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Relativistic correction to J/ψ production at hadron colliders

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Relativistic corrections to the color-singlet J/ψ hadroproduction at the Tevatron and LHC are calculated up to $\mathcal{O}(v^2)$ in nonrelativistic QCD (NRQCD). The short-distance coefficients are obtained by matching full QCD with NRQCD results for the subprocess $g + g \to J/\psi + g$. The long-distance matrix elements are extracted from observed J/ψ hadronic and leptonic decay widths up to $\mathcal{O}(v^2)$. Using the CTEQ6 parton distribution functions, we calculate the leading-order production cross sections and relativistic corrections for the process $p + \bar{p}(p) \to J/\psi + X$ at the Tevatron and LHC. We find that the enhancement of $\mathcal{O}(v^2)$ relativistic corrections to the cross sections over a wide range of large transverse momentum p_t is negligible, only at a level of about 1%. This tiny effect is due to the smallness of the correction to short-distance coefficients and the suppression from long-distance matrix elements. These results indicate that relativistic corrections cannot help to resolve the large discrepancy between leading-order prediction and experimental data for J/ψ production at the Tevatron.

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I. INTRODUCTION

Nonrelativistic QCD (NRQCD) [1] is an effective field theory to describe production and decay of heavy quarkonium. In this formalism, inclusive production cross sections and decay widths of charmonium and bottomonium can be factored into short-distance coefficients, indicating the creation or annihilation of a heavy quark pair, and long-distance matrix elements, representing the evolvement of a free quark pair into a bound state. The short-distance part can be calculated perturbatively in powers of coupling constant α_s, while the nonperturbative matrix elements, which are scaled as v, the typical velocity of heavy quark or antiquark in the meson rest frame, can be estimated by nonperturbative methods or models, or extracted from experimental data.

One important aspect of NRQCD is the introduction of the color-octet mechanism, which allows the intermediate heavy quark pair to exist in a color-octet state at short distances and evolve into the color-singlet bound state at long distances. This mechanism has been applied successfully to absorb the infrared divergences in P-wave [1–3] and D-wave [4,5] decay widths of heavy quarkonia. In Ref. [6], the color-octet mechanism was introduced to account for the J/ψ production at the Tevatron, and the theoretical prediction of production rate fits well with experimental data. However, the color-octet gluon fragmentation predicts that the J/ψ is transversely polarized at large transverse momentum p_t, which is in contradiction with the experimental data [7]. (For a review of these issues, one can refer to Refs. [8–10]). Moreover, in Refs. [11,12] it was pointed out that the color-octet long-distance matrix elements of J/ψ production may be much smaller than previously expected, and accordingly this may reduce the color-octet contributions to J/ψ production at the Tevatron.

In the past a couple of years, in order to resolve the large discrepancy between the color-singlet leading-order (LO) predictions and experimental measurements [13–15] of J/ψ production at the Tevatron, the next-to-leading-order (NLO) QCD corrections to this process have been performed, and a large enhancement of an order of magnitude for the cross section at large p_t is found [16,17]. But this still cannot make up the large discrepancy between the color-singlet contribution and data. Similarly, the observed double charmonium production cross sections in e^+e^- annihilation at B factories [18,19] also significantly differ from LO theoretical predictions [20]. Much work has been done and it seems that those discrepancies could be resolved by including NLO QCD corrections [21–24] and relativistic corrections [25,26]. One may wonder if the relativistic correction could also play a role to some extent in resolving the long standing puzzle of J/ψ production at the Tevatron.

In this paper we will estimate the effect of relativistic corrections to the color-singlet J/ψ production based on NRQCD. The relativistic effects are characterized by the relative velocity v with which the heavy quark or antiquark moves in the quarkonium rest frame. According to the velocity scaling rules of NRQCD [27], the matrix elements of operators can be organized into a hierarchy in the small parameter v. We calculate the short-distance part perturbatively up to $\mathcal{O}(v^2)$. In order to avoid model dependence in determining the long-distance matrix elements, we extract the matrix elements of up to dimension-8 four fermion operators from observed decay rates of J/ψ [28]. We find that the relativistic effect on the color-singlet J/ψ production at both the Tevatron and LHC is tiny and negligible,

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and relativistic corrections cannot offer much help to resolve the puzzle associated with charmonium production at the Tevatron, and other mechanisms should be investigated to clarify the problem.

The rest of the paper is organized as follows. In Sec. II, the NRQCD factorization formalism and matching condition of full QCD and NRQCD effective field theory at long distances are described briefly, and then detailed calculations are given, including the perturbative calculation of the short-distance coefficient, the long-distance matrix elements extracted from experimental data, and the parton-level differential cross section convolution with the parton distribution functions (PDF). In Sec. III, numerical results of differential cross sections over transverse momentum p_t at the Tevatron and LHC are given and discussions are made for the enhancement effects of relativistic corrections. Finally the summary of this work is presented.

