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Study of $\bar{B} \to \Lambda_c \bar{\Lambda}_c$ and $\bar{B} \to \Lambda_c \bar{\Lambda}_c \bar{K}$

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We study the doubly charmful two-body and three-body baryonic B decays $\bar{B} \to \Lambda_c^+ \bar{\Lambda}_c^-$ and $\bar{B} \to \Lambda_c^+ \bar{\Lambda}_c^- \bar{K}$. As pointed out before, a naive estimate of the branching ratio $\mathcal{O}(10^{-8})$ for the latter decay is too small by 3 to 4 orders of magnitude compared to experiment. Previously, it has been shown that a large enhancement for the $\Lambda_c^+ \bar{\Lambda}_c^- \bar{K}$ production can occur due to a charmoniumlike resonance (e.g. $X(4630)$ discovered by Belle) with a mass near the $\Lambda_c \bar{\Lambda}_c$ threshold. Motivated by the *BABAR*'s observation of a resonance in the $\Lambda_c \bar{K}$ system with a mass of order 2930 MeV, we study in this work the contribution to $\bar{B} \to \Lambda_c^+ \bar{\Lambda}_c^- \bar{K}$ from the intermediate state $\Xi_c(2980)$ which is postulated to be a first positive-parity excited D-wave charmed baryon state. Assuming that a soft $q\bar{q}$ quark pair is produced through the σ and π meson exchanges in the configuration for $\bar{B} \to \Xi_c(2980)\bar{\Lambda}_c$ and $\Lambda_c \bar{\Lambda}_c$, it is found that branching ratios of $\bar{B} \to \Lambda_c^+ \bar{\Lambda}_c^- \bar{K}$ and $\bar{B} \to \Lambda_c^+ \bar{\Lambda}_c^-$ are of order 3.5×10^{-4} and 5×10^{-5}, respectively, in agreement with experiment except that the prediction for the $\Lambda_c \bar{\Lambda}_c K^-$ is slightly smaller. In conjunction with our previous analysis, we conclude that the enormously large rate of $\bar{B} \to \Lambda_c^+ \bar{\Lambda}_c^- \bar{K}$ arises from the resonances $\Xi_c(2980)$ and $X(4630)$.

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I. INTRODUCTION

There are several unique features in baryonic B decays. First, a peak near the threshold area of the dibaryon invariant mass spectrum has been observed in many baryonic B decays. Second, three-body decays usually have rates larger than their two-body counterparts; that is, $\mathcal{B}(B \to \mathbf{B}\bar{\mathbf{B}}'M) \gg \mathcal{B}(B \to \mathbf{B}\bar{\mathbf{B}}')$. This phenomenon can be understood in terms of the threshold effect, namely, the invariant mass of the dibaryon is preferred to be close to the threshold. The configuration of the two-body decay $B \to \mathbf{B}\bar{\mathbf{B}}'$ is not favorable since its invariant mass is m_B. In $B \to \mathbf{B}\bar{\mathbf{B}}'M$ decays, the effective mass of the baryon pair is reduced as the emitted meson can carry away much energy. The low mass threshold effect can be understood in terms of a simple short-distance picture [1]. For singly charmful baryonic B decays, experimentally we have $\mathcal{B}(B^- \to \Lambda_c^+ \bar{p} \pi^- \pi^0) > \mathcal{B}(B^- \to \Lambda_c^+ \bar{p} \pi^-) > \mathcal{B}(\bar{B}^0 \to \Lambda_c^+ \bar{p})$ [2,3] and $\mathcal{B}(\bar{B}^0 \to D^{(*)+} \pi^- p \bar{p}) > \mathcal{B}(\bar{B}^0 \to D^{(*)0} p \bar{p})$ [4]. Therefore, we have a pattern like

$$\mathcal{B}(\bar{B} \to \mathbf{B_{(c)}}\bar{\mathbf{B}}'MM') > \mathcal{B}(\bar{B} \to \mathbf{B_{(c)}}\bar{\mathbf{B}}'M) \gg \mathcal{B}(\bar{B} \to \mathbf{B_{(c)}}\bar{\mathbf{B}}'), \tag{1}$$

where $\mathbf{B_c}$ denotes a charmed baryon.

The experimental measurements for doubly charmful B decays are summarized in Table I. For $B \to \Xi_c \bar{\Lambda}_c$ decays, we extract their branching ratios using $\mathcal{B}(\Xi_c^0 \to \Xi^- \pi^+) = 1.3\%$ and $\mathcal{B}(\Xi_c^+ \to \Xi^- \pi^+ \pi^+) = 3.9\%$ [8,9], respectively,

$$\begin{aligned}
\mathcal{B}(B^- \to \Xi_c^0 \bar{\Lambda}_c^-) &= (2.0 \pm 0.6^{+1.1}_{-0.5}) \times 10^{-3} \\
&\qquad \text{(average of } BABAR \text{ and Belle)}, \\
\mathcal{B}(\bar{B}^0 \to \Xi_c^+ \bar{\Lambda}_c^-) &= (3.8 \pm 3.1^{+8.7}_{-2.4}) \times 10^{-4} < 1.4 \times 10^{-3} \\
&\qquad (BABAR), \\
&= (2.4 \pm 1.2^{+5.3}_{-1.5}) \times 10^{-3} \text{ (Belle)},
\end{aligned} \tag{2}$$

where the second errors originate from the uncertainties in $\mathcal{B}(\Xi_c^0 \to \Xi^- \pi^+)$ ranging from 0.83% to 1.74% and $\mathcal{B}(\Xi_c^+ \to \Xi^- \pi^+ \pi^+)$ from 1.2% to 10.1% [10]. Theoretically, it is expected that the charged and neutral B decays to $\Xi_c \bar{\Lambda}_c$ should have similar rates. Experimentally, this feature should be tested by the forthcoming measurements. Since $\mathcal{B}(\bar{B} \to \Lambda_c \bar{p}) \approx 2 \times 10^{-5}$ [3], we have another pattern

$$\begin{aligned}
\mathcal{B}(\bar{B} \to \mathbf{B_c}\bar{\mathbf{B}}_\mathbf{c}') &\sim 10^{-3} \gg \mathcal{B}(\bar{B} \to \mathbf{B_c}\bar{\mathbf{B}}') \\
&\sim 10^{-5} \gg \mathcal{B}(\bar{B} \to \mathbf{B}\bar{\mathbf{B}}') \lesssim 10^{-7}
\end{aligned} \tag{3}$$

for two-body baryonic B decays.

Since the doubly charmed baryonic decay $\bar{B} \to \Xi_c \bar{\Lambda}_c$ proceeds via $b \to cs\bar{c}$, while $\bar{B} \to \Lambda_c \bar{p}$ via a $b \to cd\bar{u}$ quark transition, the Cabibbo-Kobayashi-Maskawa (CKM) mixing angles for them are the same in magnitude but opposite in sign. One may wonder why the $\mathbf{B_c}\bar{\mathbf{B}}_\mathbf{c}'$ mode has a rate 2 orders of magnitude larger than $\mathbf{B_c}\bar{\mathbf{B}}$. According to the conjecture made by Hou and Soni [11], one has to reduce the energy release and at the same time allow for baryonic ingredients to be present in the final state in order to have larger baryonic B decays. Hence, it is expected that

TABLE I. Branching ratios of doubly charmful two-body (in units of 10^{-5}) and three-body (in units of 10^{-4}) baryonic B decays.

