PHYSICAL REVIEW D **79,** 106002 (2009)

String coupling and interactions in type IIB matrix model

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(Received 24 December 2008; published 13 May 2009)

We investigate the interactions of closed strings in a IIB matrix model. The basic interaction of the closed superstring is realized by the recombination of two intersecting strings. Such interaction is investigated in a IIB matrix model via two-dimensional noncommutative gauge theory in the IR limit. By estimating the probability of the recombination, we identify the string coupling g_s in the IIB matrix model. We confirm that our identification is consistent with matrix string theory.

DOI: 10.1103/PhysRevD.79.106002 PACS numbers: 11.25.−w, 11.25.Yb, 11.25.Db, 11.27.+d

I. INTRODUCTION

The IIB matrix model [1] is considered as a candidate of the nonperturbative formulation of superstring theory. The relation between the IIB matrix model and perturbative string theory is shown in [2]. Perturbative string theory is also contained in Dijkgraaf, Verlinde, and Verlinde's matrix string theory [3]. The strong coupling region of two-dimensional supersymmetric Yang-Mills theory is described by the perturbative superstring theory. In the strong coupling limit, a free Green-Schwarz string theory is obtained. On the other hand, a weak coupling region is described by the perturbative Yang-Mills theory. In addition, there is an intermediate region which is described by the type IIB supergravity solution in the large N limit [4].

The aim of this paper is to identify the string coupling g_s in the IIB matrix model. The hint for the identification comes from the matrix string theory. The gauge coupling of the two-dimensional Yang-Mills theory has the $[\text{length}]^{-1}$ of the world sheet and it is related to the string coupling as $g_{\rm YM}^{-2} = \alpha' g_s^2$. The g_s has the dimension [length] on the world sheet. Thus, our task is to search for the dimensionful parameter on the world sheet.

Before searching for g_s, we have to construct the world sheets in the IIB matrix model. They are constructed as two-dimensional classical backgrounds [2] in the IR limit. The string length is identified there and free multiple closed strings are obtained. Vertex operators of type IIA superstring are constructed from the IIB matrix model on these backgrounds in [5]. The relation between type IIA superstring theory and the IIB matrix model is also proposed in [6] in a different way.

In this paper, we consider the interaction of perturbative strings. The basic interactions are the transitions from two strings into one string or vice versa. These interactions are introduced in the formulation of superstring field theory in the light-cone gauge [7]. 2 strings → 1 string interactions are represented by the recombination of intersecting strings locally. See Fig. 1.

Although this process locally represents the recombination of 2 strings → 2 strings, the final state is connected globally. Thus, after the recombination, we obtain a single closed string. In the matrix model, the instability of this system comes from the off-diagonal modes. The system with the larger intersection angle ϕ is more unstable than that with the smaller intersection angle. In other words, the decay rate of two closed strings is decided by the intersection angle ϕ. We can replace the angle ϕ with another parameter q which is related to ϕ as $\phi = 2\tan^{-1} q$. Since we choose the horizontal axis in Fig. 1 as the coordinate of the world-sheet σ and the vertical axis as the transverse scalar field $\phi_i(\sigma)$, the parameter q denotes the slope of the tilted strings. In the two-dimensional gauge theory, q is the dimensionful parameter. We clarify the q dependence of the instability. Since the configuration decays into the single closed string through the recombination process, we estimate q dependence of the recombination probability. In the string perturbation theory, the recombination probability is proportional to g_s^2 to the leading order.

In our investigation, we derive the action of multiple strings from the IIB matrix model. We identify the unstable mode for the intersecting strings which are the solutions of this action. From this mode, we estimate the probability of the recombination. By comparing the result to that of the perturbative string theory, we identify the string coupling g_s in the IIB matrix model.

In Sec. II, we derive the action of multiple closed superstrings. In Sec. III A, we carry out the fluctuation analysis for the intersecting closed strings. We identify the unstable

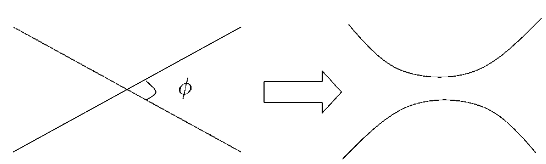

FIG. 1. Two strings intersect at an angle ϕ. The recombination happens at the intersection point.

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mode. In Sec. III B, we calculate the probability of the recombination from the unstable mode. By comparing our result with that of the perturbative superstring theory, we identify the string coupling g_s. In Sec. III C, we estimate the probability of the recombination in matrix string theory. Section IV is devoted to the conclusion. In Appendix A, our notation of light-cone coordinates is given. In Appendix B 1, the differential equations for the fluctuation modes are solved. In Appendix B 2, the Schrödinger equation which controls the time evolution of the probability is solved.