II. PRODUCTION CROSS SECTION IN NRQCD FACTORIZATION

According to NRQCD factorization [1], the inclusive cross section for the hadroproduction of J/ψ can be written as

$$\frac{d\sigma}{dt}(g+g\to J/\psi+g)=\sum_n\frac{F_n}{m_c^{d_n-4}}\langle 0|\mathcal{O}_n^{J/\psi}|0\rangle. \quad (1)$$

The short-distance coefficients F_n describe the production of a heavy quark pair $Q\bar{Q}$ from the gluons, which come from the initial state hadrons, and are usually expressed in kinematic invariants. m_c is the mass of charm quark. The long-distance matrix elements $\langle 0|\mathcal{O}_n^{J/\psi}|0\rangle$ with mass dimension d_n describe the evolution of $Q\bar{Q}$ into J/ψ. The subscript n represents the configuration in which the $c\bar{c}$ pair can be for the J/ψ Fock state expansion, and it is usually denoted as $n={}^{2S+1}L_J^{[1,8]}$. Here, S,L, and J stand for spin, orbital, and total angular momentum of the heavy quarkonium, respectively. Superscript 1 or 8 means the color-singlet or color-octet state.

For the color-singlet ${}^3S_1$ $c\bar{c}$ production, there are only two matrix elements contributing up to $\mathcal{O}(v^2)$: the leading-order term $\langle 0|\mathcal{O}^{J/\psi}({}^3S_1^{[1]})|0\rangle$ and the relativistic correction term $\langle 0|\mathcal{P}^{J/\psi}({}^3S_1^{[1]})|0\rangle$. Therefore the differential cross section takes the following form:

$$\frac{d\sigma}{dt}(g+g\to J/\psi+g)=\frac{F({}^3S_1^{[1]})}{m_c^2}\langle 0|\mathcal{O}^{J/\psi}({}^3S_1^{[1]})|0\rangle+\frac{G({}^3S_1^{[1]})}{m_c^4}\langle 0|\mathcal{P}^{J/\psi}({}^3S_1^{[1]})|0\rangle+\mathcal{O}(v^4), \quad (2)$$

and the explicit expressions of the matrix elements are [1]

$$\langle 0|\mathcal{O}^{J/\psi}({}^3S_1^{[1]})|0\rangle=\langle 0|\chi^\dagger\sigma^i\psi(a_\psi^\dagger a_\psi)\psi^\dagger\sigma^i\chi|0\rangle,$$
$$\langle 0|\mathcal{P}^{J/\psi}({}^3S_1^{[1]})|0\rangle=\left\langle 0\left|\frac{1}{2}\left[\chi^\dagger\sigma^i\psi(a_\psi^\dagger a_\psi)\psi^\dagger\sigma^i\times\left(-\frac{i}{2}\overleftrightarrow{\mathbf{D}}\right)^2\chi+\text{H.c.}\right]\right|0\right\rangle, \quad (3)$$

where ψ annihilates a heavy quark, χ creates a heavy antiquark, $a_\psi^\dagger$ and a_ψ are operators creating and annihilating J/ψ in the final state, and $\overleftrightarrow{\mathbf{D}}=\overrightarrow{\mathbf{D}}-\overleftarrow{\mathbf{D}}$.

In order to determine the short-distance coefficients $F({}^3S_1^{[1]})$ and $G({}^3S_1^{[1]})$, the matching condition of full QCD and NRQCD is needed:

$$\frac{d\sigma}{dt}(g+g\to J/\psi+g)|_{\text{pert QCD}}=\frac{F({}^3S_1^{[1]})}{m_c^2}\langle 0|\mathcal{O}^{J/\psi}({}^3S_1^{[1]})|0\rangle+\frac{G({}^3S_1^{[1]})}{m_c^4}\langle 0|\mathcal{P}^{J/\psi}({}^3S_1^{[1]})|0\rangle|_{\text{pert NRQCD}}. \quad (4)$$

The differential cross section for the production of charmonium J/ψ on the left-hand side of Eq. (4) can be calculated in perturbative QCD. On the right-hand side the long-distance matrix elements are extracted from experimental data. Quantities on both sides of the equation are expanded at leading order of α_s and next-to-leading order of v^2. Then the short-distance coefficients $F({}^3S_1^{[1]})$ and $G({}^3S_1^{[1]})$ can be obtained by comparing the terms with powers of v^2 on both sides.

A. Perturbative short-distance coefficients

We now present the calculation of relativistic correction to the process $g+g\to J/\psi+g$. In order to determine the $\mathcal{O}(v^2)$ contribution in Eq. (2), the differential cross section on the left-hand side of Eq. (4) or equivalently the QCD amplitude should be expanded up to $\mathcal{O}(v^2)$. We use FeynArts [29] to generate Feynman diagrams and amplitudes, FeynCalc [30] to handle amplitudes, and FORTRAN to evaluate the phase space integrations. A typical Feynman diagram for the process is shown in Fig. 1.

