text{Im}[\Pi_A(s)]}{\pi(s-q^2)}ds = \mathcal{B}\sum_k c_k^A(q)\langle 0|\mathcal{O}_k|0\rangle. \quad (7)$$

This sum rule and tricks are used to find M_A, which should vary little with M_B. A gap between M_A^2 and s_o is needed for accuracy. If the gap is too large, the solution is unphysical, which is important for our present work, as we discuss below.

B. Hybrid charmonium

Here we give a brief review of the calculation of the correlator and the results for the QCD sum rule for a pure hybrid charmonium 1^{--} state, which could possibly be the $\Psi'(2S)$. The current J_{HH} (which we called J_H in Ref. [1]) for a heavy quark hybrid meson with $J^{\text{PC}} = 1^{--}$ is

$$J_{HH}^{\mu} = \bar{\Psi}\Gamma_\nu G^{\mu\nu}\Psi, \quad (8)$$

where Ψ is the heavy quark field, $\Gamma_\nu = C\gamma_\nu$, γ_ν is the usual Dirac matrix, C is the charge conjugation operator, and the gluon color field is

$$G^{\mu\nu} = \sum_{a=1}^{8}\frac{\lambda_a}{2}G_a^{\mu\nu}, \quad (9)$$

with λ_a the SU(3) generator ($\text{Tr}[\lambda_a\lambda_b] = 2\delta_{ab}$). The correlator

$$\Pi_{HH}^{\mu\nu}(x) = \langle 0|T[J_{HH}^{\mu}(x)J_{HH}^{\nu}(0)]|0\rangle, \quad (10)$$

after a Fourier transform, was evaluated using the leading two operators in the operator product expansion, shown in Figs. 5 and 6. The scalar correlator Π^S is defined by $\Pi^{\mu\nu}(p) = (p_\mu p_\nu/p^2 - g^{\mu\nu})\Pi^V(p) + (p_\mu p_\nu/p^2)\Pi^S(p)$.

The leading term in the OPE for Π_{HH}^S, Π_{1HH}^S, corresponds to the diagram in Fig. 5. It is quite complicated, and the result is given in Ref. [1]. After the Borel transform, $\Pi_{1HH}^S(M_B)$ is given in the appendix, Eq. (A1).

The second order term, corresponding to the operator with a gluon condensate shown in Fig. 6, has a scalar part $\Pi_{2HH}^S(M_B)$ given in the appendix, Eq. (A2).

In Ref. [1] a solution to the QCD sum rule was found for a charmonium hybrid state at the mass of $\Psi'(2S)$, from which one would at first conclude that the $\Psi'(2S)$ is a pure hybrid 1^{--} meson. However, in order to satisfy the criterion that the solution is almost independent of the Borel mass a value of $s_o = 60.0\ \text{GeV}^2$ was needed. This would imply that the next excited state was 7 to 8 GeV, which is not consistent with the first state at only 3.66 GeV. Note that lattice QCD calculations found the first charmonium hybrid at about 1 GeV higher than our solution [9,10], which is also consistent with the $\Psi'(2S)$ not being a pure hybrid.

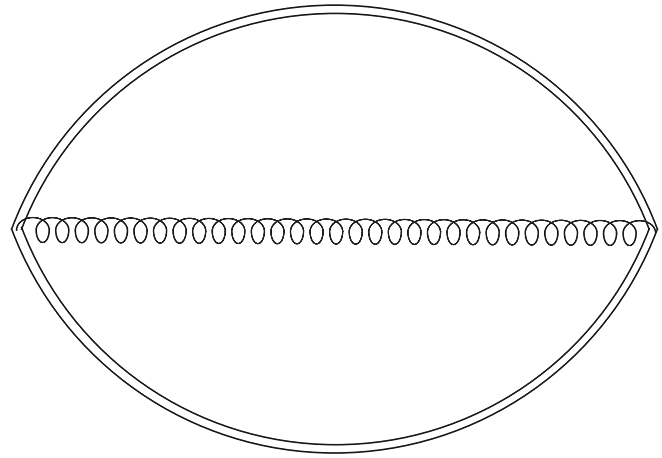

FIG. 5. Lowest-order term in sum rule.

