Minimally allowed neutrinoless double beta decay rates within an anarchical framework

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Neutrinoless double beta decay ($\beta\beta0\nu$) is the only realistic probe of the Majorana nature of the neutrino. In the standard picture, its rate is proportional to m_{ee}, the e-e element of the Majorana neutrino mass matrix in the flavor basis. I explore minimally allowed m_{ee} values within the framework of mass matrix anarchy where neutrino parameters are defined statistically at low energies. Distributions of mixing angles are well defined by the Haar integration measure, but masses are dependent on arbitrary weighting functions and boundary conditions. I survey the integration measure parameter space and find that for sufficiently convergent weightings, m_{ee} is constrained between (0.01–0.4) eV at 90% confidence. Constraints from neutrino mixing data lower these bounds. Singular integration measures allow for arbitrarily small m_{ee} values with the remaining elements ill-defined, but this condition constrains the flavor structure of the model's ultraviolet completion. $\beta\beta0\nu$ bounds below $m_{ee} \sim 5 \times 10^{-3}$ eV should indicate symmetry in the lepton sector, new light degrees of freedom, or the Dirac nature of the neutrino.

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I. INTRODUCTION

The observation of nonzero neutrino mass and mixing via flavor oscillations is the first terrestrial evidence of physics beyond the standard model (SM) of particle physics. See [1–3] for a review of neutrino physics. This discovery provides useful insight into the nature of new high scale phenomena and introduces many important questions. Chief among these is the charge conjugation properties (Dirac versus Majorana) of the neutrino. First, one may enlarge the SM field content to include one or more gauge singlets N (right-handed neutrinos) and give the neutrino a Dirac mass similar to the other fermions via coupling to the neutral SM Higgs boson: $\nu N H^0$. On the other hand, possessing no unbroken gauge quantum numbers, the neutrino may have a Majorana mass term which couples the neutrino to its charge conjugate ν^c, and thus renders the neutrino equal to its own antiparticle. This breaks all nontrivial global symmetries. In particular, a Majorana neutrino mass violates lepton number by two units. The Dirac versus Majorana nature of the neutrino is an important issue that must be addressed experimentally.

Neutrinoless double beta decay ($\beta\beta0\nu$) is an as yet unobserved lepton number violating (LNV) process that would unambiguously identify the Majorana nature of the neutrino [4]. In fact, except in rare circumstances [5], $\beta\beta0\nu$ is the only realistic hope of probing LNV in the near future [6,7]. Given the importance of this process, it is useful to explore its theoretical expectations over a broad range of scenarios. In the standard picture, with no new light LNV degrees of freedom below the TeV scale, $\beta\beta0\nu$ proceeds primarily via Majorana neutrino exchange. In this case, its amplitude is proportional to m_{ee}, the e-e element of the neutrino mass matrix in the flavor basis where the charged leptons are diagonal. While this relationship is generally spoiled by new physics [5], it should still effectively hold for sufficiently small m_{ee} values. While it is true that the smallest $\beta\beta0\nu$ rates may be dominated by high dimensional nonrenormalizable operators there is an important feedback mechanism into m_{ee}. This follows from the extended black box theorem [8] where a one to one relationship is derived between m_{ee} and the effective $m_{ee}^{\rm eff}$ that governs $\beta\beta0\nu$ in the vanishing limit, such that $m_{ee} = 0 \leftrightarrow m_{ee}^{\rm eff} = 0$. Thus, it is reasonable to assume that, when searching for small $\beta\beta0\nu$ rates, it is enough to study the behavior of m_{ee}. This is only loose motivation for the present analysis as it is impossible to extract the exact rate below which this reasoning holds without assuming properties of the neutrino mass's ultraviolet completion. In what follows I will assume the dominance of the light Majorana neutrino exchange mechanism parametrized by m_{ee}. This is the most popular case. Additionally, since other new physics has yet to be discovered, it is the minimal mechanism currently implied by direct observation. Current experimental limits constrain the $\beta\beta0\nu$ half-life below $\sim 10^{25}$ yr, corresponding to $m_{ee} < 0.35$ eV at 90% confidence[1] [12–15]. Next generation experiments are poised to extend this reach by roughly an order of magnitude to $m_{ee} < 0.05$ eV [14,16,17].