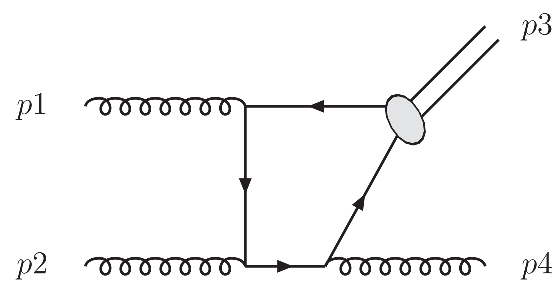

FIG. 1. Typical Feynman diagram for ${}^3S_1^{[1]}$ $c\bar{c}$ hadroproduction at LO.

The momenta of quark and antiquark in the lab frame are [26,31,32]:

$$\tfrac{1}{2}P + q = L(\tfrac{1}{2}P_r + q_r), \qquad \tfrac{1}{2}P - q = L(\tfrac{1}{2}P_r - q_r), \tag{5}$$

where L is the Lorenz boost matrix from the rest frame of the J/ψ to the frame in which it moves with four momentum P. $P_r = (2E_q, \mathbf{0})$, $E_q = \sqrt{m_c^2 + |\vec{q}|^2}$, and $2q_r = 2(0, \vec{q})$ is the relative momentum between heavy quark and antiquark in the J/ψ rest frame. The differential cross section on the left-hand side of Eq. (4) is

$$\frac{d\sigma}{dt}(g + g \to J/\psi + g)|_{\text{pert QCD}} = \frac{1}{16\pi s^2}\bar{\sum}|\mathcal{M}(g + g \to J/\psi + g)|^2\langle 0|\mathcal{O}^{J/\psi}(^3S_1^{[1]})|0\rangle, \tag{6}$$

where $\langle 0|\mathcal{O}^{J/\psi}(^3S_1^{[1]})|0\rangle$ is the matrix element evaluated at tree level, and the summation/average of the color and polarization degrees of freedom for the final/initial state has been implied by the symbol $\bar{\sum}$. The amplitude for the color-singlet process $g(p_1) + g(p_2) \to J/\psi(p_3 = P) + g(p_4)$ is

$$\mathcal{M}(g + g \to J/\psi + g) = \sqrt{\frac{1}{E_q}}\text{Tr}[\mathcal{C}^{[1]}\Pi^{(1)}\mathcal{M}^{am}], \tag{7}$$

where $\mathcal{M}^{am}$ denotes the parton-level amplitude amputated of the heavy quark spinors. The factor $\sqrt{\frac{1}{E_q}}$ comes from the normalization of the composite state $|^3S_1^{[1]}\rangle$ [5]. Here the covariant projection operator method [33,34] is adopted. For a color-singlet state, the color projector $\mathcal{C}^{[1]} = \frac{\delta_{ij}}{\sqrt{N_c}}$. The covariant spin-triplet projector $\Pi^{(1)}$ in (7) is defined by

$$\Pi^{(1)} = \sum_{s\bar{s}} v(s)\bar{u}(\bar{s})\left\langle\frac{1}{2}, s; \frac{1}{2}, \bar{s}|1, S_z\right\rangle, \tag{8}$$

with its explicit form

$$\Pi^{(1)} = \frac{1}{\sqrt{2}(E_q + m_c)}\left(\frac{\not{P}}{2} - \not{q} - m_c\right) \times \frac{\not{P} - 2E_q}{4E_q}\not{\epsilon}\left(\frac{\not{P}}{2} + \not{q} + m_c\right), \tag{9}$$

where the superscript (1) denotes the spin-triplet state and ϵ_ρ is the polarization vector of the spin 1 meson. The Lorentz-invariant Mandelstam variables are defined by

$$\begin{aligned} s &= (p_1 + p_2)^2 = (p_3 + p_4)^2, \\ t &= (p_1 - p_3)^2 = (p_2 - p_4)^2, \\ u &= (p_1 - p_4)^2 = (p_2 - p_3)^2, \end{aligned} \tag{10}$$

and they satisfy

$$s + t + u = P^2 = 4E_q^2 = 4(m_c^2 + |\vec{q}|^2). \tag{11}$$

Furthermore, the covariant spinors are normalized relativistically as $\bar{u}u = -\bar{v}v = 2m_c$.