This result, as well as the heavy quark puzzles and RHIC experiments discussed in Sec. II, were the main motivation for the present work, in which we seek a solution for a mixed charmonium and hybrid charmonium state.

C. Mixed charmonium-hybrid charmonium states

Recognizing that there is strong mixing between a heavy quark meson and a hybrid heavy quark meson with the same quantum numbers (as shown below), and the fact that our pure hybrid charmonium solution was not a physical state, we now attempt to find the lowest $J^{\text{PC}} = 1^{--}$ char-

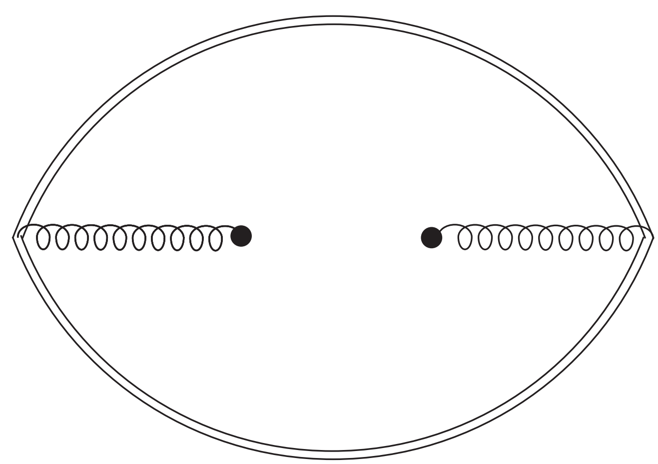

FIG. 6. Gluon condensate term in sum rule.

monium state with a sizable admixture of a charmonium meson and a hybrid charmonium meson. An appropriate mixed vector ($J^{PC} = 1^{--}$) charmonium, hybrid charmonium current to use in QCD sum rules is

$$J^\mu = bJ^\mu_H + \sqrt{1-b^2}J^\mu_{HH} \tag{11}$$

with

$$J^\mu_H = \bar{q}^a_c \gamma^\mu q^a_c, \tag{12}$$

where J^μ_H is the standard current for a 1^{--} charmonium state, and J^μ_{HH} is the heavy charmonium hybrid current given above in Eqs. (8) and (9).

Therefore the correlator for the mixed state:

$$\Pi^{\mu\nu}_{H\text{-}HH}(x) = \langle 0|T[J^\mu(x)J^\nu(0)]|0\rangle \tag{13}$$

is

$$\begin{aligned}\Pi^{\mu\nu}_{H\text{-}HH}(x) &= b^2\Pi^{\mu\nu}_H(x) + (1-b^2)\Pi^{\mu\nu}_{HH}(x)\\ &\quad + 2b\sqrt{1-b^2}\Pi^{\mu\nu}_{HHH}(x),\\ \Pi^{\mu\nu}_H(x) &= \langle 0|T[J^\mu_H(x)J^\nu_H(0)]|0\rangle,\\ \Pi^{\mu\nu}_{HH}(x) &= \langle 0|T[J^\mu_{HH}(x)J^\nu_{HH}(0)]|0\rangle,\\ \Pi^{\mu\nu}_{HHH}(x) &= \langle 0|T[J^\mu_H(x)J^\nu_{HH}(0)]|0\rangle. \end{aligned}\tag{14}$$

For heavy quarks the gluon condensate is proportional to the quark condensate, and the renormalization group equations for the Wilson coefficients of the operator product expansions of Π_H, Π_{HH}, and Π_{HHH} are similar for the terms considered here [8].

The heavy hybrid correlator, $\Pi^{\mu\nu}_{HH}(q^2)$ was presented in the previous section. The operator product expansion for the standard heavy quark correlator, $\Pi^{\mu\nu}_H(q^2)$ (rhs) is given by the diagrams shown in Fig. 7.

