QCD evolution of the transverse momentum dependent correlations

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We study the nonsinglet evolution for the twist-three quark-gluon correlation functions associated with the transverse momentum odd quark distributions. Different from that for the leading-twist quark distributions, these evolution equations involve more general twist-three functions beyond the correlation functions themselves. They provide important information on nucleon structure and can be studied in the semi-inclusive hadron production in deep inelastic scattering and Drell-Yan lepton pair production in the pp scattering process.

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I. INTRODUCTION

Semi-inclusive hadronic processes have attracted much theoretical interest in recent years, where the so-called transverse momentum dependent (TMD) parton distributions and fragmentation functions can be studied [1–9]. These functions generalize the original Feynman parton picture, where the partons only carry longitudinal momentum fractions of the parenting (final state) hadrons. They provide important information on nucleon structure and are crucial in understanding the novel spin phenomena, such as the single transverse spin asymmetry (SSA) [1–3,9–11].

Important aspects of the TMD parton distributions have been explored in the last few years, such as the gauge property and the crucial role of the initial and final state interactions for the nonzero Sivers quark distribution leading to the SSA in semi-inclusive hadron production in deep inelastic scattering (SIDIS) and Drell-Yan lepton pair production processes [1–4]. Further study has shown that the transverse momentum dependent approach is consistent with the twist-three quark-gluon correlation approach for the SSA phenomena [12–16]. In particular, these two approaches are unified to describe the same physics in the overlap region where both apply [17].

At the leading order, there are eight independent TMD quark distributions, depending on the polarizations of the nucleon and the quark [7,8]. Three of them are called $k_\perp$-even distributions, where $k_\perp$ is the quark transverse momentum. After being integrated over transverse momentum, these $k_\perp$-even distributions lead to the integrated leading-twist quark distributions: the spin averaged, the longitudinal polarized, and the transversity quark distributions [18]. The remaining five are called $k_\perp$-odd distributions and vanish in the quark correlation matrix upon integral over the transverse momentum. These TMD quark distributions are defined from the following matrix:

$$\mathcal{M}^{\alpha\beta}(x,\vec{k}_\perp) = P^+\int\frac{d\xi^-}{2\pi}e^{ix\xi^-P^+}\int\frac{d^2b_\perp}{(2\pi)^2}e^{-i\vec{b}_\perp\cdot\vec{k}_\perp} \times\langle PS|\bar{\Psi}_v^\beta(0)\Psi_v^\alpha(\xi^-,0,\vec{b}_\perp)|PS\rangle, \tag{1}$$

where x is the longitudinal momentum fraction of the proton carried by the quark and $\vec{k}_\perp$ is the transverse momentum. The nucleon momentum is defined as $P = (P^+, 0^-, 0_\perp)$, and S is the polarization vector. The field operator $\Psi_v(\xi)$ is defined as $\Psi_v(\xi) \equiv \mathcal{L}_v(-\infty;\xi)\psi(\xi)$, where $\psi(\xi)$ is the quark field and $\mathcal{L}_v$ is the gauge link [3]. We have chosen it going to $-\infty$, indicating that we can adopt the definition for the TMD quark distributions for the Drell-Yan process [1–3]. The $k_\perp$-odd TMD quark distributions can be obtained by the following expansion of $\mathcal{M}$ [7,8]:

$$\begin{aligned}\mathcal{M} = \frac{1}{2M}\Big[&g_{1T}(x,k_\perp)\gamma_5\not{P}(\vec{k}_\perp\cdot\vec{S}_\perp)\\ &+ f_{1T}^\perp(x,k_\perp)\epsilon^{\mu\nu\alpha\beta}\gamma_\mu P_\nu k_\alpha S_\beta\\ &+ h_{1L}(x,k_\perp)\lambda i\sigma_{\mu\nu}\gamma_5 P^\mu k_\perp^\nu + h_1^\perp(x,k_\perp)\sigma^{\mu\nu}k_\mu P_\nu\\ &+\frac{1}{M}h_{1T}^\perp(x,k_\perp)i\sigma_{\mu\nu}\gamma_5 P^\mu\Big(\vec{k}_\perp\cdot\vec{S}_\perp k_\perp^\nu - \frac{1}{2}\vec{k}_\perp^2 S_\perp^\nu\Big)\Big],\end{aligned} \tag{2}$$

where M is the nucleon mass. The interpretations of the four $k_\perp$-odd TMD quark distributions are as follows: g_{1T} and $f_{1T}^\perp$ represent longitudinal polarized and unpolarized quark distributions in a transversely polarized nucleon, respectively; and h_{1L} and $h_1^\perp$ represent transversely polarized quark distributions in longitudinal polarized and unpolarized nucleon targets, respectively. The last $k_\perp$-odd TMD quark distribution $h_{1T}^\perp$ represents a correlated transversely polarized quark distribution in a transversely polarized nucleon target. They are called $k_\perp$-odd distributions, because they will vanish if we integrate out the matrix $\mathcal{M}$ over the quark transverse momentum. However, if we weight the integral with transverse momentum, the first four distributions will lead to a set of quark-gluon correlation functions at the twist-three level, whereas the last one leads to a twist-four quark-gluon correlation function. These correlation functions are transverse momentum moments of the corresponding $k_\perp$-odd TMD quark distributions.