Let $\mathcal{M}$ be short for the amplitude $\mathcal{M}(g + g \to J/\psi + g)$ in Eq. (7), and it can be expanded in powers of v or equivalently $|\vec{q}|$. That is

$$\begin{aligned} \mathcal{M} &= \epsilon_\rho\mathcal{M}^\rho \\ &= \epsilon_\rho\left(\mathcal{M}^\rho(0) + \frac{1}{2}q^\alpha q^\beta\frac{\partial^2\mathcal{M}^\rho}{\partial q^\alpha\partial q^\beta}\bigg|_{q=0}\right) + \mathcal{O}(q^4), \end{aligned} \tag{12}$$

where high order terms in four momentum q have been omitted. Terms of odd powers in q vanish because the heavy quark pair is in an S-wave configuration. Note that the polarization vector ϵ_ρ also depends on q, but it only has even powers of four momentum q, and their expressions may be found e.g. in the appendix of Ref. [35]. Therefore expansion on q^2 of ϵ_ρ can be carried out after amplitude squaring. The following substitute is adopted:

$$q^\alpha q^\beta = \frac{1}{3}|\vec{q}|^2\left(-g^{\alpha\beta} + \frac{P^\alpha P^\beta}{P^2}\right) \equiv \frac{1}{3}|\vec{q}|^2\Delta^{\alpha\beta}. \tag{13}$$

This substitute should be understood to hold in the integration over relative momentum $\vec{q}$ and in the S-wave case. Here, $|\vec{q}|^2$ can be identified as [33,36]

$$\begin{aligned} |\vec{q}|^2 &= \frac{|\langle 0|\chi^\dagger\boldsymbol{\sigma}(-\frac{i}{2}\overleftrightarrow{\mathbf{D}})^2\psi|\psi(^3S_1^{[1]})\rangle|}{|\langle 0|\chi^\dagger\boldsymbol{\sigma}\psi|\psi(^3S_1^{[1]})\rangle|} \\ &= \frac{\langle 0|\mathcal{P}^{J/\psi}(^3S_1^{[1]})|0\rangle}{\langle 0|\mathcal{O}^{J/\psi}(^3S_1^{[1]})|0\rangle}[1 + \mathcal{O}(v^4)]. \end{aligned} \tag{14}$$

Then the amplitude squared defined in Eq. (6) up to $\mathcal{O}(v^2)$ is

$$\begin{aligned} \sum|\mathcal{M}|^2 = {} & \mathcal{M}^\rho(0)\mathcal{M}^{\lambda*}(0)\sum\epsilon_\rho\epsilon_\lambda^* \\ & + \frac{1}{6}|\vec{q}|^2\bigg[\left(\Delta^{\alpha\beta}\frac{\partial^2\mathcal{M}^\rho}{\partial q^\alpha\partial q^\beta}\right)\bigg|_{q=0}\mathcal{M}^{\lambda*}(0) \\ & + \left(\Delta^{\alpha\beta}\frac{\partial^2\mathcal{M}^{\lambda*}}{\partial q^\alpha\partial q^\beta}\right)\bigg|_{q=0}\mathcal{M}^\rho(0)\bigg]\sum\epsilon_\rho\epsilon_\lambda^* \\ & + \mathcal{O}(v^4). \end{aligned} \tag{15}$$

The heavy quark and antiquark are taken to be on shell, which means that $P \cdot q = 0$, and then gauge invariance is maintained. The polarization sum in Eq. (15) is

$$\sum\epsilon_\rho\epsilon_\lambda^* = -g_{\rho\lambda} + \frac{P_\rho P_\lambda}{P^2}. \tag{16}$$

It is clearly seen that the polarization sum above only contains even order powers of four momentum q, therefore it will make a contribution to the relativistic correction at $\mathcal{O}(v^2)$ in the first term on the right-hand side of Eq. (15) when the contraction over indices ρ and λ is carried out. However, since the second term on the right-hand side of Eq. (15) already has a term proportional to q^2, i.e. $|\vec{q}|^2$, the four momentum q can be set to zero throughout the index contraction. Then we have

$$\sum|\mathcal{M}|^2 = A + B|\vec{q}|^2 + \mathcal{O}(v^4), \tag{17}$$

where A and B are independent of $|\vec{q}|$. By comparing Eqs. (4) and (6), we obtain the short-distance coefficients shown explicitly below. The leading-order one is

$$\begin{aligned}\frac{F(^3S_1^{[1]})}{m_c^2} &= \frac{1}{16\pi s^2}\frac{1}{64}\frac{1}{4}\frac{1}{2N_c}\frac{1}{3}A\\ &= \frac{1}{16\pi s^2}\frac{1}{64}\frac{1}{4}\frac{1}{2N_c}\frac{1}{3}(4\pi\alpha_s)^3 5120 m_c[16(s^2+ts+t^2)m_c^4 - 4(2s^3+3ts^2+3t^2s+2t^3)m_c^2\\ &\quad + (s^2+ts+t^2)^2]/[9(s-4m_c^2)^2(t-4m_c^2)^2(s+t)^2],\end{aligned} \tag{18}$$