The leading term for the quark correlator in momentum space, with M_C the charm quark mass, is

$$\begin{aligned}\Pi^{\mu\nu}_{H1}(p) &= g_v^2\int\frac{d^4k}{(2\pi)^4}\,\mathrm{Tr}[S(k)\Gamma^\mu_5 S(p-k)\Gamma^{\nu T}_5],\\ S(k) &= \frac{\not{k}+M_C}{k^2-M_C^2}, \qquad \Gamma^\mu_5 = \gamma^\mu\gamma_5.\end{aligned}\tag{15}$$

Noting that the charmonium quark condensate is very small, and that the gluon condensate term and all higher-dimensional terms are also small, $\Pi^{\mu\nu}_{H1}(p)$ dominates the heavy quark correlator, $\Pi^{\mu\nu}_H(p)$. Carrying out the momentum integral in Eq. (15) and extracting the scalar correlator we find

$$\Pi^S_H(p) = i\frac{3g_v^2}{(4\pi)^2}\int_o^1 d\alpha\,\frac{6p^4 - 23p^2M_C^2}{(\alpha-\alpha^2)p^2 - M_C^2}. \tag{16}$$

Carrying out the Borel transform we find

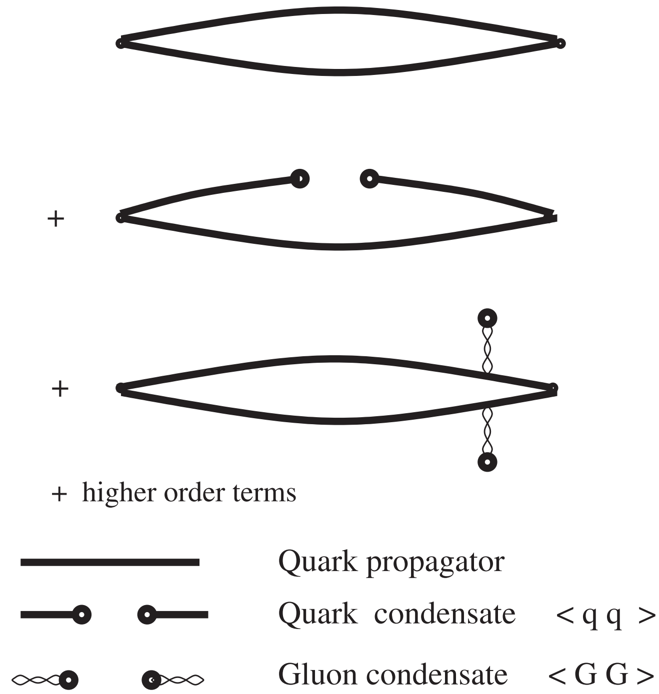

FIG. 7. Heavy quark meson diagrams.

$$\begin{aligned}\Pi^S_H(M_B) &= \frac{3}{2\pi^2}M_C^4\exp^{-2z}\Big[\frac{13}{4}K_o(2z)\\ &\quad + \frac{1}{2}K_1(2z) + 3K_2(2z)\Big],\end{aligned}\tag{17}$$

with $z = M_C^2/M_B^2$.

Finally, for the $\Pi^{\mu\nu}_{HHH}$ term, the dominant diagram is shown in Fig. 8, in which the gluon from the J_{HH} operator is coupled to a quark, leading to the J_H operator. This is essentially the perturbative plus nonperturbative *H-HH* matrix element without condensates.

Using the external field method, the leading term of $\Pi^{\mu\nu}_{HHH}$, corresponding to Fig. 8, is

$$\begin{aligned}\Pi^{\mu\nu}_{HHH1}(p) &= -i\frac{g_v^2}{4}\int\frac{d^4k}{(2\pi)^4}\,\mathrm{Tr}\Big[\frac{[\sigma_{\kappa\delta},(\not{k}+M_C)]_+}{(k^2-M_C^2)}\\ &\quad\times\frac{C\gamma_\lambda(\not{p}-\not{k}+M_C)(C\gamma_\mu)^T}{(p-k)^2-M_C^2}\Big]\\ &\quad\times \mathrm{Tr}[G^{\nu\lambda}(0)G^{\kappa\delta}(0)].\end{aligned}\tag{18}$$

