Concerning gauge field fluctuations around classical configurations

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We treat the fluctuations of non-Abelian gauge fields around a classical configuration by means of a transformation from the Yang-Mills gauge field to a homogeneously transforming field variable. We use the formalism to compute the effective action induced by these fluctuations in a static background without Wu-Yang ambiguity.

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The response of quanta to classical gauge fields is a fundamental issue of continuing phenomenological and theoretical interest in Abelian and non-Abelian field theories. The canonical example is the production of electron-positron pairs in strong photon fields [1] which among other effects is about to be tested with ultra strong light sources [2]. It is one thing if the aforementioned quanta are matter particles like fermions or scalars and another if they are fluctuations of the gauge field around its expectation value: As the entire gauge field does not transform homogeneously under gauge transformations, at variance with the matter fields, keeping track of gauge invariance when handling the fluctuations has an extra twist to it. At leading order this is only important for self-interacting, i.e., non-Abelian fields. One way is to use the Faddeev-Popov approach, especially in conjunction with the background field method [3]. Here, we will use another approach based on a transformation from the vector gauge field A_μ^a to an antisymmetric tensor variable $B_{\mu\nu}^a$, passing via a first-order formulation. $B_{\mu\nu}^a$ transforms homogeneously under gauge transformations. We derive the general formalism and then compute the effective Lagrangian for an example. Before we delve into the non-Abelian case we first take a look at the Abelian, where $B_{\mu\nu}$ is even gauge invariant.

Abelian. The partition function of quantum electrodynamics coupled to an external source J^μ is given by

$$Z = \int [dA][d\psi][d\bar{\psi}]\exp\bigg\{i\int d^4x\bigg[-\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \bar{\psi}(i\not{D} - m)\psi - A_\mu J^\mu\bigg]\bigg\}, \tag{1}$$

where $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$ represents the field tensor, A_μ the gauge field, $D_\mu = \partial_\mu - igA_\mu$ the covariant derivative, g the coupling constant, m the fermion mass, and $\psi/\bar{\psi}$ the fermion fields. Integrating out A_μ leads to

$$Z \cong \int [d\psi][d\bar{\psi}]\exp\bigg\{i\int d^4x[\bar{\psi}(i\not{\partial} - m)\psi]\bigg\} \times \exp\bigg\{i\int d^4x d^4y\bigg[-\frac{1}{2}(g\bar{\psi}\gamma_\mu\psi - J_\mu)(x) \times \Gamma^{\mu\nu}(x-y)(g\bar{\psi}\gamma_\nu\psi - J_\nu)(y)\bigg]\bigg\}, \tag{2}$$

where $\Gamma^{\mu\nu}(x-y)$ stands for the photon propagator in some gauge. $\cong$ indicates that the normalization was changed in that step. The terms in the second exponential describe single-photon exchange and couple the fermions to the background. The stationarity condition for $\bar{\psi}$ yields the Dirac equation for ψ in the background $\mathcal{A}_\mu$, $(i\not{\mathcal{D}} - m)\psi = 0$, where $\mathcal{D}_\mu$ stands for the covariant derivative on $\mathcal{A}_\mu$. The latter is the solution of the stationarity condition for A_μ in the action in Eq. (1),

$$\partial_\mu \mathcal{F}^{\mu\nu} = J^\nu - g\bar{\psi}\gamma^\nu\psi, \tag{3}$$

where $\mathcal{F}_{\mu\nu}$ is the field tensor on the classical solution.

We replace the gauge field by an antisymmetric tensor field $B_{\mu\nu}$. This is achieved by multiplying Eq. (1) by a Gaussian integral over $B_{\mu\nu}$, followed by a shift of $B_{\mu\nu}$ by the dual field tensor $\tilde{F}^{\mu\nu} = \epsilon^{\mu\nu\kappa\lambda}F_{\kappa\lambda}/2$,

$$Z \cong \int [dA][dB][d\psi][d\bar{\psi}]\exp\bigg\{i\int d^4x\bigg[-\frac{1}{4}B_{\mu\nu}B^{\mu\nu} - \frac{1}{2}B_{\mu\nu}\tilde{F}^{\mu\nu} + \bar{\psi}(i\not{D} - m)\psi - A_\mu J^\mu\bigg]\bigg\}. \tag{4}$$

