PHYSICAL REVIEW D **79,** 114025 (2009)

Unstable rotational states of string models and width of a hadron

G. S. Sharov*

Tver State University, 170002, Sadovyj per. 35, Tver, Russia

(Received 6 April 2009; published 23 June 2009)

Rotational states (planar uniform rotations) of various string hadron models are tested for stability with respect to small disturbances. These models include an open or closed string carrying n massive points (quarks), and their rotational states result in a set of quasilinear Regge trajectories. It is shown that rotations of the linear string baryon model q-q-q and the similar states of the closed string are unstable, because spectra of small disturbances for these states contain complex frequencies, corresponding to exponentially growing modes of disturbances. Rotations of the linear model are unstable for any values of points' masses, but for the closed string we have the threshold effect. This instability is important for describing excited hadrons; in particular, it increases predictions for their width Γ. Predicted large values Γ for N, Δ and strange baryons in comparison with experimental data result in unacceptability of the linear string model q-q-q for describing these baryon states.

DOI: 10.1103/PhysRevD.79.114025 PACS numbers: 12.40.−y, 12.39.Mk

I. INTRODUCTION

The Nambu-Goto string (relativistic string) simulates strong interaction between quarks at large distances in various string models of mesons, baryons [1–12] [Figs. 1(a)–1(e)], and glueballs [13–16] [Figs. 1(f)–1(i)]. This string has linearly growing energy (energy density is equal to the string tension γ) and describes a nonperturbative contribution of the gluon field and QCD confinement mechanism.

Such a string with massive ends in Fig. 1(a) may be regarded as the meson string model [2]. String models of the baryon were suggested in the following four topologically different variants [3]: Fig. 1(b) the quark-diquark model q-qq [5] [on the classic level it coincides with the meson model in Fig. 1(a)], Fig. 1(c) the linear configuration q-q-q [6], Fig. 1(d) the "three-string" model or Y configuration [3,7], and Fig. 1(e) the "triangle" model or Δ configuration [8].

All cited string hadron models generate linear or quasilinear Regge trajectories in the limit of large energies for excited states of mesons and baryons [4,5,9,10]

$$J \simeq \alpha_0 + \alpha' E^2, \tag{1.1}$$

if we use rotational states of these systems (classical planar uniform rotations). Here J and E are the angular momentum and energy of a hadron state (or rotational state of a string model), respectively, and the slope $\alpha' \simeq 0.9$ GeV^{-2}. For the meson model in Fig. 1(a) and for the baryon models in Figs. 1(b) and 1(c) this slope and the string tension γ are connected by the Nambu relation [1]

$$\alpha' = \frac{1}{2\pi\gamma}. \tag{1.2}$$

For rotational states of the linear baryon configuration [Fig. 1(c)] the middle mass is at the rotational center. In papers [6,11] we have shown in numerical experiments that the mentioned states are unstable with respect to small disturbances.

The string baryon model Y [Fig. 1(d)] for its rotational states demonstrates the Regge asymptotics (1.1) with the slope [7] $\alpha' = 1/(3\pi\gamma)$. To obtain $\alpha' \simeq 0.9$ GeV^{-2} in trajectories (1.1) we are to assume that the effective string tension γ_Y in this model differs from γ in models in Figs. 1(a)–1(c) (the fundamental string tension) and equals $\gamma_Y = \frac{2}{3}\gamma$ [4,9]. Moreover, rotations of the Y string con-

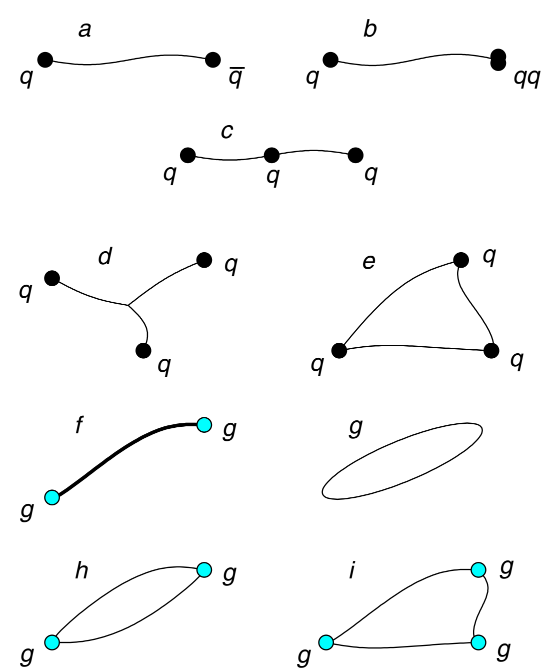

FIG. 1 (color online). String models of mesons, baryons and glueballs.

*german.sharov@mail.ru

 © 2009 The American Physical Society

figuration are also unstable with respect to small disturbances on the classic level [11,12].

The string baryon model triangle or Δ generates a set of rotational states with different topology [8]. The so-called triangle states were applied for describing excited baryon states on the Regge trajectories [4,9], but in this case (like for the model Y) we are to take another effective string tension $\gamma_\Delta = \frac{3}{8}\gamma$.

Different string models shown in Figs. 1(f)–1(i) were used for describing glueballs (bound states of gluons) and other exotic hadrons [13–16] predicted in QCD. String models of glueballs include the open string with enhanced tension $\gamma_{\rm adj} = \frac{9}{4}\gamma$ (the adjoint string) and two constituent gluons at the end points [14,17] in Fig. 1(f); the closed string without masses [Fig. 1(g)] [13,15,18] and the closed string carrying massive points [Figs. 1(h) and 1(i)] [16].

The problem of stability for rotational states with respect to small disturbances is very important for choosing the most adequate string model for baryons, glueballs and other exotic hadrons [4,11,12,16]. Note that instability of classical rotations for some string configuration does not mean that the considered string model must be totally prohibited. All excited hadron states (objects of modeling) are resonances; they are unstable with respect to strong decays. So they have rather large width Γ. On the level of string models these decays are described as string breaking with probability, proportional to the string length ℓ [19,20]. The corresponding width $\Gamma = \Gamma_{\rm br} \sim \ell$.

If classical rotations of a string configuration are unstable, this instability gives the additional contribution $\Gamma_{\rm inst}$ to width Γ. This effect is one of the manifestations of rotational instability. It can restrict applicability of some string models, if the total width Γ predicted by this model (below we suppose $\Gamma = \Gamma_{\rm br} + \Gamma_{\rm inst}$) essentially exceeds experimental data.

The stability problem for rotational states is solved for the string with massive ends [Figs. 1(a) and 1(b)]. Analytical investigation of small disturbances demonstrated that rotational states of this system are stable, and there is the spectrum of quasirotational states in the linear vicinity of these stable rotations [11,21].

For string baryon models q-q-q, Y and Δ evolution of small disturbances of rotational states was investigated in numerical experiments [11,12]. These calculations demonstrated instability of rotations for the linear model q-q-q and for the Y configuration. However, some aspects of this instability are not studied yet. In particular, we are to estimate analytically increments of instability for all models and to investigate its influence on properties of excited hadrons.

In this paper dynamics of a string with massive points is described in Sec. II. In Sec. III small disturbances of rotational states for the linear model [Fig. 1(c)] are studied analytically and the stability problem for these states is solved. In Sec. IV the similar problem is solved for central rotational states (with a massive point at the rotational center) of the closed string carrying n pointlike masses [Fig. 1(e) and 1(h) or 1(i)]. In Sec. V we study how rotational instability enlarges width of excited hadrons on Regge trajectories.

II. DYNAMICS OF A STRING WITH MASSIVE POINTS

Dynamics of an open or closed string carrying n pointlike masses $m_1, m_2, \dots m_n$ is determined by the action [4,8,16]

$$A = -\gamma \int_D \sqrt{-g}\, d\tau d\sigma - \sum_{j=1}^{n} m_j \int \sqrt{\dot{x}_j^2(\tau)}\, d\tau. \qquad (2.1)$$

Here γ is the string tension, g is the determinant of the induced metric $g_{ab} = \eta_{\mu\nu}\partial_a X^\mu \partial_b X^\nu$ on the string world surface $X^\mu(\tau, \sigma)$ embedded in Minkowski space $R^{1,3}$, $\eta_{\mu\nu} = \mathrm{diag}(1, -1. -1, -1)$, and the speed of light $c = 1$.

A world surface of the closed string mapping into $R^{1,3}$ from the domain

$$D = \{(\tau, \sigma)\colon \tau \in R, \sigma_0(\tau) < \sigma < \sigma_n(\tau)\}$$

is divided into n world sheets by the world lines of massive points

$$x_j^\mu(\tau) = X^\mu(\tau, \sigma_j(\tau)), \qquad j = 0, 1, \dots, n.$$

Two of these functions $x_0(\tau)$ and $x_n(\tau)$ describe the same trajectory of the nth massive point, and their equality forms the closure condition

$$X^\mu(\tau, \sigma_0(\tau)) = X^\mu(\tau^*, \sigma_n(\tau^*)) \qquad (2.2)$$

on the tubelike world surface [8,22]. These equations may contain two different parameters τ and τ^*, connected via the relation $\tau^* = \tau^*(\tau)$. This relation should be included in the closure condition (2.2).

For the string baryon model q-q-q (an open string with $n = 3$ masses) the domain D in Eq. (2.1) has the form $\sigma_1(\tau) < \sigma < \sigma_3(\tau)$. This domain and the world surface are divided into two sheets by the line $\sigma = \sigma_2(\tau)$. Naturally, there is no closure condition in this model.