and the relativistic correction term is

$$\begin{aligned}\frac{G(^3S_1^{[1]})}{m_c^4} &= \frac{1}{16\pi s^2}\frac{1}{64}\frac{1}{4}\frac{1}{2N_c}\frac{1}{3}B\\ &= \frac{1}{16\pi s^2}\frac{1}{64}\frac{1}{4}\frac{1}{2N_c}\frac{1}{3}(4\pi\alpha_s)^3(-2560)[2048(3s^2+2ts+3t^2)m_c^{10} - 256(5s^3-2ts^2-2t^2s+5t^3)m_c^8\\ &\quad - 320(3s^4+10ts^3+10t^2s^2+10t^3s+3t^4)m_c^6 + 16(21s^5+63ts^4+88t^2s^3+88t^3s^2+63t^4s+21t^5)m_c^4\\ &\quad - 4(7s^6+18ts^5+23t^2s^4+28t^3s^3+23t^4s^2+18t^5s+7t^6)m_c^2\\ &\quad - st(s+t)(s^2+ts+t^2)^2]/[27m_c(4m_c^2-s)^3(4m_c^2-t)^3(s+t)^3].\end{aligned} \tag{19}$$

Each of the factors has its own origin: $1/16\pi s^2$ is proportional to the inverse square of the Møller's invariant flux factor, $1/64$ and $1/4$ are the color average and spin average of initial two gluons, respectively, $1/2N_c$ comes from the color-singlet long-distance matrix element definition in Eq. (3) with $N_c = 3$, $1/3$ is the spin average for total spin $J = 1$ states, and $(4\pi\alpha_s)^3$ quantifies the coupling in the QCD interaction vertices. Furthermore the variable u has been expressed in terms of s and t through Eq. (11). To verify our results, we find that those in Ref. [31] discussed for J/ψ photoproduction are consistent with ours under replacement $(4\pi\alpha)e_c^2 \to (4\pi\alpha_s)$, and the result in Ref. [37] agrees with ours at leading order after performing the polarization summation.

B. Nonperturbative long-distance matrix elements

The long-distance matrix elements may be determined by potential model [25,36] or lattice calculations [38], and by phenomenological extraction from experimental data. Here we first extract the decay matrix elements from experimental data. Up to NLO QCD and v^2 relativistic corrections, decay widths of the color-singlet J/ψ to light hadrons (LH) and e^+e^- can be expressed analytically as follows [33]:

$$\begin{aligned}\Gamma[J/\psi\to \mathrm{LH}] &= \frac{F_{\mathrm{LH}}(^3S_1^{[1]})}{m_c^2}\langle H|\mathcal{O}^{J/\psi}(^3S_1^{[1]})|H\rangle + \frac{G_{\mathrm{LH}}(^3S_1^{[1]})}{m_c^4}\langle H|\mathcal{P}^{J/\psi}(^3S_1^{[1]})|H\rangle,\\ \Gamma[J/\psi\to e^+e^-] &= \frac{F_{e^+e^-}(^3S_1^{[1]})}{m_c^2}\langle H|\mathcal{O}^{J/\psi}(^3S_1^{[1]})|H\rangle + \frac{G_{e^+e^-}(^3S_1^{[1]})}{m_c^4}\langle H|\mathcal{P}^{J/\psi}(^3S_1^{[1]})|H\rangle,\end{aligned} \tag{20}$$

where the short-distance coefficients are [33]

$$\begin{aligned}F_{\mathrm{LH}}(^3S_1^{[1]}) &= \frac{(N_c^2-1)(N_c^2-4)}{N_c^3}\frac{(\pi^2-9)}{18}\alpha_s^3(2m_c)\times\left[1+(-9.46C_F+4.13C_A-1.161N_f)\frac{\alpha_s}{\pi}\right]\\ &\quad + 2\pi e_Q^2\left(\sum_{i=1}^{Nf}Q_i^2\right)\alpha_e^2\left(1-\frac{13}{4}C_F\frac{\alpha_s}{\pi}\right),\\ G_{\mathrm{LH}}(^3S_1^{[1]}) &= -\frac{5(19\pi^2-132)}{729}\alpha_s^3(2m_c),\\ F_{e^+e^-}(^3S_1^{[1]}) &= \frac{2\pi e_Q^2\alpha_e^2}{3}\left[1-4C_F\frac{\alpha_s(2m_c)}{\pi}\right],\\ G_{e^+e^-}(^3S_1^{[1]}) &= -\frac{8\pi e_Q^2\alpha_e^2}{9}.\end{aligned} \tag{21}$$

Then, the production matrix elements can be related to the decay matrix elements through vacuum saturation approximation

$$\begin{aligned}\langle 0|\mathcal{O}^{J/\psi}({}^3S_1^{[1]})|0\rangle &= (2J+1)\langle H|\mathcal{O}^{J/\psi}({}^3S_1^{[1]})|H\rangle \\ &\quad\times[1+\mathcal{O}(v^4)] \\ &= 3\langle H|\mathcal{O}^{J/\psi}({}^3S_1^{[1]})|H\rangle[1+\mathcal{O}(v^4)]. \quad (22)\end{aligned}$$