In this paper, we study the first four $k_\perp$-odd TMD distributions defined in Eq. (2). The transverse momentum moments of these distributions define the following correlation functions in nucleon[1]:

$$\int d^2k_\perp \frac{\vec{k}_\perp^2}{2\pi M^2} f_{1T}^\perp(x, k_\perp) = T_F(x),$$
$$\int d^2k_\perp \frac{\vec{k}_\perp^2}{2M^2} g_{1T}^\perp(x, k_\perp) = \tilde{g}(x), \tag{3}$$

$$\int d^2k_\perp \frac{\vec{k}_\perp^2}{2\pi M^2} h_1^\perp(x, k_\perp) = T_F^{(\sigma)}(x),$$
$$\int d^2k_\perp \frac{\vec{k}_\perp^2}{2M^2} h_{1L}^\perp(x, k_\perp) = \tilde{h}(x). \tag{4}$$

From the definitions Eqs. (1) and (2), it is straightforward to show that the correlation functions T_F and $T_F^{(\sigma)}$ can be defined from the following quark-gluon correlation matrix [4]:

$$M_{F\alpha\beta}^\mu(x) \equiv \int \frac{dy^-}{2\pi}\frac{dy_1^-}{2\pi} e^{ixP^+y^-} \times \langle PS|\bar{\psi}_\beta(0) g F^{+\mu}(y_1^-)\psi_\alpha(y^-)|PS\rangle, \tag{5}$$

where μ is a transverse index, $F_{+\mu}$ the gluon field tensor, and the gauge links between different fields have been suppressed for simplicity. The expansion of the above quark-gluon correlation matrix contains the contributions from $T_F(x)$ and $T_F^{(\sigma)}(x)$,

$$M_{F\alpha\beta}^\mu(x) = \frac{M}{2}[T_F(x)\epsilon_\perp^{\nu\mu} S_{\perp\nu}\not{p} + T_F^{(\sigma)}(x) i\gamma_\perp^\mu \not{p}]. \tag{6}$$

We notice $T_F(x)$ and $T_F^{(\sigma)}(x)$ are diagonal parts of the general quark-gluon correlation functions $T_F(x_1, x_2)$ and $T_F^{(\sigma)}(x_1, x_2)$ which are responsible for the single spin asymmetry in the hadronic process [13,19], i.e., $T_F(x) \equiv T_F(x, x)$ and $T_F^{(\sigma)}(x) \equiv T_F^{(\sigma)}(x, x)$.[2] Similarly, the correlation functions $\tilde{g}$ and $\tilde{h}$ can be calculated from the following matrix expansion [4]:

$$\tilde{M}_{F\alpha\beta}^\mu(x) = \frac{M}{2}[\tilde{g}(x) S_\perp^\mu \gamma_5 \not{p} + \tilde{h}(x)\lambda\gamma_5\gamma_\perp^\mu\not{p}], \tag{7}$$

where $\tilde{M}_{F\alpha\beta}^\mu$ is defined as

$$\tilde{M}_{F\alpha\beta}^\mu(x) = \int \frac{d\xi^-}{2\pi} e^{i\xi^- xP^+}\langle PS|\bar{\psi}_\beta(0)\Big\{iD_\perp{}^\mu(\xi^-) - \int_{\xi^-}^{-\infty} d\zeta^- g F^{+\mu}(\zeta^-)\Big\}\psi_\alpha(\xi^-)|PS\rangle. \tag{8}$$

Applying the time-reversal invariance, we find the above definition of $\tilde{g}$ to be the same as that in [15], except a normalization factor of 2.