II. THE EFFECTIVE ACTION OF MULTIPLE STRINGS

The interaction among multiple strings is described in various ways in string theory. Perturbatively, transitions from two strings into one string or vice versa are the basic process. We aim to identify such interactions in the IIB matrix model. Since this interaction is proportional to g_s at the tree level, we can identify g_s in the IIB matrix model through this process.

Let us start from the action

$$S = -\frac{1}{g^2}\mathrm{Tr}\left(\frac{1}{4}[A^\mu, A^\nu][A_\mu, A_\nu] + \frac{1}{2}\bar{\psi}\Gamma^\mu[A_\mu, \psi]\right), \tag{2.1}$$

where ψ is a ten-dimensional Majorana-Weyl spinor and A_μ ($\mu = 0, 1, \ldots, 9$) and ψ are $N \times N$ Hermitian matrices. By expanding the action (2.1) around a two-dimensional noncommutative background,

$$[p_\mu, p_\nu] = i\theta_{\mu\nu}, \tag{2.2}$$

we obtain an $\mathcal{N} = 8$ two-dimensional $U(n)$ noncommutative gauge theory [8–10]:

$$S = -\frac{\theta}{8\pi g^2}\int d^2x \mathrm{tr}(F^2_{\tilde{\mu}\tilde{\nu}} + 2(D_{\tilde{\mu}}\phi_i)^2 + [\phi_i, \phi_j][\phi_i, \phi_j] + 2\bar{\psi}\Gamma^{\tilde{\mu}}D_{\tilde{\mu}}\psi + 2\bar{\psi}\Gamma_i[\phi_i, \psi])_*, \tag{2.3}$$

where $\tilde{\mu}, \tilde{\nu} = 0, 1$ and $i, j = 2, \ldots, 9$.

The $*$ product is described by

$$a * b = \exp\left(\frac{iC^{\mu\nu}}{2}\frac{\partial^2}{\partial\xi^\mu\partial\eta^\nu}\right)a(x+\xi)b(x+\eta)|_{\xi=\eta=0}, \tag{2.4}$$

where $C^{\mu\nu}$ is defined as the inverse of $\theta_{\mu\nu}$.

By taking the commutative (IR) limit, we obtain a commutative gauge theory

$$S = -\frac{\theta}{8\pi g^2}\int d^2x \mathrm{tr}(F^2_{\tilde{\mu}\tilde{\nu}} + 2(D_{\tilde{\mu}}\phi_i)^2 + [\phi_i, \phi_j][\phi_i, \phi_j] + 2\bar{\psi}\Gamma^{\tilde{\mu}}D_{\tilde{\mu}}\psi + 2\bar{\psi}\Gamma_i[\phi_i, \psi] + O(\theta)). \tag{2.5}$$

Diagonal components are relevant degrees of freedom in the IR limit. We interpret the diagonal elements of the field ϕ_i in (2.5) as the coordinates of the fundamental strings. This two-dimensional Yang-Mills theory is related to a low energy effective theory of D strings by the S-duality transformation, as is shown in Fig. 2 of our previous paper [2].

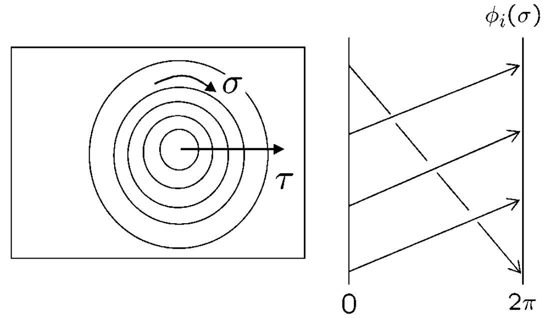

FIG. 2. The string world-sheet coordinate. We map the coordinate from (x_0, x_1) into (τ, σ).

We map the world-sheet coordinate from R^2 into $R^1 \times S^1$ as

$$z \equiv x_0 + ix_1 = e^{\tau + i\sigma}. \tag{2.6}$$

Since the only gauge invariant quantity is a set of the eigenvalues of the matrices ϕ_i, if we go around the circle S^1, the eigenvalues can be interchanged. See Fig. 2. We can consider a string of length n by the identification $\phi_i(\sigma) = \phi_i(\sigma + 2\pi n)$.

