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$AdS_4 \times \mathbb{CP}^3$ superstring and $D = 3$ $\mathcal{N} = 6$ superconformal symmetry

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Motivated by the isomorphism between $osp(4|6)$ superalgebra and $D = 3$ $\mathcal{N} = 6$ superconformal algebra we consider the superstring action on the $AdS_4 \times \mathbb{CP}^3$ background parametrized by $D = 3$ $\mathcal{N} = 6$ super-Poincare and $\mathbb{CP}^3$ coordinates supplemented by the coordinates corresponding to dilatation and superconformal generators. The relation between the degeneracy of fermionic equations of motion and the action κ-invariance in the framework of the supercoset approach is also discussed.

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I. INTRODUCTION

The idea of gauge/string correspondence has been elaborated since the early days of string theory. During the last decade significant progress has been attained in understanding the duality [1–3] between the $D = 4$ $\mathcal{N} = 4$ super-Yang-Mills theory and string theory on an $AdS_5 \times S^5$ background. A recently novel example of the gauge/string correspondence has been proposed [4] involving the superconformal $D = 3$ $\mathcal{N} = 6$ Chern-Simons-matter theory [5] with the gauge group $U(N) \times U(N)$ and level k and M-theory on an $AdS_4 \times (S^7/\mathbb{Z}_k)$ background. In the t'Hooft limit N, $k \to \infty$ with $\lambda = N/k$ fixed the field theory can be effectively described by the IIA superstring on an $AdS_4 \times \mathbb{CP}^3$ background.

For both dualities one of the main unsolved problems is to quantize corresponding superstring models. The full action for the Green-Schwarz (GS) superstring on $AdS_5 \times S^5$ was constructed in [7,8] on the symmetry grounds using that $AdS_5 \times S^5$ is the maximally supersymmetric background of Type IIB supergravity and that all bosonic and fermionic degrees of freedom fit into the supercoset space $PSU(2,2|4)/(SO(1,4) \times SO(5))$. It was then discovered [9] that such full action is classically integrable extending the previous result [10] for the bosonic model. This stimulated application of the methods developed for the investigation of integrable systems [11]. However, the nonlinearity of the superstring action even after the exclusion of the pure gauge degrees of freedom still precludes from solving the quantization problem and motivates application of the approximate methods [13,14].

To obtain the superstring action on an $AdS_4 \times \mathbb{CP}^3$ background including the fermions it has been suggested in [16,17] to apply the supercoset method of [7]. The main observation is that the bosonic degrees of freedom fit into the bosonic body $(Sp(4)/SO(1,3)) \times (SO(6)/U(3))$ of the supercoset space $OSp(4|6)/(SO(1,3) \times U(3))$ that also allows to accommodate 24 fermions equal in number to the supersymmetries preserved by the $AdS_4 \times \mathbb{CP}^3$ background. It was shown [16] that such superstring action involving 24 fermions is invariant under the 8-parameter κ-symmetry transformations and is classically integrable.

Similarly to the $AdS_5 \times S^5$ superstring the original superstring action on $AdS_4 \times \mathbb{CP}^3$ was given in the AdS basis for the Cartan forms with the appropriate choice of the supercoset element. The isomorphism between the AdS_4 algebra and conformal algebra in $1 + 2$ dimensions suggests considering also the superstring action in the conformal basis [20]. Choosing the $OSp(4|6)/(SO(1,3) \times U(3))$ supercoset representative parametrized by the $D = 3$ $\mathcal{N} = 6$ superspace coordinates, $\mathbb{CP}^3$ coordinates and those associated with the dilatation and superconformal generators yields the action with manifest $D = 3$ $\mathcal{N} = 6$ super-Poincare symmetry that is the subgroup of the symmetry group on the field theory side of the duality [4,23].

It should be noted that despite the fact that the supercoset action on the $AdS_4 \times \mathbb{CP}^3$ background has clear group-theoretical structure, involves the correct number of physical degrees of freedom and is classically integrable, unlike the supercoset action on $AdS_5 \times S^5$, it cannot describe all possible superstring motions, as was already observed in [16]. To study such string configurations the explicit form of the action depending on all 32 fermionic variables is needed that in turn requires to elaborate on the full superspace solution of the IIA supergravity on $AdS_4 \times \mathbb{CP}^3$ [24]. However, whether such full-fledged action for the IIA superstring on $AdS_4 \times \mathbb{CP}^3$ inherits the integrability property remains unknown.

In Sec. II we discuss the properties of the superstring action on the supercoset space $OSp(4|6)/(SO(1,3) \times U(3))$. In particular, the equations of motion for the fermions are cast into the form close to that derived in the conventional GS approach [25], and it is proved that in the general case 8 of 24 equations are trivial. So that the κ-symmetry already manifests itself at the level of the equations of motion. This degeneracy of the equations of motion for the fermions traces back to the form of the anticommutators of the fermionic generators of $D = 3$ $\mathcal{N} = 6$ superconformal algebra. We also give the representation for the κ-symmetry transformations, that allows to gauge away $\frac{1}{3}$ of the fermionic degrees of freedom, in the

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form amenable for comparison with the GS κ-symmetry transformations that remove $\frac{1}{2}$ of the fermions.

In Sec. III we derive the explicit expressions for the Cartan forms in the conformal basis starting from the $OSp(4|6)/(SO(1,3)\times U(3))$ supercoset element and use them to write the superstring action in the form with manifest $D=3$ $\mathcal{N}=6$ super-Poincare symmetry. The possibility of fixing the gauge freedom related to the 8-parameter κ-symmetry is discussed.

In the Appendixes the relevant properties of spinors and γ-matrices in $D=2+3$, $D=1+3$ and $D=1+2$ dimensions are summarized, and details on the isomorphism between the $osp(4|6)$ superalgebra and $D=3$ $\mathcal{N}=6$ superconformal algebra are given.

II. SUPERSTRING ACTION IN THE SUPERCOSET APPROACH: EQUATIONS OF MOTION AND κ-SYMMETRY

The starting point is the $OSp(4|6)/(SO(1,3)\times U(3))$ supercoset element $\mathcal{G}$ that is used to define the left-invariant Cartan 1-forms

$$\mathcal{G}^{-1}d\mathcal{G} = G_{\underline{mn}}(d)M^{\underline{mn}} + \Omega_{\hat{i}}{}^{\hat{j}}(d)V_{\hat{j}}{}^{\hat{i}} + F^{\alpha}_{a}(d)O^{a}_{\alpha} + \bar{F}^{\alpha a}(d)\bar{O}_{\alpha a}. \quad (1)$$

The bosonic 1-forms $G_{\underline{mn}}(d)$, $\underline{m}$, $\underline{n}=0',0,\dots,3$ are associated with the $so(2,3)\sim sp(4)$ generators $M^{\underline{mn}}$ that can be split into the $so(1,3)$ generators $M^{m'n'}$, $m',n'=0,\dots,3$ and the $so(2,3)/so(1,3)$ coset generators $M^{0'm'}$ that corresponds to representing the $so(2,3)$ algebra as the AdS_4 one. Accordingly the 1-forms $G_{m'n'}(d)$ define the $so(1,3)$ connection and $G_{0'm'}(d)$ the AdS_4 veirbein. Analogously the Cartan forms $\Omega_{\hat{i}}{}^{\hat{j}}(d)$, $\hat{i},\hat{j}=1,\dots,4$

$$\Omega_{\hat{i}}{}^{\hat{j}} = \begin{pmatrix} \Omega_a{}^b & \Omega_a{}^4 \\ \Omega_4{}^b & \Omega_4{}^4 \end{pmatrix}, \qquad \Omega_4{}^4 = -\Omega_a{}^a \quad (2)$$

can be split into the 1-forms $\Omega_a{}^b(d)$ corresponding to the $u(3)$ generators $V_a{}^b$ and the 1-forms $\Omega_4{}^a(d)$, $\Omega_a{}^4(d)$ related to the $su(4)/u(3)$ coset generators $V_a{}^4$, $V_4{}^a$. These forms define the $u(3)$ connection and the $\mathbb{CP}^3$ vielbein, respectively. The fermionic 1-forms $F^{\alpha}_a(d)$ and $\bar{F}^{\alpha a}(d)$ are related to the $osp(4|6)$ odd generators O^a_{α}, $\bar{O}_{\alpha a}$ carrying the $D=2+3$ Majorana spinor index $\alpha=1,\dots,4$ and transforming in the vector representation of $SO(6)$ that decomposes as $\mathbf{3}\oplus\bar{\mathbf{3}}$ with respect to $SU(3)$ (see Appendix B). By construction the Cartan forms (1) satisfy the Maurer-Cartan (MC) equations that can be schematically written as

$$d\omega_{\mathcal{A}} + \frac{1}{2}\omega_{\mathcal{B}}(d)\wedge\omega_{\mathcal{C}}(d)f^{\mathcal{CB}}{}_{\mathcal{A}} = 0, \quad (3)$$

where $f^{\mathcal{CB}}{}_{\mathcal{A}}$ are the structure constants of the $osp(4|6)$ superalgebra.

Under the discrete automorphism Υ of the $osp(4|6)$ superalgebra the $so(1,3)$ and $u(3)$ generators are inert, while the remaining bosonic generators change the sign $\Upsilon(M^{0'm'})=-M^{0'm'}$, $\Upsilon(V_a{}^4)=-V_a{}^4$, $\Upsilon(V_4{}^a)=-V_4{}^a$. The fermionic generators transform as $\Upsilon(O^a_{\alpha})=iO^a_{\alpha}$, $\Upsilon(\bar{O}_{\alpha a})=-i\bar{O}_{\alpha a}$. These transformations of the $osp(4|6)$ generators induce transformations of the associated Cartan forms and serve as the guide to construct the $\mathbb{Z}_4$-invariant superstring action

$$\mathcal{S} = -\frac{1}{2}\int d^2\xi\sqrt{-g}g^{ij}(G_{i0'}{}^{m'}G_{j0'm'} + \Omega_{ia}{}^4\Omega_{j4}{}^a) + \mathcal{S}_{WZ}, \quad (4)$$

where the Wess-Zumino term is given by the wedge product of the fermionic Cartan forms [26,27]

$$\mathcal{S}_{WZ} = \frac{i}{2}\varepsilon^{ij}\int d^2\xi F^{\alpha}_{ia}C'_{\alpha\beta}\bar{F}^{\beta a}_{j}. \quad (5)$$

Two summands entering the kinetic term correspond to the AdS_4 and $\mathbb{CP}^3$ parts of the background. The WZ term involves the $D=1+3$ charge conjugation matrix $C'_{\alpha\beta}$.