The above four correlation functions are subsets of more general twist-three quark-gluon correlation functions [18,20]: $G_D(x, y)$, $\tilde{G}_D(x, y)$, $H_D(x, y)$, and $E(x, y)$. They are defined by parametrizing the following correlation matrix [18]:

$$\begin{aligned} M_{D\alpha\beta}^\mu(x, x_1) &\equiv \int \frac{dy^-}{2\pi}\frac{dy_1^-}{2\pi} e^{ixP^+y^-} e^{i(x_1-x)P^+y_1^-} \\ &\quad\times \langle P, S|\bar{\psi}_\beta(0) i D_\perp^\mu(y_1^-)\psi_\alpha(y^-)|P, S\rangle \\ &= \frac{M}{2P^+}[G_D(x, x_1) i\epsilon_\perp^{\mu\nu} S_{\perp\nu}\not{p} + \tilde{G}_D(x, x_1) S_\perp^\mu \gamma_5 \not{p} \\ &\quad + H_D(x, x_1)\lambda\gamma_5\gamma_\perp^\mu\not{p} + E_D(x, x_1)\gamma_\perp^\mu\not{p}]. \end{aligned} \tag{9}$$

These functions are called D-type twist-three quark-gluon correlation functions, because they are defined by the covariant derivatives. We can also define the F-type correlation functions [20] by replacing $D_\perp$ with $F^{+\perp}$ in the above matrix,

$$\begin{aligned} M_{F\alpha\beta}^\mu(x, x_1) &\equiv \int \frac{dy^-}{2\pi}\frac{dy_1^-}{2\pi} e^{ixP^+y^-} e^{i(x_1-x)P^+y_1^-} \\ &\quad\times \langle P, S|\bar{\psi}_\beta(0) g F_+{}^\mu(y_1^-)\psi_\alpha(y^-)|P, S\rangle \\ &= \frac{M}{2}[T_F(x, x_1)\epsilon_\perp^{\nu\mu} S_{\perp\nu}\not{p} + \tilde{T}_F(x, x_1) i S_\perp^\mu \gamma_5 \not{p} \\ &\quad + \tilde{T}_F^{(\sigma)}(x, x_1) i\lambda\gamma_5\gamma_\perp^\mu\not{p} + T_F^{(\sigma)}(x, x_1) i\gamma_\perp^\mu\not{p}]. \end{aligned} \tag{10}$$

By using the equation of motion [16,20] it was found that these two types of correlation functions are related to each other,

$$G_D(x, x_1) = P\frac{1}{x - x_1} T_F(x, x_1), \tag{11}$$

$$\tilde{G}_D(x, x_1) = P\frac{1}{x - x_1}\tilde{T}_F(x, x_1) + \delta(x - x_1)\tilde{g}(x), \tag{12}$$

where P stands for the principal value prescription. A similar expression holds for H_D and E_D with $T_F^{(\sigma)}$ and $\tilde{T}_F^{(\sigma)}$.

These twist-three functions and their contributions to the inclusive DIS and Drell-Yan lepton pair productions have been under intense investigation in the last two decades (see, for example, [18]). The above four correlation functions Eqs. (3) and (4), however, will enter in the transverse momentum weighted cross sections in the semi-inclusive hadron production in DIS and Drell-Yan lepton pair production in pp collisions [7,8,21]. They will provide addi-

[1]Here we emphasize that the TMD quark distributions follow their definitions in the Drell-Yan process. If we choose those for the semi-inclusive DIS process, the two equations associated with $T_F(x)$ and $T_F^{(\sigma)}(x)$ will change signs.

[2]For the convenience of our presentation, we have changed the normalization for $T_F(x_1, x_2)$ and $T_F^{(\sigma)}(x_1, x_2)$ by a factor of $1/2\pi M$ as compared to those in [17,19].

tional information on the quark-gluon correlations in the nucleon and will be complementary to those studied in the inclusive DIS and Drell-Yan processes. Recent experimental developments will help to pin down these contributions and build strong physics associated with these correlation functions [22].

One of the important questions remaining to be answered is the scale evolution for these correlation functions. The evolution equation controls the energy dependence of the associated observables [23]. For example, with the evolution equations, we will be able to compare the single spin asymmetries coming from the same quark-gluon correlation function $T_F(x)$ in hadronic processes at different energy experiments. The general structure of the evolution equations for the twist-three quark-gluon correlation functions has been known in the literature [24]. However, the above correlation functions Eqs. (3) and (4) are special projections of the general twist-three quark-gluon correlations, and their evolutions are not directly available from the already known results [24]. Earlier attempts [25] have been made to derive the evolution equations for the correlation functions of Eqs. (3) and (4), but were not complete. On the other hand, from the large transverse momentum quark Sivers function calculated in [17], we would already obtain the evolution equation for $T_F(x)$, since the collinear divergence in that calculation will lead to the splitting function of $T_F(x)$. This splitting function was confirmed by a complete calculation of next-to-leading order QCD correction to the transverse momentum weighted spin asymmetry in Drell-Yan lepton pair production [26]. More comprehensive evolution equations for $T_F(x)$, together with those for the three-gluon correlation functions which are relevant to the single spin asymmetry observables, have recently been derived in [27]. In this paper, we will extend these studies to calculate the scale evolutions for the above four quark-gluon correlation functions, but restrict ourselves to the nonsinglet case for the splitting functions.