Equations of motion for both open and closed strings with massive points result from the action (2.1) and its variation. If we use invariance of the action (2.1) with respect to nondegenerate reparametrizations $\tau = \tau(\tilde{\tau}, \tilde{\sigma})$, $\sigma = \sigma(\tilde{\tau}, \tilde{\sigma})$ and choose the coordinates τ, σ satisfying the orthonormality conditions on the world surface

$$(\partial_\tau X \pm \partial_\sigma X)^2 = 0, \qquad (2.3)$$

the equations of motion are reduced to the simplest form [4,8]. They include the string motion equation

$$\frac{\partial^2 X^\mu}{\partial \tau^2} - \frac{\partial^2 X^\mu}{\partial \sigma^2} = 0, \qquad (2.4)$$

and equations for two types of massive points: for end points of the model q-q-q

$$m_1 \frac{d}{d\tau}\frac{\dot{x}_1^\mu(\tau)}{\sqrt{\dot{x}_1^2(\tau)}} - \gamma[X'^\mu + \dot{\sigma}_1(\tau)\dot{X}^\mu]|_{\sigma=\sigma_1} = 0, \quad (2.5)$$

$$m_3 \frac{d}{d\tau}\frac{\dot{x}_3^\mu(\tau)}{\sqrt{\dot{x}_3^2(\tau)}} + \gamma[X'^\mu + \dot{\sigma}_3(\tau)\dot{X}^\mu]|_{\sigma=\sigma_3} = 0; \quad (2.6)$$

and for the middle point in the mentioned model or points on a closed string

$$m_j \frac{d}{d\tau}\frac{\dot{x}_j^\mu(\tau)}{\sqrt{\dot{x}_j^2(\tau)}} + \gamma[X'^\mu + \dot{\sigma}_j(\tau)\dot{X}^\mu]|_{\sigma=\sigma_j-0} - \gamma[X'^\mu + \dot{\sigma}_j(\tau)\dot{X}^\mu]|_{\sigma=\sigma_j+0} = 0, \quad (2.7)$$

$$m_n \frac{d}{d\tau}\frac{\dot{x}_0^\mu(\tau)}{\sqrt{\dot{x}_0^2(\tau)}} + \gamma[X'^\mu(\tau^*, \sigma_n) - X'^\mu(\tau, 0)] = 0. \quad (2.8)$$

Here $\dot{X}^\mu \equiv \partial_\tau X^\mu$, $X'^\mu \equiv \partial_\sigma X^\mu$, and the scalar product $(\xi, \zeta) = \eta_{\mu\nu}\xi^\mu\zeta^\nu$.

In Eq. (2.8) for nth massive point we fix

$$\sigma_0(\tau) = 0, \qquad \sigma_n(\tau) = 2\pi \quad (2.9)$$

without loss of generality with the help of substitutions $\tau \pm \sigma = f_\pm(\tilde{\tau} \pm \tilde{\sigma})$, keeping conditions (2.3) (conformal flatness of the induced metric g_{ab}) [8,22].

For the open string model q-q-q we can fix the similar conditions at the ends [4,6] in Eqs. (2.5) and (2.6):

$$\sigma_1(\tau) = 0, \qquad \sigma_3(\tau) = \pi. \quad (2.10)$$

Equations (2.2), (2.3), (2.4), (2.7), and (2.8) with the continuity condition for X^μ at $\sigma = \sigma_j$ describe all motions of the closed string with n pointlike masses. For the string baryon model q-q-q this system includes Eqs. (2.3), (2.4), (2.5), (2.6), and (2.7) for $j = 2$.

III. ROTATIONAL STATES AND THEIR STABILITY FOR LINEAR MODEL

Rotational states of the linear string model q-q-q are planar uniform rotations of the rectilinear string segment with the middle quark at the rotational center. These rotations may be described by the following exact solution of Eqs. (2.3), (2.4), (2.5), (2.6), and (2.7) [11]:

$$X^\mu(\tau, \sigma) \equiv \underline{X}^\mu = \Omega^{-1}[\omega\tau e_0^\mu + \cos(\omega\sigma + \phi_1)\cdot e^\mu(\tau)]. \quad (3.1)$$

Here $\sigma \in [0, \pi]$, Ω is the angular velocity, e_0, e_1, e_2, e_3 is the orthonormal tetrad in Minkowski space $R^{1,3}$, and

$$e^\mu(\tau) = e_1^\mu \cos\omega\tau + e_2^\mu \sin\omega\tau \quad (3.2)$$

is the unit spacelike rotating vector directed along the string. Values ω (dimensionless frequency) and ϕ_1 are connected with the constant speeds v_j of the ends

$$v_1 = \cos\phi_1, \quad v_3 = -\cos(\pi\omega + \phi_1), \quad \frac{m_j\Omega}{\gamma} = \frac{1 - v_j^2}{v_j}, \quad (3.3)$$

where $j = 1, 3$. The central massive point of the q-q-q system is at rest (in the corresponding frame of reference) at the rotational center. Its inner coordinate is

$$\sigma_2(\tau) = \underline{\sigma_2} = \frac{\pi - 2\phi_1}{2\omega} = \text{const.} \quad (3.4)$$

Rotational states (3.1) of the model q-q-q was tested for stability in Ref. [11] in numerical experiments. They demonstrated instability of rotations (3.1). Here we prove this result analytically, generalizing the approach applied for the string with massive ends in Refs. [11,21].

Let us consider a slightly disturbed motion of the system q-q-q in the linear vicinity of the rotational state (3.1). This disturbed motion is described by the general solution of Eq. (2.4)

$$X^\mu(\tau, \sigma) = \tfrac{1}{2}[\Psi_{j+}^\mu(\tau + \sigma) + \Psi_{j-}^\mu(\tau - \sigma)]. \quad (3.5)$$

Here $j = 1$ for $\sigma \in [0, \sigma_2]$ and $j = 2$ for $\sigma \in [\sigma_2, \pi]$, and functions $\Psi_{j\pm}^\mu(\tau)$ have isotropic derivatives

$$\dot{\Psi}_{j+}^2 = \dot{\Psi}_{j-}^2 = 0 \quad (3.6)$$

as a consequence of the orthonormality conditions (2.3). The functions $\Psi_{j\pm}^\mu$ are smooth, and the world surface (3.5) (smooth if $\sigma \neq \sigma_2$) is continuous at the line $\sigma = \sigma_2(\tau)$. This condition in terms Eq. (3.5) takes the form

$$\Psi_{1+}^\mu(+_2) + \Psi_{1-}^\mu(-_2) = \Psi_{2+}^\mu(+_2) + \Psi_{2-}^\mu(-_2), \quad (3.7)$$

where $(\pm_2) \equiv (\tau \pm \sigma_2(\tau))$.

We use underlined symbols for describing the particular exact solution (3.1) for the rotational states. For example, we denote

$$\underline{\Psi}_{1\pm}^\mu(\tau) = \underline{\Psi}_{2\pm}^\mu(\tau) = \Omega^{-1}[e_0^\mu\omega\tau + e^\mu(\tau \pm \phi_1/\omega)] \quad (3.8)$$

the functions in Eq. (3.5) corresponding to the states (3.1): $\underline{X}^\mu = \frac{1}{2}[\underline{\Psi}_{j+}^\mu(\tau + \sigma) + \underline{\Psi}_{j-}^\mu(\tau - \sigma)]$.

To describe any small disturbances of the rotational motion, that is, motions close to states (3.1) we consider vector functions $\Psi_{j\pm}^\mu$ close to $\underline{\Psi}_{j\pm}^\mu$ (3.8) in the form

$$\Psi_{j\pm}^\mu(\tau) = \underline{\Psi}_{j\pm}^\mu(\tau) + \psi_{j\pm}^\mu(\tau). \quad (3.9)$$

The disturbance $\psi_{j\pm}^\mu(\tau)$ is supposed to be small, so we omit squares of $\psi_{j\pm}$ when we substitute the expression (3.9) into dynamical equations (2.5), (2.6), (2.7), and (3.7). In other words, we work in the first linear vicinity of the states (3.1). Both functions $\dot{\Psi}_{j\pm}^\mu$ and $\dot{\underline{\Psi}}_{j\pm}^\mu$ in expression (3.9) must satisfy the condition (3.6) resulting from Eq. (2.3); hence in the first order approximation on $\dot{\psi}_{j\pm}$

the following scalar product equals zero:

$$(\dot{\underline{\Psi}}_{j\pm}, \dot{\psi}_{j\pm}) = 0. \tag{3.10}$$

For disturbed (quasirotational) motions of the model q-q-q the inner coordinate $\sigma_2(\tau)$ of the middle massive point differs from the constant value $\underline{\sigma_2}$ (3.4) and should include the following small correction δ_2:

$$\sigma_2(\tau) = \underline{\sigma_2} + \delta_2(\tau). \tag{3.11}$$

If we substitute expressions (3.9) and (3.11) with (3.5) into the continuity condition (3.7) and three equations (2.5), (2.6), and (2.7) (with $j = 2$) for massive points, we obtain equalities for summands with $\underline{\Psi}^{\mu}_{j\pm}$ and four equations for small disturbances $\psi^{\mu}_{j\pm}(\tau)$ in the first linear approximation:

$$\begin{aligned}
\psi^{\mu}_{1+}(+_2) + \psi^{\mu}_{1-}(-_2) &= \psi^{\mu}_{2+}(+_2) + \psi^{\mu}_{2-}(-_2),\\
\dot\psi^{\mu}_{1+} + \dot\psi^{\mu}_{1-} - \underline{U}^{\mu}_1(\underline{U}_1, \dot\psi^{\mu}_{1+} + \dot\psi^{\mu}_{1-}) &= Q_1(\psi^{\mu}_{1+} - \psi^{\mu}_{1-}),\\
\dot\psi^{\mu}_{2+}(+) + \dot\psi^{\mu}_{2-}(-) - \underline{U}^{\mu}_3(\underline{U}_3, \dot\psi^{\mu}_{2+}(+) + \dot\psi^{\mu}_{2-}(+)) &= Q_3[\psi^{\mu}_{2-}(-) - \psi^{\mu}_{2+}(+)],\\
\dot\psi^{\mu}_{1+}(+_2) + \dot\psi^{\mu}_{1-}(-_2) - 2a_0[\dot\delta_2 e^{\mu}(\tau) + \omega\delta_2 \acute{e}^{\mu}(\tau)] &= \frac{e^{\mu}_0}{2a_0}[(\dot{\underline{\Psi}}_{1+}(+_2), \dot\psi_{1-}(-_2)) + (\dot{\underline{\Psi}}_{1-}(-_2), \dot\psi_{1+}(+_2))]\\
&\quad + 2Q_2[\psi^{\mu}_{2+}(+_2) - \psi^{\mu}_{1+}(+_2)].
\end{aligned} \tag{3.12}$$