While it is true that $\beta\beta0\nu$ rates are below current sensitivities,[2] it is possible that they will be discovered

[1]The translation between measured half-life and m_{ee} is not straightforward, as it depends critically on isotope dependent nuclear matrix element calculations where uncertainties currently range an order of magnitude [9–11]. This is likely to improve within the next several years.

[2]A positive signal was reported by a subset of the Heidelberg-Moscow group with a half-life near 1.19×10^{25} yr at 4.2σ confidence [18]. I neglect this observation in what follows awaiting confirmation, except to point out that their extracted m_{ee} is well accommodated by the anarchy model of neutrino mass.

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by the next round of experiments. In terms of the measured oscillation parameters, we are only beginning to explore the interesting range dominated by the atmospheric mass squared difference within the inverted and quasidegenerate spectral hierarchies [19]. If $\beta\beta0\nu$ observation is "right around the corner," in which case m_{ee} is relatively large, it is unlikely that its rate is suppressed by an approximate flavor symmetry. However, if bounds are pushed significantly lower, it is quite reasonable a small m_{ee} is protected by an appropriate symmetry mechanism [20]. It is natural to wonder how small it can be without the introduction of imposed mass matrix structure. To this end, it is instructive to take the "no structure" limit and consider m_{ee} bounds, assuming the anarchy hypothesis [21,22]. Similar reasoning was applied much earlier in the history of neutrino physics to probe the potential for large mixing angles [23]. In this scenario, the underlying neutrino mass model is sufficiently complicated, such that the mass matrix appears random at low energies in any basis. In other words, there is effectively no difference between the three light neutrino states. This leads to a distribution of observables that must be treated statistically. In [21,24], it was shown that the large mixing angles and small hierarchies of the neutrino sector are consistent with anarchy, provided θ_{13} is not too small, whereas the Cabibbo-Kobayashi-Maskawa matrix is inconsistent with anarchy, as expected by pure inspection of its structure.[3] In this analysis, the marginalized mixing angle distribution functions are well defined in terms of the Haar measure invariant under the $U(3)$ group. It is not as straightforward to consider questions involving mass eigenvalues since one may include arbitrary $U(3)$ invariant weighting functions into the integration measure [21] and boundary conditions. Here, I survey this added layer of ambiguity and derive the smallest allowed mass matrix element consistent with anarchy.

In what follows, I analyze expectations for m_{ee} within the anarchy picture of neutrino mass generation. The goal is to determine how small/large one mass matrix element may be from the others within an anarchical framework. In Sec. II, I introduce the formalism and notation employed throughout the analysis. Using the Kolmogorov-Smirnov (KS) goodness of fit test, I scan the parameter space of measures defined by both simple polynomial and divergent $U(3)$ invariant functions with "spherical" boundary conditions in subsection II A. Here, I also comment on modifications induced by the use of nontrivial boundary conditions. In subsection II B, I connect these results to the case of realistic neutrino mixing. I conclude in Sec. III with a summary of my results and comment on the impact of future experimental data.

[3]These claims were questioned and explored by analysis discussed in [25,26] but the overall consistency between anarchy and current neutrino data still holds.

II. MASS MATRIX ANARCHY AND $\beta\beta0\nu$

I parametrize the complex, symmetric, three neutrino Majorana mass matrix as

$$m_{\alpha\beta} = ma_{\alpha\beta} = mr_{\alpha\beta}e^{i\phi_{\alpha\beta}}, \qquad (2.1)$$

where the dimensionless complex parameters $a_{\alpha\beta} \equiv (a)_{\alpha\beta}$ define the structure of the matrix and are constructed to have a magnitude $r_{\alpha\beta}$ and phase $\phi_{\alpha\beta}$. The latter contains the familiar Majorana and Dirac phases of the neutrino mixing matrix in various linear combinations. Three of these six phases may be rotated away as unphysical with appropriate transformations, but are included in this analysis without loss of generality. See for example [27,28] and references therein. A dimensionful factor of m is pulled out to carry the scale of neutrino masses. There is ambiguity in the factorization of m and $r_{\alpha\beta}$. To be concrete, I define m such that the resulting $r_{\alpha\beta}$ matrix elements have maximal magnitudes defined by boundary conditions subject to anarchy constraints. The average $r_{\alpha\beta}$ values should be near unity. In other words, $r_{\alpha\beta}$ is a generally $\mathcal{O}(1)$ matrix up to some deviations described by the anarchy hypothesis. The overall neutrino mass scale m is inferred from experiment and included into the analysis by hand. Currently, m is bounded at 0.05 eV from below by the atmospheric mass squared difference [15,29] and from above at roughly 1 eV by cosmological data [15]. In the remainder of this paper I refer to these as the hierarchical and quasidegenerate limits, respectively. These are realized when the lightest mass eigenvalue squared is less than (hierarchical) or greater than (quasidegenerate) the mass squared differences. In what follows, within the anarchy picture, one may only extract limits on the deviations of $r_{\alpha\beta}$ from its average value, or equivalently, bound the ratio of mass matrix elements.