The superstring Lagrangian is constructed out of the world-sheet projections of Cartan 1-forms; thus to find its variation it is necessary to consider the variations of relevant 1-forms. Using the general formula for the variation of a form

$$\delta F(d) = d(i_\delta F(d)) + i_\delta(dF(d)) \quad (6)$$

in the second summand one substitutes the MC equations

$$dG^{0'm'} - 2G^{m'}{}_{n'}(d)\wedge G^{0'n'}(d) - iF^{\alpha}_a(d)\wedge\gamma^{0'm'}_{\alpha\beta}\bar{F}^{\beta a}(d)=0, \quad (7)$$

$$d\Omega_a{}^4 + i\Omega_{+a}{}^b(d)\wedge\Omega_b{}^4(d) - \varepsilon_{abc}\bar{F}^{\alpha b}(d)\wedge C_{\alpha\beta}\bar{F}^{\beta c}(d)=0, \quad (8)$$

$$d\Omega_4{}^a + i\Omega_4{}^b(d)\wedge\Omega_{+b}{}^a(d) + \varepsilon^{abc}F^{\alpha}_b(d)\wedge C_{\alpha\beta}F^{\beta}_c(d)=0, \quad (9)$$

$$dF^{\alpha}_a + \frac{1}{2}F^{\beta}_a(d)\wedge G_{\underline{mn}}(d)\gamma^{\underline{mn}}{}_{\beta}{}^{\alpha} + i\Omega_{-a}{}^b(d)\wedge F^{\alpha}_b(d) + i\varepsilon_{acb}\Omega_4{}^c(d)\wedge\bar{F}^{\alpha b}(d) = 0, \quad (10)$$

$$d\bar{F}^{\alpha a} + \frac{1}{2}\bar{F}^{\beta a}(d)\wedge G_{\underline{mn}}(d)\gamma^{\underline{mn}}{}_{\beta}{}^{\alpha} + i\bar{F}^{\alpha b}(d)\wedge\Omega_{-b}{}^a(d) - i\varepsilon^{acb}\Omega_c{}^4(d)\wedge F^{\alpha}_b(d) = 0, \quad (11)$$

where $G_{\underline{mn}}(d)\gamma^{\underline{mn}}{}_{\alpha}{}^{\beta} = 2G_{0'm'}(d)\gamma^{0'm'}{}_{\alpha}{}^{\beta} + G_{m'n'}(d)\gamma^{m'n'}{}_{\alpha}{}^{\beta}$ and $\Omega_{\pm a}{}^b(d) = \Omega_a{}^b(d) \pm \delta^b_a\Omega_c{}^c(d)$ are the $so(2,3)$ and $u(3)\oplus u(1)$ connections. Then the variation of the 1-forms entering the action (4) acquires the form

$$\delta G^{0'm'}(d) = dG^{0'm'}(\delta) + 2G^{m'}{}_{n'}(d)G^{0'n'}(\delta) - 2G^{m'}{}_{n'}(\delta)G^{0'n'}(d) + iF^{\alpha}_{a}(d)\gamma^{0'm'}_{\alpha\beta}\bar{F}^{\beta a}(\delta) - iF^{\alpha}_{a}(\delta)\gamma^{0'm'}_{\alpha\beta}\bar{F}^{\beta a}(d), \tag{12}$$

$$\delta\Omega_a{}^4(d) = d\Omega_a{}^4(\delta) - i\Omega_{+a}{}^b(d)\Omega_b{}^4(\delta) + i\Omega_{+a}{}^b(\delta)\Omega_b{}^4(d) + 2\varepsilon_{abc}\bar{F}^{\alpha b}(d)C_{\alpha\beta}\bar{F}^{\beta c}(\delta), \tag{13}$$

$$\delta\Omega_4{}^a(d) = d\Omega_4{}^a(\delta) - i\Omega_4{}^b(d)\Omega_{+b}{}^a(\delta) + i\Omega_4{}^b(\delta)\Omega_{+b}{}^a(d) - 2\varepsilon^{abc}F^{\alpha}_b(d)C_{\alpha\beta}F^{\beta}_c(\delta), \tag{14}$$

$$\delta F^{\alpha}_a(d) = dF^{\alpha}_a(\delta) - \frac{1}{2}F^{\beta}_a(d)G_{\underline{mn}}(\delta)\gamma^{\underline{mn}}{}_{\beta}{}^{\alpha} + \frac{1}{2}F^{\beta}_a(\delta)G_{\underline{mn}}(d)\gamma^{\underline{mn}}{}_{\beta}{}^{\alpha} - i\Omega_{-a}{}^b(d)F^{\alpha}_b(\delta) + i\Omega_{-a}{}^b(\delta)F^{\alpha}_b(d) - i\varepsilon_{acb}\Omega_4{}^c(d)\bar{F}^{\alpha b}(\delta) + i\varepsilon_{acb}\Omega_4{}^c(\delta)\bar{F}^{\alpha b}(d), \tag{15}$$

$$\delta\bar{F}^{\alpha a}(d) = d\bar{F}^{\alpha a}(\delta) - \frac{1}{2}\bar{F}^{\beta a}(d)G_{\underline{mn}}(\delta)\gamma^{\underline{mn}}{}_{\beta}{}^{\alpha} + \frac{1}{2}\bar{F}^{\beta a}(\delta)G_{\underline{mn}}(d)\gamma^{\underline{mn}}{}_{\beta}{}^{\alpha} - i\bar{F}^{\alpha b}(d)\Omega_{-b}{}^a(\delta) + i\bar{F}^{\alpha b}(\delta)\Omega_{-b}{}^a(d) + i\varepsilon^{acb}\Omega_c{}^4(d)F^{\alpha}_b(\delta) - i\varepsilon^{acb}\Omega_c{}^4(\delta)F^{\alpha}_b(d). \tag{16}$$

Since of the utmost importance is the κ-invariance of the superstring action (4) we concentrate on the fermionic contribution to the variation of the action

$$\delta\mathcal{S}|_f = \int d^2\xi(\bar{\mathcal{F}}^{\alpha\hat{a}}_{\{+\}i}V^{ij}_{+}M_{j\alpha\hat{a}}{}^{\beta\hat{b}}C_{\beta\gamma}\mathcal{F}^{\gamma}_{\{+\}\hat{b}}(\delta) + \bar{\mathcal{F}}^{\alpha\hat{a}}_{\{-\}i}V^{ij}_{-}M_{j\alpha\hat{a}}{}^{\beta\hat{b}}C_{\beta\gamma}\mathcal{F}^{\gamma}_{\{-\}\hat{b}}(\delta)), \tag{17}$$

where

$$M_{i\alpha\hat{a}}{}^{\beta\hat{b}} = \begin{pmatrix} -i\delta^b_aG_{i0'm'}\gamma^{0'm'}{}_{\alpha}{}^{\beta} & \delta^{\beta}_{\alpha}\varepsilon_{acb}\Omega_{i4}{}^c \\ -\delta^{\beta}_{\alpha}\varepsilon^{acb}\Omega_{ic}{}^4 & -i\delta^a_bG_{i0'm'}\gamma^{0'm'}{}_{\alpha}{}^{\beta} \end{pmatrix}. \tag{18}$$

The expression (17) analogous to the GS superstring case [25] involves the world-sheet projectors

$$V^{ij}_{\pm} = \frac{1}{2}(\sqrt{-g}g^{ij} \pm \varepsilon^{ij}) \tag{19}$$

obeying the relations

$$V^{ij}_{+} + V^{ij}_{-} = \sqrt{-g}g^{ij}, \qquad V^{ik}_{\pm}g_{kl}V^{jl}_{\pm} = 0,$$
$$V^{ik}_{\pm}g_{kl}V^{lj}_{\pm} = \sqrt{-g}V^{ij}_{\pm}, \qquad V^{ij}_{\pm}V^{kl}_{\pm} = V^{kj}_{\pm}V^{il}_{\pm}. \tag{20}$$

The fermionic variation parameters and the world-sheet projections of Cartan forms have been grouped as follows:

$$\mathcal{F}^{\ \alpha}_{\{\pm\}\hat{a}}(\delta) = \begin{pmatrix} F^{\alpha}_{(\pm)a}(\delta) \\ \bar{F}^{\alpha a}_{(\mp)}(\delta) \end{pmatrix} \tag{21}$$

and

$$\bar{\mathcal{F}}^{\ \alpha\hat{a}}_{\{\pm\}i} = \begin{pmatrix} \bar{F}^{\alpha a}_{(\mp)i} \\ F^{\alpha}_{(\pm)ia} \end{pmatrix}. \tag{22}$$

They include chiral in the $D = 1 + 3$ dimensional sense spinors $F^{\alpha}_{(\pm)a}(\delta)$, $F^{\alpha}_{(\pm)ia}$ and their conjugates $\bar{F}^{\alpha a}_{(\mp)}(\delta)$, $\bar{F}^{\alpha a}_{(\mp)i}$ [28]. The chiral projectors are defined as

$$P^{\alpha}_{+\beta} = \frac{1}{2}(\delta^{\alpha}_{\beta} + C^{\alpha\gamma}C'_{\gamma\beta}),$$
$$P^{\alpha}_{-\beta} = (P^{\alpha}_{+\beta})^* = \frac{1}{2}(\delta^{\alpha}_{\beta} - C^{\alpha\gamma}C'_{\gamma\beta}) \tag{23}$$

and satisfy the requisite properties

$$P_+ + P_- = I, \qquad P_{\pm}P_{\pm} = P_{\pm},$$
$$P_+P_- = P_-P_+ = 0, \tag{24}$$

because of the relation

$$C^{\alpha\beta}C'_{\beta\gamma}C^{\gamma\delta}C'_{\delta\epsilon} = \delta^{\alpha}_{\epsilon}. \tag{25}$$

The definition (23) is justified by the fact that $C^{\alpha\beta}C'_{\beta\gamma}$ is related to the $4d$ matrix $\Gamma^{5}_{\alpha}{}^{\beta}$

$$C^{\alpha\beta}C'_{\beta\delta} = iC'^{\alpha\beta}\Gamma^{5}_{\beta}{}^{\gamma}C'_{\gamma\delta}. \tag{26}$$