Here

$$Q_j = \frac{\gamma}{m_j}\sqrt{\dot x^2_j(\tau)} = \frac{\gamma a_0}{m_j}\sqrt{1 - v_j^2}, \qquad a_0 = \frac{\omega}{\Omega}, \tag{3.13}$$

vector functions

$$\underline{U}^{\mu}_j(\tau) = (1 - v_j^2)^{-1/2}[e^{\mu}_0 - \epsilon_j v_j \acute{e}^{\mu}(\tau)],$$
$$\epsilon_1 = -1, \qquad \epsilon_3 = 1,$$

are unit velocity vectors of the moving massive points, and

$$\acute{e}^{\mu}(\tau) = -e^{\mu}_1 \sin\omega\tau + e^{\mu}_2 \cos\omega\tau$$

is the unit rotating vector, orthogonal to $e(\tau)$ (3.2).

If we consider projections (scalar products) of 4 equations (3.12) onto 4 basic vectors e_0, $e(\tau)$, $\acute{e}(\tau)$, e_3 and add Eqs. (3.10) we obtain the system of 20 differential equations with deviating arguments with respect to 17 unknown functions: $\delta_2(\tau)$ and 16 projections

$$\begin{aligned}
\psi^0_{j\pm} &\equiv (e_0, \psi_{j\pm}), & \psi^3_{j\pm} &\equiv (e_3, \psi_{j\pm}),\\
\psi_{j\pm} &\equiv (e, \psi_{j\pm}), & \acute\psi_{j\pm} &\equiv (\acute e, \psi_{j\pm}).
\end{aligned} \tag{3.14}$$

Four projections of Eqs. (3.12) onto direction e_3 (orthogonal to the rotational plane e_1, e_2) form the closed subsystem with respect to 4 functions (3.14) $\psi^3_{j\pm}$:

$$\begin{aligned}
\psi^3_{1+}(+_2) + \psi^3_{1-}(-_2) &= \psi^3_{2+}(+_2) + \psi^3_{2-}(-_2),\\
\dot\psi^3_{1+}(\tau) + \dot\psi^3_{1-}(\tau) &= Q_1[\psi^3_{1+}(\tau) - \psi^3_{1-}(\tau)],\\
\dot\psi^3_{2+}(+) + \dot\psi^3_{2-}(-) &= Q_3[\psi^3_{2-}(-) - \psi^3_{2+}(+)],\\
\dot\psi^3_{1+}(+_2) + \dot\psi^3_{1-}(-_2) &= 2Q_2[\psi^3_{2+}(+_2) - \psi^3_{1+}(+_2)].
\end{aligned} \tag{3.15}$$

We search solutions of this homogeneous system in the form of harmonics

$$\psi^3_{j\pm} = B^3_{j\pm} e^{-i\xi\tau}. \tag{3.16}$$

This substitution results in the linear homogeneous system of 4 algebraic equations with respect to 4 amplitudes $B^3_{j\pm}$. The system has nontrivial solutions if and only if its determinant equals zero:

$$\begin{vmatrix}
i\xi + Q_1 & i\xi - Q_1 & 0 & 0\\
0 & 0 & (i\xi - Q_3)e^{-2i\pi\xi} & i\xi + Q_3\\
i\xi - 2Q_2 & i\xi e^{2i\underline{\sigma_2}\xi} & -2Q_2 & 0\\
e^{-i\underline{\sigma_2}\xi} & e^{i\underline{\sigma_2}\xi} & -e^{-i\underline{\sigma_2}\xi} & -e^{i\underline{\sigma_2}\xi}
\end{vmatrix} = 0.$$

This equation is reduced to the form

$$\begin{aligned}
&Q_2[(Q_1Q_3 - \xi^2)\sin\pi\xi + (Q_1 + Q_3)\xi\cos\pi\xi]\\
&\quad + \xi(Q_1\tilde c_1 - \xi\tilde s_1)(Q_3\tilde c_3 - \xi\tilde s_3) = 0,
\end{aligned} \tag{3.17}$$

where

$$\tilde c_1 = \cos\underline{\sigma_2}\xi, \qquad \tilde s_1 = \sin\underline{\sigma_2}\xi,$$
$$\tilde c_3 = \cos(\pi - \underline{\sigma_2})\xi, \qquad \tilde s_3 = \sin(\pi - \underline{\sigma_2})\xi.$$

The spectrum of transversal (with respect to the e_1, e_2 plane) small fluctuations of the string for the considered rotational state contains frequencies ξ which are roots of Eq. (3.17). We search complex roots $\xi = \xi_1 + i\xi_2$ of this equation.

In Fig. 2(a) the thick and thin lines are zero level lines correspondingly present real and imaginary part of the left-hand side $f(\xi) = f(\xi_1 + i\xi_2)$ of Eq. (3.17) for given values Q_j. Roots of this equation are shown as cross points of a thick line with a thin line. If the values (3.13) Q_j are given, one can determine values ω, $\underline{\sigma_2}$, v_j, m_j/γ from Eqs. (3.3), (3.11), and (3.13). In particular, values ω, Q_1, Q_3 are connected by the relation

$$\omega(Q_1 + Q_3) = (\omega^2 - Q_1Q_3)\tan\pi\omega, \tag{3.18}$$

resulting from the mentioned equations.

Analysis of roots of Eq. (3.17) for various values Q_j, m_j and v_j shows that for all values of mentioned parameters all these frequencies are real numbers (cross points lie on the real axis), and therefore amplitudes of such fluctuations do not grow with growing time t.

Note that any complex frequency $\xi = \xi_1 + i\xi_2$ with positive imaginary part ξ_2 results in exponential growth of the corresponding amplitude of disturbances

$$\psi_k^3 = B_k^3 \exp(-i\xi_1\tau)\cdot\exp(\xi_2\tau).$$

In this case the considered state will be unstable [11,12].

To study small disturbances in the e_1, e_2 plane we consider projections (scalar products) of Eqs. (3.12) onto 3 vectors e_0, $e(\tau)$, $\acute{e}(\tau)$. They form the system of 12 differential equations with deviating arguments with respect to 9 unknown functions $\psi_{j\pm}$, $\acute{\psi}_{j\pm}$, δ_2, if functions $\psi^0_{j\pm}$ are excluded via the orthonormality condition, Eqs. (3.10):

$$\dot{\psi}^0_{j\pm} = \pm c_1(e, \dot{\psi}_{j\pm}) - v_1(\acute{e}, \dot{\psi}_{j\pm}), \qquad c_1 = \cos\underline{\sigma}_2\omega.$$

Only 9 from these 12 equations are independent ones. When we search solutions of this system in the form of harmonics (3.16)

$$\psi_{j\pm} = B_{j\pm}e^{-i\xi\tau}, \quad \acute{\psi}_{j\pm} = \acute{B}_{j\pm}e^{-i\xi\tau}, \quad 2a_0\delta_2 = \Delta_2 e^{-i\xi\tau}, \tag{3.19}$$

we obtain the homogeneous system of 9 algebraic equations with respect to 9 amplitudes $B_{j\pm}$, $\acute{B}_{j\pm}$, Δ_2 (it is convenient to use the linear combinations of them $A_{j\pm} = -i\xi B_{j\pm} - \omega\acute{B}_{j\pm}$, $\acute{A}_{j\pm} = -i\xi\acute{B}_{j\pm} + \omega B_{j\pm}$):

$$K_1^+A_{1+} + K_1^-A_{1-} = \acute{K}_1^+\acute{A}_{1+} + \acute{K}_1^-\acute{A}_{1-}, \qquad (1 - i\xi Q_1^\xi)A_{1+} + (1 + i\xi Q_1^\xi)A_{1-} = \omega Q_1^\xi(\acute{A}_{1-} - \acute{A}_{1+}),$$
$$(v_1\Gamma_1 - \omega Q_1^\xi)(A_{1-} - A_{1+}) = (1 - i\xi Q_1^\xi)\acute{A}_{1+} + (1 + i\xi Q_1^\xi)\acute{A}_{1-}, \qquad c_1(A_{1+} - A_{2+}) - v_1(\acute{A}_{1+} - \acute{A}_{2+}) = 0,$$
$$c_1(A_{1-} - A_{2-}) + v_1(\acute{A}_{1-} - \acute{A}_{2-}) = 0, \qquad K_1^+E_3^+A_{2+} + K_1^-E_3^-A_{2-} = \acute{K}_1^+E_3^+\acute{A}_{2+} + \acute{K}_1^-E_3^-\acute{A}_{2-},$$
$$K_2^+E_3^+A_{2+} + K_2^-E_3^-A_{2-} = \acute{K}_2^+E_3^+\acute{A}_{2+} + \acute{K}_2^-E_3^-\acute{A}_{2-},$$
$$(c_1 - 2\omega v_1 Q_2^\xi)E_2^+A_{1+} + c_1E_2^-A_{1-} + v_1E_2^-\acute{A}_{1-} - i\xi\Delta_2 = E_2^+[(v_1 + 2\omega c_1 Q_2^\xi)\acute{A}_{1+} - 2\omega Q_2^\xi(v_1A_{2+} + c_1\acute{A}_{2+})],$$
$$\acute{K}_1^+E_2^+A_{1+} + \acute{K}_1^-E_2^-A_{1-} + K_1^+E_2^+\acute{A}_{1+} + K_1^-E_2^-\acute{A}_{1-} = -(\xi^2 + \omega^2)\Delta_2.$$