Postulating the gauge invariance of the BF term requires a gauge invariant $B_{\mu\nu}$. The stationarity conditions for A_μ and $B_{\mu\nu}$,

$$\partial_\mu \tilde{\mathcal{B}}^{\mu\nu} = J^\nu - g\bar{\psi}\gamma^\nu\psi \quad \text{and} \quad \mathcal{B}^{\mu\nu} = -\tilde{\mathcal{F}}^{\mu\nu}, \tag{5}$$

combine into Eq. (3), where $\mathcal{B}_{\mu\nu}$ is the classical value of $B_{\mu\nu}$. Integrating out A_μ in Eq. (4) yields

$$Z \cong \int [dB][d\psi][d\bar{\psi}]\delta(\partial_\mu\tilde{B}^{\mu\nu} - J^\nu + g\bar{\psi}\gamma^\nu\psi) \times \exp\bigg\{i\int d^4x\bigg[-\frac{1}{4}B_{\mu\nu}B^{\mu\nu} + \bar{\psi}(i\not{\partial} - m)\psi\bigg]\bigg\}, \tag{6}$$

which required no gauge fixing and yields a local result. The first of Eq. (5) is now strictly enforced; it does not merely give the most probable configuration, but the only allowed configuration. This constraint can be used to eliminate $B_{\mu\nu}$ from the partition function, such that

$$B_{\mu\nu} = \frac{1}{2}\epsilon_{\kappa\mu\nu\lambda}\int d^4y S^\kappa(x-y)(J^\lambda - g\bar{\psi}\gamma^\lambda\psi)(y), \tag{7}$$

where in momentum space $\overset{\circ}{S}{}^{\kappa}(p) = ip^{\kappa}p^{-2}$ with an appropriate pole prescription. Replacing $B_{\mu\nu}$ in the exponent of Eq. (6) by Eq. (7) leads to Eq. (2) with $\Gamma^{\mu\nu}$ transverse, which corresponds to the Landau gauge.

The δ constraint in Eq. (6) eliminated $B_{\mu\nu}$, resulting in a theory of interacting fermions, reproducing Eq. (2), but without fixing a gauge.

Non-Abelian. Consider the generating functional for a Yang-Mills (YM) field A^a_μ coupled to an external source J^a_μ,

$$Z = \int [dA] \exp\left[\mathrm{i} \int d^4x \left(-\frac{1}{4} F^a_{\mu\nu} F^{a\mu\nu} - A^a_\mu J^{a\mu} \right) \right], \quad (8)$$

where $F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + g f^{abc} A^b_\mu A^c_\nu$ stands for the field tensor, g for the coupling constant, and f^{abc} for the antisymmetric structure constant of the gauge group. The corresponding classical equations of motion read

$$\mathcal{D}^{ab}_\mu \mathcal{F}^{b\mu\nu} = J^{a\nu}, \quad (9)$$

where $\mathcal{D}^{ab}_\mu$ represents the covariant derivative $D^{ab}_\mu = \delta^{ab}\partial_\mu + g f^{acb} A^c_\mu$ and $\mathcal{F}^a_{\mu\nu}$ the field tensor $F^a_{\mu\nu}$ both on the classical solution $\mathcal{A}^a_\mu$ for the gauge field. We will now split the gauge field according to $A^a_\mu = \mathcal{A}^a_\mu + a^a_\mu$, and introduce an antisymmetric tensor field in the same way as in the previous section. Doing so yields Z in the so-called first-order formalism [4]

$$Z \cong \int [da][dB] \exp\left[\mathrm{i} \int d^4x \left(-\frac{1}{4} B^a_{\mu\nu} B^{a\mu\nu} - \frac{1}{2} \tilde{B}^a_{\mu\nu} F^{a\mu\nu} - A^a_\mu J^{a\mu} \right) \right]. \quad (10)$$

