PHYSICAL REVIEW D **79,** 106003 (2009)

Classification of QCD defects via holography

Alexander S. Gorsky and Valentin I. Zakharov
Institute of Theoretical and Experimental Physics, B. Cheremushkinskaya ul. 25, 117259 Moscow, Russia

Ariel R. Zhitnitsky
Department of Physics and Astronomy, University of British Columbia, Vancouver, BC V6T1Z1, Canada
(Received 11 March 2009; published 14 May 2009)

We discuss classification of defects of various codimensions within a holographic model of pure Yang-Mills theories or gauge theories with fundamental matter. We focus on their role below and above the phase transition point as well as their weights in the partition function. The general result is that objects which are stable and heavy in one phase are becoming very light (tensionless) in the other phase. We argue that the θ dependence of the partition function drastically changes at the phase transition point, and therefore it correlates with stability properties of configurations. We also explore the possibility that novel stable glueballlike particles, with mass which scales like N_c and which are analogous to carbon Fullerenes, may exist in nature on the QCD scale. Some possible applications for studying the QCD vacuum properties above and below the phase transition are also discussed.

DOI: 10.1103/PhysRevD.79.106003 PACS numbers: 11.25.Mj

I. INTRODUCTION

The gauge/string duality proved to be effective in the description of various aspects of gauge theories at weak and strong coupling. In the $N = 4$ supersymmetric Yang-Mills theory (SYM) one can discuss a precise comparison of the gauge theory results with the sigma model or supergravity calculation since the relevant geometry $\mathrm{AdS}_5 \times S^5$ is well established. However the situation in the theory with less supersymmetry (SUSY) is much more complicated and the explicit background for the pure YM or QCD is not found yet. The most useful dual model of pure YM at nonzero temperature [1] is based on a stack of N_c $D4$ branes which are wrapped around a compact coordinate and at large N_c provide the geometry of the black hole in AdS_5. Adding the probe N_f $D8$-$\bar{D}8$ branes one obtains the dual picture for QCD [2] which reproduces the chiral Lagrangian and captures many qualitative aspects of the strong coupling physics.

In this paper we shall discuss extended objects in QCD from the dual perspective. In the dual model the natural objects to consider are probe D branes. The Sakai-Sugimoto model is based on the IIA side of the string theory hence there are stable $D0$, $D2$, $D4$, $D6$, $D8$ D branes as well as NS5 brane which we shall discuss. We shall try to get a qualitative picture concerning the role of the various defects which is insensitive to the details of the metric. The background involves two periodic coordinates, an angular coordinate on the black hole cigar x_4, and the Euclidean time coordinate τ which branes can wrap around.

In the previous studies some identifications of the defects have been made. The $D0$ brane extended along x_4 was identified as the YM instanton [3]. The $D2$ brane wrapped around both periodic coordinates was identified as the magnetic string [4]. The $D6$ brane wrapped around the compact S^4 part of the dual geometry and τ was considered in pure YM [5] and dual QCD [6,7] where it has the interpretation of the domain wall separating two vacua. The $D4$ brane wrapped around the S^4 and extended along τ has the interpretation of the "baryonic vertex" in pure YM and in the Sakai-Sugimoto model [2]. Some interplay between the D branes and the Z_N domain walls has been discussed in the holographic picture in [8–10]; see also [11] for some other identifications.

There are a few generic facts concerning the branes and their intersections. Let us enumerate a few of them relevant for the main text:

(i) a p brane behaves as an instanton on the $(p + 4)$-brane worldvolume [12],
(ii) a p brane parallel to the $(p + 2)$ brane gets melted into a homogeneous field [13],
(iii) a p brane transverse to the $p + 2$ brane behaves as the monopole on the $(p + 2)$ worldvolume,
(iv) branes in external fields can expand into higher dimensional branes via the Myers effect [14].

Our goal here is to look at a variety of defects using a universal classification scheme. We will be mostly interested in behavior of the corresponding configurations when the confinement-deconfinement phase transition is crossed and we shall emphasize some universal properties of the D defects. It is very likely that some of the defects to be discussed here are very important for physics, yet some of them could be irrelevant. Therefore, we anticipate that some important/interesting defects will be further discussed and studied in great detail in future. It is not the goal of the present paper to go into a deep detailed analysis of each particular configuration. Instead, our goal is the classification of the D defects, with emphasis on the change of their properties across the phase transition.

To be more specific, our basic tool is the dual description of the deconfinement phase transition as the Hawking-Page phase transition [1], in which case the two metrics with the same asymptotics get interchanged. The wrapping around x_4 is stable above the phase transition $T > T_c$ while it is unstable below the critical temperature $T < T_c$; see definitions and details below. And vice versa, the wrapping around τ is stable at small temperatures $T < T_c$ and unstable at high temperatures $T > T_c$.

Since the nonperturbative physics is sensitive to the θ term we shall also discuss the θ dependence from the dual perspective. We argue that the behavior of the D defects is strongly correlated with the θ dependence when the phase transition is crossed at T_c. Such a drastic change in θ dependence at the T_c has already been noticed in the literature [3,4,15–17]. We note also that some drastic changes in θ behavior are also supported by the numerical lattice results [18–22]; see also a review article [23], which unambiguously suggests that topological fluctuations (related to the θ behavior) are strongly suppressed in deconfined phase, and this suppression becomes more severe with increasing N_c. Here we shall present some additional examples which support this picture.