Here $Q_j^\xi = Q_j/(\xi^2 - \omega^2)$, $E_j^\pm = \exp(\mp i\xi\underline{\sigma}_j)$,

$$K_1^\pm = c_1\omega \mp iv_1\xi, \qquad \acute{K}_1^\pm = \pm v_1\omega + ic_1\xi,$$
$$K_2^\pm = \omega Q_3^\xi \sin\pi\omega - (1 \pm i\xi Q_3^\xi)\cos\pi\omega,$$
$$\acute{K}_2^\pm = \mp\omega Q_3^\xi\cos\pi\omega - (\pm 1 + i\xi Q_3^\xi)\sin\pi\omega.$$

Nontrivial solutions of this system exist if the condition similar to Eq. (3.17) takes place. It may be reduced to the following equation:

$$\frac{\xi}{Q_2}\cdot\frac{\xi^2 - \omega^2}{\xi^2 + \omega^2} = \sum_{j=1,3}\frac{(q_j - \xi^2)\tilde{c}_j - 2Q_j\xi\tilde{s}_j}{(q_j - \xi^2)\tilde{s}_j + 2Q_j\xi\tilde{c}_j}. \tag{3.20}$$

Here $q_j = Q_j^2(1 + v_j^{-2})$.

Figures 2(b) and 2(c) demonstrate roots $\xi = \xi_n$ of Eq. (3.20), corresponding to frequencies of small oscillations of the rotating system q-q-q in the rotational plane. Unlike Eq. (3.17), describing oscillations in the z or e_3 direction, Eq. (3.20) always has two imaginary roots $\xi = \pm i\xi_2^*$. The positive imaginary roots $\xi = i\xi_2^*$, $\xi_2^* > 0$ are marked with a circle in Figs. 2(b) and 2(c).

Other roots of Eq. (3.20) are real ones. In Figs. 2(a) and 2(b) values Q_j, m_j are the same, and the mass relation here is m_1:m_2:$m_3 \simeq$ 1:1.85:1; for the case in Fig. 2(c) it is m_1:m_2:$m_3 \simeq$ 1:10.5:4.2.

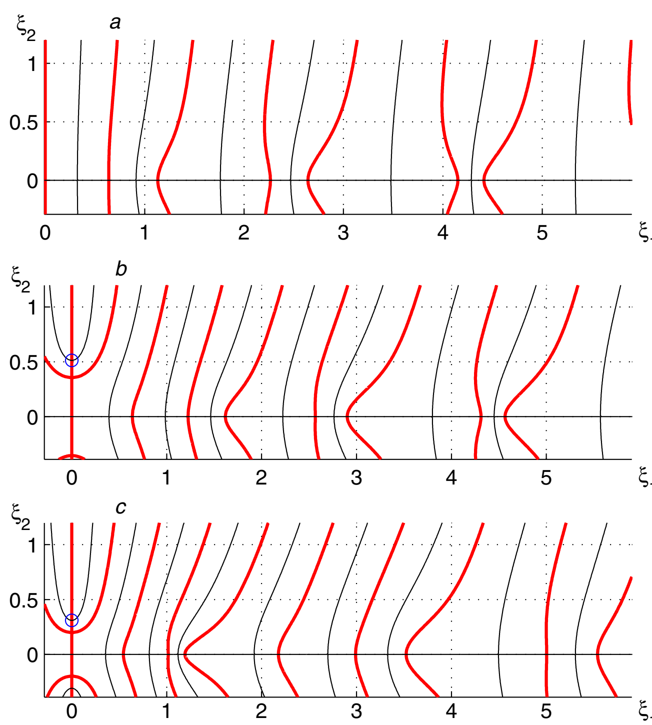

FIG. 2 (color online). Zero level lines for real part (thick) and imaginary part (thin) (a) for Eq. (3.17) with $Q_1 = Q_2 = Q_3 = 1$; (b) for Eq. (3.20) with $Q_1 = Q_2 = Q_3 = 1$; (c) for Eq. (3.20), $Q_1 = 1$, $Q_2 = 0.2$, $Q_3 = 0.4$.

The positive imaginary root $\xi = i\xi_2^*$ of Eq. (3.20) may be found after substituting $\xi = i\xi_2$:

$$\frac{\xi_2}{Q_2}\cdot\frac{\xi_2^2+\omega^2}{\omega^2-\xi_2^2} = \sum_{j=1,3}\frac{q_j+\xi_2^2+2Q_j\xi_2\tanh\check{\sigma}_j\xi_2}{(q_j+\xi_2^2)\tanh\check{\sigma}_j\xi_2+2Q_j\xi_2}.$$

Here $\check{\sigma}_1 = \underline{\sigma}_2$, $\check{\sigma}_3 = \pi - \underline{\sigma}_2$. Evidently, the required value ξ^* exists in the interval $(0, \omega)$. An arbitrary disturbed motion of the q-q-q configuration contains exponentially growing modes in its spectrum, in particular,

$$\psi_{j\pm} = B_{j\pm}\exp(\xi_2^*\tau). \tag{3.21}$$

So the rotational motion (3.1) is unstable with respect to small disturbances. Evolution of this instability was numerically analyzed in Ref. [11].

IV. ROTATIONAL STATES FOR CLOSED STRING

For the case of a closed string rotational states (planar uniform rotations of the string with massive points) were described in Refs. [8,16] in the following form:

$$\underline{X}^\mu = e_0^\mu a_0(\tau - \theta\sigma) + u(\sigma)\cdot e^\mu(\tau) + \tilde{u}(\sigma)\cdot \acute{e}^\mu(\tau). \tag{4.1}$$

Here the function

$$u(\sigma) = \begin{cases} A_1\cos\omega\sigma + B_1\sin\omega\sigma, & \sigma\in[0,\sigma_1],\\ A_2\cos\omega\sigma + B_2\sin\omega\sigma, & \sigma\in[\sigma_1,\sigma_2],\\ \dots & \\ A_n\cos\omega\sigma + B_n\sin\omega\sigma, & \sigma\in[\sigma_{n-1},2\pi],\end{cases}$$

and $\tilde{u}(\sigma) = \tilde{A}_j\cos\omega\sigma + \tilde{B}_j\sin\omega\sigma$, $\sigma\in[\sigma_{j-1},\sigma_j]$ are continuous, but their derivatives have discontinuities at $\sigma = \sigma_j \equiv \underline{\sigma}_j = \text{const}$ (positions of masses m_j); the closure condition (2.2) takes the form

$$\tau^* = \tau + 2\pi\theta, \qquad \theta = \text{const}, \tag{4.2}$$

and the values (3.13) $\gamma m_j^{-1}\sqrt{\dot{X}^2(\tau,\underline{\sigma}_j)} = Q_j$ are constants.

Expression (4.1) satisfies Eq. (2.4) and describes uniform rotations of the closed string with masses if conditions (2.2), (2.3), (2.7), and (2.8) and the condition of continuity

$$X^\mu(\tau,\sigma_j(\tau)-0) = X^\mu(\tau,\sigma_j(\tau)+0) \tag{4.3}$$

are fulfilled. Substituting Eq. (4.1) into these conditions we obtain the system of equations [16]. This system connects parameters ω, θ, $\underline{\sigma}_j$, A_j, $\tilde{A}_j$, B_j, $\tilde{B}_j$ characterizing rotational states (4.1).

A set of rotational states (4.1) of the closed string is divided into 3 classes [16]: "hypocycloidal states," "linear states" and "central states." Hypocycloidal states exist if $\theta\neq 0$; in this case segments of rotating string, connecting massive points, are segments of a hypocycloid [8,16]. Hypocycloid is the curve drawing by a point of a circle (with radius r) rolling inside another fixed circle with larger radius R. Here $r/R = \frac{1}{2}(1-|\theta|)$.

Linear states take place if $\theta = 0$, solution (4.1) describes rotating n times folded closed string with rectilinear segments, and all masses m_j move at nonzero velocities v_j at the ends of the segments. Central states also correspond to the case $\theta = 0$, but differ from linear ones by a massive point (or some of them) placed at the rotational center.