A homogeneously transforming $B_{\mu\nu} \to U B_{\mu\nu} U^\dagger$, leads to a gauge invariant action, for $J^a_\mu \equiv 0$. Integrating out a^a_μ we find

$$Z \cong \int [dB] \mathrm{Det}^{-(1/2)} \mathbb{B} \exp\left\{ \mathrm{i} \int d^4x \left[-\frac{1}{4} B^a_{\mu\nu} B^{a\mu\nu} - \frac{1}{2} \tilde{B}^a_{\mu\nu} \mathcal{F}^{a\mu\nu} - \mathcal{A}^a_\mu J^{a\mu} + \frac{1}{2} (\mathcal{D}^{ac}_\kappa \tilde{B}^{c\kappa\mu} - J^{a\mu}) (\mathbb{B}^{-1})^{ab}_{\mu\nu} (\mathcal{D}^{bd}_\lambda \tilde{B}^{d\lambda\nu} - J^{b\nu}) \right] \right\}, \quad (11)$$

where $\mathbb{B}^{bc}_{\mu\nu} = g \tilde{B}^a_{\mu\nu} f^{abc}$ and

$$\mathrm{Det}^{-(1/2)} \mathbb{B} \cong \int [d\zeta] \exp\left[-\frac{\mathrm{i}}{2} \int d^4x (\zeta^{a\mu} \mathbb{B}^{ab}_{\mu\nu} \zeta^{b\nu}) \right].$$

With the decomposition $B^a_{\mu\nu} = b^a_{\mu\nu} - \tilde{\mathcal{F}}^a_{\mu\nu}$ and making use of Eq. (9), we obtain

$$Z = \mathcal{Z} \int [db] \mathrm{Det}^{-(1/2)} (\mathbb{b} + \mathbb{F}) \exp\left(\mathrm{i} \int d^4x \left\{ -\frac{1}{4} b^a_{\mu\nu} b^{a\mu\nu} + \frac{1}{2} (\mathcal{D}^{ac}_\kappa \tilde{b}^{c\kappa\mu}) [(\mathbb{b} + \mathbb{F})^{-1}]^{ab}_{\mu\nu} (\mathcal{D}^{bd}_\lambda \tilde{b}^{d\lambda\nu}) \right\} \right), \quad (12)$$

where $\mathcal{Z} = \exp[\mathrm{i} \int d^4x (-\frac{1}{4} \mathcal{F}^a_{\mu\nu} \mathcal{F}^{a\mu\nu} - \mathcal{A}^a_\mu J^{a\mu})]$, $\mathbb{b}^{bc}_{\mu\nu} = g \tilde{b}^a_{\mu\nu} f^{abc}$, and $\mathbb{F}^{bc}_{\mu\nu} = g \mathcal{F}^a_{\mu\nu} f^{abc}$.

Carrying out a gauge transformation U of the background $J_\mu \to U J_\mu U^\dagger$ leads to $\mathcal{D}_\mu \to U \mathcal{D}_\mu U^\dagger$ and $\mathcal{F}_{\mu\nu} \to U \mathcal{F}_{\mu\nu} U^\dagger$. The gauge transformations U that then appear in Z can be removed by the same unitary transformation of the integration variable $b_{\mu\nu} \to U b_{\mu\nu} U^\dagger$. Consistently, $\mathcal{F}_{\mu\nu} + \tilde{b}_{\mu\nu} = B_{\mu\nu} \to U B_{\mu\nu} U^\dagger$. $\mathcal{Z}$ is unaffected. Let us call these type IB gauge transformations.

The generating functional is invariant as long as the total $B_{\mu\nu} = b_{\mu\nu} - \tilde{\mathcal{F}}_{\mu\nu}$ transforms homogeneously. This remains true especially for what one could call a type IIB transformation, where the background is left invariant and the fluctuation field accounts for the entire transformation, $b_{\mu\nu} \to U(b_{\mu\nu} - \tilde{\mathcal{F}}_{\mu\nu})U^\dagger + \tilde{\mathcal{F}}_{\mu\nu}$. After such a transformation, however, the transformed $b^a_{\mu\nu}$ field is in general not a pure fluctuation field anymore; it obtains an expectation value $-U\tilde{\mathcal{F}}_{\mu\nu}U^\dagger + \tilde{\mathcal{F}}_{\mu\nu}$. A redecomposition into a true expectation value and true fluctuations would reverse this transformation.