Decay	*BABAR* [5]	Belle [6,7]
$B^- \to \Xi_c^0(\to \Xi^- \pi^+)\bar{\Lambda}_c^-$	$2.08 \pm 0.65 \pm 0.29 \pm 0.54$	$4.8^{+1.0}_{-0.9} \pm 1.1 \pm 1.2$
$\bar{B}^0 \to \Xi_c^+(\to \Xi^- \pi^+ \pi^+)\bar{\Lambda}_c^-$	$1.50 \pm 1.07 \pm 0.20 \pm 0.39 < 5.6$	$9.3^{+3.7}_{-2.8} \pm 1.9 \pm 2.4$
$\bar{B}^0 \to \Lambda_c^+ \bar{\Lambda}_c^-$		$2.2^{+2.2}_{-1.6} \pm 1.3 < 6.2$
$B^- \to \Lambda_c^+ \bar{\Lambda}_c^- K^-$	$11.4 \pm 1.5 \pm 1.7 \pm 6.0$	$6.5^{+1.0}_{-0.9} \pm 1.1 \pm 3.4$
$\bar{B}^0 \to \Lambda_c^+ \bar{\Lambda}_c^- \bar{K}^0$	$3.8 \pm 3.1 \pm 0.5 \pm 2.0 < 15$	$7.9^{+2.9}_{-2.3} \pm 1.2 \pm 4.1$

$$\Gamma(B \to \mathbf{B_1}\bar{\mathbf{B}}_2) = |\text{CKM}|^2/f(\text{energy release}) = |\text{CKM}|^2/(Q \text{ value}), \quad (4)$$

where CKM stands for the relevant CKM angles. For charmful modes, one will expect

$$\mathcal{B}(\bar{B}^0 \to \Lambda_c^+ \bar{p}) = |V_{ud}/V_{cs}|^2 \mathcal{B}(\bar{B}^0 \to \Xi_c^+ \bar{\Lambda}_c^-) \times (\text{dynamical suppression}), \quad (5)$$

where the dynamical suppression arises from the larger energy release in $\Lambda_c^+ \bar{p}$ than in $\Xi_c \bar{\Lambda}_c$. This is because no hard gluon is needed to produce the energetic $\Xi_c \bar{\Lambda}_c$ pair in the latter decay, while two hard gluons are needed for the former process [8]. Therefore, $\Lambda_c \bar{p}$ is suppressed relative to $\Xi_c \bar{\Lambda}_c$ due to a dynamical suppression from $\mathcal{O}(\alpha_s^4) \sim 10^{-2}$. These qualitative statements have been confirmed by the realistic calculations of $\bar{B} \to \Xi_c \bar{\Lambda}_c$ in [8] and $\bar{B} \to \Lambda_c \bar{p}$ in [12].

For $\bar{B}^0 \to \Lambda_c^+ \bar{\Lambda}_c^-$, we expect a branching ratio of order 10^{-5} from the estimate of $\mathcal{B}(\bar{B}^0 \to \Lambda_c^+ \bar{\Lambda}_c^-) \simeq |V_{cd}/V_{cs}|^2 \mathcal{B}(\bar{B}^0 \to \Xi_c^+ \bar{\Lambda}_c^-)$ and from $|V_{cd}/V_{ud}|^2 \mathcal{B}(\bar{B}^0 \to \Lambda_c \bar{p})/(\text{dynamical suppression})$. Hence, the expected branching ratio obtained from the naive extrapolation from $\mathcal{B}(\bar{B}^0 \to \Xi_c^+ \bar{\Lambda}_c^-)$ and from $\mathcal{B}(\bar{B}^0 \to \Lambda_c \bar{p})$ is in accordance with experiment.

The three-body doubly charmed baryonic decay $\bar{B} \to \Lambda_c \bar{\Lambda}_c \bar{K}$ has been observed at B factories with the branching ratio of order (10^{-3}–10^{-4}) [5,7]. Since this mode is color suppressed and its phase space is highly suppressed, the naive estimate of $\mathcal{B}(\bar{B} \to \Lambda_c \bar{\Lambda}_c \bar{K}) \sim 10^{-8}$ from Fig. 1(a) is too small by 3 to 4 orders of magnitude compared to experiment. It was originally conjectured in [8] that the great suppression for the $\Lambda_c^+ \bar{\Lambda}_c^- \bar{K}$ production can be alleviated provided that there exists a hidden charm bound state $X_{c\bar{c}}$ with a mass near the $\Lambda_c \bar{\Lambda}_c$ threshold [see Fig. 1(b)], of order $4.6 \sim 4.7$ GeV. This possibility is motivated by the observation of many new charmonium-like resonances with masses around 4 GeV starting with $X(3872)$ and so far ending with $Z(4430)$ by *BABAR* and Belle. This new state that couples strongly to the charmed baryon pair can be searched for in B decays and in $p\bar{p}$ and e^+e^- collisions by studying the mass spectrum of $D^{(*)}\bar{D}^{(*)}$ or $\Lambda_c \bar{\Lambda}_c$. However, an initial investigation of the $\Lambda_c \bar{\Lambda}_c$ spectrum in the $\bar{B} \to \Lambda_c \bar{\Lambda}_c \bar{K}$ decays by Belle did not reveal any new resonance with a mass near the $\Lambda_c \bar{\Lambda}_c$ threshold (see Fig. 3 in version 2 of [7]). Nevertheless, the situation was dramatically changed recently. Using initial-state radiation, Belle has reported a near-threshold enhancement in the $e^+e^- \to \Lambda_c^+ \Lambda_c^-$ exclusive cross section [13]. With an assumption of a resonance origin for the observed peak, called the $X(4630)$, Belle obtained $m = 4634^{+8+5}_{-7-8}$ MeV and $\Gamma = 92^{+40+10}_{-24-21}$ MeV. Interestingly, these values are consistent within errors with the mass and width of the $Y(4660)$ with $J^{PC} = 1^{--}$ found in $\psi(2S)\pi\pi$ decays [14].

Other possibilities for the enhancement of $\Lambda_c \bar{\Lambda}_c \bar{K}$ rates include final-state interactions and $\Lambda_c \bar{K}$ resonances. For the first possibility, the weak decay $\bar{B} \to D^{(*)}\bar{D}_s^{(*)}$ followed by the rescattering of $D^{(*)}\bar{D}_s^{(*)}$ to $\Lambda_c \bar{\Lambda}_c \bar{K}$ has been considered in [15]. For the second possibility, *BABAR* has recently studied possible intermediate states in $\bar{B} \to \Lambda_c \bar{\Lambda}_c \bar{K}$ and found a resonance in the $\Lambda_c \bar{K}$ invariant mass distribu-

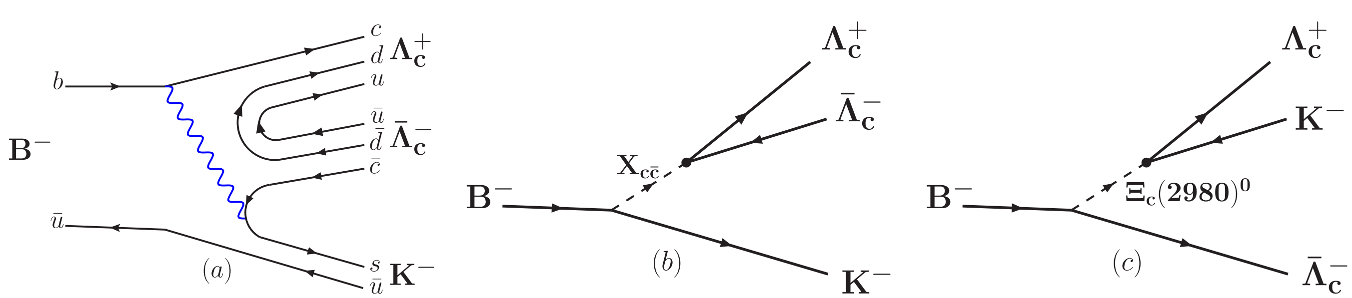

FIG. 1 (color online). $B^- \to \Lambda^+ \bar{\Lambda}^- K^-$ as proceeding through (a) the internal W-emission diagram, (b) the dominant charmonium-like resonance $X_{c\bar{c}}$, and (c) the resonant state of D-wave $\Xi_c(2980)^0$. The blob in (b) and (c) shows where the strong decays take place.

tion [5]

$$m = 2931 \pm 3 \pm 5\ \text{MeV}, \qquad \Gamma = 36 \pm 7 \pm 11\ \text{MeV}. \tag{6}$$

This could be interpreted as a single Ξ_c^0 resonance. An examination of the Ξ_c spectroscopy suggests that this resonance can be identified with $\Xi_c(2980)$ [16]

$$\begin{aligned} \Xi_c(2980)^+\colon m_{\Xi_c'} &= 2974 \pm 5\ \text{MeV}, \\ \Gamma &= 33 \pm 8\ \text{MeV}, \\ \Xi_c(2980)^0\colon m_{\Xi_c'} &= 2974 \pm 4\ \text{MeV}, \\ \Gamma &= 31 \pm 11\ \text{MeV}. \end{aligned} \tag{7}$$

In this work, we shall consider the $\Lambda_c \bar{K}$ resonant contribution to $\bar{B} \to \Lambda_c \bar{\Lambda}_c \bar{K}$ from $\Xi_c(2980)$ to see if it can lead to the anomalously large rate for this decay mode [Fig. 1(c)]. Besides, we also examine $\bar{B}^0 \to \Lambda_c^+ \bar{\Lambda}_c^-$ to give a concrete prediction. This paper is organized as follows. The formalism is given in Sec. II followed by a numerical analysis. We then give a discussion on physical results and conclude the paper in Sec. IV.