To derive (17) we have also used the following properties of chiral projectors (23):

$$P^{\alpha}_{\pm\beta}C_{\alpha\gamma}P^{\gamma}_{\mp\delta} = 0, \qquad P^{\alpha}_{\pm\beta}\gamma^{0'm'}_{\alpha\gamma}P^{\gamma}_{\pm\delta} = 0. \tag{27}$$

The variation (17) determines the equations of motion for the fermions

$$V^{ij}_{\pm}M^{T\alpha\hat{a}}_{j}{}_{\beta\hat{b}}\bar{\mathcal{F}}^{\beta\hat{b}}_{\{\pm\}i} = 0. \tag{28}$$

To find whether all of the Eqs. (28) are nontrivial we need to compute the rank of $M^{T\alpha\hat{a}}_{j}{}_{\beta\hat{b}}$

$$M^{T\alpha\hat{a}}_{j}{}_{\beta\hat{b}} = \begin{pmatrix} -i\delta^a_bG_{j0'm'}\gamma^{0'm'\alpha}{}_{\beta} & \delta^{\alpha}_{\beta}\varepsilon^{acb}\Omega_{jc}{}^4 \\ -\delta^{\alpha}_{\beta}\varepsilon_{acb}\Omega_{j4}{}^c & -i\delta^b_aG_{j0'm'}\gamma^{0'm'\alpha}{}_{\beta} \end{pmatrix} \tag{29}$$

on shell of the Virasoro constraints

$$\frac{\delta\mathcal{S}}{\delta g^{ij}(\xi)} = G_{i0'm'}G_{j0'}{}^{m'} + \frac{1}{2}(\Omega_{ia}{}^4\Omega_{j4}{}^a + \Omega_{ja}{}^4\Omega_{i4}{}^a) - \frac{1}{2}g_{ij}g^{kl}(G_{k0'm'}G_{l0'}{}^{m'} + \Omega_{ka}{}^4\Omega_{l4}{}^a) = 0. \tag{30}$$

In Eqs. (28) the $2d$ vector index of $M^{T\alpha\hat{a}}_{j}{}_{\beta\hat{b}}$ is acted by the world-sheet projectors (19) so that only one of its components out of two is independent. This can be illustrated, for instance, by the action of $V_\pm^{ij}$ on a vector F_j that can be presented as

$$V_\pm^{ij} F_j = V_\pm^i F_{\pm\tau}, \qquad V_\pm^i = \frac{1}{2}\begin{pmatrix} \frac{1}{\sqrt{-g}g^{\tau\sigma}\mp 1} \\ \sqrt{-g}g^{\tau\tau} \end{pmatrix}, \\ F_{\pm\tau} = \sqrt{-g}g^{\tau\tau}F_\tau + (\sqrt{-g}g^{\tau\sigma} \pm 1)F_\sigma. \tag{31}$$

Similarly the result of $V_\pm^{ij}$ projector action on the Virasoro constraints (30) reads

$$V_\pm^{ik} V_\pm^{jl} \frac{\delta\mathcal{S}}{\delta g^{kl}(\xi)} = V_\pm^i V_\pm^j (G_{\pm\tau 0'm'} G_{\pm\tau 0'}{}^{m'} + \Omega_{\pm\tau a}{}^4 \Omega_{\pm\tau 4}{}^a) \\ = 0. \tag{32}$$

Then one observes that the matrix $G_{\pm\tau 0'm'}\gamma^{0'm'\alpha}{}_\beta$ is nonsingular

$$G_{\pm\tau 0'm'}\gamma^{0'm'\alpha}{}_\beta G_{\pm\tau 0'n'}\gamma^{0'n'\beta}{}_\gamma = G_{\pm\tau\pm\tau}\delta^\alpha_\gamma, \tag{33}$$

where $G_{\pm\tau\pm\tau} = G_{\pm\tau 0'm'}G_{\pm\tau 0'}{}^{m'}$. This allows by the rank preserving transformation to bring M^T to the triangular form

$$\begin{pmatrix} -i\delta^a_b G_{\pm\tau 0'm'}\gamma^{0'm'\alpha}{}_\beta & \delta^\alpha_\beta \varepsilon^{acb}\Omega_{\pm\tau c}{}^4 \\ 0 & i\frac{\Omega_{\pm\tau a}{}^4\Omega_{\pm\tau 4}{}^b}{G_{\pm\tau\pm\tau}} G_{\pm\tau 0'm'}\gamma^{0'm'\alpha}{}_\beta \end{pmatrix}. \tag{34}$$

Since the rank of the 3×3 matrix $\Omega_{\pm\tau a}{}^4\Omega_{\pm\tau 4}{}^b$ is unity [29], the rank of M^T equals $4\times 3 + 4\times 1 = 16$. As a result 8 out of 24 equations (28) are trivial and this implies via the second Noether theorem the 8-parameter fermionic symmetry of the action (4). The crucial distinction of the supercoset string model [16] from the GS superstring on flat background [25] and on $AdS_5\times S^5$ [7] is that the κ-symmetry can gauge away only $\frac{1}{3}$ of the fermions rather than $\frac{1}{2}$. This is attributed to the fact that the action (4) could be obtained by the partial κ-symmetry gauge fixing from the full action containing 32 fermionic degrees of freedom, and the 8-parameter fermionic symmetry of (4) is the remnant of the 16-parameter symmetry of that full action [31].

It is worthwhile to note that the matrix M can be obtained starting from the matrix of the anticommutators of the fermionic generators of the $osp(4|6)$ superalgebra

$$\begin{pmatrix} \{\bar{O}_{\alpha a}, O^b_\beta\} & \{\bar{O}_{\alpha a}, \bar{O}_{\beta b}\} \\ \{O^a_\alpha, O^b_\beta\} & \{O^a_\alpha, \bar{O}_{\beta b}\} \end{pmatrix}. \tag{35}$$

Substituting the explicit expressions for its entries (see Appendix B)

$$\begin{pmatrix} -i\delta^b_a\gamma^{mn}_{\alpha\beta}M_{\underline{mn}} + 2C_{\alpha\beta}(V_a{}^b - \delta^b_a V_c{}^c) & 2C_{\alpha\beta}\varepsilon_{acb}V_4{}^c \\ -2C_{\alpha\beta}\varepsilon^{acb}V_c{}^4 & -i\delta^a_b\gamma^{mn}_{\alpha\beta}M_{\underline{mn}} - 2C_{\alpha\beta}(V_b{}^a - \delta^a_b V_c{}^c) \end{pmatrix} \tag{36}$$

and replacing the $so(2,3)/so(1,3)$ and $su(4)/u(3)$ coset generators by the Cartan forms $M_{0'm'} \to G_{0'm'}(d)$, $V_a{}^4 \to \Omega_a{}^4(d)$, $V_4{}^a \to \Omega_4{}^a(d)$ yields up to the overall factor the entries of the matrix M. This is of course the anticipated result since the action variation is determined by the variation of Cartan forms that in turn depends on the structure constants of the $osp(4|6)$ superalgebra. However, this observation could be of more use when applied backwards: starting from the matrix composed of the anticommutators of the fermionic generators of the isometry superalgebra for some superbackground, whose bosonic part can be presented as the coset space, one can study the degeneracy of such a matrix to find whether the corresponding string model, constructed using the supercoset approach, will be κ-invariant.

Let us consider the κ-invariance property of the action (4) in more detail. An equal number of physical and pure gauge fermions in the GS superstring implied that the same matrix with the space-time spinor indices was present both in the equations of motion for the fermions and the κ-symmetry transformation rules. However, in the present case M^T cannot directly appear in the κ-transformations because it is required that the matrix of the rank 8 single out the requisite number of independent transformation parameters. Such a matrix can be constructed as the second-order polynomial in the world-sheet projections of Cartan 1-forms [16]

$$K_{ij\hat{a}\beta}^{\alpha\hat{b}} = \begin{pmatrix} \delta^\alpha_\beta(G_{i0'}{}^{m'}G_{j0'm'}\delta^b_a + \Omega_{ia}{}^4\Omega_{j4}{}^b) & iG_{i0'm'}\gamma^{0'm'\alpha}{}_\beta\varepsilon_{acb}\Omega_{j4}{}^c \\ -iG_{i0'm'}\gamma^{0'm'\alpha}{}_\beta\varepsilon^{acb}\Omega_{jc}{}^4 & \delta^\alpha_\beta(G_{i0'}{}^{m'}G_{j0'm'}\delta^a_b + \Omega_{i4}{}^a\Omega_{jb}{}^4) \end{pmatrix} \tag{37}$$

and is used in the κ-symmetry transformation rules for the fermionic 1-forms

$$\mathcal{F}^\alpha_{\{-\}\hat{a}}(\delta_\kappa) = V_-^{ij}V_-^{kl}K_{jl\hat{a}\beta}^{\alpha\hat{b}}\varkappa^\beta_{\{-\}\hat{b}ik}, \qquad \mathcal{F}^\alpha_{\{+\}\hat{a}}(\delta_\kappa) = V_+^{ij}V_+^{kl}K_{jl\hat{a}\beta}^{\alpha\hat{b}}\tilde{\varkappa}^\beta_{\{+\}\hat{b}ik}. \tag{38}$$

As the bosonic forms are inert under the κ-symmetry [32]

$$G_{0'm'}(\delta_\kappa) = 0, \qquad \Omega_4{}^a(\delta_\kappa) = 0, \qquad \Omega_a{}^4(\delta_\kappa) = 0, \tag{39}$$

the κ-variation of the action (4) obtained by the substitution of (38) into (17) is compensated by the variation of the auxiliary $2d$ metric

$$\delta_\kappa(\sqrt{-g}g^{ij}) = 2i(\bar{\mathcal{F}}^{\alpha\hat{a}}_{\{-\}k} V_-^{kl} G_{l0'm'} \gamma^{0'm'}_{\alpha\beta} V_+^{ii'} V_+^{jj'} \varkappa^\beta_{\{-\}\hat{a}i'j'} + \bar{\mathcal{F}}^{\alpha\hat{a}}_{\{+\}k} V_+^{kl} G_{l0'm'} \gamma^{0'm'}_{\alpha\beta} V_-^{ii'} V_-^{jj'} \tilde{\varkappa}^\beta_{\{+\}\hat{a}i'j'}). \tag{40}$$