Imposing the anarchy hypothesis implies that the matrix elements of Eq. (2.1) are distributed randomly in any basis or, more precisely, the probability distribution of each $a_{\alpha\beta}$ is invariant under arbitrary unitary rotations. Starting from the diagonal mass basis, the flavor basis (as well as any other physical basis) is found by a random rotation, and one may calculate the probability distribution of any matrix structure. Notice that the obvious distinction of the diagonal mass basis renders it a very improbable structure which may be understood in terms of a sampled ensemble of matrices. The small chance of landing on it via random rotation does not preclude its existence.

Assuming three light neutrinos, the condition of $U(3)$ invariance imposes strict conditions on the total distribution function $G(a_{\alpha\beta})$ universally associated with each element. This quantity may only be a function of

$$\det(a) = \epsilon_{ijk}r_{1i}r_{2j}r_{3k}e^{i(\phi_{1i}+\phi_{2j}+\phi_{3k})}$$

and the purely radial quantity

$$\mathrm{tr}\,(a^\dagger a) = r_{11}^2 + r_{22}^2 + r_{33}^2 + 2r_{12}^2 + 2r_{13}^2 + 2r_{23}^2.$$

Working in these polar coordinates, the marginalized probability distribution of any single element's magnitude, say $|a_{11}| = r_{11} \equiv r$, is obtained by integrating over the other radial coordinates and all phases, subject to some (also invariant) integration limits $l = l[\mathrm{tr}(a^\dagger a), \det(a)]$. Thus, the r-coordinate distribution may be written in terms of the arbitrary functionals G and l, as

$$g(r) = \int_0^{l[\mathrm{tr}(a^\dagger a),\det(a)]} G[\mathrm{tr}(a^\dagger a), \det(a)] \prod_{\alpha\leq\beta,\beta\neq 1} r_{\alpha\beta} dr_{\alpha\beta} \times \prod_{\alpha\leq\beta} d\phi_{\alpha\beta}. \tag{2.2}$$

At this point, G may be thought of as a weighting function in the integration measure. As a probability, $g(r)$ is positive and normalized as $\int_0^1 g(r) r dr = 1$. This implies that matrix elements do not have independent distribution functions, so that the probability of structures defined by magnitudes $r_{\alpha\beta}$ cannot be expressed as a product of $g(r_{\alpha\beta})$. Rather, one must calculate the generalized version Eq. (2.2). The cumulative probability distribution that the radial magnitude is less than some r is given by

$$F(r) = \int_0^r g(r')r'dr'. \tag{2.3}$$

With this, the C confidence interval limits are found from

$$\max[F(r), 1 - F(r)] - \frac{C+1}{2} = 0, \tag{2.4}$$

which yields two solutions interpreted as the extreme r values allowed at C confidence. One may trivially generalize this procedure to n neutrino states with no qualitative changes. It reflects the mechanism behind the KS goodness of fit test and yields the same results for one free variable.