II. THE DERIVATION OF SCALE EVOLUTION EQUATIONS

The scale evolution of these twist-3 correlation functions can be studied in terms of the higher twist collinear factorization approach in the covariant gauge as well as in the light-cone gauge. In our calculations, we will choose the light-cone gauge: $A^+ = 0$. There are several advantages for this choice. First, the quark Sivers function was previously calculated in the covariant gauge [17]. Our calculation in the light-cone gauge will provide an important cross-check for the results. Second, the light-cone gauge is more convenient to calculate the evolution equations for $\tilde{g}$ and $\tilde{h}$. In particular, the evolution equations for $T_F(x)$ and $\tilde{g}(x)$ can be calculated simultaneously. The only difference is that for $T_F(x)$ we have to take a pole contribution for some diagrams, whereas for $\tilde{g}(x)$ we will not take the pole (see the discussions below). Third, we can further choose a particular boundary condition in the light-cone gauge [3], which will greatly simplify the derivation. We have also checked that the final results do not depend on the boundary condition. According to the quark distribution definition we have chosen above, it is convenient to choose the retarded boundary condition, i.e., $A_\perp(-\infty^-) = 0$. With this choice, the gauge link associated with the TMD quark distributions in Eq. (1) becomes a unit [3], and correspondingly the quark-gluon correlation functions $T_F(x)$ and $\tilde{g}$ can be written as

$$T_F(x) = \int \frac{dy^-}{8\pi^2 M} e^{ixP^+y^-} \times \langle PS|\bar{\psi}(0)\not{n}\epsilon_\perp^{\nu\mu} S_{\perp\nu} i\partial_{\perp\mu}\psi_\alpha(y^-)|PS\rangle, \quad (13)$$

$$\tilde{g}(x) = \int \frac{dy^-}{4\pi M} e^{ixP^+y^-} \langle PS|\bar{\psi}_\beta(0)\gamma_5\not{n} S_{\perp\mu} i\partial_\perp^\mu \psi_\alpha(y^-)|PS\rangle, \quad (14)$$

in the light-cone gauge with retarded boundary condition. Similar expressions hold for the other two correlation functions, $T_F^{(\sigma)}$ and $\tilde{h}$. In the following calculations, we will focus on the derivation for the evolution functions for T_F and $\tilde{g}$, especially for T_F, and those for $T_F^{(\sigma)}$ and $\tilde{h}$ can be obtained accordingly.

We follow the general approach to calculate the evolution equations for these correlation functions. For example, as a perturbative expansion, we have

$$T_F(x) = T_F^{(0)}(x) + T_F^{(1)}(x) + \cdots, \quad (15)$$

where $T_F^{(0)}$ is the "bare" leading order correlation function, $T_F^{(1)}$ the next-to-leading correction, and the left terms are even higher order expansions. The leading order evolution of $T_F(x)$ comes from the collinear divergence of the calculation for $T_F^{(1)}$. The contributions include the virtual and real diagrams. The virtual diagrams can be calculated following that for the spin-average leading-twist quark distributions, and the results are the same at this order. The real diagram contributions are much more involved. In particular, we have to take into account the contributions from the operators $(\bar{\psi}\partial_\perp\psi)$ and $(\bar{\psi}A_\perp\psi)$ [20], because they are at the same order. Especially, because of the contribution from $A_\perp$, the evolution of the above correlation functions will involve more general twist-three functions such as $G_D(x, x_1)$ and $\tilde{G}_D(x, x_1)$ or $T_F(x, x_1)$ and $T_F^{(\sigma)}(x, x_1)$. This is an important feature for the scale evolution of the higher twist distributions, such as that of the g_T structure function [24].

We plot the Feynman diagram contributions from the real gluon radiations in Fig. 1, where Fig. 1(a) is the contribution from the partial derivative on the quark field, and Figs. 1(b)–1(d) are those from $A_\perp$ contributions. The

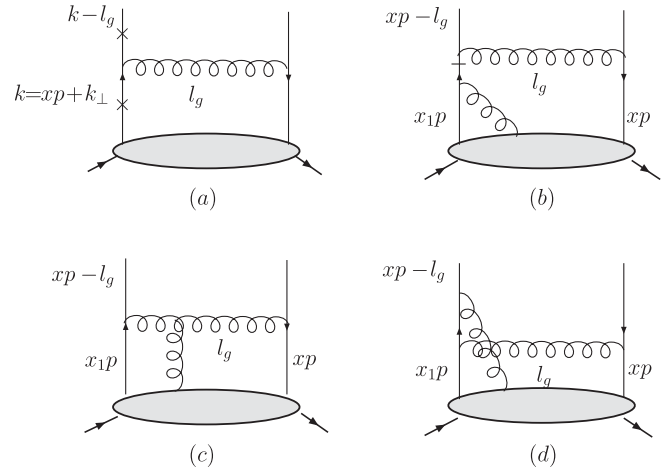

FIG. 1. Real gluon radiation contributions to the evolution equations for the twist-three quark-gluon correlation functions $T_F(x)$, $\tilde{g}(x)$, $T_F^{(\sigma)}(x)$, and $\tilde{h}(x)$.

virtual corrections only contribute to the partial derivative part, and their contributions are easy to carry out.