After a Borel transform and extracting the scalar component of $\Pi^{\mu\nu}_{HHH1}$, one finds that

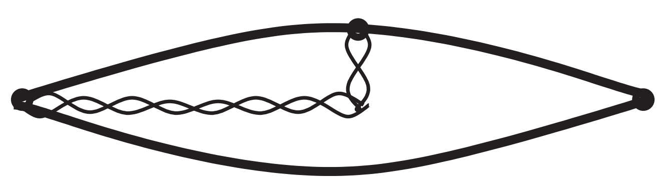

FIG. 8. Meson-hybrid meson lowest-order diagram.

$$\Pi^S_{HHH1}(M_B) \simeq \pi^2 \Pi^S_H(M_B). \quad (19)$$

Therefore the right-hand side of our scalar correlator is

$$\Pi^S_{H\text{-}HH}(M_B)_{\rm rhs} = (b^2 + 2\pi^2 b\sqrt{1. - b^2})\Pi^S_H(M_B) + (1. - b^2)\Pi^S_{HH}(M_B). \quad (20)$$

The left-hand side of the sum rule has the usual form (see Eq. (7))

$$\Pi^S_{H\text{-}HH}(M_B)_{\rm lhs} = Fe^{-M^2_{H\text{-}HH}/M_B^2} + e^{-s_o/M_B^2}(K0 + K1M_B^2 + K2M_B^4 + K3M_B^6), \quad (21)$$

with s_o, $K0$, $K1$, $K2$, $K3$ parameters used to fit the continuum. Note that the meson and hybrid meson states associated with the H and HH operators are normalized independently, and the operators have different dimensions. We renormalize by calculating $NHH = \int dM_B \Pi^S_H(M_B)/\int dM_B \Pi^S_{HH}(M_B)$. Henceforth, for $\Pi^S_{HH}(M_B)$ we use $NHH \times \Pi^S_{HH}(M_B)$.

As in Ref. [1], we obtain the expression for the mass of the mixed heavy meson-hybrid heavy meson by taking the ratio of the derivative of the sum rule with respect to $1/M_B^2$ to the sum rule, giving

$$M^2_{H\text{-}HH} = \{[s_o(K0 + K1M_B^2 + K2M_B^4 + K3M_B^6) + K1M_b^4 + 2K2M_B^6 + 3K3M_B^8]e^{-(s_o/M_B^2)} + \partial_{1/M_B^2}\Pi^S_{H\text{-}HH}\} \times \times\{(K0 + K1M_B^2 + K2M_B^4 + K3M_B^6)e^{-(s_o/M_B^2)} - \Pi^S_{H_HH}\}^{-1}. \quad (22)$$

A key parameter in our numerical fits is the value of b. The solution for $b = -0.7$ was most successful in fitting the criteria for finding the mixed hybrid state using QCD sum rules. The range of b for which a satisfactory solution is obtained is $b = -0.7 \pm 0.1$, with the result for $b = -.7$ shown in Fig. 9.

We find the mass of the lowest-energy mixed charmonium-hybrid charmonium to be about the energy of the $\Psi'(2S)$ state, 3.69 GeV, with $s_o = 20$ GeV^2, $b = -0.7 \Rightarrow$ 50%–50% charmonium-hybrid charmonium. It satisfies the criteria for about a 15% accuracy. The values of the other parameters are $K0 = -15.9$, $K1 = 0.224$, $K2 = -0.000\,15$, $K3 = 0.000\,09$. The only solutions satisfying the sum rule criteria are those with the value of b about $-.7 \pm .1$, so that we find the state to be about a 50%–50% meson-hybrid meson. As we shall see, this gives a solution to the $\rho - \pi$ puzzle.

D. Mixed upsilon-hybrid upsilon states

The calculation of the mixed upsilon-hybrid upsilon meson mass is the same as that of the mixed charmonium-hybrid charmonium mass using QCD sum rules, except the charm quark mass (which we took as $M_C^2 = 1.8$ GeV^2) is replaced by the bottom quark mass (which we take as $M_b^2 = 25.0$ GeV^2). In fact, the QCD sum rule method is more accurate for the calculation of upsilon states, since the bottom quark condensate is much smaller than the charm quark condensate, and the operator product expansion converges faster.