A. Simple polynomial measures

Consider a one-dimensional marginalized probability distribution function, limited by $l = \mathrm{Tr}(a^\dagger a) \leq 3^2$. This choice is not unique but does make physical sense, as it implies that each matrix element lives within a three-dimensional sphere. This is clear upon rotation to the diagonal mass eigenbasis. I chose a nonunit radius for convenience to reflect the physical definition of the mass m, but this may be changed, provided a corresponding scaling of r. With this choice, the universal $\mathcal{O}(1)$ matrix structures indicative of anarchy saturates the upper integration bound. This is easy to understand, as sample volumes at large radii dominate that at smaller radii. Thus, $r_{\alpha\beta} \in [0, 3]$, with the most probable values naively expected near one. It is reasonable to assume the weighting functional may be expanded in a double Taylor series as $G[\mathrm{tr}(a^\dagger a), \det(a)] = \sum_{p,q}^{\infty} c_{pq} \mathrm{tr}(a^\dagger a)^p \det(a)^q$. Upon integrating over the complex phases it is clear that all nonzero powers of $\det(a)$ vanish by symmetry. However, this is only true with $\phi_{\alpha\beta}$ independent integration limits. A nontrivial dependence on $\det(a)$ is possible within this framework but requires a departure from the physically motivated spherical boundary conditions. An example case of polynomial dependence reveals that such limits have little impact on the marginalized distribution function for small values of r, which is the primary concern of this analysis.

Thus, it is enough to consider only linear combinations of

$$\begin{aligned} G_p[\mathrm{tr}(a^\dagger a), \det(a)] &= \mathrm{Tr}(a^\dagger a)^p \\ &= (r^2 + r_{22}^2 + r_{33}^2 + 2r_{12}^2 + 2r_{23}^2 \\ &\quad + 2r_{13}^2)^p. \end{aligned} \tag{2.5}$$

In this case, it is simple to obtain the cumulative distribution function

$$F_p(\tilde{r}) = (6+p)\tilde{r}^2\left[\frac{5}{5+p} - \frac{10\tilde{r}^2}{4+p} + \frac{10\tilde{r}^4}{3+p} - \frac{5\tilde{r}^6}{2+p} + \frac{\tilde{r}^8}{1+p} - \frac{120\tilde{r}^{10+2p}}{(6+p)(5+p)(4+p)(3+p)(2+p)(1+p)}\right] \tag{2.6}$$

via the procedure outlined in Eq. (2.2) and (2.3) in terms of the scaled magnitude $\tilde{r} \equiv r/3$ that lives in the unit interval. This is defined for all $p > -6$. It is easy to see that all other pole divergences cancel pairwise out of this expression.

Combining Eq. (2.6) and (2.4) yields limits on the ratio of matrix elements r. Upon multiplication by the current experimentally allowed neutrino mass scale range $0.05\ \mathrm{eV} < m < 1\ \mathrm{eV}$, one may obtain the two sided 90% confidence limits of m_{ee} (or any other matrix element) as a function of p within the anarchy framework. This is shown in the gray contour of Fig. 1 along with the current and future $\beta\beta 0\nu$ reach denoted by horizontal dotted and dashed lines, respectively. For convenience, the 90% allowed region is decomposed into the two extreme cases of hierarchical and quasidegenerate mass spectra bounded by the blue dashed and red dotted-dashed curves, respectively. Given a particular neutrino mass scale, the allowed m_{ee} range spans scarcely an order of magnitude. Much of the breadth of the totally allowed gray region in Fig. 1 comes from uncertainty in m. Therefore, an absolute neutrino mass measurement should significantly tighten the anarchy prediction of m_{ee}.

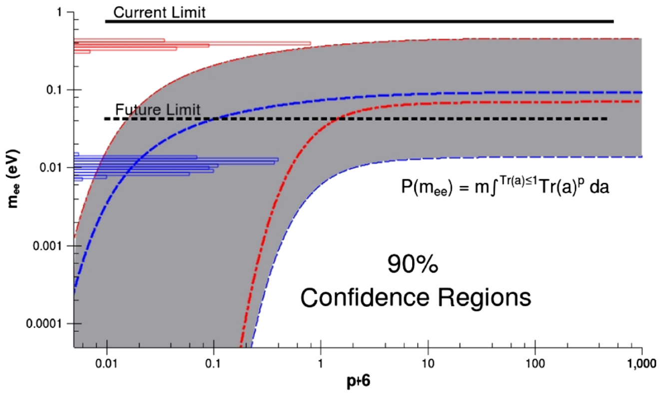

FIG. 1 (color online). Anarchy allowed m_{ee} limits at 90% confidence as a function of $p+6$ defined by Eq. (2.5) (see text for details). Shaded region is total confidence interval while the blue (dashed) and red (dash-dotted) bounds indicate the hierarchical and quasidegenerate neutrino mass spectra, respectively. $m_{ee} \to 0$ as $p \to -6$. Red (upper) and blue (lower) horizontal histograms show allowed upper and lower m_{ee} bounds for a measure series expansion sample space for quasidegenerate and hierarchical spectra, respectively. Current and future $\beta\beta 0\nu$ bounds are also shown.