We perform the collinear expansion for the hard scattering part to calculate the contribution from Fig. 1(a). The linear $k_\perp$ expansion term combining with the quark field will lead to the quark-gluon correlation function $T_F(x)$ and $\tilde{g}(x)$ in Eqs. (11) and (12). In the collinear expansion in terms of $k_\perp$, we can fix the transverse momentum of the probing quark (l_q) or the radiated gluon (l_g), because of momentum conservation and we are integrating over them to obtain $T_F(x)$ and $\tilde{g}(x)$. We have also checked that they will generate the same result. In the following calculations, we choose l_g being fixed in the collinear expansion. This will avoid the collinear expansion of the on-shell condition for the radiated gluon and simplify the derivations.

For the $A_\perp$ contribution, we notice that $F^{+\mu} = \partial^+ A_\perp^\mu$ in the light-cone gauge. Therefore, one can relate the corresponding soft matrix to the correlation function $T_F(x, x_1)$ in the following way:

$$\begin{aligned}
&\frac{i}{x - x_1 + i\epsilon}\int \frac{dy^- dy_1^-}{4\pi} e^{ixP^+y^-} e^{i(x-x_1)P^+y_1^-} \\
&\quad\times \langle PS|\bar{\psi}_\beta(0^-)\not{n}\epsilon_\perp^{\nu\mu} S_{\perp\nu} g F^+{}_\mu(y_1^-)\psi_\alpha(y^-)|PS\rangle \\
&= \int \frac{dy^- dy_1^-}{4\pi} P^+ e^{ixP^+y^-} e^{i(x-x_1)P^+y_1^-} \\
&\quad\times \langle PS|\bar{\psi}_\beta(0^-)\not{n}\epsilon_\perp^{\nu\mu} S_{\perp\nu} g A_{\perp\mu}(y_1^-)\psi_\alpha(y^-)|PS\rangle. \qquad (16)
\end{aligned}$$

In the above formula, the soft gluon pole appearing in the first line comes from the partial integration. The prescription of this pole has been determined because we have chosen the retarded boundary condition. For the same reason, we have to regulate the light-cone propagator in a consistent manner, and the gluon propagator in Fig. 1(c) in the light-cone gauge with retarded boundary condition is given by [3]

$$D^{\alpha\beta}(l) = \frac{-i}{l^2 + i\epsilon}\left(g^{\alpha\beta} - \frac{l^\alpha n^\beta + n^\alpha l^\beta}{l\cdot n + i\epsilon}\right), \qquad (17)$$

where l is the gluon propagator momentum entering the quark-gluon vertex in Fig. 1(c).

Adding the contributions from the partial derivative and $A_\perp$, we find that the real diagram contributions to $T_F^{(1)}$ can be summarized as

$$\begin{aligned}
T_F^{(1)}(x_B)|_{\rm real} &= \int dx d^2 l_{g\perp} \frac{\partial}{\partial k_\perp^\mu}\{[\hat{H}(k, l_g)\not{p}]\times l_{q\perp}^\mu\}|_{k_\perp=0} \\
&\quad\times T_F(x, x) + \int dx dx_1 d^2 l_{g\perp} \\
&\quad\times \{[\hat{H}_\mu(xP, x_1P, l_g)\not{p}]\times l_{q\perp}^\mu\}\frac{1}{\pi}\frac{i}{x - x_1 + i\epsilon} \\
&\quad\times T_F(x, x_1), \qquad (18)
\end{aligned}$$

where the transverse spin vector has been integrated out, and the transverse index μ is not meant to be summed up. $l_{q\perp}$ is the probing quark transverse momentum. In the above equation, $\hat{H}(k, l_g)$ represents the hard partonic part in Fig. 1(a) with transverse momentum dependence on $k_\perp$, and $\hat{H}_\mu(xP, x_1P, l_g)$ is the hard part for Figs. 1(b)–1(d) with transverse polarized gluon $A_{\perp\mu}$ insertion where all momenta are collinear. We have to include both contributions to obtain a complete result.