For all classes of rotational states (4.1) the string rotates at the angular velocity $\Omega = \omega/a_0$, the value a_0 connected with speeds v_j of massive points by the following equations, resulting from Eqs. (3.13):

$$a_0 = \frac{m_1Q_1}{\gamma\sqrt{1-v_1^2}} = \dots = \frac{m_nQ_n}{\gamma\sqrt{1-v_n^2}}. \tag{4.4}$$

A. Central rotational states

In this section we investigate the central rotational states (4.1) with $n = 3$ massive points where the mass m_3 is at the center and other masses m_1 and m_2 rotate at the ends of rectilinear segments. These states look like the states (3.1) of the linear model q-q-q, but have the additional string segment (the string is closed) and another numeration of massive points.

These central states (4.1) with $n = 3$ have the form

$$\underline{X}^\mu = e_0^\mu a_0\tau + u(\sigma)\cdot e^\mu(\tau). \tag{4.5}$$

They are described by the following parameters, determined by Eqs. (2.2), (2.3), (2.7), (2.8), and (4.3): $\theta = 0$, $\tilde{u}(\sigma) = 0$, $A_1 = 0$, $\underline{\sigma}_2 - \underline{\sigma}_1 = \pi$, $A_2 = 2\check{S}_1\check{C}_1B_1$, $B_2 = (\check{S}_1^2 - \check{C}_1^2)B_1$, $A_3 = -\check{S}B_1$, $B_3 = \check{C}B_1$; $v_1 = \check{S}_1$, $v_2 = \sin(2\pi - \underline{\sigma}_2)\omega$; the relation between values ω, Q_1, Q_2

$$\omega(Q_1+Q_2) = (\omega^2 - Q_1Q_2)\tan\pi\omega,$$

is similar to Eq. (3.18). Here

$$\check{C}_j = \cos\omega\underline{\sigma}_j, \qquad \check{S}_j = \sin\omega\underline{\sigma}_j, \qquad \check{C}\equiv\check{C}_3, \qquad \check{S}\equiv\check{S}_3.$$

If we present the considered central rotational state (4.5) in the form (3.5), the derivatives of corresponding functions $\underline{\Psi}_{j\pm}^\mu$, $j = 1, 2, 3$, are

$$\begin{aligned}\underline{\dot{\Psi}}_{1\pm}^\mu(\tau) &= a_0[e_0^\mu \pm e^\mu(\tau)],\\ \underline{\dot{\Psi}}_{2\pm}^\mu(\tau) &= a_0[e_0^\mu + 2v_1\check{C}_1\acute{e}^\mu(\tau) \pm (2v_1^2-1)e^\mu(\tau)],\\ \underline{\dot{\Psi}}_{3\pm}^\mu(\tau) &= a_0[e_0^\mu - \check{S}\acute{e}^\mu(\tau) \pm \check{C}e^\mu(\tau)].\end{aligned} \tag{4.6}$$

To test for stability central rotational states (4.5) we use the approach suggested in Sec. III for states (3.1) of the linear baryon model. In particular, to describe any small disturbances of the state (4.5) we consider vector functions $\Psi_{j\pm}^\mu$ close to the functions (4.6) in the form (3.9), where the disturbance $\psi_{j\pm}^\mu(\tau)$ is supposed to be small. Both functions (4.6) and (3.9) must satisfy the condition (3.6); hence in the first order approximation the condition (3.10) takes place.

For disturbed motions the equalities $\sigma_j = \underline{\sigma}_j = \text{const}$ and (4.2) $\tau^* = \tau + 2\pi\theta = \tau$, generally speaking, are not imposed but are instead replaced with the conditions

$$\sigma_j(\tau) = \underline{\sigma}_j + \delta_j(\tau), \qquad \tau^* = \tau + \delta(\tau), \tag{4.7}$$

where $\delta_1(\tau)$, $\delta_2(\tau)$ and $\delta(\tau)$ are small disturbances.

Substituting expressions (3.9) and (4.7) with Eq. (4.6) into Eqs. (2.7) and (2.8), the closure condition (2.2) and the continuity condition (4.3) we obtain in the first linear approximation the following system of equations for small disturbances $\psi^\mu_{j\pm}$, δ_j, δ, similar to the system (3.12):

$$\dot\psi^\mu_{j+} + \dot\psi^\mu_{j-} - \dot\psi^\mu_{j^*+} - \dot\psi^\mu_{j^*-} + 2F_j(e^\mu\dot\delta_j + \omega\acute{e}^\mu\delta_j) = 0, \qquad \dot\psi^\mu_{3+} + \dot\psi^\mu_{3-} - \dot\psi^\mu_{1+}(\tau) - \dot\psi^\mu_{1-}(\tau) + 2a_0e^\mu_0\dot\delta = 0,$$
$$\frac{d}{d\tau}\{\dot\psi^\mu_{j+} + \dot\psi^\mu_{j-} + F_j(e^\mu\dot\delta_j + \omega\acute{e}^\mu\delta_j) + G_jg^\mu_j\} = Q_j[\dot\psi^\mu_{j-} - \dot\psi^\mu_{j+} + \dot\psi^\mu_{j^*+} - \dot\psi^\mu_{j^*-}], \tag{4.8}$$
$$\frac{d}{d\tau}\{\dot\psi^\mu_{1+}(\tau) + \dot\psi^\mu_{1-}(\tau) + [\varphi_{1+}(\tau) - \varphi_{1-}(\tau)]e^\mu_0\} = Q_3[\dot\psi^\mu_{3-} - \dot\psi^\mu_{3+} + \dot\psi^\mu_{1+}(\tau) - \dot\psi^\mu_{1-}(\tau) + 2\omega a_0\acute{e}^\mu\delta].$$

Here $j = 1, 2$, $j^* \equiv j + 1$, and we denote projections

$$\varphi_{j\pm} \equiv (e, \dot\psi_{j\pm}), \qquad \acute\varphi_{j\pm} \equiv (\acute{e}, \dot\psi_{j\pm}); \tag{4.9}$$

the following arguments are omitted in Eqs. (4.8): (τ) for δ, δ_j, e^μ and $\acute{e}^\mu$, for $\dot\psi^\mu_{j\pm}$ and their projections (4.9)

$$\dot\psi^\mu_{j\pm} \equiv \dot\psi^\mu_{j\pm}(\tau \pm \sigma_j), \qquad \dot\psi^\mu_{j^*\pm} \equiv \dot\psi^\mu_{j^*\pm}(\tau \pm \sigma_j),$$

in particular, $\dot\psi^\mu_{3\pm} \equiv \dot\psi^\mu_{3\pm}(\tau \pm 2\pi)$.

Other notations in Eqs. (4.8) are

$$\tilde\sigma_3 = 2\pi - \underline{\sigma}_2 = \pi - \underline{\sigma}_1, \quad C_3 = \cos\tilde\sigma_3\omega, \quad F_1 = 2\check{C}_1a_0,$$
$$F_2 = -2C_3a_0, \quad g^\mu_j = e^\mu_0 - (-1)^j v_j\acute{e}^\mu(\tau),$$
$$G_1 = \varphi_{1+} - \varphi_{1-} - v_1(\acute\varphi_{1+} + \acute\varphi_{1-} - 2\omega a_0\delta_1)/\check{C}_1,$$
$$G_2C_3 = \check{C}_2[\varphi_{2-} - \varphi_{2+}] + \check{S}_2[\acute\varphi_{2+} + \acute\varphi_{2-}] + 2\omega v_2a_0\delta_2.$$

Scalar products of Eqs. (4.8) onto the vector e_3 (orthogonal to the rotational plane e_1, e_2) form the closed subsystem from 6 equations with respect to 6 functions $(\psi_{j\pm}, e_3)$. It corresponds to the system (3.15) and has nontrivial solutions in the form (3.16) $B^3_{j\pm}e^{-i\xi\tau}$ if and only if the corresponding determinant equals zero. This conditions is reduced to the equation

$$4Q_1Q_2Q_3\tilde{s}^2 + 2(Q_1Q_2 + Q_2Q_3 + Q_1Q_3)\tilde{s}\,\tilde{c}\,\xi = (Q_1\tilde{s}_3\tilde{s}_2 + Q_2\tilde{s}_1\tilde{s}_{23} + Q_3\tilde{s}^2)\xi^2 - \tilde{s}_1\tilde{s}_3\tilde{s}\xi^3. \tag{4.10}$$

Here

$$\tilde{s}_j = \sin\underline{\sigma}_j\xi, \qquad j = 1, 2, \qquad \tilde{s}_3 = \sin\tilde\sigma_3\xi,$$
$$\tilde{s} = \sin\pi\xi, \qquad \tilde{c} = \cos\pi\xi, \qquad \tilde{s}_{23} = \sin(2\pi - \sigma_1)\xi.$$

Analysis of the real and imaginary parts of Eq. (4.10) demonstrates that all its roots (frequencies of small oscillations in the e_3 direction) are real numbers. They behave like roots of Eq. (3.17) [see Fig. 2(a)].