The polynomial structure of K requires the parameters of the κ-transformation $\varkappa^\beta_{\{-\}\hat{a}ij}$, $\tilde{\varkappa}^\beta_{\{+\}\hat{a}ij}$ to carry the pair of the world-sheet vector indices instead of one as in the GS case and to satisfy the (anti)self-duality constraints in each index

$$\frac{1}{\sqrt{-g}} g_{ij} V_+^{jk} \varkappa^\beta_{\{-\}\hat{a}kl} = \frac{1}{\sqrt{-g}} g_{lj} V_+^{jk} \varkappa^\beta_{\{-\}\hat{a}ik} = \varkappa^\beta_{\{-\}\hat{a}il}, \tag{41}$$

and

$$\frac{1}{\sqrt{-g}} g_{ij} V_-^{jk} \tilde{\varkappa}^\beta_{\{+\}\hat{a}kl} = \frac{1}{\sqrt{-g}} g_{lj} V_-^{jk} \tilde{\varkappa}^\beta_{\{+\}\hat{a}ik} = \tilde{\varkappa}^\beta_{\{+\}\hat{a}il}. \tag{42}$$

On the constraint shell defined by the Virasoro constraints (30) the rank of the κ-transformations equals 8 so that only $\frac{1}{3}$ parameters act nontrivially. Note that in the κ-symmetry transformation rules (38) the $2d$ vector indices of the matrix K are contracted with the world-sheet projectors $V_\pm^{ij}$ so only one independent component of 4 remains. Thus to find the rank of $K_{\pm\tau\pm\tau\hat{a}\beta}{}^{\alpha\hat{b}}$ one can solve the eigenvalue problem that amounts to computing the determinant of $K - \lambda I$ using its block structure

$$\det(K - \lambda I) = \det\begin{pmatrix} A^{\alpha b}_{a\beta} & B^\alpha_{a\ \beta b} \\ C^{\alpha ab}{}_\beta & D^{\alpha a}{}_{\beta b} \end{pmatrix} = \det A \det(D - CA^{-1}B), \tag{43}$$

where

$$\begin{aligned} A^{\alpha b}_{a\beta} &= \delta^\alpha_\beta A_a{}^b, \\ A_a{}^b &= (G_{\pm\tau\pm\tau} - \lambda)\delta^b_a + \Omega_{\pm\tau a}{}^4 \Omega_{\pm\tau 4}{}^b, \\ D^{\alpha a}{}_{\beta b} &= \delta^\alpha_\beta((G_{\pm\tau\pm\tau} - \lambda)\delta^a_b + \Omega_{\pm\tau 4}{}^a \Omega_{\pm\tau b}{}^4), \\ B^\alpha_{a\ \beta b} &= iG_{\pm\tau 0'm'} \gamma^{0'm'\alpha}{}_\beta \varepsilon_{acb} \Omega_{\pm\tau 4}{}^c, \\ C^{\alpha ab}{}_\beta &= -iG_{\pm\tau 0'm'} \gamma^{0'm'\alpha}{}_\beta \varepsilon^{acb} \Omega_{\pm\tau c}{}^4. \end{aligned} \tag{44}$$

The addition of λI renders the matrix $A_a{}^b$ nonsingular, $\det A = -\lambda(G_{\pm\tau\pm\tau} - \lambda)^2$, and its inverse is given by

$$A^{-1}{}_b{}^a = \frac{1}{\lambda(G_{\pm\tau\pm\tau} - \lambda)}(\lambda\delta^a_b + \Omega_{\pm\tau b}{}^4 \Omega_{\pm\tau 4}{}^a). \tag{45}$$

Then the calculation yields that

$$\det(K - \lambda I) = \lambda^{16}(\lambda - 2G_{\pm\tau\pm\tau})^8 = 0. \tag{46}$$

One finds that 8 of 24 eigenvalues of K are nonzero proving that its rank indeed equals 8. So that the matrices M and K are complementary in the sense that $\mathrm{rank}M + \mathrm{rank}K = 24$.

III. SUPERSTRING ACTION IN THE CONFORMAL BASIS

The introduction of the (1 + 2)-dimensional superconformal group generators (B13) and (B23) (see Appendix B) implies via (1) the introduction of the corresponding 1-forms in the conformal basis

$$\begin{aligned} &\Delta(d) = G_{0'3}(d), \qquad \hat{\omega}_m(d) = -(G_{0'm}(d) + G_{3m}(d)), \\ &\hat{c}_m(d) = G_{3m}(d) - G_{0'm}(d), \qquad m = 0, 1, 2 \end{aligned} \tag{47}$$

and

$$F^\alpha_a(d) = \begin{pmatrix} \hat{\omega}^\mu_a \\ \hat{\chi}_{\mu a} \end{pmatrix}, \qquad \bar{F}^{\alpha a}(d) = \begin{pmatrix} \bar{\hat{\omega}}^{\mu a} \\ \bar{\hat{\chi}}^a_\mu \end{pmatrix}. \tag{48}$$

So that the expression (1) acquires the form

$$\begin{aligned} \mathcal{G}^{-1}d\mathcal{G} = {} & G_{mn}(d)M^{mn} + \hat{\omega}_m(d)P^m + \hat{c}_m(d)K^m \\ & + \Delta(d)D + \Omega_a{}^b(d)V_b{}^a + \Omega_a{}^4(d)V_4{}^a \\ & + \Omega_4{}^a(d)V_a{}^4 + \Omega_4{}^4(d)V_4{}^4 + \hat{\omega}^\mu_a(d)Q^a_\mu \\ & + \bar{\hat{\omega}}^{\mu a}(d)\bar{Q}_{\mu a} + \hat{\chi}_{\mu a}(d)S^{\mu a} + \bar{\hat{\chi}}^a_\mu(d)\bar{S}^\mu_a. \end{aligned} \tag{49}$$

It follows from (4) and the definition (47) and (48) that the Cartan forms $\hat{\omega}^m(d)$, $\hat{c}^m(d)$, $\Delta(d)$ and $\hat{\omega}^\mu_a(d)$, $\bar{\hat{\omega}}^{\mu a}(d)$, $\hat{\chi}_{\mu a}(d)$, $\bar{\hat{\chi}}^a_\mu(d)$ enter the superstring action. Relevant MC equations in the conformal basis read

$$
\begin{aligned}
d\hat{\omega}^m - 2\Delta(d)\wedge\hat{\omega}^m(d) - 2G^m{}_n(d)\wedge\hat{\omega}^n(d) + 2i\hat{\omega}^\mu_a(d)\wedge\sigma^m_{\mu\nu}\bar{\hat{\omega}}^{\nu a}(d) &= 0,\\
d\hat{c}^m + 2\Delta(d)\wedge\hat{c}^m(d) - 2G^m{}_n(d)\wedge\hat{c}^n(d) + 2i\hat{\chi}_{\mu a}(d)\wedge\tilde{\sigma}^{m\mu\nu}\bar{\hat{\chi}}^a_\nu(d) &= 0,\\
d\Delta - \hat{\omega}^m(d)\wedge\hat{c}_m(d) - i(\bar{\hat{\omega}}^{\mu a}(d)\wedge\hat{\chi}_{\mu a}(d) + \hat{\omega}^\mu_a(d)\wedge\bar{\hat{\chi}}^a_\mu(d)) &= 0,\\
d\Omega_a{}^4 + i\Omega_{+a}{}^b(d)\wedge\Omega_b{}^4(d) - 2\varepsilon_{abc}\bar{\hat{\omega}}^{\mu b}(d)\wedge\bar{\hat{\chi}}^c_\mu(d) &= 0,\\
d\Omega_4{}^a + i\Omega_4{}^b(d)\wedge\Omega_{+b}{}^a(d) + 2\varepsilon^{abc}\hat{\omega}^\mu_b(d)\wedge\hat{\chi}_{\mu c}(d) &= 0,
\end{aligned} \tag{50}
$$

and

$$
\begin{aligned}
&d\hat{\omega}^\mu_{\hat{a}} - \Delta(d)\wedge\hat{\omega}^\mu_{\hat{a}}(d) + \frac{1}{2}\hat{\omega}^\nu_{\hat{a}}(d)\wedge G_{mn}(d)\sigma^{mn}{}_\nu{}^\mu\\
&\quad + \hat{\omega}^m(d)\wedge\tilde{\sigma}^{\mu\nu}_m\hat{\chi}_{\nu\hat{a}}(d) + i\Omega_{\hat{a}}{}^{\hat{b}}(d)\wedge\hat{\omega}^\mu_{\hat{b}}(d) = 0,
\end{aligned} \tag{51}
$$

$$
\begin{aligned}
&d\hat{\chi}_{\mu\hat{a}} + \Delta(d)\wedge\hat{\chi}_{\mu\hat{a}}(d) + \frac{1}{2}G_{mn}(d)\wedge\sigma^{mn}{}_\mu{}^\nu\hat{\chi}_{\nu\hat{a}}(d)\\
&\quad - \hat{c}_m(d)\wedge\sigma^m_{\mu\nu}\hat{\omega}^\nu_{\hat{a}}(d) + i\Omega_{\hat{a}}{}^{\hat{b}}(d)\wedge\hat{\chi}_{\mu\hat{b}}(d) = 0,
\end{aligned} \tag{52}
$$

where the fermionic 1-forms have been grouped

$$
\hat{\omega}^\mu{}_{\hat{a}}(d) = \begin{pmatrix}\hat{\omega}^\mu_a\\ \bar{\hat{\omega}}^{\mu a}\end{pmatrix}, \qquad \hat{\chi}_{\mu\hat{a}}(d) = \begin{pmatrix}\hat{\chi}_{\mu a}\\ \bar{\hat{\chi}}^a_\mu\end{pmatrix} \tag{53}
$$

according to the decomposition of the $SO(6)$ vector representation into the $SU(3)$ irreducible parts. The elements of the matrix $\Omega_{\hat{a}}{}^{\hat{b}}(d)$ are the components of the $su(4)$ Cartan forms (2)

$$
\Omega_{\hat{a}}{}^{\hat{b}}(d) = \begin{pmatrix}\Omega_a{}^b - \delta^b_a\Omega_c{}^c & \varepsilon_{acb}\Omega_4{}^c\\ -\varepsilon^{acb}\Omega_c{}^4 & -\Omega_b{}^a + \delta^a_b\Omega_c{}^c\end{pmatrix}. \tag{54}
$$

It is antisymmetric with respect to the metric

$$
H_{\hat{a}\hat{b}} = \begin{pmatrix}0 & \delta^b_a\\ \delta^a_b & 0\end{pmatrix} \tag{55}
$$

thus having 15 independent components.