The paper is organized as follows. In Sec. II we describe our model based on the N_c $D4$ branes with one compact worldvolume coordinate. In our main Sec. III we classify D defects treated in the probe approximation when they do not deform the dual geometry. In Sec. IV we discuss some composite objects which combine different types of D branes. Finally, in Sec. V we introduce matter fields in our system which are treated as probes $N_f \ll N_c$. Section VI is our conclusions.

II. DESCRIPTION OF THE MODEL

A natural starting point to discuss the dual holographic geometry is provided by a set of N_c $D4$ branes wrapped around a compact dimension [1]. We shall consider the pure gauge sector first and then add flavor $D8$ branes along the lines of the Sakai-Sugimoto model [2].

We shall assume the large N_c limit and consider the supergravity approximation. In this approximation the geometry looks as $M_{10} = R_{3,1} \times D \times S^4$ and the corresponding metric reads as

$$ds^2 = \left(\frac{u}{R_0}\right)^{3/2}(-dt^2 + \delta_{ij}dx^i dx^j + f(u)dx_4^2) + \left(\frac{u}{R_0}\right)^{-3/2} \times \left(\frac{du^2}{f(u)} + u^2 d\Omega_4^2\right)$$

$$e^{\Phi} = \left(\frac{u}{R_0}\right)^4, \qquad F_4 = \frac{3N_c\epsilon_4}{4\pi}, \qquad f(u) = 1 - \left(\frac{u_\Lambda}{u}\right)^3, \tag{1}$$

where $R_0 = (\pi g_s N_c)^{1/3}$ and $R = \frac{4\pi}{3}(\frac{R_0^3}{u_\Lambda})^{1/2}$. The coupling constant of Yang-Mills theory is related to the radius of the compact dimension R as follows:

$$g_{\rm YM}^2 = \frac{8\pi^2 g_s l_s}{R}.$$

At zero temperature, the theory is in the confinement phase and in the (u, x_4) coordinates we have the geometry of a cigar with the tip at $u = u_\Lambda$. The $D4$ branes are located along our $(3+1)$ geometry and are extended along the internal x_4 coordinate. The key point is that in the nonzero temperature case there are two backgrounds with similar asymptotic topology of $R^3 \times S_\tau^1 \times S^1 \times S^4$, where τ is the Wick-rotated time coordinate $\tau = it$, $\tau \propto \tau + \beta$. One background corresponds to the analytic continuation of the metric described above while the second background corresponds to interchange of τ and x_4, that is, the warped factor is attached to the τ coordinate and the cigar geometry emerges in the (τ, u) plane instead of the (x_4, u) plane which now exhibits the cylinder geometry (see Fig. 1). It was shown in [1] by calculation of the free energies that above T_c the latter background dominates.

That is, above phase transition the wrapping around the internal x_4 circle is topologically stable, while the wrapping around the Euclidean time coordinate is unstable. This is opposite to the stability pattern of the two wrappings below the phase transition.

Another issue which we shall be interested in concerns the θ dependence of the worldvolume theories on various probe branes. It can be traced from the Chern-Simons (CS) terms involving the interaction with the Ramond-Ramond (RR) one-form C_1

$$\delta L = \int C_1 \wedge e^{\mathcal{F}}, \tag{2}$$

where $\mathcal{F}$ is the gauge two-form. Taking into account that

$$\theta = \int dx_4 C_1, \tag{3}$$

one immediately recognizes that the θ dependence of the worldvolume theories on the defects correlates with the wrapping around the x_4 coordinate. Moreover it is clear that the θ dependence of a single defect is poorly defined in the confined phase since the wrapping around x_4 is topologically unstable and one could discuss the θ dependence of a kind of a condensate of the defects.

To model QCD one adds N_f $D8$-$\bar{D}8$ pairs localized at points in x_4 coordinate. There are a few qualitative phenomena to be mentioned. First, the chiral symmetry breaking is described geometrically in terms of the connectedness of the $D8$-$\bar{D}8$ pair. It was argued that the restoration of the chiral symmetry and the deconfinement phase transition generically take place at different temperatures [24]. Another essential point concerns the baryonic degrees of freedom. Geometrically baryons are identified as the $D4$ branes wrapped around the compact

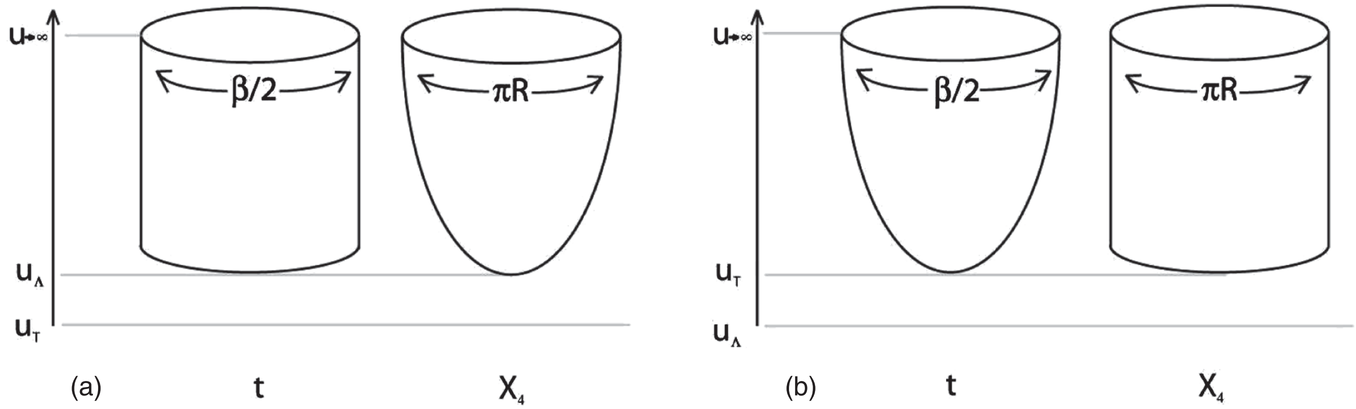

FIG. 1. Metric at (a) $T < T_c$ (low temperature phase) and (b) $T > T_c$ (high temperature phase).