II. FORMALISM

The Cabibbo-allowed two-body doubly charmed baryonic B decays $\bar{B} \to \Xi_c(2980)\bar{\Lambda}_c$ and Cabibbo-suppressed decay $B \to \Lambda_c \bar{\Lambda}_c$ receive contributions from the internal W emission (Fig. 2) and weak annihilation. The latter contribution can be safely neglected as it is not only quark mixing but also helicity suppressed. As mentioned in the introduction, we shall consider the $\Lambda_c \bar{K}$ resonant contribution to $\bar{B} \to \Lambda_c \bar{\Lambda}_c \bar{K}$ from $\Xi_c(2980)$ using the narrow width approximation

$$\mathcal{B}(\bar{B} \to \Lambda_c \bar{\Lambda}_c \bar{K}) = \mathcal{B}(\bar{B} \to \Xi_c(2980)\bar{\Lambda}_c)\mathcal{B}(\Xi_c(2980) \to \Lambda_c \bar{K}). \tag{8}$$

Since for an energetic charmed baryon its momentum is carried mostly by the charmed quark, the two-body doubly charmful baryonic B decays can proceed without a hard gluon. In other words, the $q\bar{q}$ pair (e.g. $q'\bar{q}'$ in Fig. 2(a) and $u\bar{u}$ in Fig. 2(b)) is likely produced from the vacuum via the soft nonperturbative interactions so that it carries the vacuum quantum numbers $^3P_0$. Following [8], we shall consider the possibility that the $q\bar{q}$ pair is produced via a light meson exchange. The $q\bar{q}$ pair created from soft nonperturbative interactions tends to be soft. To be specific, we assume the exchange of the σ, π^0, and π^- between the soft $q\bar{q}$ quark pair and the spectator as shown in Fig. 3. It should be stressed that Fig. 3 here differs from Fig. 5 of [8] as Ξ_c in the latter is a ground-state S-wave cascade charmed baryon, while $\Xi_c(2980)$ in the former is an excited charmed baryon. Hence, a repeat of the analysis in [8] will not provide any information on $B \to \Xi_c(2980)\bar{\Lambda}_c$.

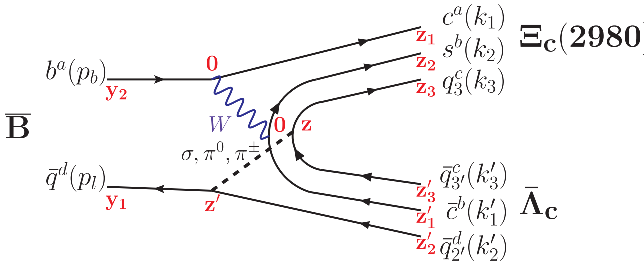

FIG. 3 (color online). $\bar{B} \to \Xi_c(2980)\bar{\Lambda}_c$, where $qq_3q_{3'}q_2 = uddu(udud)$ for σ, π^0 $(\pi^\pm)$ exchange in B^- decays, and $qq_3q_{3'}q_2 = duud(dudu)$ for σ, π^0 $(\pi^\pm)$ exchange in $\bar{B}^0$ decays.

To obtain the amplitudes of $B^- \to \Xi_c(2980)^0\Lambda_c^-$ and $\bar{B}^0 \to \Lambda_c^+\bar{\Lambda}_c^-$, we start from the short-distance effective Hamiltonian given by

$$\mathcal{H}_{\text{eff}} = \frac{G_F}{\sqrt{2}} V_{cb} V_{cq}^* (c_1 O_1 + c_2 O_2), \tag{9}$$

where $O_1 = (\bar{c}b)(\bar{q}c)$ and $O_2 = (\bar{c}c)(\bar{q}b)$ with $q = s$ for $\Xi_c(2980)^0$, $q = d$ for Λ_c^+, and $(\bar{q}q') \equiv \bar{q}\gamma_\mu(1-\gamma_5)q'$. We shall use the Wilson coefficients $c_1 = 1.169$ and $c_2 = -0.367$. The Lagrangian for meson-quark interactions reads

$$\begin{aligned} \mathcal{L}_{\sigma qq} &= g_\sigma(\bar{u}u + \bar{d}d)\sigma, \\ \mathcal{L}_{\pi^0 qq} &= g_{\pi^0}(\bar{u}i\gamma_5 u - \bar{d}i\gamma_5 d)\pi^0, \\ \mathcal{L}_{\pi^\pm qq} &= g_\pi(\bar{u}i\gamma_5 d\pi^+ + \bar{d}i\gamma_5 u\pi^-), \end{aligned} \tag{10}$$

where g_i $(i = \sigma, \pi^0, \pi)$ is the coupling constant, and $g_\pi = \sqrt{2}g_{\pi^0}$ from isospin symmetry. The amplitude of $B^- \to \Xi_c(2980)^0\bar{\Lambda}_c^-$ in Fig. 3 thus has the form

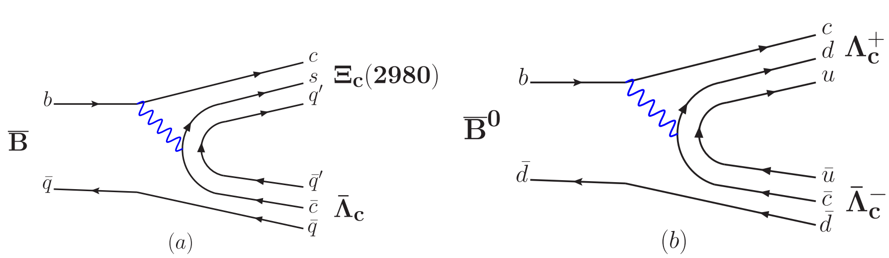

FIG. 2 (color online). (a) $\bar{B} \to \Xi_c(2980)\bar{\Lambda}_c$ and (b) $\bar{B}^0 \to \Lambda_c^+\bar{\Lambda}_c^-$ as proceeding via internal W-emission diagrams. In (a), $qq' = du$ and ud for B^- and $\bar{B}^0$ decays, respectively.

$$\mathcal{A} = \mathcal{A}_\sigma + \mathcal{A}_{\pi^0} + \mathcal{A}_{\pi^\pm}. \tag{11}$$

In the case of σ exchange, the amplitude reads

$$\begin{aligned} i\mathcal{A}_\sigma = {} & \frac{G_F}{\sqrt{2}} V_{cb} V_{cs}^* (c_1 - c_2) \int d^4 z d^4 z' (i g_\sigma)^2 \langle \sigma(z)\sigma(z')\rangle \\ & \times (-1) \Gamma_{\alpha\rho} \Gamma_{\beta\delta} \Gamma^\sigma_{\gamma\gamma'} \Gamma^\sigma_{\eta\eta'} \\ & \times \langle \Xi_c(2980)^0 | \bar{c}^a_\alpha(0) \bar{s}^b_\beta(0) \bar{d}^c_\gamma(z)|0\rangle \\ & \times \langle \bar{\Lambda}_c^- | c^b_\delta(0) u^d_{\eta'}(z') d^c_{\gamma'}(z)|0\rangle \langle 0|\bar{u}^d_\eta(z) b^a_\rho(0)|B^-\rangle, \end{aligned} \tag{12}$$

with the Latin superscripts denoting the color indices, the Greek subscripts the Dirac indices, and $z_1 = z_1' = z_2 = y_2 = 0$, $z_3 = z_3' = z$, $y_1 = z_2' = z'$ in the position space for the constitute quarks. The propagator for the σ meson exchange is given by

$$\langle \sigma(z)\sigma(z')\rangle = \int \frac{d^4 p}{(2\pi)^4} \frac{i}{p^2 - m_\sigma^2 + i m_\sigma \Gamma_\sigma} e^{-ip\cdot(z-z')}. \tag{13}$$

We note that the factor of (-1) in Eq. (12) comes from quark reordering, $\Gamma_{\alpha\rho} = [\gamma_\mu(1-\gamma_5)]_{\alpha\rho}$, $\Gamma_{\beta\delta} = [\gamma^\mu(1-\gamma_5)]_{\beta\delta}$ from $\mathcal{H}_{\rm eff}$ in Eq. (9), and $\Gamma^\sigma_{\gamma\gamma'} = \Gamma^\sigma_{\eta\eta'} = 1$ from $\mathcal{L}_{\sigma qq}$ in Eq. (10). Note that the relevant Wilson coefficient is $(c_1 - c_2)$ rather than $a_2 = c_2 + c_1/3$ due to the totally antisymmetric color indices in the baryon wave function and in the antitriplet operator $(O_1 - O_2)$, which is indeed the case found in the pole model calculation [17].