To obtain explicit expressions for the Cartan forms in the conformal basis we consider the following $OSp(4|6)/(SO(1,3)\times U(3))$ supercoset element [33]

$$
\mathcal{G} = e^{x_m P^m + \theta^\mu_a Q^a_\mu + \bar{\theta}^{\mu a}\bar{Q}_{\mu a}} e^{\eta_{\mu a}S^{\mu a} + \bar{\eta}^a_\mu\bar{S}^\mu_a} e^{z^a V_a{}^4 + \bar{z}_a V_4{}^a} e^{\varphi D}. \tag{56}
$$

The bosonic real coordinates x^m and φ parametrize AdS_4, while 3 complex coordinates z^a and their conjugate $\bar{z}_a$ parametrize $\mathbb{CP}^3$. The anticommuting coordinates can be divided into θ^μ_a, $\bar{\theta}^{\mu a}$ related to the Poincare supersymmetry and $\eta_{\mu a}$, $\bar{\eta}^a_\mu$ related to the conformal supersymmetry. Then the calculation yields for the Cartan forms associated with the $so(2,3)/so(1,3)$ coset generators

$$
\begin{aligned}
\hat{\omega}^m(d) &= e^{-2\varphi}\omega^m(d),\\
\omega^m(d) &= dx^m - id\theta^\mu_a\sigma^m_{\mu\nu}\bar{\theta}^{\nu a} + i\theta^\mu_a\sigma^m_{\mu\nu}d\bar{\theta}^{\nu a},
\end{aligned} \tag{57}
$$

$$
\begin{aligned}
\hat{c}^m(d) &= e^{2\varphi}c^m(d),\\
c^m(d) &= -id\eta_{\mu a}\tilde{\sigma}^{m\mu\nu}\bar{\eta}^a_\nu + i\eta_{\mu a}\tilde{\sigma}^{m\mu\nu}d\bar{\eta}^a_\nu\\
&\quad - 2\left(d\theta_{\mu a} + \frac{1}{4}\zeta_{\mu a}(d)\right)\tilde{\sigma}^{m\mu\nu}\bar{\eta}^a_\nu(\bar{\eta}^b_\rho\eta^\rho_b)\\
&\quad + 2\eta_{\mu a}\tilde{\sigma}^{m\mu\nu}\left(d\bar{\theta}^a_\nu + \frac{1}{4}\bar{\zeta}^a_\nu(d)\right)(\bar{\eta}^b_\rho\eta^\rho_b),
\end{aligned} \tag{58}
$$

$$
\Delta(d) = d\varphi + id\theta^\mu_a\bar{\eta}^a_\mu + id\bar{\theta}^{\mu a}\eta_{\mu a}, \tag{59}
$$

where

$$
\begin{aligned}
\zeta^\mu_a(d) &= -\tilde{\sigma}^{m\mu\nu}\omega_m(d)\eta_{\nu a},\\
\bar{\zeta}^{\mu a}(d) &= -\tilde{\sigma}^{m\mu\nu}\omega_m(d)\bar{\eta}^a_\nu,
\end{aligned} \tag{60}
$$

and for those associated with the $so(1,3)$ generators

$$
\begin{aligned}
G^{mn} &= -i\left(d\theta^\mu_a + \frac{1}{2}\zeta^\mu_a(d)\right)\sigma^{mn}{}_\mu{}^\nu\bar{\eta}^a_\nu\\
&\quad - i\left(d\bar{\theta}^{\mu a} + \frac{1}{2}\bar{\zeta}^{\mu a}(d)\right)\sigma^{mn}{}_\mu{}^\nu\eta_{\nu a}.
\end{aligned} \tag{61}
$$

For the $su(4)$ Cartan form matrix (54) we find

$$
\Omega_{\hat{a}}{}^{\hat{b}}(d) = \Omega_{\mathbf{b}\hat{a}}{}^{\hat{b}}(d) + \Omega_{\mathbf{f}\hat{a}}{}^{\hat{b}}(d). \tag{62}
$$

The bosonic contribution is given by

$$
\begin{aligned}
\Omega_{\mathbf{b}\hat{a}}{}^{\hat{b}}(d) &= iT_{\hat{a}}{}^{\hat{c}}d\bar{T}_{\hat{c}}{}^{\hat{b}}\\
&= \begin{pmatrix}\Omega_{\mathbf{b}a}{}^b - \delta^b_a\Omega_{\mathbf{b}c}{}^c & \varepsilon_{acb}\Omega_{\mathbf{b}4}{}^c\\ -\varepsilon^{acb}\Omega_{\mathbf{b}c}{}^4 & -\Omega_{\mathbf{b}b}{}^a + \delta^a_b\Omega_{\mathbf{b}c}{}^c\end{pmatrix},
\end{aligned} \tag{63}
$$

where the unitary matrix T equals

$$
\begin{aligned}
T_{\hat{a}}{}^{\hat{b}} &= \begin{pmatrix}\delta^b_a\cos|z| + \bar{z}_a z^b\frac{(1-\cos|z|)}{|z|^2} & i\varepsilon_{acb}z^c\frac{\sin|z|}{|z|}\\ -i\varepsilon^{acb}\bar{z}_c\frac{\sin|z|}{|z|} & \delta^a_b\cos|z| + z^a\bar{z}_b\frac{(1-\cos|z|)}{|z|^2}\end{pmatrix},\\
|z|^2 &= z^a\bar{z}_a.
\end{aligned} \tag{64}
$$

So that the explicit form of the entries of $\Omega_{\mathbf{b}\hat{a}}{}^{\hat{b}}(d)$ is given by

$$\Omega_{\mathbf{b}a}{}^b(d) = i\frac{(1-\cos|z|)}{|z|^2}(\bar{z}_a dz^b - d\bar{z}_a z^b) - i\bar{z}_a z^b \times \frac{(1-\cos|z|)^2}{|z|^4}(dz^c\bar{z}_c - z^c d\bar{z}_c),$$

$$\Omega_{\mathbf{b}a}{}^4(d) = d\bar{z}_a\frac{\sin|z|}{|z|} + \bar{z}_a\frac{\sin|z|(1-\cos|z|)}{2|z|^3} \times (dz^c\bar{z}_c - z^c d\bar{z}_c) + \bar{z}_a\left(\frac{1}{|z|} - \frac{\sin|z|}{|z|^2}\right)d|z|,$$

$$\Omega_{\mathbf{b}4}{}^a(d) = dz^a\frac{\sin|z|}{|z|} + z^a\frac{\sin|z|(1-\cos|z|)}{2|z|^3} \times (z^c d\bar{z}_c - dz^c\bar{z}_c) + z^a\left(\frac{1}{|z|} - \frac{\sin|z|}{|z|^2}\right)d|z|. \quad (65)$$

The fermionic contribution can be presented as

$$\Omega_{\mathbf{f}\hat{a}}{}^{\hat{b}}(d) = (T\Psi(d)\bar{T})_{\hat{a}}{}^{\hat{b}},$$
$$\Psi_{\hat{a}}{}^{\hat{b}}(d) = \begin{pmatrix} \Psi_a{}^b - \delta_a^b\Psi_c{}^c & \varepsilon_{acb}\Psi_4{}^c \\ -\varepsilon^{acb}\Psi_c{}^4 & -\Psi_b{}^a + \delta_b^a\Psi_c{}^c \end{pmatrix}, \quad (66)$$

where the entries of $\Psi_{\hat{a}}{}^{\hat{b}}(d)$ equal

$$\Psi_a{}^b(d) = 2\left(d\theta_a^\mu + \frac{1}{2}\zeta_a^\mu(d)\right)\bar{\eta}_\mu^b - 2\left(d\bar{\theta}^{\mu b} + \frac{1}{2}\bar{\zeta}^{\mu b}(d)\right)\eta_{\mu a} - \delta_a^b\left(\left(d\theta_c^\mu + \frac{1}{2}\zeta_c^\mu(d)\right)\bar{\eta}_\mu^c - \left(d\bar{\theta}^{\mu c} + \frac{1}{2}\bar{\zeta}^{\mu c}(d)\right)\eta_{\mu c}\right),$$

$$\Psi_a{}^4(d) = 2\varepsilon_{abc}\left(d\bar{\theta}^{\mu b} + \frac{1}{2}\bar{\zeta}^{\mu b}(d)\right)\bar{\eta}_\mu^c,$$

$$\Psi_4{}^a(d) = -2\varepsilon^{abc}\left(d\theta_b^\mu + \frac{1}{2}\zeta_b^\mu(d)\right)\eta_{\mu c}. \quad (67)$$

The expressions for the fermionic Cartan forms can be brought to the form

$$\begin{pmatrix} \hat{\omega}_a^\mu \\ \bar{\hat{\omega}}^{\mu a} \end{pmatrix} = e^{-\varphi}T_{\hat{a}}{}^{\hat{b}}\begin{pmatrix} \omega_b^\mu \\ \bar{\omega}^{\mu b} \end{pmatrix}, \qquad \omega_b^\mu(d) = d\theta_b^\mu + \zeta_b^\mu(d),$$
$$\bar{\omega}^{\mu b}(d) = d\bar{\theta}^{\mu b} + \bar{\zeta}^{\mu b}(d) \quad (68)$$

and

$$\begin{pmatrix} \hat{\chi}_{\mu a} \\ \bar{\hat{\chi}}_\mu^a \end{pmatrix} = e^{\varphi}T_{\hat{a}}{}^{\hat{b}}\begin{pmatrix} \chi_{\mu b} \\ \bar{\chi}_\mu^b, \end{pmatrix} \quad (69)$$

where

$$\chi_{\mu a}(d) = d\eta_{\mu a} + 2i\bar{\eta}_\mu^b d\theta_b^\nu\eta_{\nu a} + 2i\eta_{\mu b}d\bar{\theta}^{\nu b}\eta_{\nu a} + i(d\theta_{\mu a} + \zeta_{\mu a}(d))(\eta_b^\nu\bar{\eta}_\nu^b),$$
$$\bar{\chi}_\mu^a(d) = d\bar{\eta}_\mu^a + 2i\eta_{\mu b}d\bar{\theta}^{\nu b}\bar{\eta}_\nu^a + 2i\bar{\eta}_\mu^b d\theta_b^\nu\bar{\eta}_\nu^a + i(d\bar{\theta}_\mu^a + \bar{\zeta}_\mu^a(d))(\eta_b^\nu\bar{\eta}_\nu^b). \quad (70)$$