S^4 and they are the instantons in the $D8$ brane worldvolume theory.

In the thermal gauge theory a natural order parameter is the vacuum expectation value (VEV) of the Polyakov loop

$$\langle W(\beta)\rangle = \left\langle \mathrm{Tr}P\exp\left(\int d\tau A_0\right)\right\rangle \tag{4}$$

which is vanishing at $T < T_c$ while $W(\beta) \neq 0$ at $T > T_c$. This implies that Z_N symmetry is unbroken at $T < T_c$ and broken at $T > T_c$. It will be useful to introduce another Z_N symmetry extending the total discrete symmetry to $Z_N \times Z_N$ where the order parameter for the second factor is

$$\langle W(R)\rangle = \left\langle \mathrm{Tr}P\exp\left(\int dx_4 A_4\right)\right\rangle. \tag{5}$$

It can serve as the order parameter analogous to the Polyakov loop since it has a nontrivial VEV at small temperatures and vanishes in the deconfinement phase. Let us emphasize that the total discrete group mentioned above differs from the same product discussed in [8]. In that paper the second factor corresponds to the S-dual magnetic center group whose order parameter is identified with the 't Hooft loop.

III. ZOO OF THE *D* DEFECTS

A. *D*0 branes

1. *D*0 *instantons*

The simplest defects to be discussed are $D0$ branes. It was argued in [3] that instantons are represented by the Euclidean $D0$ branes wrapped around x_4. In that case it was argued that the instanton is well-defined above T_c as it corresponds to the geometry of the cylinder. On the other hand, any finite number of instantons are ill-defined below T_c because of the $D0$-brane instability.

The θ dependence of the $D0$ action is captured by the CS term on its worldline. The change of the instanton role at the transition point corresponds to the change from the Witten-Veneziano to t' Hooft mechanisms of the solution to the $U(1)$ problem. In QCD-like brane setup $D0$ branes wrapped around x_4 intersect with the flavor branes and induce the sources on the flavor brane worldvolumes. The unbroken Z_N symmetry (5) introduced above implies that $D0$ instantons if they have nontrivial holonomy [25] exist in N-tuples symmetrically in the τ circle.

This picture can also be readily understood in the quantum-field theory terms since an estimation for T_c in the Λ_{QCD} units can be given [16]. Indeed, the wrapping around x_4 corresponds to the well-defined small-size instanton and one can use the standard instanton calculus to estimate the critical temperature T_c and the θ behavior above T_c:

$$\begin{aligned} &V_{\mathrm{inst}}(\theta) \sim \cos\theta \cdot e^{-\alpha N(((T-T_c)/(T_c)))},\\ &\qquad 1 \gg \left(\frac{T-T_c}{T_c}\right) \gg 1/N,\\ &\chi(T) \sim \frac{\partial^2 V_{\mathrm{inst}}(\theta)}{\partial\theta^2} \sim e^{-\alpha N(((T-T_c)/(T_c)))} \to 0,\\ &\qquad \alpha \sim 1, \qquad N \gg 1. \end{aligned} \tag{6}$$

Such a behavior implies that the dilute gas approximation at large N_c is justified even in close vicinity of T_c as long as $(T - T_c)/T_c \gg 1/N_c$. Such a sharp behavior of the topological susceptibility $\chi(T)$ is supported by numerical lattice results [18–22] which unambiguously suggest that the topological fluctuations are strongly suppressed in the deconfined phase, and this suppression becomes more severe with increasing N_c. These general features observed in the lattice simulations have a very simple explanation within quantum field theory (QFT) framework as Eq. (6) shows, as well as in holographic model of QCD [3,16].

Finally, let us address the following question: what happens with our $D0$ instantons in the deconfined phase, immediately at $T > T_c$? We know that at sufficiently large temperatures $(T - T_c)/T_c \gg 1/N$ the configuration becomes a stable instanton in four dimensions with the size $\rho \sim (\pi T)^{-1}$. The density of the instantons is exponentially suppressed $\sim \cos\theta \cdot e^{-\alpha N(((T-T_c)/(T_c)))}$, magnetic charges of the constituents (if they exist, see Sec. IV) are completely screened such that it makes no sense to speak about individual constituents in this regime. However, for finite N there is a window of temperatures $0 < (T - T_c)/T_c \leq 1/N$ when the magnetic degrees of freedom are not completely screened yet. This window which shrinks to a point at $N = \infty$ is obviously beyond analytical control. However, these magnetic degrees of freedom could be extremely important in the window $0 < (T - T_c)/T_c \leq 1/N$. It is tempting to assume that these magnetic degrees of freedom are a trace of fractional instanton constituents which are likely to exist in confined phase; see discussions in Sec. IV.

2. $D0$ particle

The orientation of the $D0$ brane worldline along the Euclidean time τ corresponds to its realization as a Kaluza-Klein (KK) particle without the θ dependence. However the Polyakov loop $\mathrm{Tr}P\exp(\int d\tau A_t)$ may develop. This configuration at $T < T_c$ is a stable scalar gluelike configuration which must have very different properties in comparison with all standard glueballs when the temperature approaches T_c from below, $(T_c - T) \to 0$. Above the critical temperature (deconfinement phase) KK modes tend to condense near the tip of the cigar because of the instability of the wrapping around τ. In the deconfinement phase KK modes behave as the instantonlike configuration (with no θ dependence) in the effective three-dimensional gauge theory. This instability may have enormous consequences for physics since an arbitrarily large number of such states can be produced in the vicinity of $T \simeq T_c$. We shall not elaborate on this issue in the present work.