For $p > -5$ the curves of Fig. 1 are relatively level yielding m_{ee} in the range (0.01–0.1) eV and (0.05–0.5) eV for the hierarchical and quasidegenerate spectra. This is easy to understand, as $\tilde{r} \leq 1$ and Eq. (2.6) may be truncated at quadratic order[4]. Hence, $F_p(r) \to \frac{5}{9}\frac{6+p}{5+p} r^2$ yielding a C confidence limit for r between $3\sqrt{(1 \pm C)\frac{5+p}{10(6+p)}}$. This approximation works very well for lower bounds but receives up to 5% corrections for the upper 90% confidence limits. At this point, it is instructive to consider the generalized n neutrino scenario for analogous weightings and boundary conditions $l = \mathrm{Tr}(a^\dagger a) \leq n^2$. In the small $\tilde{r} = r/n$ approximation, the cumulative distribution function is given by $F_p^{(n)} \approx r^2 \frac{(n-1)(n+2)}{2n^2}\{\frac{2p+n^2+n}{2p+n^2+n-2}\}$, which yields confidence limits between $n\sqrt{(1 \pm C)\frac{2p+n^2+n-2}{(n-1)(n+2)(2p+n^2+n)}}$. At large n, the matrix elements become independent with distributions bounded by $\sqrt{1 \pm C}$.

I now consider general well-behaved measure functions as series expansions of $\mathrm{Tr}(a^\dagger a)^p$ weighted by arbitrary coefficients c_p, which by a trivial extension of Eq. (2.6) results in

$$F(\tilde{r}) = \frac{1}{\sum_{m=0}^{\infty}\frac{c_m}{m+6}} \sum_{p=0}^{\infty} \frac{c_p}{6+p} F_p(\tilde{r}). \tag{2.7}$$

[4]This is only true for $p > -5$ as, in this limit, the quadratic terms of Eq. (2.6) diverge and thus cancel. In this case one must truncate at quartic order.

Here, one coefficient is always redundant, as it may be factored out of both the numerator and denominator and ultimately canceled. Because of this normalization condition, only those terms with comparable weightings having large destructive/constructive interference contribute to deviations from the limit contours of Fig. 1. This can be seen in the small $\tilde{r}$ approximation $F(\tilde{r}) \approx 5\tilde{r}^2 \sum_{p=0}^{\infty}\frac{c_p}{p+5} / \sum_{m=0}^{\infty}\frac{c_m}{m+6}$. Uniformly scanning the parameters of Eq. (2.7), truncated at $p = 4$ within the range $c_p \in [-50, 50]$, I sample the m_{ee} bounds obtained from 10^{10} sample functions. Results are shown in the blue (lower) and red (upper) horizontal histograms of Fig. 1 for the hierarchical and quasidegenerate mass scales, respectively. These arbitrarily normalized distributions are highly dependent on the sampling procedure. Consequently, the upper and lower endpoints are the only useful quantities. Because of the small $\tilde{r}$ approximate behavior, the distribution of upper bounds is much wider than that of the lower. The behaviors of these histograms are in analogy with the work of [21] where random mass matrices were generated and studied using a linear measure function and cubic boundary conditions. In that case, mass eigenvalues were histogrammed as opposed to the combined quantity m_{ee}.