In terms of the Cartan forms in the conformal basis (47) and (48), the superstring action (4) acquires the form

$$\mathcal{S} = -\frac{1}{2}\int d^2\xi\sqrt{-g}g^{ij}\left(\frac{1}{4}(\hat{\omega}_i^m + \hat{c}_i^m)(\hat{\omega}_{mj} + \hat{c}_{mj}) + \Delta_i\Delta_j + \Omega_{ia}{}^4\Omega_{j4}{}^a\right) - \frac{1}{2}\varepsilon^{ij}\int d^2\xi(\hat{\omega}_{ia}^\mu\varepsilon_{\mu\nu}\bar{\hat{\omega}}_j^{\nu a} + \hat{\chi}_{i\mu a}\varepsilon^{\mu\nu}\bar{\hat{\chi}}_{j\nu}^a). \quad (71)$$

It has a rather complicated structure with the kinetic term containing contributions up to the 8th power in the fermions and the WZ term up to the 6th power. Note, however, that similarly to the AdS$_5 \times S^5$ superstring anticommuting coordinates θ_a^μ, $\bar{\theta}^{\mu a}$ related to the Poincare supersymmetry enter expressions for the Cartan forms utmost quadratically, and the nonlinear fermionic contribution is due to $\eta_{\mu a}$, $\bar{\eta}_\mu^a$ related to the conformal supersymmetry. For the AdS$_5 \times S^5$ superstring there have been proposed the κ-symmetry gauges that entirely remove the coordinates η so that the action becomes quadratic [35] or quartic in the fermions [21,36,37]. This seems to be the simplest known form of the AdS$_5 \times S^5$ superstring action. In the case under consideration it is impossible to gauge away all 12 coordinates η by the 8-parameter κ-symmetry transformation. Among the $SO(1,2)$ covariant gauges one can consider the gauge

$$\eta_{\mu a} = \begin{pmatrix} \eta_{\mu A} \\ \eta_{\mu 3} \end{pmatrix}, \qquad \eta_{\mu A} = 0, \quad (72)$$

where the index A corresponds to the fundamental representation of $SU(2)$, that removes 8 coordinates η. In this case the following entries of the matrix (66) $\Psi_1{}^2 = \Psi_2{}^1 = \Psi_4{}^3 = \Psi_3{}^4 = 0$ turn to zero, and the kinetic term of the superstring action (71) becomes utmost of the sixth order in the fermions. The gauge

$$\theta_a^\mu = \begin{pmatrix} \theta_A^\mu \\ \theta_3^\mu \end{pmatrix}, \qquad \theta_3^\mu = 0, \qquad \eta_{\mu 3} = 0 \quad (73)$$

removes an equal number of θ and η coordinates [38]. In this gauge vanish the components of the Cartan forms $\Psi_{1,2}{}^4 = \Psi_4{}^{1,2} = \Psi_{1,2}{}^3 = \Psi_3{}^{1,2} = 0$ and $\omega_3^\mu = 0$, $\chi_{\mu 3} = 0$. More substantial simplification can be attained, e.g., by considering the noncovariant condition

$$\eta_{1a} = 0 \quad (74)$$

that partially fixes the κ-symmetry gauge freedom. In such a case $\hat{c}^1 = 0$, while other components of the Cartan forms (58) become quadratic in fermions and also $\chi_{1a} = \bar{\chi}_1^a = 0$ so that the kinetic term of the action (71) contains the fermionic contributions up to the fourth power and the WZ term up to the second power. Then the remaining freedom can be used to turn to zero extra Cartan form components.

IV. CONCLUSION

In the present paper we have considered in the framework of the supercoset approach the superstring action on the AdS$_4 \times \mathbb{CP}^3$ background [16] in the conformal basis for the Cartan 1-forms motivated by the isomorphism between the $osp(4|6)$ superalgebra and $D = 3$ $\mathcal{N} = 6$ superconformal algebra. We have obtained the expressions for the Cartan forms explicitly covariant under the $D = 3$ $\mathcal{N} = 6$ super-Poincare transformations starting from the $OSp(4|6)/(SO(1,3) \times U(3))$ supercoset representative parametrized by the coordinates associated with the $D = 3$ $\mathcal{N} = 6$ superconformal generators. These results can be used to establish a more transparent relation to the field theory side of the Aharony-Bergman-Jafferis-Maldacena duality [4].

We have also derived the $SO(1,3) \times SU(3)$ covariant expression for the matrix M that enters the equations of motion for the fermions and have shown that in the general case its rank equals 16 implying via the second Noether theorem the 8-parameter κ-symmetry of the action. The form of the matrix M can be found by inspecting the anticommutation relations of the fermionic generators of $osp(4|6)$ superalgebra. The complementary matrix K that enters the κ-symmetry transformation rules is quadratic in the world-sheet projections of Cartan forms rather than linear as for the GS superstring, and we have proved in the $SO(1,3) \times SU(3)$ covariant way that the rank of K equals 8. These results outline the similarities and differences of the supercoset formulation for the superstring on $\mathrm{AdS}_4 \times \mathbb{CP}^3$ background and the conventional GS one.

It was suggested in [16] that the $OSp(4|6)/(SO(1,3) \times U(3))$ supercoset action could be obtained by partial gauge fixing of the κ-symmetry in the full superstring action on an $\mathrm{AdS}_4 \times \mathbb{CP}^3$ background. However, it is interesting to note that such supercoset action *per se* may be viewed as belonging to the family of the models of pointlike [40–42] and extended [43–45] objects in extended superspaces describing the Bogomol'nyi-Prasad-Sommerfield states preserving exotic [46] fractions of the space-time supersymmetry. Here the role of extra superspace variables complementing the super-Poincare ones is played by the bosonic z^a, $\bar{z}_a$, φ and fermionic $\eta_{\mu a}$, $\bar{\eta}^a_\mu$ coordinates.

As the extension of the presented results one can examine the supercoset action invariance under the full $D = 3$ $\mathcal{N} = 6$ superconformal transformations, and derive the corresponding Noether charges and calculate their algebra. It is of interest by fixing the gauge freedom to seek for the simplest form of the action to be compared with that for the $\mathrm{AdS}_5 \times S^5$ superstring. Novel insights into the structure of the action and the quantization problem could also be gained by working out the first-order formulation in analogy with the GS superstring in a flat background [48] and elaborating on the twistor transform [49]. We hope to address these issues in future.

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APPENDIX A: SPINORS AND γ-MATRICES

The $D = 2 + 3$ spinor indices are raised and lowered by means of the antisymmetric charge conjugation matrix and its inverse

$$\psi^\alpha = C^{\alpha\beta}\psi_\beta, \qquad \psi_\alpha = C_{\alpha\beta}\psi^\beta. \tag{A1}$$

The spinor ψ^α is composed of a pair of the $(1 + 2)$-dimensional spinors

$$\psi^\alpha = \begin{pmatrix} \phi^\mu \\ \varphi_\nu \end{pmatrix}, \qquad \psi_\alpha = \begin{pmatrix} \varphi_\mu \\ -\phi^\nu \end{pmatrix} \tag{A2}$$

and the charge conjugation matrix and its inverse admit the representation in terms of the 2×2 unit matrix

$$C_{\alpha\beta} = \begin{pmatrix} 0 & \delta^\nu_\mu \\ -\delta^\mu_\nu & 0 \end{pmatrix}, \qquad C^{\alpha\beta} = \begin{pmatrix} 0 & -\delta^\mu_\nu \\ \delta^\nu_\mu & 0 \end{pmatrix}. \tag{A3}$$

The position of indices of the 2-component spinors can be changed as follows:

$$\begin{aligned} \phi^\mu &= \varepsilon^{\mu\nu}\phi_\nu, & \varphi_\mu &= \varepsilon_{\mu\nu}\varphi^\nu, \\ \varepsilon_{\mu\nu}\varepsilon^{\nu\lambda} &= \delta^\lambda_\mu, & \varepsilon^{12} &= \varepsilon_{21} = 1. \end{aligned} \tag{A4}$$

The Majorana condition in $1 + 2$ dimensions

$$(\varphi^\mu)^\dagger \sigma^{0\mu}{}_\nu = \varepsilon_{\nu\mu}\varphi^\mu \tag{A5}$$

amounts to the reality of the spinor components in the chosen basis, where $\tilde{\sigma}^{0\mu\nu} = \delta^{\mu\nu}$. Accordingly in $2 + 3$ dimensions the Majorana condition

$$(\psi^\alpha)^\dagger (\tilde{\gamma}^{0'}\gamma^0)^\alpha{}_\beta = C_{\beta\alpha}\psi^\alpha \tag{A6}$$

is satisfied for the spinors composed of a pair of the $(1 + 2)$-dimensional Majorana spinors. Because of the relation $(\tilde{\gamma}^{0'}\gamma^0)^\alpha{}_\beta C^{\beta\gamma} = -\delta^{\alpha\gamma}$ it also amounts to the component by component reality of a spinor.