B. $D2$ branes

1. $D2$ string

The magnetic string is the probe $D2$ brane wrapped around S_1 parametrized by x_4 and its tension is therefore proportional to the effective radius $R(u)$ [4]. At small temperatures this wrapping is topologically unstable and the $D2$ brane tends to shrink to the tip where its tension vanishes. This is the large-N_c counterpart of the effect of dissolving of p branes inside $p + 2$ branes [13]. We see that in this way one immediately reproduces the observed property of tensionlessness of the magnetic string in the confining phase which however becomes tensionful above the critical temperature T_c of the deconfinement phase transition.

The θ term in the magnetic string worldsheet Lagrangian is induced by the CS term

$$L_{\mathrm{CS}} = \int d^3x C_1 \wedge F, \qquad (7)$$

that is, configurations with the flux on the worldsheet amount to the nontrivial four-dimensional topological charge. It was also argued that the magnetic strings amount to a negative contribution to the total energy of plasma at $T > T_c$ [4]. It could explain the negative-sign contribution of the lattice magnetic strings into the energy of plasma [26]. Because of the instability of the "thermal" cigar, the magnetic string becomes effectively a particlelike object in the Euclidean three-dimensional [4], in agreement with the lattice studies [27]. See also Refs. [28,29] for related discussions in deconfined phase at $T > T_c$.

2. $D2$ domain walls

There are two types of domain walls in R^3 built from $D2$ branes. Consider first a $D2$ brane localized in the x_4 coordinate. It corresponds to a domain wall in four dimensions and has no θ dependence. The theory on the domain wall involves the periodic real scalar field which corresponds to the position of the $D2$ brane on the x_4 circle as well as a scalar corresponding to its radial coordinate. Such domain walls are expected to exist in N-tuples in the deconfinement phase because of the unbroken magnetic Z_N symmetry (5), that is, one expects natural SU(N) gauge theory on their worldvolume. Its tension is small in the deconfinement phase and it behaves as the string in the three-dimensional effective description. It has no θ dependence.

The second type of $D2$ domain wall extended in x_4 involves a nontrivial θ dependent term. Because of the unbroken electric Z_N symmetry in the confinement phase such domain walls are expected to exist in N-tuples in this phase symmetrically on the τ circle. The worldsheet theory in the deconfinement phase involves only one real scalar corresponding to its radial coordinate. These $D2$ domain walls may play an important dynamical role supporting the constituents with fractional topological charges; see Sec. IV.

3. Space-filling $D2$ brane

One could also consider the $D2$ branes localized both in τ and x_4 directions. Such space-filling $D2$ branes are expected to exist in N-tuples in both phases because in each phase the $D2$ brane has one unbroken Z_N symmetry. Hence the effective gauge theory should have SU(N) gauge theory in both phases. These branes could play an important role in the effective three-dimensional description of the deconfinement phase. Their worldvolume theory naturally involves one complex and one real periodic scalars.

4. $D2$-$\bar{D}2$ pair

One could also discuss the $D2$ brane extended along the radial coordinate. Such a configuration is an analogue of the $D8$ brane in the Sakai-Sugimoto model which is extended along the radial coordinate and has U-shape form in the chirally broken phase. In the holographic QCD case, one actually has a $D8$-$\bar{D}8$ pair whose connectedness indicates nonvanishing of the chiral condensate. In the case of the U-shaped $D2$ brane we have no flavor brane hence the chirality cannot be defined in the conventional way. However, there is a configuration which is readily prepared to provide the chiral condensate if a $D8$ brane is introduced into the system. If such a U-shaped $D2$ brane is extended along τ it behaves as string while in the opposite case as the domain wall. In both cases the tension of the object is finite. It is tempting to speculate that some kind of chiral symmetry breaking happens in pure gauge theory (without fermions) being localized at lower dimensional defects rather than in the entire space.

It is also tempting to speculate that some of the $D2$ branes discussed in this subsection could be mapped into the low-dimensional, chirality-related structures observed on the lattices.

C. *D*4 branes

1. *D*4 *particle*

There are several possible embeddings of $D4$ branes. One possibility concerns $D4$ wrapped around S^4 and extended along the Euclidean time τ. The familiar example of such wrapping in the QCD-like geometry [2] has the interpretation of baryon if matter fields in the form of $D8$ branes are present in the system. The key point here is that due to the CS term

$$\int d^5 x C_3 \wedge F \sim N_c, \tag{8}$$

the "electric charge" N_c is induced on the $D4$ brane, that is, one has to add N_c open strings. It is a static topologically stable configuration. In the QCD-like case these open strings end on the $D8$ brane yielding the baryonic state.

In the pure YM case there are no flavor branes, that is, one has to add additional low-dimensional branes to compensate charge and make a gauge invariant object. The most simple way to achieve this goal is to add N_c $D0$ branes yielding the $D0$-$D4$ open strings. Hence we get the $D4$ particle which does not feel the θ term and is well-defined below the critical temperature $T < T_c$.