Since we found that the $\Psi'(2S)$ is a mixed charmonium state (see previous subsection) and as we noted earlier the separation in energy between the $\psi'(2S)$ and $J/\psi(1S)$ states is nearly the same as the separation energy of the $\Upsilon(2S)$ and the $\Upsilon(1S)$ states (see Fig. 1), we would expect that the $\Upsilon(2S)$ is a 50–50 mixture of upsilon and hybrid upsilon. This is not our result, as we shall now see.

From the QCD sum rule one obtains the expression given in Eq. (22), except the charm quark mass is replaced by the bottom quark mass in the expressions for the right-hand side of the correlator. The parameters s_o, $K0$, $K1$, $K2$, $K3$ are chosen to fit the continuum, and the mixing parameter b is also chosen to give a solution in which the mixed upsilon state mass is almost independent of the Borel mass. The result is shown in Fig. 10.

We find the energy of the lowest mixed upsilon meson and hybrid upsilon meson state to be at 10.4 GeV, approximately the energy of the $\Upsilon(3S)$ state (see Fig. 1.). The parameters are $s_o = 120$ GeV^2, $K0 = -50\,000$, $K1 = 70\,500$, $K2 = -605.$, $K3 = -0.5165$, and $b \simeq -0.7$ for a good solution. Thus we predict that the $\Upsilon(3S)$ state is a

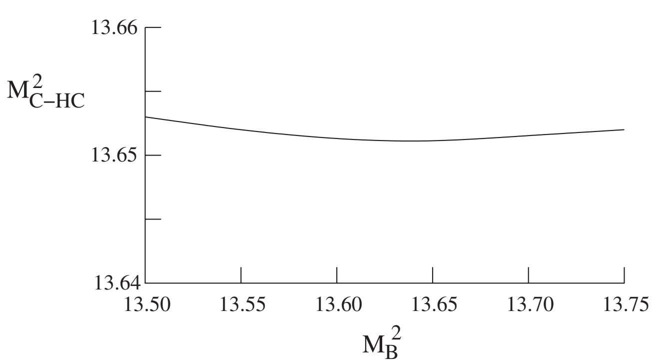

FIG. 9. Mixed charmonium-hybrid charmonium mass = 3.69 GeV.

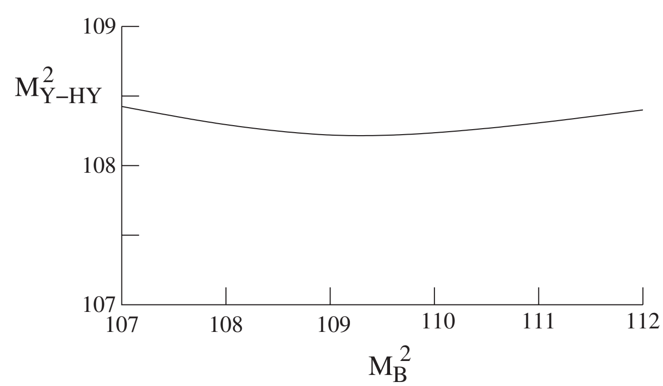

FIG. 10. Mixed upsilon-hybrid upsilon mass = 10.4 GeV.

50%–50% admixture of an upsilon meson and a hybrid upsilon meson. As we shall now see, from this we have obtained a solution to the Vogel $\Delta n = 2$ puzzle.

IV. MIXED MESON-HYBRID MESON, HEAVY QYARK DECAY PUZZLES, AND OCTET MODEL

In this section we show that the solutions for the mixed nature of the charmonium $\psi'(2S)$ and bottomonium $\Upsilon(3S)$ states provide explanations for the decay puzzles and a basis for the calculation of the nonperturbative matrix elements needed for the octet model used in RHIC calculations.

A. The $\rho - \pi$ puzzle

First note that the matrix element $\langle\pi\rho|O|\psi'(c\bar{c}, 2S)\rangle$ for $\rho - \pi$ decay of $|c\bar{c}(2S)\rangle$ is given by the PQCD diagram shown in Fig. 11.