Combining the above equations and the experimental data in [28], i.e., $\Gamma[J/\psi \to \mathrm{LH}] = 81.7$ KeV and $\Gamma[J/\psi \to e^+e^-] = 5.55$ KeV and excluding the NLO QCD radiative corrections in (21), we get the solutions accurate at leading order in α_s

$$\begin{aligned}\langle 0|\mathcal{O}^{J/\psi}({}^3S_1^{[1]})|0\rangle &= 0.868\ \mathrm{GeV}^3, \\ \langle 0|\mathcal{P}^{J/\psi}({}^3S_1^{[1]})|0\rangle &= 0.190\ \mathrm{GeV}^5,\end{aligned} \quad (23)$$

and the enhanced matrix elements accurate up to NLO in α_s can be obtained by including NLO QCD radiative corrections in (21)

$$\begin{aligned}\langle 0|\mathcal{O}^{J/\psi}({}^3S_1^{[1]})|0\rangle &= 1.64\ \mathrm{GeV}^3, \\ \langle 0|\mathcal{P}^{J/\psi}({}^3S_1^{[1]})|0\rangle &= 0.320\ \mathrm{GeV}^5.\end{aligned} \quad (24)$$

The strong coupling constant evaluated at the charm quark mass scale is $\alpha_s(2m_c) = 0.250$ for $m_c = 1.5$ GeV. The other input parameters are chosen as follows: the QCD scale parameter $\Lambda_{\mathrm{QCD}} = 392$ MeV, the number of quarks with mass less than the energy scale m_c is $N_f = 3$, color factor $C_F = 4/3$ and $C_A = 3$, the electric charge of the charm quark is $e_Q = 2/3$, Q_i are the electric charges of the light quarks and fine structure constant $\alpha_e = 1/137$. Our numerical values for the production matrix elements $\langle 0|\mathcal{O}^{J/\psi}({}^3S_1^{[1]})|0\rangle$ and $\langle 0|\mathcal{P}^{J/\psi}({}^3S_1^{[1]})|0\rangle$ are accurate up to NLO in v^2 with uncertainties due to experimental errors and higher order corrections.

C. Cross sections for $p + \bar{p}(p) \to J/\psi + X$ and phase space integration

Based on the results obtained for the subprocess $g + g \to J/\psi + g$ we further calculate the production cross sections and relativistic corrections in the process $p + \bar{p}(p) \to J/\psi + X$, which involves hadrons as the initial states. In order to get the cross sections at the hadron level, the partonic cross section defined in Eq. (6) has to be convoluted with the parton distribution function (PDF) $f_{g/p}(x_\pm)$, where $x_\pm$ denotes the fraction of the proton or antiproton beam energy carried by the gluons.

We will work in the $p\bar{p}$ center-of-mass (CM) frame and denote the $p\bar{p}$ energy by $\sqrt{S}$, the rapidity of J/ψ by y_C, and that of the gluon jet by y_D. The differential cross section of $p + \bar{p}(p) \to J/\psi + X$ can be written as [39]

$$\frac{d^3\sigma(p+\bar{p}(p)\to J/\psi + X)}{dp_t^2 dy_C dy_D} = x_+ f_{g/p}(x_+) x_- f_{g/\bar{p}(p)}(x_-) \frac{d\sigma(g+g\to J/\psi+g)}{dt}, \quad (25)$$

where

$$x_\pm = \frac{m_t^C \exp(\pm y_C) + m_t^D \exp(\pm y_D)}{\sqrt{S}}, \quad (26)$$

with the transverse mass $m_t^{C,D} = \sqrt{m_{C,D}^2 + p_t^2}$, the meson mass $m_C = 2m_c$, the gluon mass $m_D = 0$, and the transverse momentum p_t. The Mandelstam variables can be expressed in terms of p_t, y_C, and y_D

$$\begin{aligned}s &= x_+ x_- S, \qquad t = -p_t^2 - m_t^C m_t^D \exp(y_D - y_C), \\ u &= -p_t^2 - m_t^C m_t^D \exp(y_C - y_D).\end{aligned} \quad (27)$$

The accessible phase space puts kinetic constraints on variables p_t, y_C, and y_D for a fixed value of two colliding hadron center-of-mass energy $\sqrt{S}$

$$\begin{aligned}0 \le p_t &\le \frac{1}{2}\sqrt{\frac{\lambda(S, m_C^2, m_D^2)}{S}}, \\ |y_C| &\le \mathrm{Arcosh}\frac{S + m_C^2 - m_D^2}{2\sqrt{S}m_t^C}, \\ -\ln\frac{\sqrt{S} - m_t^C\exp(-y_C)}{m_t^D} &\le y_D \le \ln\frac{\sqrt{S} - m_t^C \exp(y_C)}{m_t^D},\end{aligned} \quad (28)$$

where $\lambda(x, y, z) = x^2 + y^2 + z^2 - 2(xy + yz + zx)$. The distribution over p_t of the differential cross section can be obtained after phase space integration.