We can combine the " $D4$ particle" with "$D4$ antiparticle" to form a gauge invariant object (see Fig. 2). The mass of this object scales as $\sim N_c$ and is much heavier than the usual glueballs. In fact, one can construct gauge invariant objects with any even number of vertices such that the total charge vanishes. It is a new family of glueballs with mass $\sim N_c V$, where V is the number of $D4$ and anti-$D4$ particles which form a desired configuration. It is amusing that this kind of structure in QCD had been previously discussed [30] which in turn was motivated by the discovery of the carbonic Fullerenes C_{60} and C_{70} in 1985, which are nanoscale objects [31]. The QCD objects, similar to the carbonic Fullerenes with femtometer scale were named buckyballs. It has been also demonstrated that the "magic" numbers for buckyballs are $V = 8, 24, 48, 120$ which correspond to the most symmetric, and likely, most stable configurations [30]. The properties of this configuration are not sensitive to θ. The possibility to discover such configurations at the Relativistic Heavy Ion Collider was discussed in [30].

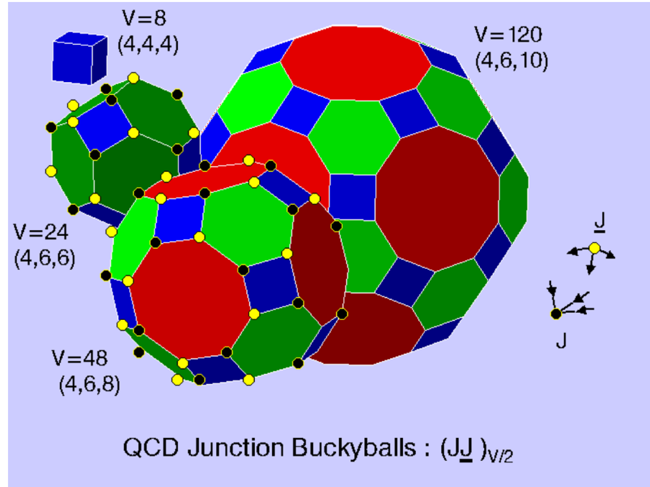

FIG. 2 (color online). Buckyballs with $N = 3$ and $V = 8, 24, 48, 120$ from [30]. The construction combines a $D4$ particle with a $D4$ antiparticle to form a gauge invariant object with zero baryon charge. The mass of this object scales as $\sim N_c$.

2. *D*4 *instanton*

If we consider a wrapped $D4$ brane extended along x_4 we get a new object, a "$D4$ instanton" which is to be distinguished from the "$D4$ particle" discussed above. The origin of this term is due to its similarity in the four-dimensional Euclidean space-time to the canonical instanton (or caloron at $T \neq 0$). This object tends to condense below the phase transition and is well-defined above the transition point, similar to the instanton. It carries a nontrivial θ dependence and the corresponding contribution to the action from the single "$D4$ instanton" looks as follows:

$$\delta S \propto \theta \int \mathrm{Tr} F \wedge F \propto C_1. \tag{9}$$

Because of the background flux dC_3 through S^4 one has a

$$N_c \int dx_4 C_1 \tag{10}$$

term in the action implying that the $D4$ instanton worldvolume is populated by N $D0$ instantons which provide the topological charge N in $D4$ worldvolume theory. Because of the unbroken electric Z_N in the confinement phase one could expect the N-tuples of $D4$ instantons to exist. The nature of large factor N_c in both Eqs. (10) and (8) is one and the same, namely, the background flux dC_3 through S^4 which is proportional to N_c. However, the physical interpretation for these two cases is quite different: in the first case it is the mass of the particle which is $\sim N_c$, while in the second case it is the action of N_c different instantons accompanied by $4N_c$ zero modes each. It is tempting to identify this $D4$ instanton with the configuration consisting of N_c different calorons with maximally nontrivial holonomies.[1] As is known, exactly the configuration consisting of N_c different calorons provides an infrared finite contribution to the partition function [32].

3. $D4$-$\bar{D}4$ *U-shaped pair*

One can also consider the U-shaped $D4$ brane extended along the radial coordinate. If it wraps S^4 it is an instanton-like object which however does not carry any θ dependence. Since there is in fact a connected $D4$-$\bar{D}4$ pair one could say that such objects amount to the chiral symmetry breaking "at a point." On the other hand one can consider U-shaped $D4$ brane localized at S^4. Such an object is a space-time filling brane which provides a kind of homogeneous "chiral symmetry breaking" in pure YM theory.

D. $D6$ branes

1. $D6$ *string*

Let us turn to $D6$ branes. Consider first the pure YM case at large N_c. If $D6$ is wrapped around $S^4 \times x_4$ it behaves as the string in the space-time whose tension is defined by the scale of S^4. Because of the wrapping around S^4 the string carries a CS term generated on its worldsheet from the CS term

$$\int d^7 x C_3 \wedge F \wedge F \tag{11}$$

which reads as $N_c \int d^3 x A \wedge F$. In the confinement phase there is a nontrivial holonomy along x_4 represented by $W(R)$[see (5)], hence the effective two-dimensional θ term on the $D6$ string is induced. Note that the induced θ term on the $D6$ string is proportional to N_c, somewhat similar to the situation discussed in [33] for the magnetic strings in $N = 1*$ theory.

Because of the wrapping around x_4 it carries an intrinsic θ dependent term

$$\int d^7 x C_1 \wedge F \wedge F \wedge F. \tag{12}$$

To provide the θ dependent contribution from this term the topological Chern number $c_3(F)$ should be nontrivial. It follows from the $D2$ branes on the $D6$ worldvolume. Hence we see that the interesting "composite" $D6$-$D2$ string has θ dependent contributions from the both components. Such $D6$ strings according to our standard arguments are individually unstable and large number of them have a tendency to condense at small temperatures $T < T_c$. The object is well-defined at large temperatures, $T > T_c$ and has finite tension. The interpretation of such objects is far from obvious, and the role they play in physics is also unclear at the moment.