We have similar expression for $\tilde{g}(x)$ splitting. The only difference is to replace $\not{p}$ with $\gamma_5\not{p}$ and a similar replacement in the hard parts in the above two terms. Moreover, because of simple Dirac algebra, the first term is the same for both $T_F(x)$ and $\tilde{g}(x)$ which comes from Fig. 1(a). Let us first discuss this contribution. In the calculations, we have to perform the collinear expansion in terms of $k_\perp^\mu$. Because of the momentum conservation, $l_q^\mu = k_\perp^\mu - l_g^\mu$, we can separate the contribution from the explicit dependence on $k_\perp^\mu$,

$$\begin{aligned}
T_F^{(1)}|_{\text{Fig. 1(a)}} &= \int dx T_F(x, x) d^2 l_{g\perp}\Big\{[\hat{H}(k, l_g)\not{p}]|_{k_\perp=0} \\
&\quad - l_{g\perp}^\mu \frac{\partial}{\partial k_\perp^\mu}[\hat{H}(k, l_g)\not{p}]|_{k_\perp=0}\Big\}. \qquad (19)
\end{aligned}$$

The first term is easy to derive, and its contribution is

$$\frac{\alpha_s}{2\pi}\int \frac{dl_{g\perp}^2}{l_{g\perp}^2}\int_{x_B}^1 \frac{dx}{x} C_F\left(\frac{1 + z^2}{1 - z}\right) T_F(x, x), \qquad (20)$$

where $z = x_B/x$, and the well-known splitting kernel appears. The collinear divergence in the transverse momentum integral $dl_{g\perp}^2$ is identified as the splitting function for $T_F(x_B)$. This splitting kernel contains the end-point divergence, which should be canceled out by the virtual diagram contributions. After taking into account the virtual contribution, the end point will be regulated by the plus function,

$$\frac{\alpha_s}{2\pi}\int\frac{dl_{g\perp}^2}{l_{g\perp}^2}\int_{x_B}^1\frac{dx}{x}C_F\left(\frac{1+z^2}{(1-z)_+}+\frac{3}{2}\delta(1-z)\right)T_F(x,x), \tag{21}$$

where the plus function follows the definition of [23]. To calculate the second term of Eq. (14), we do the collinear expansion of the hard scattering part $\hat{H}(k,l_g)$ with $l_{g\perp}$ fixed. The transverse momentum $k_\perp$ flow can go through the quark line in Fig. 1(a), labeled as a $\times$ in the diagram. We can further simplify the derivation by using the following identity:

$$\frac{\partial}{\partial k^\alpha}\frac{i}{\not k}=\frac{i}{\not k}i\gamma^\alpha\frac{i}{\not k}, \tag{22}$$

which essentially represents the application of the Ward identity. Applying the above identity, we can relate the $k_\perp$ expansion in the quark propagator and quark line to that with a transverse polarized gluon insertion with zero momentum attachment. These contributions will be combined with those from Figs. 1(c) and 1(d). We discuss them below.

As we mentioned above, the contributions from Fig. 1(a) are the same for the evolutions of $T_F(x)$ and $\tilde{g}(x)$. Therefore, the above results apply for that of $\tilde{g}(x)$ too. However, the contributions from Figs. 1(b)–1(d) are different for $T_F(x)$ and $\tilde{g}(x)$. This is because for this part, we have to take a pole contribution to obtain the splitting for $T_F(x)$, whereas for $\tilde{g}(x)$ we do not take a pole contribution. We first discuss their contributions to the evolution of $T_F(x)$ correlation function. Depending on the value of $x_g = x - x_1$ when we take the pole, these poles are called soft ($x_g = 0$) and hard ($x_g \neq 0$) poles, respectively. The hard pole contribution only comes from the light-cone propagator in Fig. 1(c), and its contribution is easy to calculate. For this, we obtain

$$T_F^{(1)}|_{\text{Fig.1(c)}}^{\text{hp}}=\frac{\alpha_s}{2\pi}\int\frac{dl_{g\perp}^2}{l_{g\perp}^2}\int_{x_B}^1\frac{dx}{x}\frac{C_A}{2}\left(\frac{1+z}{1-z}\right)T_F(xz,x). \tag{23}$$

We emphasize that for the hard pole contribution the explicit factor $1/(x-x_1)$ has been included in the above result, which is finite because $x \neq x_1$.

On the other hand, the soft pole contribution comes from the explicit pole in Eq. (13) which leads to a delta function $\delta(x_1 - x)$. Because this pole results into zero gluon momentum insertion to the diagrams, we can combine these contributions with the second term in Eq. (14) as we mentioned above. Therefore, we can add them together,

$$-\int dxT_F(x,x)d^2l_{g\perp}l_{g\perp}^\mu\left\{\frac{\partial}{\partial k_\perp^\mu}[\hat{H}(k,l_g)\not p]|_{k_\perp=0}-[\hat{H}_\mu(xP,xP,l_g)\not p]\right\}. \tag{24}$$