Let us consider small disturbances concerning to the e_1, e_2 plane. Projections (scalar products) of Eqs. (4.8) onto 3 vectors e_0, $e(\tau)$, $\acute{e}(\tau)$ form the system of 18 differential equations with deviating arguments. It contains only 15 independent equations with respect to 15 unknown functions of τ ($\varphi_{j\pm}$, $\acute\varphi_{j\pm}$, $j = 1, 2, 3$, δ_1, δ_2, δ), if we exclude projections $(\psi_{j\pm}, e^0)$ via the following relations resulting from Eqs. (3.10):

$$(\dot\psi_{1\pm}, e^0) = \mp\varphi_{1\pm}, \qquad (\dot\psi_{3\pm}, e^0) = \check{S}\acute\varphi_{3\pm} \mp \check{C}\varphi_{3\pm},$$
$$(\dot\psi_{2\pm}, e^0) = \pm(1 - 2v_1^2)\varphi_{2\pm} - 2v_1\check{C}_1\acute\varphi_{2\pm}.$$

When we search solutions of this system in the form of harmonics (3.19) with addition $2a_0\delta_j = \Delta_je^{-i\xi\tau}$, $2a_0\delta = \Delta e^{-i\xi\tau}$, we obtain the homogeneous system of 15 algebraic equations with respect to 15 amplitudes $B_{j\pm}$, $\acute{B}^0_{j\pm}$, Δ_1, Δ_2, Δ. Nontrivial solutions of this system exist if the corresponding determinant vanishes. This condition in the symmetric case of equal masses

$$m_1 = m_2, \qquad \sigma_1 = \frac{\pi}{2}, \tag{4.11}$$

resulting in equalities $v_1 = v_2$, $Q_1 = Q_2$, is reduced to the following equation (it is factorized):

$$(\check{C}_1\tilde{s}_1\xi - \check{S}_1\omega\tilde{c}_1)(\check{C}_1^2\tilde{s}_1\xi^2 - \check{S}_\pi\omega\tilde{c}_1\xi - Z_1\tilde{s}_1) = 0, \tag{4.12}$$

$$\tilde{c}\xi(\xi^2 - \omega^2)(\check{C}_1^3\tilde{s}\xi^3 - 3\check{S}_1\check{C}_1^2\omega\tilde{c}\xi^2 - Z_3\check{C}_1\tilde{s}\xi + Z_1\check{S}_1\omega\tilde{c}) = 4Q_3(\check{C}_1^2\tilde{c}_1\xi^2 + \check{S}_\pi\omega\tilde{s}_1\xi - Z_1\tilde{c}_1)[\check{C}_1\xi^3\cos\tfrac{3}{2}\pi\xi + 2(\check{S}_1\omega\tilde{c}\tilde{s}_1 + Q_3\check{C}_1\tilde{s}\tilde{c}_1)\xi^2 + (\check{C}_1\omega\tilde{c}\tilde{c}_1 + 2Q_3\check{S}_1\tilde{s}\tilde{s}_1)\omega\xi + \check{S}_1\omega^3\tilde{c}\tilde{s}_1]. \tag{4.13}$$

Here $Z_1 = \omega^2(1 + \check{S}_1^2)$, $Z_3 = \omega^2(1 + 3\check{S}_1^2)$, and $\check{S}_\pi = \sin\pi\omega$.

Analysis of Eq. (4.12) (decomposing into two factors) for complex $\xi = \xi_1 + i\xi_2$ shows that for all values ω all its roots are real numbers. Their behavior is similar to that for roots of Eqs. (3.17) and (4.10).

But roots of Eq. (4.13) have other properties. These roots $\xi = \xi_1 + i\xi_2$ are shown in Figs. 3(a)–3(c) as cross points of thin and thick zero level lines for the real and imaginary parts of Eq. (4.13) (like in Fig. 2) for specified

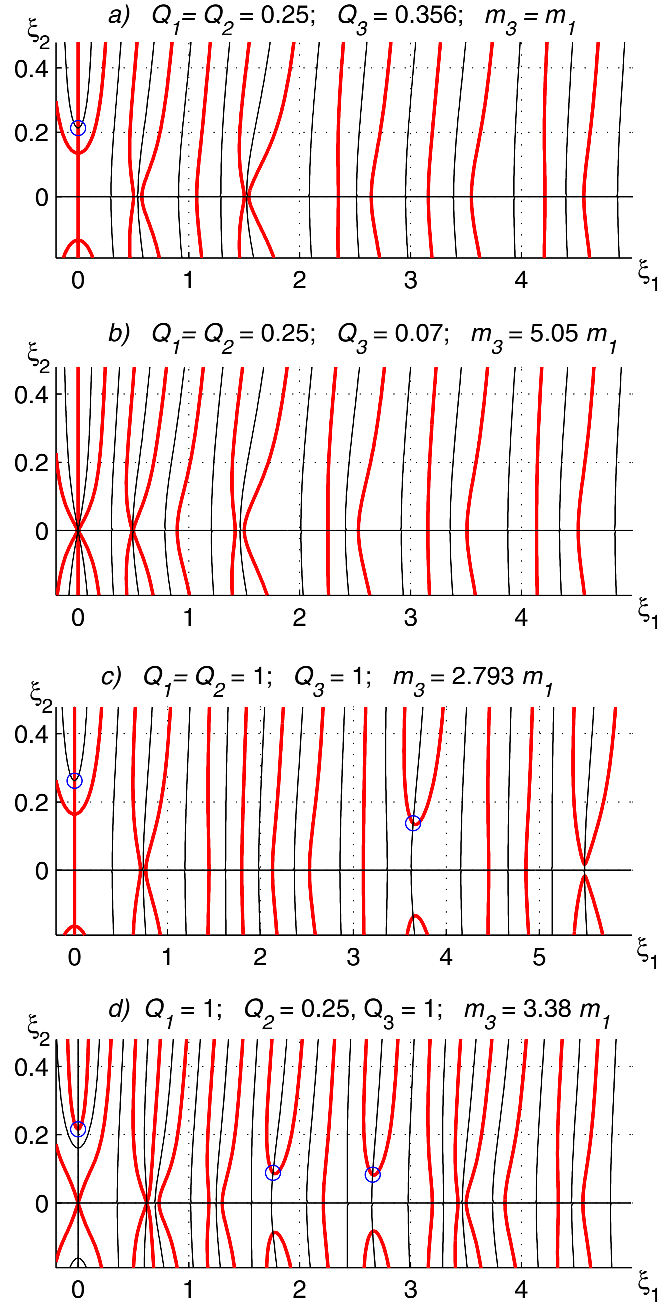

FIG. 3 (color online). Frequencies of small disturbances for central states (4.5); values Q_j and m_3 are specified.

values Q_j and m_j. In Fig. 3(d) the similar spectrum for the case of different masses m_1:m_2:$m_3 \simeq 1$:10.5:3.38 is presented. Here we used the generalization of Eqs. (4.12) and (4.13), considered in Ref. [23].

Figure 3 demonstrates that the spectrum of small disturbances of the central rotational states (4.5) has complex frequencies (marked with a circle); in particular, Eq. (4.13) has complex roots $\xi = \xi_1 + i\xi_2$ with positive imaginary parts $\xi_2 > 0$, if the central mass m_3 is nonzero and does not exceed some critical value

$$0 < m_3 < m_{3\mathrm{cr}}. \qquad (4.14)$$

Under this condition the complex roots generate exponentially growing modes of disturbances $|\psi| \sim \exp(\xi_2\tau)$, and the considered rotational state is unstable with respect to small disturbances.

The critical value $m_{3\mathrm{cr}}$ (and the corresponding $Q_{3\mathrm{cr}}$) is determined from the condition of vanishing all complex roots of Eq. (4.13) (with $\mathrm{Im}\xi > 0$) for $m_3 > m_{3\mathrm{cr}}$. Thus, we obtain the threshold effect in stability properties: the state is stable if $m_3 > m_{3\mathrm{cr}}$ and it is unstable in the case $m_3 \le m_{3\mathrm{cr}}$. This effect is observed only for the central rotational states (4.5) of the closed string; the similar states (3.1) of the linear string model *q-q-q* are unstable with any positive value of central mass.

Note that in the case $m_3 = m_{3\mathrm{cr}}$ the rotational state (4.5) is unstable because the vanishing complex root transforms into double real root ξ. The corresponding mode of small disturbances grows as $|\psi| \sim \tau\exp(i\xi\tau)$.

Under condition (4.14) for all values $Q_1 = Q_2$ the pure imaginary root $\xi^* = i\xi_2^*$ ($\xi_2^* > 0$) exists. For the states with $\omega \le 0.5$, $Q_1 \le 0.25$, Eq. (4.13) has no other complex roots except $\xi^* = i\xi_2^*$. The value ξ_2^* tends to 0 at $m_3 \to 0$ and at $m_3 \to m_{3\mathrm{cr}} \equiv m_{3\mathrm{cr}}^*$, and it reaches the maximum for values m_3 close to m_1. For the case $Q_1 = Q_2 = 1/4$ the value $m_{3\mathrm{cr}}^* \simeq 5.05 m_1$ [Fig. 3(b)].

The critical value $Q_{3\mathrm{cr}}^* = \gamma a_0/m_{3\mathrm{cr}}^*$, corresponding to vanishing the root ξ^*, may be calculated, if we substitute $\xi = i\xi_2$ into Eq. (4.13) and analyze its behavior at $\xi_2 \to 0$:

$$\check{S}_1\omega\xi_2 + \phi(\xi_2) = 2Q_3\frac{\check{C}_1(1+\cosh\pi\xi_2)\xi_2 + \check{S}_1\omega\sinh\pi\xi_2}{\cosh\pi\xi_2}.$$

The function $\phi(\xi_2) = \mathcal{O}(\xi_2^3) = \phi_3\xi_2^3 + \phi_5\xi_2^5 + \cdots$ is positive for $\xi_2 > 0$ (contains only positive summands), so the root $\xi_2 = \xi_2^*$ of this equation exists only under the condition $2Q_3(2\check{C}_1 + \pi\check{S}_1\omega) > \check{S}_1\omega$ resulting in the following expression for the critical value:

$$Q_{3\mathrm{cr}}^* = \frac{1}{2\pi + 4\check{C}_1(\omega\check{S}_1)^{-1}} = \frac{1}{2\pi + 2Q_1^{-1}}. \qquad (4.15)$$

For values $\omega > 1/2$ the structure of complex roots of Eq. (4.13) is more complicated. Additional complex roots may exist in certain interval of values m_3. So the critical value $m_{3\mathrm{cr}}$ (4.14) corresponds to vanishing all these roots.