To write down the matrix elements in Eq. (12) that are related to the wave functions of the B meson and charmed baryons, we first assign the four-momenta of B^-, $\Xi_c(2980)^0$, $\bar{\Lambda}_c$, and their constitute quarks as

$$\begin{aligned} & B\colon\ p_B = (p_B^+, p_B^-, \vec{0}_\perp), && \begin{cases} p_b = ((1-\xi)p_B^+, (1-\xi)p_B^-, \vec{0}_\perp), \\ p_l = (\xi p_B^+, \xi p_B^-, \vec{0}_\perp), \end{cases} \\ & \Xi_c(2980)\colon\ P = (p^+, p^-, \vec{0}_\perp), && \begin{cases} k_1 = (x_1 p^+, p^-, \vec{k}_{1\perp}), \\ k_2 = (x_2 p^+, 0, \vec{k}_{2\perp}), \\ k_3 = (x_3 p^+, 0, \vec{k}_{3\perp}), \end{cases} \\ & \bar{\Lambda}_c\colon\ P' = (p'^+, p'^-, \vec{0}_\perp), && \begin{cases} k_1' = (p'^+, x_1' p'^-, \vec{k}'_{1\perp}), \\ k_2' = (0, x_2' p'^-, \vec{k}'_{2\perp}), \\ k_3' = (0, x_3' p'^-, \vec{k}'_{3\perp}), \end{cases} \end{aligned} \tag{14}$$

where x_i (x_i') is the momentum fraction of the quark i in the charmed baryon $\Xi_c(2980)$ (Λ_c), and $\vec{k}'_{i\perp}$ the corresponding transverse momenta. Note that the light-cone momenta $p_B^\pm$ are equal to m_B in the B rest frame when the light quark masses are neglected. As discussed in the appendix, we will assume that $\Xi_c(2980)$ is a first positive-parity excitation with $J^P = \frac{1}{2}$, $L_\ell = 2$, and $J_\ell = 1$, where L_ℓ and J_ℓ are the orbital and total angular momenta of the two light quarks of $\Xi_c(2980)$. In terms of the explicit four-momenta in Eq. (14), the matrix elements involving B^-, D-wave $\Xi_c(2980)^0$, and S-wave Ξ_c and $\bar{\Lambda}_c$ are given by

$$\begin{aligned} \langle 0|\bar{u}^d_\eta(z') b^a_\rho(0)|B^-(p_B)\rangle = {} & -i\frac{\delta^{da}}{3}\frac{f_B}{4}[(\not{p}_B + m_B)\gamma_5]_{\rho\eta} \int_0^1 d\xi e^{-ip_l\cdot z'} \Phi_B(\xi), \\ \langle \Xi_c(2980)^0(P)|\bar{c}^a_\alpha(0)\bar{s}^b_\beta(0)\bar{d}^c_\gamma(z)|0\rangle = {} & \frac{\epsilon^{abc}}{6}\frac{f_{\Xi_c(2980)}}{4}[\bar{u}(P)\gamma_5\gamma_\mu]_\alpha \frac{1}{\sqrt{3}}\left\{C^{-1}\left[\sqrt{\frac{3}{20}}(\tilde{\not{k}}\tilde{K}^\mu + \tilde{\not{K}}\tilde{k}^\mu) - \sqrt{\frac{2}{30}}\tilde{k}\cdot\tilde{K}\left(\gamma^\mu - \frac{P^\mu}{m_{\Xi_c'}}\right)\right]\right. \\ & \left.\times(\not{P} + m_{\Xi_c'})\right\}_{\gamma\beta} \frac{2}{\beta^2}\int [dx][d^2k_\perp] e^{ik_3\cdot z}\Psi_{\Xi_c(2980)}(x_1, x_2, x_3, \vec{k}_{1\perp}, \vec{k}_{2\perp}, \vec{k}_{3\perp}), \\ \langle \Xi_c^0(P)|\bar{c}^a_\alpha(0)\bar{s}^b_\beta(0)\bar{d}^c_\gamma(z)|0\rangle = {} & \frac{\epsilon^{abc}}{6}\frac{f_{\Xi_c}}{4}[\bar{u}(P)]_\alpha[C^{-1}\gamma_5(\not{P} + m_{\Xi_c})]_{\gamma\beta}\int [dx][d^2k_\perp] e^{ik_3\cdot z}\Psi_{\Xi_c}(x_1, x_2, x_3, \vec{k}_{1\perp}, \vec{k}_{2\perp}, \vec{k}_{3\perp}), \end{aligned} \tag{15}$$

$$\begin{aligned} \langle \bar{\Lambda}_c(P')|c^b_\delta(0) u^d_{\eta'}(z') d^c_{\gamma'}(z)|0\rangle = {} & \frac{\epsilon^{bdc}}{6}\frac{f_{\Lambda_c}}{4}[\bar{v}(P')]_\delta[(\not{P}' - m_{\Lambda_c})\gamma_5 C]_{\eta'\gamma'}\int [dx'] \\ & \times [d^2k'_\perp] e^{i(k_2'\cdot z' + k_3'\cdot z)}\Psi_{\Lambda_c}(x_1', x_2', x_3', \vec{k}'_{1\perp}, \vec{k}'_{2\perp}, \vec{k}'_{3\perp}), \end{aligned}$$

with the decay constants f_B, $f_{\Xi_c(2980)}$, f_{Λ_c}, the charge conjugation matrix C, and

$$[dx^{(\prime)}] = dx_1^{(\prime)} dx_2^{(\prime)} dx_3^{(\prime)} \delta\Big(1 - \sum_{i=1}^{3} x_i^{(\prime)}\Big), \qquad [d^2 k_\perp^{(\prime)}] = d^2 k_{1\perp}^{(\prime)} d^2 k_{2\perp}^{(\prime)} d^2 k_{3\perp}^{(\prime)} \delta^2(\vec{k}_{1\perp}^{(\prime)} + \vec{k}_{2\perp}^{(\prime)} + \vec{k}_{3\perp}^{(\prime)}),$$
$$k = \frac{1}{2}(k_2 - k_3), \qquad K = \frac{1}{2}(k_2 + k_3 - 2k_1), \tag{16}$$

where $\tilde{A} \equiv A - P(P \cdot A)/m_{\Xi_c'}^2$ for $A = k$ or K. Recall that k_2 and k_3 are the four-momenta of the two light quarks in $\Xi_c(2980)$. The wave functions of Ξ_c and Λ_c are taken from [18]. The derivation of the structures of the matrix elements involving the D-wave $\Xi_c(2980)$ is shown in the appendix.

The amplitude A_σ in Eq. (12) then becomes

$$\mathcal{A}_\sigma = \frac{ig_\sigma^2}{18 \times 4^3} \frac{G_F}{\sqrt{2}} V_{cb} V_{cs}^* (c_1 - c_2) f_B f_{\Xi_c(2980)} f_{\Lambda_c} \frac{2}{\sqrt{3}\beta^2} \int d\xi \int [dx][d^2k_\perp][dx'][d^2k'_\perp](2\pi)^4 \delta^4(k_3 + k'_3 + k'_2 - p_l)$$
$$\times \frac{1}{(k_3 + k'_3)^2 - m_\sigma^2 + im_\sigma\Gamma_\sigma} \Phi_B(\xi) \Psi_{\Xi_c(2980)}(x_1, x_2, x_3, \vec{k}_{1\perp}, \vec{k}_{2\perp}, \vec{k}_{3\perp}) \Psi_{\Lambda_c}(x'_1, x'_2, x'_3, \vec{k}'_{1\perp}, \vec{k}'_{2\perp}, \vec{k}'_{3\perp}) \bar{u}\gamma_5\gamma_\mu$$
$$\times \Gamma[(\not{p}_B + m_B)\gamma_5]\Gamma^\sigma[(\not{P}' - m_{\Lambda_c})\gamma_5 C]\Gamma^\sigma\Big\{C^{-1}\Big[\sqrt{\frac{3}{20}}(\tilde{\not{k}}\tilde{K}^\mu + \tilde{\not{K}}\tilde{k}^\mu) - \sqrt{\frac{2}{30}}\tilde{k}\cdot\tilde{K}\Big(\gamma^\mu - \frac{P^\mu}{m_{\Xi_c'}}\Big)\Big](\not{P} + m_{\Xi_c'})\Big\}\Gamma v, \tag{17}$$

where the delta function $\delta^4(k_3 + k'_3 + k'_2 - p_l)$ in light cone is presented as

$$\delta^4(k_3 + k'_3 + k'_2 - p_l) = -2\frac{1}{p^+}\frac{1}{p'^-}\delta\Big(x_3 - \frac{\xi p_B^+}{p^+}\Big)\delta\Big(x'_3 + x'_2 - \frac{\xi p_B^-}{p'^-}\Big)\delta^2(\vec{k}'_{2\perp} + \vec{k}_{3\perp} + \vec{k}'_{3\perp}). \tag{18}$$