From this equation, we find that the contribution from Fig. 1(b) cancels out that from the $k_\perp$ expansion on the quark line with momentum k, because they have the same color factor but opposite signs. The Fig. 1(d) and the $k_\perp$ expansion on the quark propagator $k - l_g$ are also the same but with a different color factor: the color factor for Fig. 1(d) is $-1/2N_c$, whereas that for Fig. 1(a) is C_F. Their total contributions will add up to a color factor $C_A/2$. The same color factor $C_A/2$ appears for Fig. 1(c). Thus, the final result for this contribution will be proportional to $C_A/2$. By applying the identity of Eq. (17) again, we can rewrite this part of the contribution as

$$\begin{aligned}-\frac{\alpha_s}{2\pi}\frac{C_A}{2}\int dxT_F(x,x)d^2l_{g\perp}\left[\frac{\partial}{\partial l_{g\perp}^\mu}\hat{H}_0(xP,l_{g\perp})\right](-l_{g\perp}^\mu)&=-\frac{\alpha_s}{2\pi}\frac{C_A}{2}\int dxT_F(x,x)d^2l_{g\perp}\hat{H}_0(xP,l_{g\perp})\\&=-\frac{\alpha_s}{2\pi}\frac{C_A}{2}\int\frac{dl_{g\perp}^2}{l_{g\perp}^2}\int_{x_B}^1\frac{dx}{x}\left(\frac{1+z^2}{1-z}\right)T_F(x,x),\end{aligned} \tag{25}$$

where $\hat{H}_0$ represents the hard scattering part without the color factor, and we have made use of the fact that the hard part $\hat{H}_0(xP,l_{g\perp}) \propto 1/l_{g\perp}^2$. Again, the same splitting kernel appears. Finally, there is also a contribution from $\tilde{T}_F(x_1,x_2)$, which only comes from the hard pole diagram Fig. 1(c) and has been given in Ref. [28]. Summing up all contributions, we obtain the scale evolution equation for the diagonal part of the quark-gluon correlation function $T_F(x_1,x_2)$,

$$\begin{aligned}\frac{\partial}{\partial\ln\mu^2}T_F(x_B,\mu^2)=\frac{\alpha_s}{2\pi}\int_{x_B}^1\frac{dx}{x}\Bigg[&C_F\left\{\frac{1+z^2}{(1-z)_+}+\frac{3}{2}\delta(1-z)\right\}T_F(x,x)\\&+\frac{C_A}{2}\left\{\frac{1+z}{1-z}T_F(xz,x)-\frac{1+z^2}{1-z}T_F(x,x)+\tilde{T}_F(xz,x)\right\}\Bigg],\end{aligned} \tag{26}$$

which is consistent with that in Refs. [26,27]. The complete evolution equation for $T_F(x)$ shall also contain contributions from the three-gluon correlation functions, which have been calculated in [27]. In the present paper, we only take into account the singlet case and neglect three-gluon correlation function contributions for all of the evolution kernels.

As we mentioned above, the contributions from Figs. 1(b)–1(d) to the evolution of $\tilde{g}(x)$ are different from that of $T_F(x)$. For $\tilde{g}(x)$ splitting, we do not take pole contributions from these diagrams. For example, we do not have cancellation

between diagrams Fig. 1(b) and collinear expansion of the quark line k of Fig. 1(a). Moreover, without taking a pole there will be an additional integral variable in the splitting function, similar to that for the evolution of the g_T structure function [24]. The $A_\perp$ contribution from Figs. 1(b)–1(d) can be transformed into T_F and $\tilde{T}_F$, or to G_D and $\tilde{G}_D$ [18]. Because we do not take a pole for the scattering amplitudes, the calculations for these diagrams are straightforward. The partial derivative contribution from Fig. 1(a) is similar to that for the $T_F(x)$ calculation. This part depends on $\tilde{g}(x)$. After adding all these contributions together, we obtain the evolution equation for $\tilde{g}(x)$,

$$\begin{aligned}\frac{\partial \tilde{g}(x_B,\mu^2)}{\partial \ln\mu^2} = \frac{\alpha_s}{2\pi}\int_{x_B}^1 \frac{dx}{x}\int_0^1 dy\Bigg\{&\tilde{g}(x)\delta(y-x)\left[C_F\left(\frac{1+z^2}{(1-z)_+}+\frac{3}{2}\delta(1-z)\right)-\frac{C_A}{2}\frac{1+z^2}{1-z}\right]\\ &+\tilde{G}_D(x,y)\left[C_F\left(\frac{x_B^2}{x^2}+\frac{x_B}{y}-\frac{2x_B^2}{xy}-\frac{x_B}{x}-1\right)+\frac{C_A}{2}\frac{(x_B^2+xy)(2x_B-x-y)}{(x_B-y)(x-y)y}\right]\\ &+G_D(x,y)\left[C_F\left(\frac{x_B^2}{x^2}+\frac{x_B}{y}-\frac{x_B}{x}-1\right)+\frac{C_A}{2}\frac{x_B^2-xy}{(y-x_B)y}\right]\Bigg\},\end{aligned} \tag{27}$$