The $(2 + 3)$-dimensional γ-matrices in the Majorana representation can be realized in terms of the $(1 + 2)$-dimensional real γ-matrices

$$\begin{aligned} \gamma^{0'}_{\alpha\beta} &= -\begin{pmatrix} \varepsilon_{\mu\nu} & 0 \\ 0 & \varepsilon^{\mu\nu} \end{pmatrix}, & \tilde{\gamma}^{0'\alpha\beta} &= -\begin{pmatrix} \varepsilon^{\mu\nu} & 0 \\ 0 & \varepsilon_{\mu\nu} \end{pmatrix}, \\ \gamma^m_{\alpha\beta} &= \begin{pmatrix} 0 & \sigma^m{}_\mu{}^\nu \\ -\sigma^{m\mu}{}_\nu & 0 \end{pmatrix}, & \tilde{\gamma}^{m\alpha\beta} &= \begin{pmatrix} 0 & \sigma^{m\mu}{}_\nu \\ -\sigma^m{}_\mu{}^\nu & 0 \end{pmatrix}, \\ \gamma^3_{\alpha\beta} &= \begin{pmatrix} \varepsilon_{\mu\nu} & 0 \\ 0 & -\varepsilon^{\mu\nu} \end{pmatrix}, & \tilde{\gamma}^{3\alpha\beta} &= \begin{pmatrix} -\varepsilon^{\mu\nu} & 0 \\ 0 & \varepsilon_{\mu\nu} \end{pmatrix}, \end{aligned} \tag{A7}$$

where

$$\begin{aligned} \sigma^m_{\mu\nu} &= (I, \sigma^1, -\sigma^3), \\ \tilde{\sigma}^{m\mu\nu} &= \varepsilon^{\mu\lambda}\varepsilon^{\nu\rho}\sigma^m_{\lambda\rho} = (I, -\sigma^1, \sigma^3). \end{aligned} \tag{A8}$$

They satisfy the Clifford algebra relations

$$\gamma^{\underline{m}}_{\alpha\beta}\tilde{\gamma}^{\underline{n}\beta\gamma} + \gamma^{\underline{n}}_{\alpha\beta}\tilde{\gamma}^{\underline{m}\beta\gamma} = -2\eta^{\underline{mn}}\delta_\alpha^\gamma,$$
$$\eta^{\underline{mn}} = (-,-,+,+,+), \qquad \tilde{\gamma}^{\underline{m}\alpha\beta} = C^{\alpha\gamma}C^{\beta\delta}\gamma^{\underline{m}}_{\gamma\delta} \tag{A9}$$

as a result of the $D = 1 + 2$ relations

$$\sigma^m_{\mu\nu}\tilde{\sigma}^{n\nu\lambda} + \sigma^n_{\mu\nu}\tilde{\sigma}^{m\nu\lambda} = -2\eta^{mn}\delta_\mu^\lambda. \tag{A10}$$

The $so(2,3)$ generators are defined as

$$\gamma^{\underline{mn}}{}_\alpha{}^\beta = \frac{1}{2}(\gamma^{\underline{m}}_{\alpha\gamma}\tilde{\gamma}^{\underline{n}\gamma\beta} - \gamma^{\underline{n}}_{\alpha\gamma}\tilde{\gamma}^{\underline{m}\gamma\beta}). \tag{A11}$$

Their explicit form in terms of the above introduced γ-matrices is found to be

$$\gamma^{mn}{}_\alpha{}^\beta = \begin{pmatrix} \sigma^{mn}{}_\mu{}^\nu & 0 \\ 0 & \tilde{\sigma}^{mn\mu}{}_\nu \end{pmatrix}, \quad \gamma^{0'm}{}_\alpha{}^\beta = \begin{pmatrix} 0 & -\sigma^m_{\mu\nu} \\ \tilde{\sigma}^{m\mu\nu} & 0 \end{pmatrix},$$
$$\gamma^{3m}{}_\alpha{}^\beta = \begin{pmatrix} 0 & \sigma^m_{\mu\nu} \\ \tilde{\sigma}^{m\mu\nu} & 0 \end{pmatrix}, \quad \gamma^{0'3}{}_\alpha{}^\beta = \begin{pmatrix} \delta_\mu^\nu & 0 \\ 0 & -\delta^\mu_\nu \end{pmatrix},$$
$$\sigma^{mn}{}_\mu{}^\nu = \frac{1}{2}(\sigma^m_{\mu\lambda}\tilde{\sigma}^{n\lambda\nu} - \sigma^n_{\mu\lambda}\tilde{\sigma}^{m\lambda\nu}), \quad \tilde{\sigma}^{mn\mu}{}_\nu = -\sigma^{mn}{}_\nu{}^\mu. \tag{A12}$$

The $(1+3)$-dimensional charge conjugation matrix that enters the WZ term and its inverse can be realized as

$$C'_{\alpha\beta} = -i\gamma^{0'}_{\alpha\beta} = i\begin{pmatrix} \varepsilon_{\mu\nu} & 0 \\ 0 & \varepsilon^{\mu\nu} \end{pmatrix},$$
$$C'^{\alpha\beta} = i\tilde{\gamma}^{0'\alpha\beta} = -i\begin{pmatrix} \varepsilon^{\mu\nu} & 0 \\ 0 & \varepsilon_{\mu\nu} \end{pmatrix}. \tag{A13}$$

Γ-matrices in $D = 1 + 3$ dimensions are defined as

$$\Gamma^{m'}{}_\alpha{}^\beta = -\gamma^{m'}_{\alpha\gamma}C'^{\gamma\beta}\colon \Gamma^m{}_\alpha{}^\beta = \begin{pmatrix} 0 & -i\sigma^m_{\mu\nu} \\ i\tilde{\sigma}^{m\mu\nu} & 0 \end{pmatrix},$$
$$\Gamma^3{}_\alpha{}^\beta = \begin{pmatrix} i\delta_\mu^\nu & 0 \\ 0 & -i\delta^\mu_\nu \end{pmatrix}. \tag{A14}$$

They obey the Clifford algebra relations

$$\Gamma^{m'}{}_\alpha{}^\gamma\Gamma^{n'}{}_\gamma{}^\beta + \Gamma^{n'}{}_\alpha{}^\gamma\Gamma^{m'}{}_\gamma{}^\beta = -2\eta^{m'n'}\delta_\alpha^\beta. \tag{A15}$$

The matrix $\Gamma^5 = \Gamma^0\Gamma^1\Gamma^2\Gamma^3$ then equals

$$\Gamma^5_\alpha{}^\beta = \begin{pmatrix} 0 & \varepsilon_{\mu\nu} \\ -\varepsilon^{\mu\nu} & 0 \end{pmatrix}. \tag{A16}$$

APPENDIX B: $osp(4|6)$ SUPERALGEBRA AS $D = 3$ $\mathcal{N} = 6$ SUPERCONFORMAL ALGEBRA

The (anti)commutation relations of the $osp(4|6)$ superalgebra can be written in the supermatrix form

$$[O_{\hat{K}\hat{L}}, O_{\hat{M}\hat{N}}\} = i(G_{\hat{L}\hat{M}}O_{\hat{K}\hat{N}} + (-)^{lm}G_{\hat{L}\hat{N}}O_{\hat{K}\hat{M}} + (-)^{kl}G_{\hat{K}\hat{M}}O_{\hat{L}\hat{N}} + (-)^{k(l+m)}G_{\hat{K}\hat{N}}O_{\hat{L}\hat{M}}), \tag{B1}$$

where

$$G_{\hat{L}\hat{M}} = \begin{pmatrix} C_{\alpha\beta} & 0 \\ 0 & i\delta_{IJ} \end{pmatrix} \tag{B2}$$

is the orthosymplectic metric composed of the $D = 2 + 3$ charge conjugation matrix $C_{\alpha\beta}$ and the unit metric δ_{IJ} in the vector representation of $SO(6)$. The supermatrix $O_{\hat{M}\hat{N}}$ has the following block structure:

$$O_{\hat{M}\hat{N}} = \begin{pmatrix} O_{\alpha\beta} & O_{\alpha J} \\ O_{I\beta} & O_{IJ} \end{pmatrix} \tag{B3}$$

with the blocks obeying the reality

$$O^*_{\alpha\beta} = O_{\alpha\beta}, \qquad O^*_{\alpha J} = -O_{\alpha J},$$
$$O^*_{I\beta} = -O_{I\beta}, \qquad O^*_{IJ} = -O_{IJ} \tag{B4}$$

and (anti)symmetry

$$O_{\hat{M}\hat{N}} = (-)^{mn}O_{\hat{N}\hat{M}}\colon O_{\alpha\beta} = O_{\beta\alpha},$$
$$O_{\alpha J} = O_{J\alpha}, \qquad O_{IJ} = -O_{JI} \tag{B5}$$

conditions. The block structure of $O_{\hat{M}\hat{N}}$ implies that the (anti)commutation relations of the $osp(4|6)$ superalgebra can be divided into 5 groups

$$[O_{\alpha\beta}, O_{\gamma\delta}] = i(C_{\alpha\gamma}O_{\beta\delta} + C_{\alpha\delta}O_{\beta\gamma} + C_{\beta\gamma}O_{\alpha\delta} + C_{\beta\delta}O_{\alpha\gamma}), \tag{B6}$$

$$[O_{IJ}, O_{KL}] = \delta_{IK}O_{JL} - \delta_{IL}O_{JK} - \delta_{JK}O_{IL} + \delta_{JL}O_{IK}, \tag{B7}$$

$$\{O_{\alpha J}, O_{\gamma L}\} = -\delta_{JL}O_{\alpha\gamma} + iC_{\alpha\gamma}O_{JL}, \tag{B8}$$

$$[O_{\alpha\beta}, O_{\gamma L}] = i(C_{\alpha\gamma}O_{\beta L} + C_{\beta\gamma}O_{\alpha L}), \tag{B9}$$

$$[O_{IJ}, O_{\gamma L}] = \delta_{IL}O_{\gamma J} - \delta_{JL}O_{\gamma I}. \tag{B10}$$

The commutation relations of the first group can be cast into the $so(2,3)$ algebra relations

$$[M^{\underline{kl}}, M^{\underline{mn}}] = \eta^{\underline{kn}}M^{\underline{lm}} - \eta^{\underline{km}}M^{\underline{ln}} - \eta^{\underline{ln}}M^{\underline{km}} + \eta^{\underline{lm}}M^{\underline{kn}} \tag{B11}$$

by the transformation $M^{\underline{kl}} = \frac{i}{4}\gamma^{\underline{kl}\alpha\beta}O_{\alpha\beta}$, $O_{\alpha\beta} = -\frac{i}{2}\gamma^{\underline{mn}}_{\alpha\beta}M_{\underline{mn}}$. Separating the generators that carry the second time direction index one arrives at the AdS_4 algebra

$$[M^{0'm'}, M^{0'n'}] = M^{m'n'},$$
$$[M^{0'k'}, M^{m'n'}] = \eta^{k'm'}M^{0'n'} - \eta^{k'n'}M^{0'm'},$$
$$[M^{k'l'}, M^{m'n'}] = \eta^{k'n'}M^{l'm'} - \eta^{k'm'}M^{l'n'} - \eta^{l'n'}M^{k'm'} + \eta^{l'm'}M^{k'n'}. \tag{B12}$$