One can also consider the U-shaped $D8$-$\bar{D}6$ pair in the confinement phase corresponding to the string with the finite tension. Because of the connectedness of the $D6$ brane, it localizes the chiral symmetry breaking on its worldvolume.

2. $D6$ *domain wall*

If a $D6$ brane wraps around τ it behaves as the domain wall which is a source of the corresponding RR form. Such a configuration has been interpreted in Ref. [5] as the domain wall which separates different vacua known to exist in gluodynamics at large N_c. Its worldvolume theory on the domain wall involves the conventional CS term and has no θ dependent term.

IV. COMPOSITE DEFECTS

In this section we present a few examples of composite defects which exhibit interesting features. Some of them may play a crucial role in understanding of the dynamics.

A. $D0$-$D2$

First, we want to address the following question: what happens to instantons in the confined phase. Naively, one could think that as the metric takes the geometry of a cigar in the (x_4, u) plane at $T < T_c$ the system becomes unstable, and therefore, there is no subject for the discussion as instantons simply disappear from the system. However, as we discussed before, one should speak about effectively zero action for formation of such objects. Therefore, many of these objects can emerge in the system without any suppression. In fact, in [16] it has been argued that this is precisely what is happening when one crosses the phase transition line from above.

Now, in order to investigate what kind of objects may emerge when the phase transition is crossed from above, we add the $D2$ domain walls (discussed in Sec. III B) localized at some points along the x_4 coordinate. In this construction an object with a fractional topological charge $1/N_c$ may emerge. Indeed, one can follow the construction of Ref. [34] for the SUSY case when N_c $D2$ branes located symmetrically split the instanton into N_c constituents stretched between pairs of domain walls. Each constituent has fractional instanton number $1/N_c$ as well as the fractional monopole number and has no reason to condense. In our system we have precisely appropriate $D2$ branes which are needed for this construction. These monopoles are instantons in the three-dimensional gauge theory on the $D2$ worldvolume theory which involves the scalar corresponding to the position of $D2$ branes on the cigar. As we mentioned previously in Sec. III A, these magnetic monopoles may play an important role in the region close to the phase transition $0 < |T - T_c|/T_c \leq 1/N_c$.

[1]An instanton at $T \neq 0$ becomes a caloron with a generically nontrivial holonomy [25].

B. $D6$-$D4$

There are several configurations involving composite $D6$-$D4$ defects. The first one to be mentioned is the combination "$D6$ string-$D4$ instanton." Since the $D4$ instanton shares all coordinates with the $D6$ string it is melted into the flux on the string of constant "electric" field. Another example of the melting concerns the combination "$D6$ domain wall-$D4$ particle" when the $D4$ particle delocalizes on the domain wall into the flux. It is interesting to note that since in the holographic QCD the $D4$ particle is identified with the baryon upon melting we get a domain wall with the baryonic density. The defect is well-defined in the confinement phase.

Another interesting possibility concerns the combination "$D6$ domain wall-$D4$ instanton." In this case a fractional $D4$ instanton can emerge if there are several domain walls localized at different positions at x_4 and the $D4$ instanton can be stretched between a pair of domain walls in the x_4 direction yielding a monopolelike objects on the domain-wall worldvolume.

C. $D2$-$D4$

An interesting situation happens if we consider the configuration of $D2$ string and $D4$ particle localized. In this case the magnetic $D2$ string can be stretched between two D particles and therefore does not wrap the x_4 circle. Hence it has finite tension equal to the distance δx_4 between two $D4$ particles and does not condense in the confinement phase. This configuration carries fractional topological charge and is θ dependent. The $D4$ particles acquire magnetic charges where the flux of the magnetic string ends.

In the case of $D2$ domain wall and $D4$ instanton, the opposite situation can happen. The $D4$ instanton worldline can be split between two $D2$ domain walls localized at different x_4 coordinates. Such configuration carries fractional topological charge as well.

D. $D2$-$D6$

The simplest configuration of this type is " $D6$ string-$D2$ string". That is we have composite string object with additional "charge" since the $D2$ induces instantonlike charge on the $D6$ brane worldvolume. Such a composite string is unstable in the confinement phase and well-defined in the deconfinement phase. Another combination "$D6$ domain wall-$D2$ string" can be stable in the confinement phase if there are several domain walls localized at different positions in x_4. The same can be said about the configuration of "$D2$ domain wall-$D6$ string" with several $D2$ walls.

V. DEFECTS IN HOLOGRAPHIC QCD

In this section we discuss defects in the holographic QCD when matter fields are included by adding N_f U-shaped $D8$ branes [2] in the probe approximation. This case can be considered as a particular example of the composite defects in pure YM theory. To study defects in holographic QCD we shall add some additional probe branes of different dimensions. In what follows we mention only a few effects which are specific for theory with the fundamental matter.

First, we can add U-shaped $D6$ branes parallel to the $D8$ branes. From the four-dimensional viewpoint they are unstable strings. Indeed the $D6$ branes share all worldsheet coordinates with the $D8$ branes, hence there are tachyonic modes in the spectrum of $D8$-$D6$ open strings and the $D6$ brane melts in the $D8$ worldvolume yielding a flux of the flavor gauge field. Since the $D6$ branes get delocalized they do not provide stringlike localization of the chiral symmetry breaking.