Next, the hybrid decay matrix element $\langle\pi\rho|O'|\psi'(c\bar{c}g, 2S)\rangle$ is given by the PQCD diagram shown in Fig. 12.

As one can see from the diagrams, these matrix elements are almost equal in magnitude. Since we find that $|\Psi'(2S)\rangle \simeq -0.7|c\bar{c}(2S)\rangle + 0.7|c\bar{c}g(2S)\rangle$, so the charmonium and hybrid charmonium approximately cancel, we obtain for all 2 hadron decays, including $\rho + \pi$ decay,

$$R = \frac{B(\Psi'(2S) \to \rho + \pi)}{B(J/\Psi(1S) \to \rho + \pi)} \ll 0.12, \tag{23}$$

which is our proposed solution to the $\rho - \pi$ puzzle.

B. σ Decays of $\Upsilon(nS)$ states puzzle

The solution to the Vogel $\Delta n = 2$ puzzle is based on the application of the glueball/sigma model, based on the study of scalar mesons and scalar glueballs [11,12], which was motivated by the BES analysis of glueball decay [13], and our solution for the lowest mixed state to be the $\Upsilon(3S)$ state. The glueball/sigma model has been used for prediction of sigma production from glue created in hadron-hadron collisions [14] and the decay of hybrid baryons [15], which is closely related to the puzzle of sigma decays from upsilon states. The key is the glueball-meson coupling theorem [16]

$$\int dx T[J^G(x)J^m(0)] \simeq -\frac{32}{9}\langle\bar{q}q\rangle, \tag{24}$$

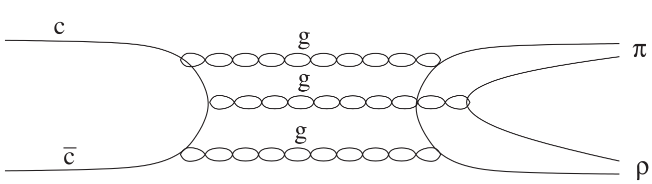

FIG. 11. PQCD diagram for charmonium decay into a π and a ρ.

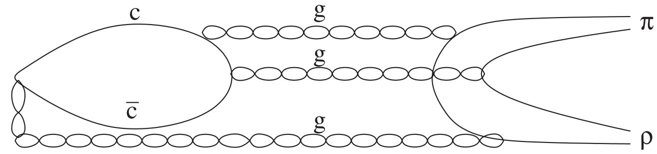

FIG. 12. PQCD diagram for hybrid charmonium decay into a π and a ρ.

where $\langle\bar{q}q\rangle \equiv$ quark condensate, which is depicted in Fig. 13.

From this one can calculate the matrix element for sigma decay from a hybrid meson, using the diagram shown in Fig. 14.

Just as scalar glueballs, such as the $f_o(1500)$, decay mainly into sigmas, the hybrid component of the $\Upsilon(3S)$ has a strong σ decay branch, while we predict that the $\Upsilon(2S)$ two-pion decay to the $\Upsilon(1S)$ would have a very small σ decay branch. Therefore, our solution for the $\Upsilon(3S)$ to be a mixed $b\bar{b}$-$b\bar{b}g$ provides a solution to the Vogel $\Delta n = 2$ puzzle. Since our states are not normalized we cannot calculate the numerical value of the cross section, a subject for future research.

C. Octet model for RHIC

As discussed in Sec. II, the octet model, depicted in Fig. 3, is the dominant mechanism for production of heavy quark states from a quark-gluon plasma produced via RHIC. Let us consider the collision of a nucleus A, e.g., lead or gold nucleus, with a similar nucleus. The differential cross section for the production of a charmonium state in a A-A collision in the color octet model is

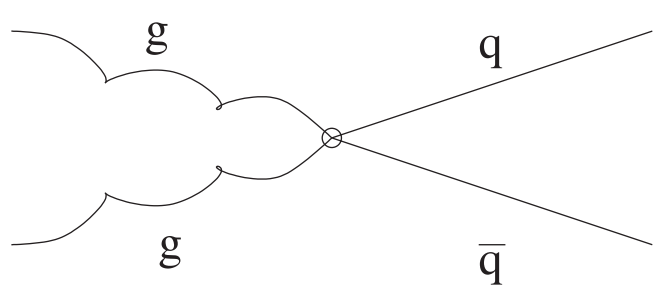

FIG. 13. Glueball-meson coupling.