After integrating over the variables with the δ functions, we are led to

$$\mathcal{A}_\sigma = \frac{ig_\sigma^2}{18 \times 4^3} \frac{G_F}{\sqrt{2}} V_{cb} V_{cs}^* (c_1 - c_2) f_B f_{\Xi_c(2980)} f_{\Lambda_c} 2(2\pi)^4 \frac{1}{2^3}(2\pi) \int_0^{p'^-/p_B^-} d\xi \int_0^{1-\xi p_B^+/p^+} \frac{dx_2}{p^+} \int_0^{\xi p_B^-/p'^-} \frac{dx'_2}{p'^-}$$
$$\times \int_0^\infty dk_{2\perp}^2 \int_0^\infty dk_{3\perp}^2 \int_0^\infty dk_{3\perp}'^2 \int_0^{2\pi} d\theta_{23} \int_0^{2\pi} d\theta_{33'} \Phi_B(\xi) \Psi_{\Xi_c(2980)}(x_1, x_2, x_3, \vec{k}_{1\perp}, \vec{k}_{2\perp}, \vec{k}_{3\perp})$$
$$\times \Psi_{\Lambda_c}(x'_1, x'_2, x'_3, \vec{k}'_{1\perp}, \vec{k}'_{2\perp}, \vec{k}'_{3\perp}) \frac{\bar{u}(a_\sigma + b_\sigma\gamma_5)v}{(k_3 + k'_3)^2 - m_\sigma^2 + im_\sigma\Gamma_\sigma}, \tag{19}$$

where

$$a_\sigma = \frac{12}{\sqrt{5}\beta^2}\Big[(k_{2\perp}^2 - k_{3\perp}^2) + \frac{m_{\Xi_c'}^2}{4}(x_2^2 - x_3^2)\Big] m_{\Xi_c'} \times (m_B + m_{\Lambda_c} + m_{\Xi_c'})(m_B + m_{\Lambda_c} - m_{\Xi_c'}),$$
$$b_\sigma = \frac{-12}{\sqrt{5}\beta^2}\Big[(k_{2\perp}^2 - k_{3\perp}^2) + \frac{m_{\Xi_c'}^2}{4}(x_2^2 - x_3^2)\Big] m_{\Xi_c'}^2 m_{\Lambda_c}, \tag{20}$$

with

$$x_1^{(\prime)} = 1 - x_2^{(\prime)} - x_3^{(\prime)}, \qquad x_3 = \frac{\xi p_B^+}{p^+}, \qquad x'_3 = \frac{\xi p_B^-}{p'^-} - x'_2,$$
$$\vec{k}_{1\perp} = -(\vec{k}_{2\perp} + \vec{k}_{3\perp}), \qquad \vec{k}'_{1\perp} = \vec{k}_{3\perp}. \tag{21}$$

Note that θ_{23} and $\theta_{33'}$ are the angles of $\vec{k}_{2\perp}$ and of $\vec{k}'_{3\perp}$ as measured against $\vec{k}_{3\perp}$, respectively. We can also obtain A_{π^0} and $A_{\pi^\pm}$ in Eq. (11) by replacing the notation of σ in Eq. (19) by π^0 and $\pi^\pm$, respectively, a_σ and b_σ by

$$a_{\pi^{0(\pm)}} = \frac{-12}{\sqrt{5}\beta^2}\Big[(k_{2\perp}^2 - k_{3\perp}^2) + \frac{m_{\Xi_c'}^2}{4}(x_2^2 - x_3^2)\Big] m_{\Xi_c'} \times (m_B - m_{\Lambda_c} + m_{\Xi_c'})(m_B + m_{\Lambda_c} - m_{\Xi_c'}),$$
$$b_{\pi^{0(\pm)}} = \frac{12}{\sqrt{5}\beta^2}\Big[(k_{2\perp}^2 - k_{3\perp}^2) + \frac{m_{\Xi_c'}^2}{4}(x_2^2 - x_3^2)\Big] m_{\Xi_c'} \times (m_B^2 + m_{\Lambda_c}^2 - m_{\Xi_c'}^2 + m_{\Xi_c'} m_{\Lambda_c}), \tag{22}$$

and $\Gamma^{\pi^0}_{\eta\eta'} = -\Gamma^{\pi^\pm}_{\eta\eta'} = -\Gamma^{\pi^0}_{\gamma\gamma'} = \Gamma^{\pi^\pm}_{\gamma\gamma'} = i\gamma_5$. The amplitude of $\bar{B}^0 \to \Lambda_c^+ \bar{\Lambda}_c^-$ is similar to that of $\bar{B} \to \Xi_c \bar{\Lambda}_c$ studied in [8] except for the CKM matrix element being replaced by $V_{cb}V_{cd}^*$, and its a_σ, b_σ, $a_{\pi^{0(\pm)}}$, $b_{\pi^{0(\pm)}}$ are given by

$$a_\sigma = -4m_{\Lambda_c} m_B(m_B + 2m_{\Lambda_c}),$$
$$b_\sigma = 4m_{\Lambda_c}(m_B - 2m_{\Lambda_c})(m_B + 2m_{\Lambda_c}),$$
$$a_{\pi^{0(\pm)}} = 4m_{\Lambda_c} m_B^2,$$
$$b_{\pi^{0(\pm)}} = -4m_{\Lambda_c} m_B(m_B - 2m_{\Lambda_c}). \tag{23}$$

Once the explicit expressions for the wave functions Φ_B,

$\Psi_{\Xi_c(2980)}$, Ψ_{Λ_c} and other parameters are given, we are ready to carry out the numerical analysis.

III. NUMERICAL ANALYSIS

To proceed with the numerical calculations, we need to specify the relevant wave functions. For the B meson, it is given by [19]

$$\Phi_B(\xi) = N_B \xi^2 (1-\xi^2) \exp\left[-\frac{1}{2}\frac{\xi^2 m_B^2}{\omega_B^2}\right], \tag{24}$$

with $\omega_B = 0.38 \pm 0.04$ GeV, where N_B is determined by the normalization

$$\int_0^1 d\xi \Phi_B(\xi) = 1. \tag{25}$$

For the charmed baryon, such as D-wave $\Xi_c(2980)$ and S-wave Λ_c, we assume that their wave functions have a similar expression [20]

$$\Psi_{\mathbf{B_c}}(x_i, \vec{k}_{i\perp}) = \frac{N_{\mathbf{B_c}}}{(2\pi\beta^2)^2} \prod_{i=1}^{3} \exp\left[-\frac{\vec{k}_{i\perp}^2 + \hat{m}_i^2}{2\beta^2 x_i}\right], \tag{26}$$

with $\beta = 0.96 \pm 0.04$ GeV and $\hat{m}_i$ the mass of the constitute quark i, where $N_{\mathbf{B_c}}$ is given by the normalization

$$\int [dx][dk_\perp^2] \Psi_{\mathbf{B_c}}(x_i, \vec{k}_{i\perp}) = \int [dx] N_{\mathbf{B_c}} \prod_{i=1}^{3} x_i \exp\left[-\frac{\hat{m}_i^2}{2\beta^2 x_i}\right] = 1. \tag{27}$$

For the decay constants, f_{Λ_c} can be related to the decay constant of the Λ_b by the relation $f_{\mathbf{B_c}} m_{\mathbf{B_c}} = f_{\Lambda_b} m_{\Lambda_b}$ [21], and we let $f_{\Xi_c(2980)} \simeq f_{\Xi_c}$ due to the lack of information on the decay constant of the D-wave charmed baryon. For other input parameters, see Table II.

TABLE II. Summary of the input parameters.