Another interesting example concerns the $D4$ instanton in holographic QCD extended along x_4 or radial coordinate. To estimate its contribution into the partition function remember that there is a CS term on the $D8$ brane worldvolume multiplied by N_c. Since a U-shaped $D4$ instanton shares all coordinates with the $D8$ branes it induces a nontrivial contribution into the CS part of the action which reads as follows:

$$\delta S_{\mathrm{CS}} = N_c \int du A_u \int d^4\, \mathrm{Tr} F \wedge F, \tag{13}$$

where we assume that $U(1)_A$ field A_u is space-time independent. It can be interpreted as the constant mode of η' meson since in the Sakai-Sugimoto model it is identified as $\int du A_u(u, x)$. Note that the constant mode of the η' mode can be thought of as the effective θ term hence the total contribution reads as $\theta_{\mathrm{eff}} N_c k$, where k is the instanton number in the flavor gauge theory. Such U-shaped $D4$ instanton is well-defined at any temperature since it does not wrap around any compact coordinate and one could speak about the pointlike contribution to the chiral symmetry breaking.

One can also discuss fractional flavor $D4$ instantons. To this aim the flavor branes have to be placed at different positions at the dual temporal circle. This can be achieved by switching on chemical potentials which correspond to nontrivial temporal holonomies of the flavor gauge fields. The eigenvalues of the flavor holonomy around the temporal circle provide the positions of the $D7$ branes on the dual circle. Hence the worldline of the $D5$ brane on the dual circle can split and we get N_f fragments of the $D5$ brane stretched between N_f $D7$ branes which carry the flavor "magnetic" and fractional instanton charges. The system can be described in QFT terms as a set of N_f "monopoles" with the total instanton charge $Q_{\mathrm{inst}} = 1$, and the total monopole charge zero, similar to the canonical caloron with nontrivial temporal holonomies; see for review [35] and references therein. Since we are working in the probe approximation $N_f \ll N_c$ the action on each "monopole" is proportional to N_c and therefore these defects are suppressed at large N_c.

Another possible phenomenon concerns a peculiar manifestation of the Myers effect in the holographic QCD. It is known that the following configuration, two parallel $D4$ branes + $D0$ branes localized on the $D4$ branes + $D0 - D0$ open strings, is unstable and decays into the so-called dyonic instanton [36]. That is, this configuration of instantons decays into the circular $D2$ brane stretched between parallel $D4$ branes and carries electric and topological charges as well as angular momentum which stabilizes the configuration. On the $D4$ worldvolume one gets a magnetically charged ring.

In our context, we can consider the set of $D8$ and $D4$ branes representing the baryons in the space-time. We assume that the worldvolume of the $D4$ brane is $S^4 \times \tau$, that is, both type of branes are localized at the x_4 coordinate. We could assume that the $D8$ branes are generically localized at different values x_{4k}, where $k + 1, \ldots N_f$ and the "baryonic" $D4$ branes are localized on the $D8$ branes. Consider two baryonic $D4$ branes and connect them by an open string which carries in the spectrum of excitations the gauge boson of the flavor group ρ meson. Similar to the $D4$-$D0$-$F1$ case we expect that the $D4$ brane is expanded into a $D6$ brane with baryonic density and the $F1$ string is melted into the isotopic charge from the initial ρ meson. Hence finally we could expect that the configuration with the baryonic charge larger than 1 gets delocalized into the "baryonic ring" carrying the isotopic charge. Note that it is known in the Skyrme model that the $B = 2$ state has a torus like ground-state geometry [37], in agreement with our arguments above. Note that such a configuration does not involve wrapping around x_4, that is, it is well-defined at $T < T_c$.

VI. CONCLUSIONS

In this paper we demonstrated that the holographic description of the pure YM or QCD-like theories implies existence, at least classically, of a plenty of defects of different codimensions. We have tried to argue that their existence is insensitive to details of the metric. However the analysis is certainly oversimplified and we have not aimed at deriving the defects' properties in detail. We have seen that various types of strings and domain walls and some more exotic composite objects are emerging.

A few claims seem to be quite generic. If the defect tends to condense (have small tension vanishing classically) at small temperatures it becomes tensionful and well-defined above the phase transition. On the other hand, all the defects apart from the S branes tend to lose one Euclidean dimension above the critical temperature. The θ dependence of the defects' worldvolume actions is present in the "condensing brane" below the phase transition and can be defined on such defects above the transition point as well on a single defect. This is an analogue of the instanton solution of the $U(1)$ problem via the Witten-Veneziano mechanism below the transition point and via the t' Hooft mechanism above this point. Our analysis implies that a similar change of mechanisms happens for defects of different dimensions as well.

The theory above the phase transition almost immediately becomes three-dimensional because of the cigar-type instability. This fits well with the lattice studies.

In particular, in holographic description we identified a few very interesting objects such as heavy buckyballs ($D4$ particles) whose masses scale as $\sim N_c$ (see Sec. III C) or $D4$ instantons whose action scales as $\sim N_c$. While such objects have been discussed previously in the literature within QFT, their future study using the holographic description may shed a new light on their nature. Another interesting example that deserves to be mentioned is the holographic description of the objects with fractional topological and magnetic charges (Sec. IV). Such objects have been discussed within QFT in the late 1970s. Future study of these objects may provide with a key to understanding the QCD vacuum structure and the nature of the phase transition.