III. NUMERICAL RESULTS AND ANALYSIS

The CTEQ6 PDFs [40] are used in our numerical calculation. We present the distribution of J/ψ production differential cross section $d\sigma/dp_t$ over p_t at the Tevatron with $\sqrt{S} = 1.96$ TeV and at the LHC with $\sqrt{S} = 14$ TeV in Figs. 2–5. The solid line represents the distribution at leading order in $\mathcal{O}(v^2)$, and the dotted line describes the relativistic correction at next-to-leading order in $\mathcal{O}(v^2)$ (excluding the leading-order result). The long-distance matrix elements are accurate up to leading order in α_s from Eq. (23) or next-to-leading order in α_s from Eq. (24). The variable p_t is set to be from 5 GeV to 30 GeV (50 GeV) for the Tevatron (LHC), and the distributions are depicted in logarithm unit along the vertical axis. All curves decrease rather rapidly as the transverse momentum p_t increases, and the leading-order $d\sigma/dp_t$ behavior is not changed by the relativistic corrections. It can be seen that the ratio of relativistic correction to leading-order term is 1% or so, and less than 2%, which is insignificant and negligible.

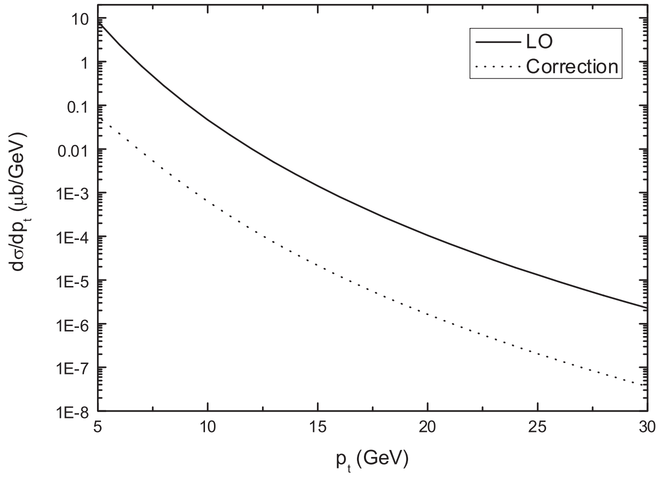

FIG. 2. The p_t distribution of $d\sigma(p + \bar{p} \rightarrow J/\psi + X)/dp_t$ at the Tevatron with $\sqrt{S} = 1.96$ TeV. The $\mathcal{O}(v^0)$ and $\mathcal{O}(v^2)$ results are represented by the solid and dotted lines, respectively.

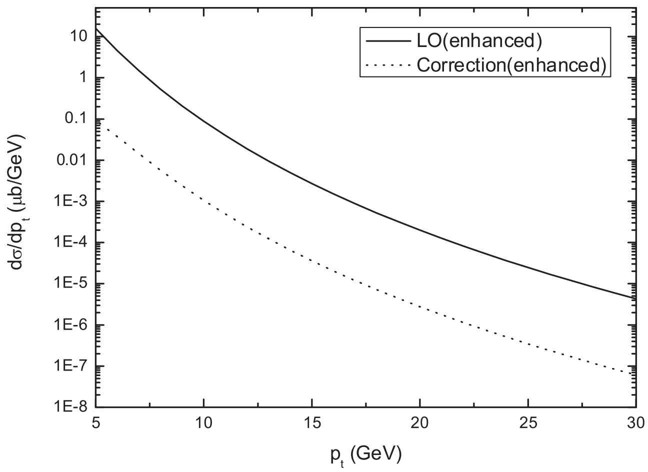

FIG. 3. The p_t distribution of $d\sigma(p + \bar{p} \rightarrow J/\psi + X)/dp_t$ (with enhanced matrix elements) at the Tevatron with $\sqrt{S} = 1.96$ TeV. The $\mathcal{O}(v^0)$ and $\mathcal{O}(v^2)$ results are represented by the solid and dotted lines, respectively.