$\omega_B = 0.38 \pm 0.04$ GeV	$f_B = 0.2$ GeV
$\beta = 0.96 \pm 0.04$ GeV	$f_{\Xi_c(2980)} \simeq f_{\Xi_c}$
$\hat{m}_s = 0.46 \pm 0.06$ GeV	$f_{\Xi_c} = 6.2 \times 10^{-3}$ GeV^2
$\hat{m}_{u(d)} = 0.26 \pm 0.04$ GeV	$f_{\Lambda_c} = 6.7 \times 10^{-3}$ GeV^2
$m_{\Xi_c(2980)} = 2.93$ GeV [5]	$\Gamma_\sigma = 0.6$ GeV
$g_\sigma = 3.35$ [8]	$\Gamma_{\pi^0} = 7.8 \times 10^{-9}$ GeV
$g_\pi = \sqrt{2} g_{\pi^0} = 4.19$ [8]	$\Gamma_{\pi^\pm} = 2.5 \times 10^{-17}$ GeV

For the two-body baryonic B decay amplitude given by

$$A(B \to \mathbf{B_c} \bar{\mathbf{B}}'_\mathbf{c}) = \bar{u}(A + B\gamma_5)v, \tag{28}$$

the decay rate reads [22]

$$\Gamma(B \to \mathbf{B_c} \bar{\mathbf{B}}'_\mathbf{c}) = \frac{p_c}{4\pi m_B^2}\{|A|^2[m_B^2 - (m_{\mathbf{B}'_\mathbf{c}} + m_{\mathbf{B_c}})^2] + |B|^2[m_B^2 - (m_{\mathbf{B}'_\mathbf{c}} - m_{\mathbf{B_c}})^2]\}, \tag{29}$$

where p_c is the center-of-mass momentum. To obtain the rate for $\bar{B} \to \Lambda_c^+ \bar{\Lambda}_c^- \bar{K}$, we shall use $\mathcal{B}(\Xi_c(2980) \to \Lambda_c \bar{K}) = 0.5$ derived from the $^3P_0$ model [23]. The calculated results for $\bar{B} \to \Xi_c \bar{\Lambda}_c$, $\bar{B}^0 \to \Lambda_c^+ \bar{\Lambda}_c^-$, and $\bar{B} \to \Lambda_c^+ \bar{\Lambda}_c^- \bar{K}$ are summarized in Table III.

IV. DISCUSSION AND CONCLUSION

In this work we have studied the doubly charmful two-body and three-body baryonic B decays $B \to \Lambda_c^+ \bar{\Lambda}_c^-$ and $\bar{B} \to \Lambda_c^+ \bar{\Lambda}_c^- \bar{K}$. For the former decay, our prediction for its branching ratio of order 5×10^{-5} (see Table III) is consistent with the extrapolation from $\mathcal{B}(\bar{B}^0 \to \Xi_c^+ \bar{\Lambda}_c^-)$ and from $\mathcal{B}(\bar{B}^0 \to \Lambda_c \bar{p})$ provided that the dynamical suppression of $\Lambda_c \bar{p}$ relative to $\Lambda_c \bar{\Lambda}_c$ is taken into account. As pointed out before, a naive estimate of the branching ratio $\mathcal{O}(10^{-8})$ for the decay $\bar{B} \to \Lambda_c^+ \bar{\Lambda}_c^- \bar{K}$ is too small by 3 to 4 orders of magnitude compared to experiment. Previously, it has been shown that a large enhancement for the $\Lambda_c^+ \bar{\Lambda}_c^- \bar{K}$ production can occur due to a charmoniumlike resonance (for example, the $X(4630)$ state discovered by Belle) with a mass near the $\Lambda_c \bar{\Lambda}_c$ threshold. Motivated by the *BABAR*'s observation of a resonance in the $\Lambda_c \bar{K}$ system with a mass of order 2930 MeV, we have studied the contribution to $B \to \Lambda_c^+ \bar{\Lambda}_c^- K$ from the intermediate state $\Xi_c(2980)$ which is postulated to be a first positive-parity excited D-wave charmed baryon state. Assuming that a soft $q\bar{q}$ quark pair is produced through the σ and π meson exchanges in the configuration for $B \to \Xi_c(2980)\bar{\Lambda}_c$, it is found that the branching ratio of $\bar{B} \to \Lambda_c^+ \bar{\Lambda}_c^- \bar{K}$ is of order 3.5×10^{-4}. This is in agreement with experiment for $\Lambda_c \bar{\Lambda}_c \bar{K}^0$, but slightly smaller for $\Lambda_c \bar{\Lambda}_c K^-$. In conjunction with the previous analysis [8], we conclude that the enormously

TABLE III. Branching ratios (in units of 10^{-4}) of doubly charmful two-body and three-body baryonic B decays, where the first and second theoretical errors come from β and ω_B, while the third and fourth errors are from $\hat{m}_{u(d)}$ and $\hat{m}_s$, respectively. Use of $\mathcal{B}(\Xi_c(2980) \to \Lambda_c \bar{K}) = 0.5$ has been made to derive the rate of $\bar{B} \to \Lambda_c \bar{\Lambda}_c \bar{K}$.

	Theory	*BABAR*	Belle	Average
$B^- \to \Xi_c^0 \bar{\Lambda}_c^-$	$10.4^{+3.8+0.3+4.2+0.3}_{-3.6-1.8-3.5-1.3}$	$16 \pm 7^{+9}_{-4}$	$37 \pm 15^{+21}_{-9}$	$20 \pm 6^{+11}_{-5}$
$\bar{B}^0 \to \Xi_c^+ \bar{\Lambda}_c^-$	$9.4^{+4.6+0.4+4.3+0.5}_{-2.6-0.8-3.0-0.4}$	$3.8 \pm 3.1^{+8.7}_{-2.4} < 14$	$24 \pm 12^{+53}_{-15}$	-
$B^- \to \Lambda_c^+ \bar{\Lambda}_c^- K^-$	$3.6^{+1.0+0.8+1.5+0.5}_{-1.0-1.0-1.2-0.7}$	11.4 ± 6.4	6.5 ± 3.7	7.7 ± 3.2
$\bar{B}^0 \to \Lambda_c^+ \bar{\Lambda}_c^- \bar{K}^0$	$3.3^{+1.2+0.8+0.9+0.2}_{-0.9-0.9-1.1-0.6}$	3.8 ± 3.0	7.9 ± 5.2	5.2 ± 3.0
$\bar{B}^0 \to \Lambda_c^+ \bar{\Lambda}_c^-$	$0.52^{+0.23+0.06+0.26+0}_{-0.11-0.03-0.15-0}$	-	$0.22^{+0.26}_{-0.21} < 0.62$	<0.62

large rate of $\bar{B} \to \Lambda_c^+ \bar{\Lambda}_c^- \bar{K}$ arises from the resonances $\Xi_c(2980)$ and $X(4630)$.

We have also presented updated results for $\bar{B} \to \Xi_c \bar{\Lambda}_c$ which are slightly smaller than our previous analysis [8] but are consistent with experiment.

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APPENDIX A

It has been conjectured that $\Xi_c(2980)$ is likely to be a first positive-parity excitation with $J^P = \frac{1}{2}^+$ [24]. Denoting the quantum numbers L_k and L_K as the eigenvalues of $\vec{L}_k^2$ and $\vec{L}_K^2$, respectively, the k-orbital momentum L_k describes relative orbital excitations of the two light quarks, and the K-orbital momentum L_K describes orbital excitations of the center of the mass of the two light quarks relative to the heavy quark [25]. The first positive-parity excitations are those states with $L_K + L_k = 2$. According to Table IV of [24], possible antitriplet candidates for $\Xi_c(2980)$ are $\tilde{\Xi}_{c1}(\frac{1}{2}^+)$, $\tilde{\Xi}''_{c0}(\frac{1}{2}^+)$, $\tilde{\Xi}''_{c1}(\frac{1}{2}^+)$, and $\tilde{\Xi}'''_{c1}(\frac{1}{2}^+)$, where the quantum number in the subscript labels J_ℓ, the total angular momentum of the two light quarks. (We use a tilde to denote states with antisymmetric orbital wave functions (i.e. $L_K = L_k = 1$) under the interchange of two light quarks.) Strong decays of these four states have been studied in [23] using the $^3P_0$ model. It turns out that $\Gamma(\tilde{\Xi}_{c1}(\frac{1}{2}^+)) \approx 3.2$ MeV is too small and $\Gamma(\tilde{\Xi}''_{c1}(\frac{1}{2}^+)) \approx 148$ MeV is too large compared to the experimental value of order 30 MeV [see Eq. (7)], while $\tilde{\Xi}''_{c0}(\frac{1}{2}^+)$ does not decay into $\Xi_c\pi$ and $\Lambda_c\bar{K}$. Therefore, the favored candidate is $\tilde{\Xi}'''_{c1}(\frac{1}{2}^+)$ which has $L_\ell = 2$ and $J_\ell = 1$.