There are many questions we have only touched upon. In particular, it would be highly interesting to understand better the role of the second order parameter "dual" to the Polyakov loop and the duality between the corresponding pairs of domain walls which gets interchanged at the phase transition. Our analysis suggests that it is probably reasonable to discuss the chiral symmetry breaking in pure YM theory induced by U-shaped defects. At first glance it might look strange, but " chiral symmetry" of the gauge boson can be defined similar to fermions. The order parameter for such a "chiral symmetry breaking" could be similar to the one recently discussed in [38].

ACKNOWLEDGMENTS

We thank The Galileo Galilei Institute for Theoretical Physics for the organization of the workshop "Nonperturbative methods in strongly coupled gauge theories" where this project was initiated. A. R. Z. was supported, in part, by the Natural Sciences and Engineering Research Council of Canada. The work of A. G. was supported in part by Grant Nos. INTAS-1000008-7865 and PICS-07-0292165. V. I. Z. would like to thank the Laboratoire de Mathematiques et Physique Theorique at Universié Francois Rabelais, Tours, France, for kind hospitality during the final stages of working on the project.

[1] E. Witten, Adv. Theor. Math. Phys. **2**, 505 (1998).
[2] T. Sakai and S. Sugimoto, Prog. Theor. Phys. **113**, 843 (2005); **114**, 1083 (2005).
[3] O. Bergman and G. Lifschytz, J. High Energy Phys. 04 (2007) 043.
[4] A. Gorsky and V. Zakharov, Phys. Rev. D **77**, 045017 (2008).
[5] E. Witten, Phys. Rev. Lett. **81**, 2862 (1998).
[6] F.L. Lin and S.Y. Wu, J. High Energy Phys. 09 (2008) 046.
[7] D.K. Hong, K.M. Lee, C. Park, and H.U. Yee, J. High Energy Phys. 08 (2008) 018.
[8] O. Aharony and E. Witten, J. High Energy Phys. 11 (1998) 018.
[9] A. Armoni, S.P. Kumar, and J.M. Ridgway, J. High Energy Phys. 01 (2009) 076.
[10] H.U. Yee, J. High Energy Phys. 04 (2009) 029.
[11] A.L. Cotrone, arXiv:0707.1483.
[12] M.R. Douglas, arXiv:hep-th/9512077.
[13] E. Gava, K.S. Narain, and M.H. Sarmadi, Nucl. Phys. **B504**, 214 (1997).
[14] R.C. Myers, J. High Energy Phys. 12 (1999) 022.
[15] D. Toublan and A.R. Zhitnitsky, Phys. Rev. D **73**, 034009 (2006).
[16] A. Parnachev and A. Zhitnitsky, Phys. Rev. D **78**, 125002 (2008).
[17] A.R. Zhitnitsky, Nucl. Phys. **A813**, 279 (2008).
[18] B. Alles, M. D'Elia, and A. Di Giacomo, Nucl. Phys. **B494**, 281 (1997); **B679**, 397(E) (2004).
[19] B. Lucini, M. Teper, and U. Wenger, J. High Energy Phys. 01 (2004) 061.
[20] B. Lucini, M. Teper, and U. Wenger, Nucl. Phys. **B715**, 461 (2005).
[21] L. Del Debbio, H. Panagopoulos, and E. Vicari, J. High Energy Phys. 09 (2004) 028.
[22] B. Lucini, M. Teper, and U. Wenger, J. High Energy Phys. 02 (2005) 033.
[23] E. Vicari and H. Panagopoulos, Phys. Rep. **470**, 93 (2009).
[24] E. Antonyan, J.A. Harvey, and D. Kutasov, Nucl. Phys. **B776**, 93 (2007).
[25] T.C. Kraan and P. van Baal, Nucl. Phys. **B533**, 627 (1998); Phys. Lett. B **428**, 268 (1998).
[26] M.N. Chernodub, K. Ishiguro, A. Nakamura, T. Sekido, T. Suzuki, and V.I. Zakharov, Proc. Sci., LAT2007 (2007) 174.
[27] M. Engelhardt, K. Langfeld, H. Reinhard, and O. Tennert, Phys. Rev. D **61**, 054504 (2000); M.N. Chernodub, A. Nakamura, and V.I. Zakharov, arXiv:0812.4633.
[28] J. Liao and E. Shuryak, Phys. Rev. C **75**, 054907 (2007).
[29] J. Liao and E. Shuryak, Phys. Rev. Lett. **101**, 162302 (2008).
[30] T. Csorgo, M. Gyulassy, and D. Kharzeev, J. Phys. G **30**, L17 (2004).
[31] H.W. Kroto, R.E. Smalley, and R.F. Curl, Nature (London) **318**, 162 (1985).
[32] D. Diakonov and V. Petrov, Phys. Rev. D **76**, 056001 (2007).
[33] R. Auzzi and S. Prem Kumar, J. High Energy Phys. 12 (2008) 077.
[34] N.M. Davies, T.J. Hollowood, V.V. Khoze, and M.P. Mattis, Nucl. Phys. **B559**, 123 (1999).
[35] F. Bruckmann, E.M. Ilgenfritz, B.V. Martemyanov, M. Muller-Preussker, D. Nogradi, D. Peschka, and P. van Baal, Nucl. Phys. B, Proc. Suppl. **140**, 635 (2005).
[36] H.Y. Chen, M. Eto, and K. Hashimoto, J. High Energy Phys. 01 (2007) 017; P.K. Townsend, C.R. Physique **6**, 271 (2005).
[37] V.B. Kopeliovich and B.E. Stern, Pis'ma Zh. Eksp. Teor. Fiz. **45**, 165 (1987) [JETP Lett. **45**, 203 (1987)].
[38] O. Aharony and D. Kutasov, Phys. Rev. D **78**, 026005 (2008).