where again $z = x_B/x$, and the definitions of G_D and $\tilde{G}_D$ follow that in [18]. The end-point singularity from $\tilde{g}(x)$ with color factor $C_A/2$ on the right-hand side of the equation is canceled out by that from $\tilde{G}_D$ at the second line. We further notice that we can replace G_D and $\tilde{G}_D$ with T_F and $\tilde{T}_F$ on the right-hand side by using the following relations between them [16,20]:

$$G_D(x,x_1) = P\frac{1}{x-x_1}T_F(x,x_1), \tag{28}$$

$$\tilde{G}_D(x,x_1) = P\frac{1}{x-x_1}\tilde{T}_F(x,x_1) + \delta(x-x_1)\tilde{g}(x), \tag{29}$$

where P stands for the principal value prescription. However, we still have the $\tilde{g}(x)$ term on the right-hand side of the equation. Although we can rewrite $\tilde{g}(x)$ in terms of $\tilde{G}_D$ and $\tilde{T}_F$ [15], that will not eliminate its dependence completely and the right-hand side will depend on $\tilde{G}_D$, $\tilde{T}_F$, and G_D instead. Therefore, the evolution of $\tilde{g}(x)$ depends on three functions: $\tilde{g}(x)$, $G_D(x,y)$, and $\tilde{G}_D(x,y)$. This feature is different from that for $T_F(x)$, where it only depends on T_F and $\tilde{T}_F$. It may indicate the nontrivial QCD dynamics associated with the evolution of the correlation function $\tilde{g}(x)$. This has also been shown in its contribution to the Drell-Yan dilepton azimuthal asymmetry in pp scattering. We leave this study for a future publication.

Since the derivation follows a similar procedure, we skip the technical details and only list the final result for the evolution equation of correlation functions $T_F^{(\sigma)}(x,x)$, $\tilde{h}(x)$. For $T_F^{(\sigma)}$, we have

$$\frac{\partial}{\partial \ln\mu^2}T_F^{(\sigma)}(x_B,\mu^2) = \frac{\alpha_s}{2\pi}\int_{x_B}^1\frac{dx}{x}\left[C_F\left\{\frac{2z}{(1-z)_+}+2\delta(1-z)\right\}T_F^{(\sigma)}(x,x)+\frac{C_A}{2}\left\{\frac{2}{1-z}T_F^{(\sigma)}(xz,x)-\frac{2z}{1-z}T_F^{(\sigma)}(x,x)\right\}\right], \tag{30}$$

which is consistent with the large transverse momentum Boer-Mulders function $h_1^\perp(x,k_\perp)$ calculated in [19]. Accordingly, we obtain the evolution equation for $\tilde{h}$,

$$\begin{aligned}\frac{\partial \tilde{h}(x_B,\mu^2)}{\partial \ln\mu^2} = \frac{\alpha_s}{2\pi}\int_{x_B}^1\frac{dx}{x}\int_0^1 dy\Bigg\{&\tilde{h}(x)\delta(y-x)\left[C_F\left(\frac{2z}{(1-z)_+}+2\delta(1-z)\right)-\frac{C_A}{2}\frac{2z}{1-z}\right]\\ &+H_D(x,y)\left[C_F\frac{2(x-y-x_B)}{y}+\frac{C_A}{2}\frac{2x_B(x_Bx+x_By-x^2-y^2)}{(x_B-y)(x-y)y}\right]\Bigg\},\end{aligned} \tag{31}$$

where the twist-three function $H_D(x,y)$ has been introduced in Ref. [18]. Similar to that of $\tilde{g}(x)$, the evolution of $\tilde{h}$ depends on $\tilde{h}$ and H_D.

III. CONCLUSION

In conclusion, we have derived the scale evolution for the transverse momentum dependent quark-gluon correlation functions associated with the four $k_\perp$-odd TMD quark distributions in the nonsinglet case. We have performed our calculations in a light-cone gauge with a particular boundary condition for the gauge potential, and we have checked that our results do not depend on these boundary conditions. Our result on the evolution of $T_F(x)$ confirms recent calculations [26,27]. The scale evolution for $\tilde{g}$ and $\tilde{h}$ reveals nontrivial QCD dynamics. We hope this will stimulate further theoretical studies.

Meanwhile, we notice that the scale evolution for the general twist-three operators has been calculated in the literature [24]. It will be interesting to compare the evolution equations for the correlation functions studied in this paper with these well-known results. Especially, the evolution of the twist-three distribution $g_T(x)$ and its contri-

bution to semi-inclusive processes deserve further investigation. We will address these issues in forthcoming papers.

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