We use the light-front approach to obtain the structure of the matrix element for the D-wave charmed baryon as shown in Eq. (15). In the light-front formalism, the charmed baryon bound state with the total momentum P, spin $J = 1/2$, and the angular momentum of the light quark pair $\vec{J}_l \equiv \vec{S}_l + \vec{L}_l$ with $\vec{S}_l \equiv \vec{S}_2 + \vec{S}_3$ and $\vec{L}_l \equiv \vec{L}_k + \vec{L}_K$ can be written as (see, for example [26–28])

$$\begin{aligned}
&|\mathbf{B}_c(P, \{L_k, L_K, L_l, S_l, J_l\}, J, , J_z)\rangle \\
&\quad = \int \{d^3p_1\}\{d^3p_2\}\{d^3p_3\} \frac{2(2\pi)^3}{\sqrt{P^+}} \delta^3(\bar{P} - \bar{p}_1 - \bar{p}_2 - \bar{p}_3) \\
&\quad \times \sum_{\lambda_i, \alpha, \beta, \gamma, a, b} \Psi^{JJ_z}_{\{L\}}(x_1, x_2, x_3, k_{1\perp}, k_{2\perp}, k_{3\perp}, \lambda_1, \lambda_2, \lambda_3) \\
&\quad \times C^{abc} F^{\{L\}}_{ff'} |c^a(p_1, \lambda_1) q^b_f(p_2, \lambda_2) q^c_{f'}(p_3, \lambda_3)\rangle, \qquad \text{(A1)}
\end{aligned}$$

where a, b, c and f, f' are color and flavor indices, respectively, $\lambda_{1,2,3}$ denote helicities, $\bar{p}_1$, $\bar{p}_2$, and $\bar{p}_3$ are the on-mass shell light-front momenta,

$$\bar{p} = (p^+, p_\perp), \qquad p_\perp = (p^1, p^2), \qquad p^- = \frac{m^2 + p_\perp^2}{p^+}, \tag{A2}$$

and

$$\begin{aligned}
&\{d^3p\} \equiv \frac{dp^+ d^2p_\perp}{2(2\pi)^3}, \qquad \delta^3(\bar{p}) = \delta(p^+)\delta^2(p_\perp), \\
&|c(p_1, \lambda_1) q_f(p_2, \lambda_2) q_{f'}(p_3, \lambda_3)\rangle \\
&\quad = b^\dagger_{c\lambda_1}(p_1) b^\dagger_{q_f\lambda_2}(p_2) b^\dagger_{q_{f'}\lambda_3}(p_3)|0\rangle, \\
&\{b_{q'\lambda'}(p'), b^\dagger_{q\lambda}(p)\} = 2(2\pi)^3 \delta^3(\bar{p}' - \bar{p}) \delta_{\lambda'\lambda} \delta_{q'q}.
\end{aligned} \tag{A3}$$

The coefficients $C^{abc} = \epsilon^{abc}/6$ and $F^{\{L\}}_{ff'}$ are normalized color factor and flavor coefficient, respectively.

In terms of the light-front relative momentum variables $(x_i, k_{i\perp})$ for $i = 1, 2, 3$ defined by

$$\begin{aligned}
&p_i^+ = x_i P^+, \qquad \sum_{i=1}^3 x_i = 1, \\
&p_{i\perp} = x_i P_\perp + k_{i\perp}, \qquad \sum_{i=1}^3 k_{i\perp} = 0,
\end{aligned} \tag{A4}$$

the momentum-space wave function $\Psi^{JJ_z}_{\{L\}}$ can be expressed as

$$\begin{aligned}
\Psi^{JJ_z}_{\{L\}}(x_i, k_{i\perp}, \lambda_i) = {} & (\Pi_{i=1}^3 \langle \lambda_i | \mathcal{R}^\dagger_M(x_i, k_{i\perp}, m_i) | s_i \rangle) \\
& \times \langle J_l S_1; j_l s_1 | J_l S_1; J J_z \rangle \\
& \times \langle S_l L_l; s_l l_l | S_l L_1; J_l j_l \rangle \\
& \times \langle L_k L_K; l_k l_K | L_k L_K; L_l l_l \rangle \\
& \times \langle S_2 S_3; s_2 s_3 | S_2 S_3; S_l s_l \rangle \\
& \times \phi_{L_k L_K, l_k l_K}(x_1, x_2, x_3, k_{1\perp}, k_{2\perp}, k_{3\perp}),
\end{aligned} \tag{A5}$$

where $\phi_{l_k l_K}(x_1, x_2, x_3, k_{1\perp}, k_{2\perp}, k_{3\perp})$ describes the momentum distribution of the constituents in the bound state with the subsystem consisting of the particles 2 and 3 in the orbital angular momentum $(L_k)_z = l_k$, $(L_K)_z = l_K$ state, $\langle J_l S_1; j_l s_1 | J_l S_1; J J_z \rangle$, and so on are Clebsch-Gordan coefficients and $\langle \lambda_i | \mathcal{R}^\dagger_M(x_i, k_{1\perp}, m_i) | s_i \rangle$ is the well normalized Melosh transform matrix element. Explicitly [29,30],

$$\begin{aligned}
\langle \lambda_i | \mathcal{R}^\dagger_M(x_i, k_{i\perp}, m_i) | s_i \rangle &= \frac{\bar{u}(k_i, \lambda) u_D(k_i, s_i)}{2m_i} \\
&= -\frac{\bar{v}(k_i, \lambda) v_D(k_i, s_i)}{2m_i} \\
&= \frac{(m_i + x_i M_0)\delta_{\lambda_i s_i} - i\vec{\sigma}_{\lambda_i s_i} \cdot \vec{k}_{i\perp} \times \vec{n}}{\sqrt{(m_i + x_i M_0)^2 + k_{i\perp}^2}},
\end{aligned} \tag{A6}$$

with $u_{(D)}$ and $v_{(D)}$ Dirac spinors in the light-front (instant) form, $\vec{n} = (0, 0, 1)$, a unit vector in the z direction, and

$$M_0^2 = \sum_{i=1}^{3} \frac{m_i^2 + k_{i\perp}^2}{x_i},$$
$$k_i = \left(\frac{m_i^2 + k_{i\perp}^2}{x_i M_0}, x_i M_0, k_{i\perp}\right) = (e_i - k_{iz}, e_i + k_{iz}, k_{i\perp}),$$
$$M_0 = e_1 + e_2 + e_3,$$
$$e_i = \sqrt{m_i^2 + k_{i\perp}^2 + k_{iz}^2} = \frac{x_i M_0}{2} + \frac{m_i^2 + k_{i\perp}^2}{2 x_i M_0},$$
$$k_{iz} = \frac{x_i M_0}{2} - \frac{m_i^2 + k_{i\perp}^2}{2 x_i M_0}. \tag{A7}$$

Note that $u_D(k_i, s_i) = u(k_i, \lambda_i)\langle\lambda_i|\mathcal{R}_M^\dagger|s_i\rangle$ and, consequently, the state $|q(k_i, \lambda_i)\rangle\langle\lambda_i|\mathcal{R}_M^\dagger|s_i\rangle$ transforms like $|q(k_i, s_i)\rangle$ under rotation, i.e. its transformation does not depend on its momentum. A crucial feature of the light-front formulation of a bound state, such as the one shown in Eq. (A1), is the frame independence of the light-front wave function [29,31]. Namely, the hadron can be boosted to any (physical) $(P^+, P_\perp)$ without affecting the internal variables $(x_i, k_{\perp i})$ of the wave function, which is certainly not the case in the instant-form formulation.