In the background field method [3,5], the gauge fixing term reads $-(\mathcal{D}^{ab}_\mu a^{b\mu})(\mathcal{D}^{ac}_\nu a^{c\nu})/(2\xi)$ and is gauge invariant under type IA gauge transformations, $\mathcal{D}_\mu \to U \mathcal{D}_\mu U^\dagger$ and $a_\mu \to U a_\mu U^\dagger$. Likewise, type IIA transformations leave the background $\mathcal{A}^a_\mu$ invariant and $a_\mu \to U D_\mu U^\dagger - \mathcal{D}_\mu$. Again here, a^a_μ after the latter transformation has, in general, an expectation value and is consequently not a pure fluctuation field anymore. The aforementioned redecomposition would undo the type IIA transformation.

In both cases the actions are manifestly gauge invariant under type I gauge transformations. Type II transformations necessitate a redecomposition into expectation value and fluctuations. In any case taking the contribution to a quantity from the background and the fluctuation field together admits finding a result invariant under both types of gauge transformations. A main difference of the $B^a_{\mu\nu}$ with respect to the A^a_μ field description is the absence of an explicit gauge fixing term and consequently of ghost terms in the former.

The stationarity condition is derived by variation with respect to $b^e_{\alpha\beta}$,

$$0 = -b^{e\alpha\beta} - \mathcal{D}^{ae}_\kappa \epsilon^{\alpha\beta\kappa\mu} [(\mathbb{b} + \mathbb{F})^{-1}]^{ab}_{\mu\nu} (\mathcal{D}^{bd}_\lambda \tilde{b}^{\lambda\nu}) - \frac{1}{2} (\mathcal{D}^{ac}_\kappa \tilde{b}^{c\kappa\mu}) [(\mathbb{b} + \mathbb{F})^{-1}]^{af}_{\mu\rho} f^{efg} \epsilon^{\alpha\beta\rho\sigma} \times [(\mathbb{b} + \mathbb{F})^{-1}]^{gb}_{\sigma\nu} (\mathcal{D}^{bd}_\lambda \tilde{b}^{\lambda\nu}). \quad (13)$$

[The determinant term does not contribute at this level as varying with respect to $\zeta^{b\nu}$ implies $\zeta^{a\mu}(\mathbb{b} + \mathbb{F})^{ab}_{\mu\nu} = 0$.] Only if it has the solution $b^a_{\mu\nu} \equiv 0$ can $b^a_{\mu\nu}$ be treated as pure fluctuation. Otherwise, the appropriate expansion point, i.e., the correct vacuum, has to be determined by finding the solution of the previous equation. Remarkably, in that case, the expansion point would be different from $\mathcal{F}^a_{\mu\nu}$, the one in the vector field formulation of YM theory. Situations where $\det\mathbb{F} = 0$, are problematic in this respect because there Eq. (13) is ill-defined at $b^a_{\mu\nu} = 0$. Among the settings belonging to this group is the trivial, i.e., background field free case. In this context this coincides with the observation that the zero field vacuum in YM theories is unstable [6]. One may wonder, how the standard high-energy perturbative treatment comes about in the present formalism. There, at least initially, YM theory looks almost Abelian. Here, for $g \to 0$ (and without background) the Gaussian made up by the last term in the generating functional (11) goes to a δ distribution and imposes several color copies of the Maxwell equation as seen in the previous section in the Abelian case. [7]

There are also nonzero configurations with $\det\mathbb{F} = 0$: $\mathbb{F}$ is in the adoint representation. Hence, each Lorentz component alone has zero eigenvalues. Therefore, to have $\det\mathbb{F} \neq 0$ one needs several Lorentz components whose eigenvectors belonging to the zero eigenvalues are misaligned. Thus, field configurations with a single Lorentz component have necessarily $\det\mathbb{F} = 0$. Among these are Coulomb fields, also those boosted onto the light cone. Their application to the description of the initial condition of heavy-ion collisions gives rise to instabilities [8].