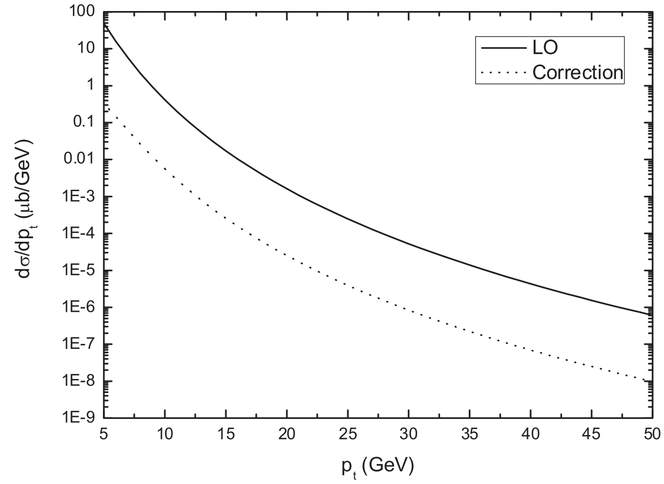

FIG. 4. The p_t distribution of $d\sigma(p + p \rightarrow J/\psi + X)/dp_t$ at the LHC with $\sqrt{S} = 14$ TeV. The $\mathcal{O}(v^0)$ and $\mathcal{O}(v^2)$ results are represented by the solid and dotted lines, respectively.

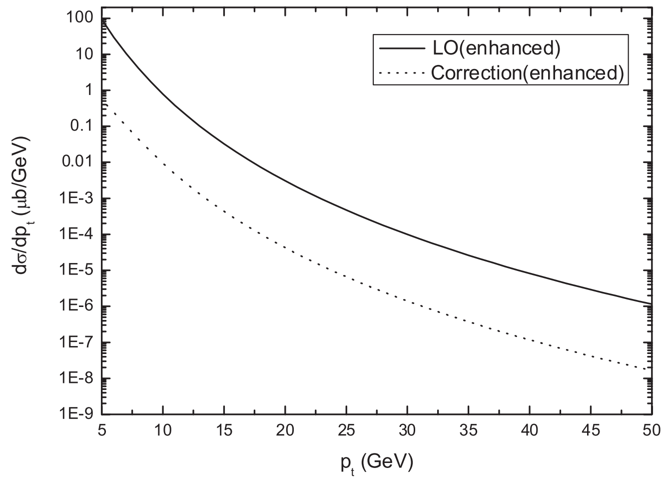

FIG. 5. The p_t distribution of $d\sigma(p + p \rightarrow J/\psi + X)/dp_t$ (with enhanced matrix elements) at the LHC with $\sqrt{S} = 14$ TeV. The $\mathcal{O}(v^0)$ and $\mathcal{O}(v^2)$ results are represented by the solid and dotted lines, respectively.

The tiny effect of relativistic corrections is partly due to the smallness of the short-distance coefficient correction. In fact, the ratio of the NLO short-distance coefficient to the LO one from Eqs. (18) and (19) can be expanded as a series of the small quantity m_c, as compared with $\sqrt{s}$, and this series reduces to a fixed small number $\frac{1}{6}$ if only the leading-order term is kept, i.e.,

$$\frac{G(^3S_1^{[1]})}{F(^3S_1^{[1]})} \rightarrow \frac{1}{6}, \quad \text{as } \frac{2m_c}{\sqrt{s}} \rightarrow 0, \qquad \frac{2m_c}{\sqrt{t}} \rightarrow 0. \tag{29}$$

Together with the suppression from long-distance matrix elements, the tiny effect of relativistic corrections can be accounted for. Our results for relativistic corrections in the process $p + \bar{p}(p) \rightarrow J/\psi + X$ are similar to that in the J/ψ photoproduction process discussed in Ref. [31]. These results may indicate that the nonrelativistic approximation in NRQCD is good for charmonium production at high energy collisions, and relativistic corrections are not important. This is in contrast to the case of double charmonium production in e^+e^- annihilation at B factories, where relativistic corrections may be significant.

IV. SUMMARY

In this paper, relativistic corrections to the color-singlet J/ψ hadroproduction at the Tevatron and LHC are calculated up to $\mathcal{O}(v^2)$ in the framework of the NRQCD factorization approach. The perturbative short-distance coefficients are obtained by matching the full QCD differential cross section with the NRQCD effective field theory calculation for the subprocess $g + g \rightarrow J/\psi + g$. The nonperturbative long-distance matrix elements are extracted from experimental data for J/ψ hadronic and leptonic decay widths up to $\mathcal{O}(v^2)$ with an approximate relation between the production matrix elements and decay matrix elements. Using the CTEQ6 parton distribution functions, we then calculate the LO production cross sections and relativistic corrections for the process $p + \bar{p}(p) \rightarrow J/\psi + X$ at the Tevatron and LHC. We find that the $\mathcal{O}(v^2)$ relativistic corrections to the differential cross sections over a wide range of large transverse momentum p_t are tiny and negligible, only at a level of about 1%. The tiny effect of relativistic corrections is due to the smallness of the short-distance coefficient correction and the suppression from long-distance matrix elements. These results may indicate that the nonrelativistic approximation in NRQCD is good for charmonium production at high energy hadron-hadron collisions, and relativistic corrections cannot offer much help to resolve the large discrepancy between the leading-order prediction and experimental data for J/ψ production at the Tevatron. Other mechanisms such as those suggested in [41–43] may need to be considered, aside from higher order QCD contributions.

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