In the case $m_3 = 0$, corresponding to $Q_3 \to \infty$, there is no massive point at the center, and we have the linear rotational state with $n = 2$. In this case Eq. (4.13) takes the form

$$\left(\xi + \omega\tan\frac{\pi\omega}{2}\tan\frac{\pi\xi}{2}\right)\xi\sin\pi\xi = 0. \qquad (4.16)$$

For all values ω it has only real roots. So the linear rotational state with $n = 2$ of the type (4.11) are stable. Stability also takes place for the case $Q_3 = 0$ ($m_3 \to \infty$).

For the case $m_1 \neq m_2$ the generalization of Eqs. (4.12) and (4.13) is rather complicated. But its roots, considered in Ref. [23] and presented in Fig. 3(d), behave similarly to the case $m_1 = m_2$; in particular, the interval (4.14) of instability exists. Generalization of the expression (4.15) for the critical value $Q^*_{3\rm cr}$ is

$$(Q^*_{3\rm cr})^{-1} = 2\pi + Q_1^{-1} + Q_2^{-1}.$$

Taking into account Eq. (4.4) $m_3 = \gamma a_0/Q_3$ we obtain the critical value of the central mass

$$m^*_{3\rm cr} = 2\pi\gamma a_0 + \frac{m_1}{\sqrt{1-v_1^2}} + \frac{m_2}{\sqrt{1-v_2^2}} \equiv E - m_3. \tag{4.17}$$

It coincides with energy of this state of the string without contribution of the mass m_3 [8,16].

We may conclude that the central rotational state is unstable if the central mass m_3 is nonzero and less than the energy of the string with other massive points.

V. INSTABILITY OF ROTATIONAL STATES AND HADRON'S WIDTH

Rotational states (3.1) of the linear string model were applied for describing orbitally excited baryons [4,9]. The similar states (4.1) and (4.5) of the closed string describe the Pomeron trajectory [16], corresponding to possible glueball states.

For rotational states (3.1) and (4.5) energy E or mass M and angular momentum J are determined by the following expressions [4,9,16]:

$$M = E = q\pi\gamma a_0 + \sum_{j=1}^{n} \frac{m_j}{\sqrt{1-v_j^2}} + \Delta E_{SL}, \tag{5.1}$$

$$J = L + S = \frac{a_0}{2\omega}\left(q\pi\gamma a_0 + \sum_{j=1}^{n} \frac{m_j v_j^2}{\sqrt{1-v_j^2}}\right) + \sum_{j=1}^{n} s_j. \tag{5.2}$$

Here $q = 1$ for the linear model, $q = 2$ for the closed string, s_j are spin projections of massive points (quarks or valent gluons), and ΔE_{SL} is the spin-orbit contribution to the energy in the following form [4]:

$$\Delta E_{SL} = \sum_{j=1}^{n} [1 - (1 - v_j^2)^{1/2}](\Omega \cdot \mathbf{s}_j).$$

If the string tension γ and values m_j and s_j are fixed, we obtain the one-parameter set of rotational states (3.1) or (4.5). Values J and E^2 for these states form the quasilinear Regge trajectory with asymptotic behavior (1.1) for large E and J [4,16] with the slope (1.2) $\alpha' = 1/(2\pi q\gamma)$.

Figure 4 presents the typical picture of Regge trajectories for N baryons with $J^P = 1/2^+, 3/2^-, 5/2^+, \ldots$ generated by the linear baryon model (solid lines) in comparison with the quark-diquark model (dashed lines).

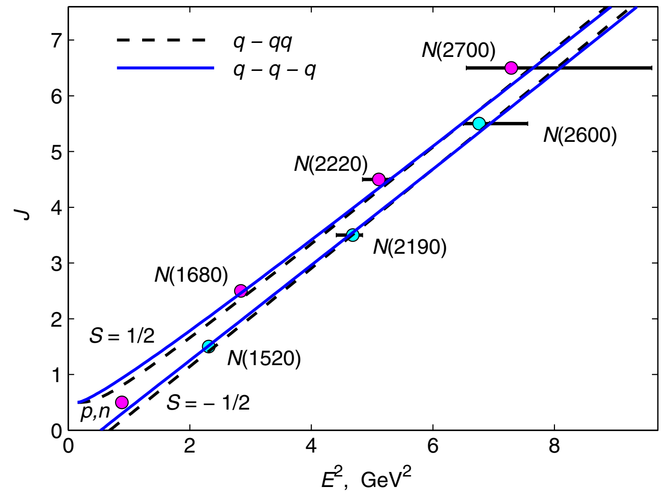

FIG. 4 (color online). Regge trajectories for rotational states (3.1) of the linear baryon model (solid lines) and the quark-diquark model (dashed lines).

Here the model parameters are taken from Ref. [4]:

$$\gamma = 0.175\ \text{GeV}^2, \qquad m_q = 130\ \text{MeV}, \qquad m_{qq} = 2m_q. \tag{5.3}$$

This tension corresponds to the slope $\alpha' \simeq 0.9\ \text{GeV}^{-2}$; effective masses of light quarks are less than constituent masses [4,9].

One can see that predictions of the linear baryon model q-q-q and the quark-diquark model q-qq are rather close under conditions (5.3). The similar picture takes place for baryons Δ and strange baryons [4,9].

We have shown in Sec. III that the rotational states (3.1) of the linear string model are unstable for all energies on the classic level. But this does not mean disappearance or terminating corresponding Regge trajectories in Fig. 4. The straight consequence of this instability is the contribution to the width of a hadron state.

String models describe only excited hadron states with large orbital momenta L. These states are unstable with respect to strong decays and have rather large width Γ. In string interpretation this width is connected with probability of string breaking; this probability is proportional to the string length ℓ [19,20]. The value ℓ is proportional to the string contribution $E_{\rm str}$ to energy E of a hadron state. For rotational states (3.1) and (4.5) this contribution to the expression (5.1) is $E_{\rm str} = q\pi\gamma a_0$.

Therefore, the component of width $\Gamma_{\rm br}$, connected with string breaking, is proportional to $E_{\rm str}$ with the factor 0.1 resulting from particle data [19,20,24]:

$$\Gamma_{\rm br} \simeq 0.1 \cdot E_{\rm str} = 0.1 \cdot q\pi\gamma a_0. \tag{5.4}$$

If a state of a string system is unstable with respect to small disturbances on the classical level, we are to take this instability into account in the form of an additional summand in width Γ of this hadron state:

$$\Gamma = \Gamma_{\rm br} + \Gamma_{\rm inst}. \qquad (5.5)$$

The contribution $\Gamma_{\rm inst}$ due to the mentioned instability is determined by the increment $\xi_2 = \xi_2^*$ of exponential growth (3.21)

$$|\psi| \sim \exp(\xi_2^* \tau) = \exp(\xi_2^* a_0^{-1} t)$$

and relation $t = a_0 \tau$ for rotational states (3.1) and (4.5). So for these states

$$\Gamma_{\rm inst} \simeq \frac{\xi_2^*}{a_0}. \qquad (5.6)$$

The values ξ_2^* and a_0 and both summands (5.4) and (5.6) of the width (5.5) depend on energy E of the hadron state. This dependence for values ξ_2^*, $\Gamma_{\rm inst}$, $\Gamma_{\rm br}$, and Γ is calculated for rotational states (3.1) of the model q-q-q, corresponding to parameters (5.3) for the N baryons in Fig. 5. These graphs are presented in Fig. 5(a) in comparison with experimental widths of N and Δ baryons [24] lying on main Regge trajectories. These widths are shown as the bar graph with dark bars for N baryons mentioned in Fig. 4 and light bars for baryons $\Delta(1232)$, $\Delta(1930)$, $\Delta(2420)$, and $\Delta(2950)$.

Note that the dimensionless value $\xi_2^* = \xi_2^*(E)$ tends to zero at $E \to E_{\rm min} = \sum m_j$, but a_0 tends to zero more rapidly, so width $\Gamma_{\rm inst}$ (5.6) tends to infinity.

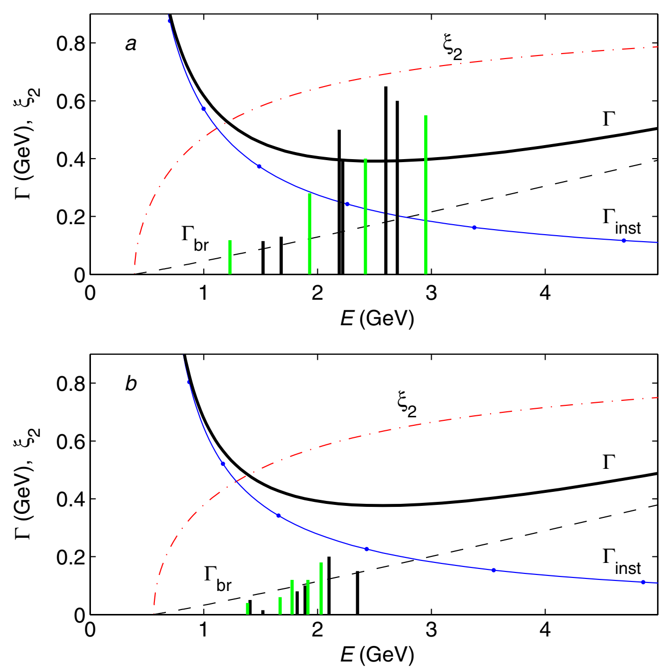

FIG. 5 (color online). Width $\Gamma = \Gamma(E)$ (5.5) (solid line), its summands $\Gamma_{\rm br}$ (5.4) (dashed line) and $\Gamma_{\rm inst}$ (5.6) (line with dots) for states (3.1) of the model $q - q - q$ (a) with parameters (5.3); (b) with $m_2 = 300$ MeV.