The customary generalization of effective actions [1] in the presence of constant external field tensors to the non-Abelian case [9] proceeds via covariantly constant fields $\mathcal{D}^{ab}_\lambda \mathcal{F}^b_{\mu\nu} = 0 \forall \lambda, \mu, \nu$. They are effectively quasi-Abelian and lead to a result analogous to the Abelian. They also have $\det\mathbb{F} = 0$. This condition is also a necessary condition for a Wu-Yang ambiguity [10] to appear in four dimensions [11]. A Wu-Yang ambiguous field tensor can be realized by different gauge field configurations, which are *not* gauge equivalent. This implies that in such cases not all information about the system or its background is contained in the field tensor. The covariant derivative contains more information than its commutator. Thus, one can also understand why the factor $\mathrm{Det}^{-1/2}\mathbb{B}$ appears as Jacobian in the measure when translating the YM generating functional Z from a vector to an antisymmetric tensor field representation [4,12]: $B^a_{\mu\nu}$ is the conjugate of $\mathcal{F}^a_{\mu\nu}$. Where the field tensor does not allow to reconstruct the system uniquely, but the vector potential would, the Jacobian becomes singular. One particular quantity which differs for gauge inequivalent gauge field realizations for the same field tensor is the YM current J^a_μ. In our case the full information about the system is communicated from the A^a_μ to the $B^a_{\mu\nu}$ representation through said J^a_μ and the (classical) covariant derivative $\mathcal{D}^{ab}_\mu$. [See Eq. (11).] A further conclusions [13] is that in the presence of a Wu-Yang ambiguous background not all observables can be expressed in terms of invariants [14] constructed merely from $\mathcal{F}^a_{\mu\nu}$.

Let us take a look at an example where the classical field has $\det\mathbb{F} \neq 0$ for which we would like to calculate the effective action induced by the fluctuations. For tractability we choose a three-dimensional Euclidean system with an $SU(2)$ gauge group. We start out with a generating functional coupled to an external source just like at the beginning of this section, separate off the fluctuations of the vector gauge field around the background, and translate into a representation based on the variables E^a_μ, the three-dimensional analogue of $B^a_{\mu\nu}$. As counterpart to Eq. (11) we find

$$Z \cong \int [dE] \mathrm{Det}^{-(1/2)}\mathbb{E} \exp\bigg\{\int d^3x \bigg[-\frac{1}{2} E^a_\mu E^a_\mu - \frac{1}{2}\tilde{E}^a_{\mu\nu}\mathcal{F}^a_{\mu\nu} - \mathcal{A}^a_\mu J^a_\mu + \frac{1}{2}(\mathcal{D}^{ac}_\kappa \tilde{E}^c_{\kappa\mu} - J^a_\mu)(\mathbb{E}^{-1})^{ab}_{\mu\nu}(\mathcal{D}^{bd}_\lambda \tilde{E}^d_{\lambda\nu} - J^b_\nu)\bigg]\bigg\}, \quad (14)$$

where $\tilde{E}^a_{\mu\nu} = \mathrm{i}\epsilon_{\kappa\mu\nu}E^a_\kappa$ and $\mathbb{E}^{bc}_{\mu\nu} = g\tilde{E}^a_{\mu\nu}\epsilon^{abc}$. The decomposition $\tilde{E}^a_{\mu\nu} = \tilde{e}^a_{\mu\nu} + \mathcal{F}^a_{\mu\nu}$ gives

$$Z = \mathcal{Z}\int [de] \mathrm{Det}^{-(1/2)}(\mathbb{e} + \mathbb{F}) \exp\bigg(\int d^3x \bigg\{ -\frac{1}{2} e^a_\mu e^a_\mu + \frac{1}{2}(\mathcal{D}^{ac}_\kappa \tilde{e}^c_{\kappa\mu})[(\mathbb{e} + \mathbb{F})^{-1}]^{ab}_{\mu\nu}(\mathcal{D}^{bd}_\lambda \tilde{e}^d_{\lambda\nu})\bigg\}\bigg). \quad (15)$$