The behavior of $F_p(\tilde{r})$ for $p \to -6$ is easy to understand, but surprising from the mass matrix anarchy viewpoint. Approaching this lower limit, one finds that $\lim_{p\to -6} F_p(\tilde{r}) \approx \tilde{r}^{2(p+6)} \approx 1 + (p+6)\ln\tilde{r}^2$, implying that the marginalized probability distribution $g(r) \to \delta(r)$. This shows that at least one mass matrix element may be arbitrarily small, provided a sufficiently divergent integration measure. Furthermore, this must hold true in any basis obtained by a random rotation from the diagonal mass basis. With this, one finds the C confidence region bounded between $e^{-(1\pm C/4(6+p))}$, which goes rapidly to zero, as seen in Fig. 1. It remains to check the behavior of the other, marginalized matrix elements in this limit. Given two magnitudes, r and s, one may calculate the marginalized probability distribution function $g_p(r, s)$ and the cumulative distribution function $F_p(r, s)$ parametrized by p in the integration measure of Eq. (2.5). In the limit $p \to -6$, when $\tilde{r}$ is small, per the above argument,

$$F_p(r, s) \approx \tilde{r}^{2(p+6)} - (p+6)\frac{r^2}{s^2} = F_p(r) - (p+6)\frac{r^2}{s^2}.$$

When r is within its C confidence limits, defined by $F_p(r) \in (1 \pm C)/2$, then $F_p(r, s) \in (1 \pm C)/2$ is also satisfied up to small perturbations. The coordinate s is virtually unconstrained. Thus, given $p \to -6$, at least one texture zero is guaranteed and all other matrix elements vary freely and are (almost) independent of the integration measure. Similar statements apply to other sufficiently divergent poles in the measure. For example, given $G[\mathrm{Tr}(a^\dagger a)] = (B^2 - \mathrm{Tr}(a^\dagger a))^p$ in the appropriate p limit, one radial magnitude goes to B while the rest are unconstrained by the theory.

B. Mass matrix anarchy and neutrino mixing

Thus far, I have explored the relation: Neutrino mixing data is consistent with the anarchy hypothesis ... Given neutrino anarchy, what are the allowed mass matrix structures in an arbitrary basis? The allowed m_{ee} range is then easily extracted. This is the proper treatment, but it does not address if/when the derived structures accommodate the neutrino data. Alternately, one might consider the anarchical m_{ee} distribution constrained by current oscillation phenomenology. This will not yield the same range as the previous analysis. As noted in [24], regarding the preferred value of the reactor mixing angle, given the consistency of neutrino data and anarchy, one would expect an expanded confidence region for unknown quantities such as the CP phases and θ_{13}. In other words, a large amount of (accidental) structure can appear among these parameters while still maintaining consistency with anarchy. In the standard parametrization, the composite mass matrix element m_{ee} magnitude is given by

$$|m_{ee}| = m|r_1 e^{i\phi_1}\cos^2\theta_{12}\cos^2\theta_{13} + r_2 e^{i\phi_2}\sin^2\theta_{12}\cos^2\theta_{13} + r_3 e^{i(\phi_3-\delta)}\sin^2\theta_{13}|, \tag{2.8}$$

where an overall mass scale m is factored out for consistency with the preceding analysis. Here r_i and ϕ_i make up the complex eigenvalues of the flavor basis mass matrix Eq. (2.1), diagonalized by the mixing angles θ_{12}, θ_{23}, θ_{13}, and δ in the usual way. In this quantity, a single Majorana phase may be removed as unphysical so the resulting expression depends on only two phase linear combinations. The mixing parameters are distributed according to the Haar measure and the radial magnitudes according to the complex Majorana mass eigenvalue measure given in [21], weighted by an invariant function analogous to Eq. (2.5). m_{ee} is the summed convolution of these distributions. It is well known that, given current data, Eq. (2.8) can vanish, provided the normal neutrino mass ordering and particular relationships among the phases and mass scale [15,30,31]. One would then expect the allowed m_{ee} range to extend lower in the normal than the inverted hierarchy.