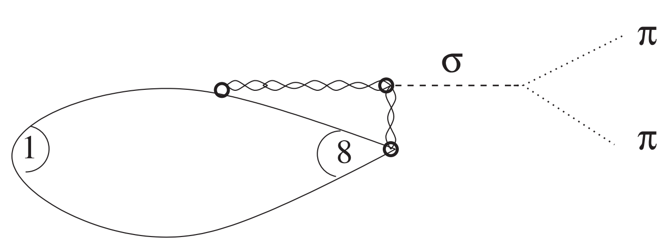

FIG. 14. Sigma decay of a hybrid meson.

$$\frac{d\sigma}{dp_T}[pp \to \psi(c\bar{c})] = \int f_{q/A} f_{q/A} \frac{d\sigma}{dt} \times [qq \to C\bar{C}(8) \to \psi(c\bar{c})],$$

$$\frac{d\sigma}{dt}[qq \to C\bar{C}(8) \to \psi(c\bar{c})] = [\text{perturbative QCD}] \times \langle 0|\mathcal{O}_8^{\psi_Q}|0\rangle, \tag{25}$$

where $f_{q/A}$ is the momentum fraction carried by a quark in the nucleus A, and $\langle 0|\mathcal{O}_8^{\psi_Q}|0\rangle$ is the NPQCD color octet matrix element. In previous applications of the model, the nonperturbative octet-singlet matrix element was taken from fits to other experiments [3,4]. We, however, can determine the NPQCD matrix elements for an octet quarkonium pair to emit a gluon (octet) and leave a physical singlet quarkonium state, which is given by $\Pi_{8,1}$. This is shown in Fig. 15.

We have seen how to evaluate this diagram, but the normalization of the states must be carried out to make a numerical estimate. We can, however, estimate ratios of matrix elements, to predict ratios of quarkonium production. As an example, from a table in Cho-Leibovich [6]

$$\langle 0|\mathcal{O}_8^{J/\psi}(1S)|0\rangle = 1.2 \times 10^{-2}\ \text{GeV}^3,$$

$$\langle 0|\mathcal{O}_8^{\psi'}(2S)|0\rangle = 0.73 \times 10^{-2}\ \text{GeV}^3, \quad \text{or}$$

$$R_C = \frac{\langle 0|\mathcal{O}_8^{\psi'}(2S)|0\rangle}{\langle 0|\mathcal{O}_8^{J/\psi}(1S)|0\rangle} \simeq 0.6. \tag{26}$$

Since in our model of the $\psi'(2S)$ state the $c\bar{c}(1)$ component should dominate, the parameter $b \simeq -.7$ gives a rough estimate of this ratio, in agreement with the Cho-Leibovich phenomenological fit.

V. CONCLUSIONS

In summary, we find that the $\psi'(2S)$ is approximately 50% charmonium and 50% hybrid charmonium; and the $\Upsilon(3S)$ is approximately 50% bottomium and 50% hybrid bottomium. This solves the $\rho - \pi$ problem for charmonium decays, and the Vogel $\Delta n = 2$ puzzle for sigma decays of upsilon states.

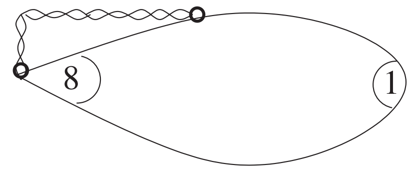

FIG. 15. $\Pi_{HHH} = \Pi_{8,1} \propto$ color octet-color singlet matrix element.