Introducing the $(1+2)$-dimensional dilatation D, momentum P^m and conformal boost K^m generators

$$D = 2M^{0'3}, \qquad P^m = -(M^{0'm} + M^{3m}), \qquad K^m = M^{3m} - M^{0'm} \tag{B13}$$

the AdS_4 algebra commutation relations transform into the $conf_3$ algebra commutation relations

$$\begin{aligned}
&[P^m, D] = -2P^m, \qquad [K^m, D] = 2K^m,\\
&[P^m, K^n] = \eta^{mn}D + 2M^{mn},\\
&[P^l, M^{mn}] = \eta^{lm}P^n - \eta^{ln}P^m,\\
&[K^l, M^{mn}] = \eta^{lm}K^n - \eta^{ln}K^m,\\
&[M^{kl}, M^{mn}] = \eta^{kn}M^{lm} - \eta^{km}M^{ln} - \eta^{ln}M^{km} + \eta^{lm}M^{kn}.
\end{aligned} \tag{B14}$$

By converting the $so(6)$ generators into the $su(4)$ generators

$$V_{\hat{i}}{}^{\hat{j}} = -\frac{i}{4}O_{IJ}\rho^{IJ}{}_{\hat{i}}{}^{\hat{j}}, \tag{B15}$$

where $\rho^{IJ}{}_{\hat{i}}{}^{\hat{j}} = \frac{1}{2}(\rho^I_{\hat{i}\hat{k}}\tilde{\rho}^{J\hat{k}\hat{j}} - \rho^J_{\hat{i}\hat{k}}\tilde{\rho}^{I\hat{k}\hat{j}})$, the commutation relations (B7) reduce to

$$[V_{\hat{i}}{}^{\hat{j}}, V_{\hat{k}}{}^{\hat{l}}] = i(\delta^{\hat{j}}_{\hat{k}}V_{\hat{i}}{}^{\hat{l}} - \delta^{\hat{l}}_{\hat{i}}V_{\hat{k}}{}^{\hat{j}}). \tag{B16}$$

The $su(4)$ generators can be split into the $u(3)$ generators $V_a{}^b$ and the $su(4)/u(3)$ coset generators $V_a{}^4$, $V_4{}^a$

$$V_{\hat{i}}{}^{\hat{j}} = \begin{pmatrix} V_a{}^b & V_a{}^4 \\ V_4{}^b & V_4{}^4 \end{pmatrix}, \qquad V_4{}^4 = -V_a{}^a. \tag{B17}$$

Then the $su(4)$ algebra commutation relations (B16) acquire the form

$$\begin{aligned}
&[V_a{}^4, V_4{}^b] = i(V_a{}^b + \delta^b_a V_c{}^c),\\
&[V_a{}^4, V_b{}^c] = -i\delta^c_a V_b{}^4,\\
&[V_4{}^a, V_b{}^c] = i\delta^a_b V_4{}^c,\\
&[V_a{}^b, V_c{}^d] = i(\delta^b_c V_a{}^d - \delta^d_a V_c{}^b).
\end{aligned} \tag{B18}$$

By contracting the $SO(6)$ vector index I of the $osp(4|6)$ fermionic generators $O_{\alpha I}$ with the $D = 6$ antisymmetric chiral γ-matrices $\rho^I_{\hat{i}\hat{j}}$ and $\tilde{\rho}^{I\hat{i}\hat{j}}$ that satisfy

$$\rho^I_{\hat{i}\hat{j}}\tilde{\rho}^{J\hat{j}\hat{k}} + \rho^J_{\hat{i}\hat{j}}\tilde{\rho}^{I\hat{j}\hat{k}} = 2\delta^{IJ}\delta^{\hat{k}}_{\hat{i}}, \tag{B19}$$

the anticommutator (B8) is brought to the form

$$\{O_{\alpha\hat{i}\hat{j}}, O^{\hat{k}\hat{l}}_{\beta}\} = i(\delta^{\hat{l}}_{\hat{i}}\delta^{\hat{k}}_{\hat{j}} - \delta^{\hat{k}}_{\hat{i}}\delta^{\hat{l}}_{\hat{j}})\gamma^{\underline{mn}}_{\alpha\beta}M_{\underline{mn}} + 2C_{\alpha\beta}(\delta^{\hat{k}}_{\hat{i}}V_{\hat{j}}{}^{\hat{l}} - \delta^{\hat{k}}_{\hat{j}}V_{\hat{i}}{}^{\hat{l}} + \delta^{\hat{l}}_{\hat{j}}V_{\hat{i}}{}^{\hat{k}} - \delta^{\hat{l}}_{\hat{i}}V_{\hat{j}}{}^{\hat{k}}). \tag{B20}$$

Performing the 3 + 1 split of the $SU(4)$ indices $\hat{i} = (a, 4)$, $\hat{j} = (b, 4)$, using the duality relations

$$\begin{aligned}
&O_{ab} = -\varepsilon_{abc}O^{4c}, \qquad O^{4a} = -\frac{1}{2}\varepsilon^{abc}O_{bc},\\
&O^{ab} = -\varepsilon^{abc}O_{4c}, \qquad O_{4a} = -\frac{1}{2}\varepsilon_{abc}O^{bc},\\
&\varepsilon_{123} = \varepsilon^{123} = 1
\end{aligned} \tag{B21}$$

that stem from the $SU(4)$ duality relations

$$O_{\hat{i}\hat{j}} = \frac{1}{2}\varepsilon_{\hat{i}\hat{j}\hat{k}\hat{l}}O^{\hat{k}\hat{l}}, \qquad \varepsilon_{\hat{i}\hat{j}\hat{k}\hat{l}} = \varepsilon_{abc4}, \tag{B22}$$

introducing the (1 + 2)-dimensional supersymmetry Q^a_μ, $\bar{Q}_{\mu a}$ and superconformal $S^{\mu a}$, $\bar{S}^\mu_a$ generators

$$O^{4a}_\alpha = \begin{pmatrix} Q^a_\mu \\ S^{\mu a} \end{pmatrix}, \qquad O_{\alpha 4a} = \begin{pmatrix} \bar{Q}_{\mu a} \\ \bar{S}^\mu_a \end{pmatrix}, \tag{B23}$$

and substituting the expressions (A12) and the definition of the $conf_3$ generators (B13) we are able to bring the relations (B20) to the anticommutation relations of $D = 3$ $\mathcal{N} = 6$ superconformal algebra in the $SU(3)$ notation

$$\begin{aligned}
&\{Q^a_\mu, \bar{Q}_{\nu b}\} = 2i\delta^a_b\sigma^m_{\mu\nu}P_m, \qquad \{S^{\mu a}, \bar{S}^\nu_b\} = 2i\delta^a_b\tilde{\sigma}^{m\mu\nu}K_m,\\
&\{Q^a_\mu, S^{\nu b}\} = 2\delta^\nu_\mu\varepsilon^{abc}V_c{}^4, \qquad \{\bar{Q}_{\mu a}, \bar{S}^\nu_b\} = -2\delta^\nu_\mu\varepsilon_{abc}V_4{}^c,\\
&\{Q^a_\mu, \bar{S}^\nu_b\} = -i\delta^a_b\delta^\nu_\mu D + i\delta^a_b\sigma^{mn}{}_\mu{}^\nu M_{mn} - 2\delta^\nu_\mu(V_b{}^a - \delta^a_b V_c{}^c),\\
&\{\bar{Q}_{\mu a}, S^{\nu b}\} = -i\delta^b_a\delta^\nu_\mu D + i\delta^b_a\sigma^{mn}{}_\mu{}^\nu M_{mn} + 2\delta^\nu_\mu(V_a{}^b - \delta^b_a V_c{}^c).
\end{aligned} \tag{B24}$$

The commutators of (B9) and (B10) define the properties of the fermionic generators under the $SO(2, 3)$ and $SO(6)$ transformations. In particular, using the definition of $D = 3$ $\mathcal{N} = 6$ generators (B13) and (B23) we get

$$\begin{aligned}
&[D, Q^a_\mu] = Q^a_\mu, \quad [D, \bar{Q}_{\mu a}] = \bar{Q}_{\mu a},\\
&[M^{mn}, Q^a_\mu] = \frac{1}{2}\sigma^{mn}{}_\mu{}^\nu Q^a_\nu, \quad [M^{mn}, \bar{Q}_{\mu a}] = \frac{1}{2}\sigma^{mn}{}_\mu{}^\nu\bar{Q}_{\nu a},\\
&[K^m, Q^a_\mu] = \sigma^m_{\mu\nu}S^{\nu a}, \quad [K^m, \bar{Q}_{\mu a}] = \sigma^m_{\mu\nu}\bar{S}^\nu_a,\\
&[D, S^{\mu a}] = -S^{\mu a}, \quad [D, \bar{S}^\mu_a] = -\bar{S}^\mu_a,\\
&[M^{mn}, S^{\mu a}] = -\frac{1}{2}S^{\nu a}\sigma^{mn}{}_\nu{}^\mu, \quad [M^{mn}, \bar{S}^\mu_a] = -\frac{1}{2}\bar{S}^\nu_a\sigma^{mn}{}_\nu{}^\mu,\\
&[P^m, S^{\mu a}] = -\tilde{\sigma}^{m\mu\nu}Q^a_\nu, \quad [P^m, \bar{S}^{\mu a}] = -\tilde{\sigma}^{m\mu\nu}\bar{Q}_{\nu a}
\end{aligned} \tag{B25}$$

and

$$[V_a{}^b, Q^c_\mu] = \frac{i}{2}\delta^b_a Q^c_\mu - i\delta^c_a Q^b_\mu, \quad [V_4{}^a, Q^b_\mu] = i\varepsilon^{abc}\bar{Q}_{\mu c}, \quad [V_a{}^b, \bar{Q}_{\mu c}] = -\frac{i}{2}\delta^b_a \bar{Q}_{\mu c} + i\delta^b_c \bar{Q}_{\mu a}, \quad [V_a{}^4, \bar{Q}_{\mu b}] = -i\varepsilon_{abc}Q^c_\mu,$$

$$[V_a{}^b, S^{\mu c}] = \frac{i}{2}\delta^b_a S^{\mu c} - i\delta^c_a S^{\mu b}, \quad [V_4{}^a, S^{\mu b}] = i\varepsilon^{abc}\bar{S}^\mu_c, \quad [V_a{}^b, \bar{S}^\mu_c] = -\frac{i}{2}\delta^b_a \bar{S}^\mu_c + i\delta^b_c \bar{S}^\mu_a, \quad [V_a{}^4, \bar{S}^\mu_b] = -i\varepsilon_{abc}S^{\mu c}. \tag{B26}$$

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