I point out that the preceding analysis could have been done from this perspective, with separated mass and mixing parameters, but was easier done in the flavor basis. Within a realistic neutrino mixing framework constrained by the data shown in Table I, the use of the convoluted Eq. (2.8) is convenient since we already know many best fit bounds from oscillation searches [15,29]. Marginalizing the convoluted m_{ee} distribution over the unknown phases as well as the neutrino mixing angle uncertainties with a polynomial weighting function, I obtain the normalized m_{ee} cumulative distribution function. Thus, this quantity is the normalized term by term convolution of angular distribution functions weighted by the Haar measure and the polynomial measured mass eigenstate distributions. The experimentally allowed mixing angle ranges may be substituted into this function. For simplicity, I consider the normal and inverted neutrino mass spectra separately in terms of only the lowest free mass eigenvalue with the others related to it by the measured mass squared differences. I find that for all nonsingular weightings, $m_{ee} > 4.4 \times 10^{-4}$ eV and $m_{ee} > 1.6 \times 10^{-2}$ eV at 90% confidence for the normal and inverted hierarchies, respectively. For the inverted hierarchy, this bound coincides roughly with the smallest possible inverted m_{ee} value obtained from Eq. (2.8) evaluated at $r_3 = 0$. As expected, the normal hierarchy bound is well below those shown in Fig. 1 due to cancellations induced by the marginalization over unknown phases. However, care must be taken when interpreting this result as one hierarchy may be preferred over another by anarchy. A numerical scan over an ensemble of mass eigenvalues, without imposing mass squared differences from oscillation data, reveals a general preference for the intermediate state to lie closer to the heavier than the lighter one. My results agree qualitatively with a similar scan done in [21]. Still, this slight effect does not suggest that anarchy favors the inverted hierarchy, which is defined in terms of mass squared differences. These results are relatively independent of the supplied weighting function, provided that it is sufficiently nonsingular.

TABLE I. Summary table of current neutrino results together with naming conventions and parameter definitions. Columns three and four list best fit central parameter values and 1σ uncertainties. These were adapted from the global oscillation analysis of [32]. The central values are used as input into the analysis of subsection II B.

Name	Parameter combination	Value	1σ uncertainty
Δm_S^2	$m_2^2 - m_1^2$	7.65×10^{-5} eV^2	0.22×10^{-5} eV^2
Δm_A^2	$\|m_3^2 - m_2^2\|$	2.40×10^{-3} eV^2	0.12×10^{-3} eV^2
$\sin\theta_S$	$\sin\theta_{12}$	0.551	0.017
$\sin\theta_A$	$\sin\theta_{23}$	0.707	0.046
$\sin\theta_R$	$\sin\theta_{13}$	0.1	<0.14

III. CONCLUSIONS

Within the framework of neutrino mass anarchy, the distribution of parameters and matrix elements must be treated statistically. Unlike a study of neutrino mixing angles and phases, which depend only on the invariant Haar measure, an analysis of mass eigenvalues/matrix elements depends critically on arbitrary integration measures and boundary conditions. For well-behaved measures, the value of any one matrix element may vary between about 0.01 eV and 0.4 eV at 90% confidence. This is well within the reach of future experiments and will be tightened with better knowledge of the overall neutrino mass scale. However, these bounds are expanded for sufficiently divergent measures, which may lead to a vanishing matrix element. This case also renders the remaining matrix elements undefined, and thus removes almost all predictability from the theory. I conclude that arbitrarily small matrix elements are allowed within the anarchy framework, but this seems counterintuitive, given the universal mass matrix structure assumed in the original formulation of the theory.

This is more puzzling when put in the framework of realistic neutrino data. $m_{ee} = 0$ implies the normal hierarchy and practically forces a $\mu - \tau$ like symmetry among the remaining elements [5,33,34]. While still consistent with anarchy (the divergent weighting imposes the texture zero and frees all other elements to vary almost unconstrained to yield the correct mass spectra and parameter relationships), this seems like a lot of accidental structure for a structureless matrix! The resolution lies in the ultraviolet completion of the theory that produces the integration measure. That is, the underlying theory yields well-defined mass parameters, but by our limited knowledge of its complicated structure, this information is communicated to low energies as an ensemble of possible choices that must be treated statistically. The nature of possible model classes selects the weighting function for us. An anarchy guaranteed texture zero implies a mechanism selecting those textures from the ensemble of ultraviolet completions. This is inconsistent with the anarchy principle from construction, as it would require a corresponding symmetry mechanism and/or parameter fine-tuning. Of course, without knowing the details of neutrino mass generation it is impossible to gauge or select uncomfortable levels of tuning or when flavor structures must arise. Still, one would not expect the metric power law parameter to venture too close to the -6 singularity. To get a handle on this, it is reasonable to assume a 10% or greater deviation which implies $m_{ee} \geq 5 \times 10^{-3}$ eV at 90% confidence. $\beta\beta0\nu$ bounds below this level should indicate nontrivial structure in the lepton flavor sector, new light LNV degrees of freedom [35–38], or the Dirac nature of the neutrino.

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