From the correlator corresponding to the mixed heavy meson and heavy hybrid meson current, the color octet-singlet matrix element can be obtained. This nonperturbative matrix element can be used for studies of the production of heavy quark states in RHIC experiments, using the octet model. It can also be used with the sigma/glueball model to predict the cross sections for sigma production from heavy quark state decays. Since the states used in the QCD sum rule method are not normalized, these numerical estimates cannot be made at the present time.

In the near future we plan to extend our calculation, so that numerical predictions of heavy quark decays and RHIC production of heavy quark states can be made. This will include possible tests of RHIC quarkonium production via sigma decays.

ACKNOWLEDGMENTS

This work was supported in part by the NSF/INT Grant No. 0529828. The author thanks Dr. Diana Parno, Dr. Seamus Riordan, Dr. Ming Liu, and Dr. Pat McGoughey; Prof. Pengnian Shen and Prof. Wei-xing Ma, and other IHEP, Beijing colleagues, and Prof. Roy Briere and Prof. Helmut Vogel for helpful discussions. We thank Prof. Y. Chen for discussions of lattice QCD in comparison to QCD sum rules for hybrid states.

APPENDIX

In Ref. [1] we found that $\Pi_{1HH}^S(M_B)$, the Borel transform of the scalar term of the main diagram for the HH correlator, shown in Figure 5, is

$$\begin{aligned}\Pi_{1HH}^S(M_B) = {}& -g_v^2 \frac{1}{2(4\pi)^2} M_Q^4 \int_0^\infty d\delta e^{-2(M_Q^2/M_B^2)(1+\delta)} \Bigg\{\Bigg[-310\frac{\delta}{1+\delta} + 638\delta - 656(1+\delta)\Bigg] K_3\Bigg(2\frac{M_Q^2}{M_B^2}(1+\delta)\Bigg) \\ & + \Bigg[-1892\frac{\delta}{1+\delta} + 606\delta - 1968(1+\delta) - 32\Bigg(4\delta - 3\frac{\delta^2}{1+\delta} + \frac{\delta^3}{3(1+\delta)^2}\Bigg)\Bigg] K_2\Bigg(2\frac{M_Q^2}{M_B^2}(1+\delta)\Bigg) \\ & + \Bigg[-4778\frac{\delta}{1+\delta} + 9442\delta - 4920(1+\delta) - 128\Bigg(4\delta - 3\frac{\delta^2}{1+\delta} + \frac{\delta^3}{3(1+\delta)^2}\Bigg)\Bigg] K_1\Bigg(2\frac{M_Q^2}{M_B^2}(1+\delta)\Bigg) \\ & + \Bigg[-1356\frac{\delta}{1+\delta} + 6284\delta - 3280(1+\delta) - 96\Bigg(4\delta - 3\frac{\delta^2}{1+\delta} + \frac{\delta^3}{3(1+\delta)^2}\Bigg)\Bigg] K_0\Bigg(2\frac{M_Q^2}{M_B^2}(1+\delta)\Bigg)\Bigg\} \\ & + \text{multiple integrals.}\end{aligned} \tag{A1}$$

The multiple integral terms in Eq. (A1) are small and are dropped. M_Q is the charm quark mass for the charmonium calculations and the bottom quark mass for the upsilon calculations. We take $M_C^2 = 1.8\ \mathrm{GeV}^2$ and $M_b^2 = 25.0\ \mathrm{GeV}^2$. The gluon condensate term is shown in Fig. 6. After the Borel transform the scalar part of this term [1], $\Pi^S_{2HH}(M_B)$, is

$$\Pi^S_{2HH}(M_B) = -ig_v^2 \frac{3}{2(4\pi)^2} M_Q^4 e^{-2(M_Q^2/M_B^2)}\bigg[11K_2\bigg(2\frac{M_Q^2}{M_B^2}\bigg) + \frac{14}{3}K_1\bigg(2\frac{M_Q^2}{M_B^2}\bigg) + 18K_0\bigg(2\frac{M_Q^2}{M_B^2}\bigg)\bigg]. \tag{A2}$$

The K_n are Bessel functions of an imaginary argument, related to Hankel functions by

$$K_n(x) = \frac{i\pi}{2} e^{-n(i\pi/2)} H_n^{(1)}(ix). \tag{A3}$$

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