In Fig. 5(b) the similar graphs for strange baryons with $m_2 \equiv m_s = 300$ MeV, $m_1 = m_3 = 130$ MeV are presented. Here dark bars show the width of baryons $\Lambda(1405)$, $\Lambda(1520)$, $\Lambda(1820), \ldots$, and light bars correspond to $\Sigma(1385)$, $\Sigma(1670)$, $\Sigma(1775)$, and $\Sigma(2030)$.

In the mass range 1–2.8 GeV the contribution $\Gamma_{\rm inst}$ (5.6) due to instability of the linear model exceeds $\Gamma_{\rm br}$ and tends to infinity at $E \to E_{\rm min}$. This behavior contradicts experimental data of baryon's width in the mentioned mass range: Γ tends to zero if $E \to E_{\rm min}$. So one may conclude that the linear baryon model q-q-q is not adequate for describing orbitally excited baryon stated as the consequence of rotational instability of this model.

We mentioned above that unstable central rotational states (4.5) of the closed string, considered in Sec. IV, may be applied for describing the Pomeron trajectory

$$J \simeq 1.08 + 0.25E^2 \qquad (5.7)$$

corresponding to possible glueball states [16,18].

Estimations of gluon masses on the base of gluon propagator in lattice calculations [25,26] yield values m_j from 700 to 1000 MeV. We suppose that gluon masses $m_j = 750$ MeV and string tension $\gamma = 0.175$ GeV^2 correspond to the value (5.3). These parameters result in Regge trajectories $J = J(E^2)$ for states (4.1) and (4.5) close to the Pomeron trajectory (5.7) [16].

In Fig. 6(a) the total width $\Gamma = \Gamma(E)$ (5.5) with its summands $\Gamma_{\rm br}$ and $\Gamma_{\rm inst}$ is presented for central rotational states (4.5). They are unstable for all energies E, if masses m_j are equal. The corresponding width $\Gamma_{\rm inst}(E)$ tends to infinity in the limit $E \to E_{\rm min} = \sum m_j$.

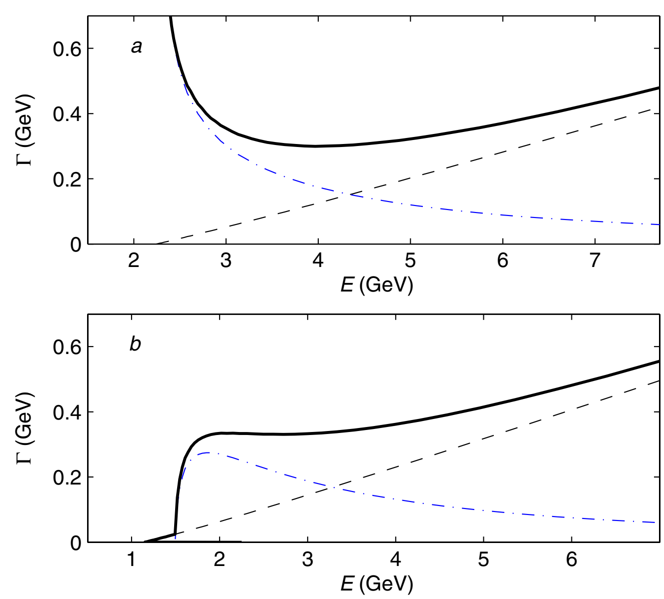

FIG. 6 (color online). Width $\Gamma(E)$ (5.5) (solid line) as the sum of $\Gamma_{\rm br}$ (5.4) (dashed line) and $\Gamma_{\rm inst}$ (5.6) (dashed-dotted line) for central states (4.5) of the closed string (a) with parameters $m_j = 750$ MeV; (b) with $m_1 = m_2 = 200$ MeV, $m_3 = 750$ MeV.

Behavior of width $\Gamma(E)$ (5.5) for central rotational states of the system with $m_3 > m_1 + m_2$ is presented in Fig. 6(b). In this case the threshold effect (4.17) exists, so the "instability" width $\Gamma_{inst}(E)$ equals zero for energies $E < E_{cr} = 2m_3$ (here it is 1.5 GeV). For $E > E_{cr}$, $\Gamma_{inst}(E)$ exceeds $\Gamma_{br}(E)$ in the certain interval, but if E grows, $\Gamma_{inst}(E)$ tends to zero and $\Gamma_{br}(E)$ increases.

VI. CONCLUSION

The stability problem was solved for classic rotational states (3.1) of the linear string baryon model and (4.5) for the closed string with massive points. It was shown for both models that the mentioned rotations are unstable, because spectra of small disturbances for these states contain complex frequencies. They are roots of Eqs. (3.20) and (4.13). These frequencies $\xi = \xi_1 + i\xi_2$ correspond to exponentially growing modes of disturbances $\psi \sim \exp(\xi_2 \tau)$ and, consequently, to instability of the mentioned rotational states.

States (3.1) of the linear string baryon model q-q-q are unstable for any values of masses m_j on the string (except for the case $m_2 = 0$: if the middle mass vanishes, the system transforms into the stable string with massive ends). But for the closed string we have the threshold effect, the central rotational states (4.5) unstable, if the central mass is nonzero and less than the critical value m_{cr} (4.17). This critical value equals the energy of the string without the central mass.

Instability of classic rotations results in some manifestations in properties of hadron states, described by the considered string model. In particular, such a model predicts additional width Γ_{inst} (5.6) of excited hadrons. To make a definite conclusion for the closed string, considered in Sec. IV, we are to have more reliable experimental data for glueballs and exotic hadrons. But for the linear string baryon model q-q-q the predicted contribution Γ_{inst} in total width Γ (5.5) in the mass range 1–3 GeV is too large in comparison with experimental data for N, Δ and strange baryons.

These predictions very weakly depend on quark masses m_j as model parameters. So we conclude that the linear string model q-q-q is unacceptable for describing these baryon states and we should refuse this model in favor of the quark-diquark and Y models. Nevertheless, we cannot exclude the q-q-q configuration as a possible structure of some baryons with anomalously large width or a variant of mixing with other configurations.

[1] Y. Nambu, Phys. Rev. D **10**, 4262 (1974).
[2] A. Chodos and C. B. Thorn, Nucl. Phys. **B72**, 509 (1974).
[3] X. Artru, Nucl. Phys. **B85**, 442 (1975).
[4] G. S. Sharov, Phys. At. Nucl. **62**, 1705 (1999).
[5] I. Yu. Kobzarev, B. V. Martemyanov, and M. G. Shchepkin, Sov. Phys. Usp. **35**, 257 1992).
[6] V. P. Petrov and G. S. Sharov, Mat. Model. **11**, 39 (1999).
[7] M. S. Plyushchay, G. P. Pronko, and A. V. Razumov, Theor. Math. Phys. **63**, 389 (1985).
[8] G. S. Sharov, Phys. Rev. D **58**, 114009 (1998).
[9] A. Inopin and G. S. Sharov, Phys. Rev. D **63**, 054023 (2001).
[10] L. D. Soloviev, Phys. Rev. D **61**, 015009 (1999).
[11] G. S. Sharov, Phys. Rev. D **62**, 094015 (2000).
[12] G. S. Sharov, Phys. At. Nucl. **65**, 906 (2002).
[13] L. D. Soloviev, Theor. Math. Phys. **126**, 203 (2001).
[14] F. J. Llanes-Estrada *et al.*, Nucl. Phys. **A710**, 45 (2002).
[15] L. A. Pando Zayas, J. Sonnenschein, and D. Vaman, Nucl. Phys. **B682**, 3 (2004).
[16] G. S. Sharov, Phys. At. Nucl. **71**, 574 (2008).
[17] G. S. Bali, Phys. Rev. D **62**, 114503 (2000).
[18] H. B. Meyer and M. Teper, Phys. Lett. B **605**, 344 (2005).
[19] I. Yu. Kobzarev, B. V. Martemyanov, and M. G. Shchepkin, Sov. J. Nucl. Phys. **48**, 344 (1988).
[20] K. S. Gupta and C. Rosenzweig, Phys. Rev. D **50**, 3368 (1994).
[21] G. S. Sharov, Theor. Math. Phys. **140**, 1109 (2004).
[22] A. E. Milovidov and G. S. Sharov, Theor. Math. Phys. **142**, 61 (2005).
[23] G. S. Sharov, arXiv:0802.3032.
[24] C. Amsler *et al.* (Particle Data Group), Phys. Lett. B **667**, 1 (2008).
[25] F. D. R. Bonnet, P. O. Bowman, D. B. Leinweber, and A. G. Williams, Phys. Rev. D **62**, 051501(R) (2000).
[26] P. J. Silva and O. Oliveira, Nucl. Phys. **B690**, 177 (2004).