Thus, in momentum space, for a constant background the fluctuation operator for e^a_μ reads

$$(G^{-1})^{bd}_{\alpha\beta} = \delta^{bd}\delta_{\alpha\beta} - (\delta^{ab}p_\kappa + \mathrm{i}g\epsilon^{afb}\mathcal{A}^f_\kappa)\epsilon_{\alpha\kappa\mu} \times (\mathbb{F}^{-1})^{ac}_{\mu\nu}(\delta^{cd}p_\lambda + \mathrm{i}g\epsilon^{ced}\mathcal{A}^e_\lambda)\epsilon_{\nu\lambda\beta}. \quad (16)$$

Based on it we would like to calculate the effective Lagrangian

$$\mathcal{L}^{(1)} = \frac{1}{2}\lim_{\varepsilon\to 0}\int \frac{d^3p}{(2\pi)^3} \ln \det \frac{\mathbb{F}G^{-1}(p)}{(\varepsilon^2\mathbb{F})G^{-1}_\varepsilon(p)}. \quad (17)$$

It is normalized with respect to the free part, which is obtained from the expression in the full background field by the rescaling $\mathcal{A}^a_\mu \mapsto \varepsilon\mathcal{A}^a_\mu$ and taking ε to zero at the end. The factors of $\mathbb{F}$ stem from the ζ^a_μ integration.

For tractability we specialize to $\mathcal{A}^1_1 = \mathcal{A}^2_2 = \mathcal{A}^3_3 = \mathcal{A}$ and zero otherwise, corresponding to $\mathcal{F}^3_{12} = \mathcal{F}^1_{23} = \mathcal{F}^2_{31} = g\mathcal{A}\mathcal{A} = \mathcal{F}$ and $J^1_1 = J^2_2 = J^3_3 = -2g^2\mathcal{A}\mathcal{A}\mathcal{A}$. We find for the determinant

$$\det(G^{-1}G_\varepsilon/\varepsilon^2) = \varepsilon^{-10}(p^4 + |2g\mathcal{F}|^2)(p^4 + |2g\varepsilon^2\mathcal{F}|^2)^{-1}.$$

For $\mathcal{L}^{(1)}$ this leads to

$$\mathcal{L}^{(1)} = \lim_{\varepsilon\to 0}\int \frac{d|p||p|^2}{(2\pi)^2}\bigg(\ln\frac{|p|^4 + |2g\mathcal{F}|^2}{|p|^4} - \ln\varepsilon^{10}\bigg).$$

The integral is IR finite and UV divergent (the last term). After removing the divergent part (and taking the now trivial $\varepsilon \to 0$ limit) $\mathcal{L}^{(1)} = |g\mathcal{F}|^{3/2}/(3\pi)$.

The $\mathcal{F}$ dependent prefactor has its origin in the covariant derivative and the momentum integral. When rescaling every momentum by $|g\mathcal{F}|^{1/2}$ the measure picks up a factor of $|g\mathcal{F}|^{d/2}$ in d dimensions. The remaining integral is field independent. In four dimensions this amounts to a factor $\sim g^2\mathcal{F}^a_{\mu\nu}\mathcal{F}^a_{\mu\nu}$. Thus, a divergent contribution from the integral can be handled by renormalization. The factor of $|g\mathcal{F}|^{d/2}$ is known from the strong field/massless limit of Abelian effective actions induced by scalars or fermions in constant fields [15].

In conclusion, we have analyzed fluctuations of gauge field around a classical configuration by means of a transformation from the inhomogeneously transforming vector gauge field to a homogeneously transforming antisymmetric tensor field. In the Abelian case this procedure yields the same result as if one integrated out A_μ in Landau gauge, with the difference that no gauge is specified.

For non-Abelian fields the fluctuation analysis proceeds also without the introduction of a gauge or ghosts. It leads to the $\sim|g\mathcal{F}|^{d/2}$ behavior of the effective action in d dimensions. We checked this explicitly for a static background without Wu-Yang ambiguity. For four dimensions this indicates a dependence $\sim g^2\mathcal{F}^a_{\mu\nu}\mathcal{F}^{a\mu\nu}$ permitting the treatment of infinite contributions by renormalization.

Additionally, in the $B^a_{\mu\nu}$ field formulation, the criterion $\det\mathbb{F} = 0$ marks background fields that give rise to instabilites, e.g., no field or a Coulomb field, which links them to Wu-Yang ambiguities.

It would be interesting to recast the present approach in the framework of the worldline formalism [16].

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