In practice it is more convenient to use the covariant form for the Melosh transform matrix element

$$\langle\lambda_1|\mathcal{R}_M^\dagger(x_1, k_{1\perp}, m_1)|s_1\rangle\langle J_l S_1; j_l s_1|J_l S_1; J J_z\rangle$$
$$= \frac{M_0}{\sqrt{2(p_1 \cdot \bar{P} + m_1 M_0)}}\bar{u}(p_1, \lambda_1)\Gamma_{J_l j_l} u_{\mathbf{B_c}}(\bar{P}, J_z),$$
$$\langle\lambda_2|\mathcal{R}_M^\dagger(x_2, k_{2\perp}, m_2)|s_2\rangle\langle\lambda_3|\mathcal{R}_M^\dagger(x_3, k_{3\perp}, m_3)|s_3\rangle$$
$$\times \langle S_2 S_3; s_2 s_3|S_2 S_3; S_l s_l\rangle$$
$$= \frac{1}{2M_0\sqrt{2(p_2 \cdot \bar{P} + m_2 M_0)(p_2 \cdot \bar{P} + m_2 M_0)}}\bar{u}(p_2, \lambda_2)$$
$$\times (\bar{\not{P}} + M_0)\gamma_5 \Gamma_{S_l s_l} v(p_3, \lambda_3), \tag{A8}$$

with $S_i = J = 1/2$ and

$$\Gamma_{00} = 1, \qquad \Gamma_{1m} = -\frac{1}{\sqrt{3}}\gamma_5 \not{\varepsilon}^*(\bar{P}, m),$$
$$\bar{P} \equiv \bar{p}_1 + \bar{p}_2 + \bar{p}_3,$$
$$\varepsilon^\mu(\bar{P}, \pm 1) = \left[\frac{2}{P^+}\vec{\varepsilon}_\perp(\pm 1)\cdot\vec{P}_\perp, 0, \vec{\varepsilon}_\perp(\pm 1)\right],$$
$$\vec{\varepsilon}_\perp(\pm 1) = \mp(1, \pm i)/\sqrt{2},$$
$$\varepsilon^\mu(\bar{P}, 0) = \frac{1}{M_0}\left(\frac{-M_0^2 + P_\perp^2}{P^+}, P^+, P_\perp\right), \tag{A9}$$

for states with $J_l, S_l = 0$ or 1, see [26,27] for the derivation the above expressions. The following identities will be useful later:

$$\langle 11; m'm''|11; 2m\rangle\langle 12; m'''m|12; 1n\rangle$$
$$\times \varepsilon_\mu(\bar{P}, m')\varepsilon_\nu(\bar{P}, m'')\varepsilon_\rho(\bar{P}, m''')$$
$$= -\sqrt{\frac{3}{20}}[\varepsilon_\mu^*(\bar{P}, m)\varepsilon_\nu(\bar{P}, n)\varepsilon_\rho(\bar{P}, m)$$
$$+ \varepsilon_\mu(\bar{P}, n)\varepsilon_\nu^*(\bar{P}, m)\varepsilon_\rho(\bar{P}, m)]$$
$$+ \sqrt{\frac{2}{30}}\varepsilon_\mu^*(\bar{P}, m)\varepsilon_\nu(\bar{P}, m)\varepsilon_\rho(\bar{P}, n),$$
$$\varepsilon_\mu^*(\bar{P}, m)\varepsilon_\nu(\bar{P}, m) = -g_{\mu\nu} + \frac{\bar{P}_\mu \bar{P}_\nu}{M_0^2}, \tag{A10}$$

where $\epsilon^*(m) = (-)^m \epsilon(-m)$ is used in the first identity, and $\tilde{A}_\mu$ is defined as $-\varepsilon_\mu^*(\bar{P}, m)[\varepsilon(\bar{P}, m)\cdot A]$ for later purposes.

Under the constraint of $1 - \sum_{i=1}^3 x_i = \sum_{i=1}^3 (k_i)_{x,y,z} = 0$, we have the expressions

$$\phi_{L_k L_K, l_k l_K}(\{x\}, \{k_\perp\}) = \left(\frac{3}{2}\right)^{3/2}\sqrt{\frac{\partial(k_{2z}, k_{3z})}{\partial(x_2, x_3)}}\varphi_{L_K l_K}(\vec{K}, \beta_K)$$
$$\times \varphi_{L_k l_k}(\vec{k}, \beta_k),$$
$$\varphi_{00}(\vec{\kappa}, \beta) = \varphi(\vec{\kappa}, \beta),$$
$$\varphi_{1m}(\vec{\kappa}, \beta) = \kappa_m \varphi_p(\vec{\kappa}, \beta), \tag{A11}$$

where $\kappa_m = \vec{\varepsilon}(\bar{P}, m)\cdot\vec{\kappa}$, or explicitly $\kappa_{m=\pm 1} = \mp(\kappa_{\perp x} \pm i\kappa_{\perp y})/\sqrt{2}$, $\kappa_{m=0} = \kappa_z$ with $\kappa_{x,y,z}$ in the rest frame of $\bar{P}$, are proportional to the spherical harmonics Y_{1m} in the momentum space, and φ, φ_p are the distribution amplitudes of S-wave and P-wave states, respectively, and the factor $(3/2)^{3/2}\sqrt{\partial(k_{2z}, k_{3z})/\partial(x_2, x_3)}$ in Eq. (A11) is a normalization factor. For a Gaussian-like wave function, one has [26,28]

$$\varphi(\vec{\kappa}, \beta) = 4\left(\frac{\pi}{\beta^2}\right)^{(3/4)}\exp\left(-\frac{\kappa_z^2 + \kappa_\perp^2}{2\beta^2}\right),$$
$$\varphi_p(\vec{\kappa}, \beta) = \sqrt{\frac{2}{\beta^2}}\varphi(\vec{\kappa}, \beta). \tag{A12}$$

By using the above equations it is straightforward to obtain

$$(\Pi_i\langle\lambda_i|\mathcal{R}_M^\dagger(x_i, k_{i\perp}, m_i)|s_i\rangle)\langle J_l S_1; j_l s_1|J_l S_1; J J_z\rangle$$
$$\times \langle S_l L_l; s_l l_l|S_l L_1; J_l j_l\rangle\langle L_k L_K; l_k l_K|L_k L_K; L_l l_l\rangle$$
$$\times \langle S_2 S_3; s_2 s_3|S_2 S_3; S_l s_l\rangle$$
$$= \frac{1}{4\sqrt{\Pi_i(p_i \cdot \bar{P} + m_i M_0)}}\bar{u}(p_1, \lambda_1)u_{\mathbf{B_c}}(\bar{P}, J_z)\bar{u}(p_2, \lambda_2)$$
$$\times (\bar{\not{P}} + M_0)\gamma_5 C\bar{u}^T(p_3, \lambda_3) \tag{A13}$$

for $J_l = S_l = L_l = L_k = L_K = 0$ and

$$(\Pi_i \langle \lambda_i | \mathcal{R}_M^\dagger(x_i, k_{i\perp}, m_i) | s_i \rangle) \langle J_l S_1; j_l s_1 | J_l S_1; J J_z \rangle \langle S_l L_l; s_l l_l | S_l L_1; J_l j_l \rangle \langle L_k L_K; l_k l_K | L_k L_K; L_l l_l \rangle \langle S_2 S_3; s_2 s_3 | S_2 S_3; S_l s_l \rangle$$
$$\times \frac{\sqrt{2} k_{l_k}}{\beta_k} \frac{\sqrt{2} K_{l_K}}{\beta_K}$$
$$= \frac{-1}{\beta_k \beta_K \sqrt{12 \Pi_i (p_i \cdot \bar{P} + m_i M_0)}} \bar{u}(p_1, \lambda_1) \gamma_5 \gamma_\mu u_{\mathbf{B_c}}(\bar{P}, J_z) \bar{u}(p_2, \lambda_2)(\bar{\not{P}} + M_0)$$
$$\times \left\{ \sqrt{\frac{3}{20}} (\tilde{\not{k}} \tilde{K}^\mu + \tilde{\not{K}} \tilde{k}^\mu) - \sqrt{\frac{2}{30}} \tilde{k} \cdot \tilde{K} \left(\gamma^\mu - \frac{\bar{P}^\mu}{M_0} \right) \right\} C \bar{u}^T(p_3, \lambda_3) \tag{A14}$$

for $L_l = 2$, $L_k = L_K = S_l = 1$. Note that the factors of $k_m = \varepsilon(\bar{P}, m) \cdot (\bar{p}_2 - \bar{p}_3)/2$, and $K_m = \varepsilon(\bar{P}, m) \cdot (\bar{p}_2 + \bar{p}_3 - 2\bar{p}_1)/2$ come from the wave function Eq. (A11) for the $L_k = L_K = 1$ case. Promoting $\bar{P} \to P$ and $M_0 \to M$ and taking Hermitian conjugation (to change the initial state to the final state) we obtain the structure of the matrix element for the S-wave and D-wave charmed baryons as shown in Eq. (15). Note that for simplicity we have taken $\beta = \beta_k = \beta_K$ in Eq. (15).

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