Since multiple free closed strings can be regarded as a moduli space of this theory, one can consider a configuration with multiple strings of various length in general,

$$\phi_i(\sigma^a) = \phi_i(\sigma^a + 2\pi w^a), \tag{2.7}$$

where $n = \sum_{a=1}^k w^a$.

For our purpose, it is enough to analyze $U(2)$ gauge theory since the recombination is a local problem which involves two strings. See Fig. 3. At the tree level, the amplitude of this interaction is proportional to g_s. After the coordinate transformation (2.6), the action (2.5) is mapped into

$$\begin{aligned} S = -\frac{\theta}{8\pi g^2}\int d\tau d\sigma |z|^2 \mathrm{tr}\bigg(& 2\left(\frac{\partial_+ A_-}{z} - \frac{\partial_- A_+}{\bar{z}} - [A_+, A_-]\right)^2 \\ &+ 4\left(\frac{\partial_+\phi_i}{z} - [A_+, \phi_i]\right)^2 + [\phi_i, \phi_j][\phi_i, \phi_j] \\ &+ 2\bar{\psi}\Gamma^+\left(\frac{\partial_+\psi}{z} - [A_+, \psi]\right) \\ &+ 2\bar{\psi}\Gamma^-\left(\frac{\partial_-\psi}{\bar{z}} - [A_-, \psi]\right) + 2\bar{\psi}\Gamma_i[\phi_i, \psi]\bigg). \end{aligned} \tag{2.8}$$

After the field redefinition,

$$\begin{aligned} A_+ &\to \frac{1}{z}\sqrt{\frac{g^2}{\theta}}A_+, \qquad A_- \to \frac{1}{\bar{z}}\sqrt{\frac{g^2}{\theta}}A_-, \\ \psi_R &\to \frac{1}{\sqrt{z}}\psi_R, \qquad \psi_L \to \frac{1}{\sqrt{\bar{z}}}\psi_L, \end{aligned} \tag{2.9}$$

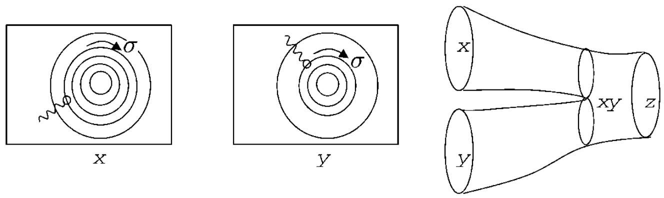

FIG. 3. Two closed strings x and y recombine at some time and finally we obtain a single closed string z.

and the rescaling,

$$\tau \to \sqrt{\frac{\theta}{g^2}}\tau, \qquad \sigma \to \sqrt{\frac{\theta}{g^2}}\sigma, \tag{2.10}$$

we obtain the effective action

$$S = -\frac{\theta}{8\pi g^2}\int_{-\infty}^{\infty} d\tau \int_0^{2\pi w} d\sigma \mathrm{tr}\bigg(\frac{g^2}{\theta|z|^2}F_{z\bar{z}}^2 + 4D_+\phi_i D_-\phi_i + \frac{\theta|z|^2}{g^2}[\phi_i, \phi_j][\phi_i, \phi_j] + 2\bar{\psi}(\Gamma^+ D_+ + \Gamma^- D_-)\psi + 2\sqrt{\frac{\theta}{g^2}}|z|\bar{\psi}\Gamma_i[\phi_i, \psi] + O(\theta)\bigg). \tag{2.11}$$

The parameter $\frac{g^2}{\theta}$ is identified with the string scale as $\alpha' \equiv \frac{g^2}{\theta}$ in [2]. ϕ_i has the target space dimension of l_s. Since the recombination is the local interaction, the length of the string does not affect the interaction. Thus, for simplicity, we can consider the two closed strings with the equal length. We parametrize the integral region of σ as $[0, 2\pi w]$.

This action is valid when two strings coincide in the target space. If two strings intersect at a point which is indeed the situation we consider in this paper, then the off-diagonal elements of this action are relevant only near the intersection point. They are meaningful only two strings are very close to each other, since otherwise these modes become massive. Thus, we can describe off-diagonal modes as local fields $A_{\mu}^{12}(\tau, \sigma)$ which are valid near the intersection point with the small intersection angle.

The definitions of light-cone coordinates $\partial_{\pm}$ and $A_{\pm}$ are summarized in Appendix A 1. In the free string limit, we obtain light-cone Green-Schwarz superstring action which consists of marginal terms. In that case, every oscillator mode decomposes into a left mover and a right mover.

III. RECOMBINATION

A. Fluctuation analysis and the recombination

In this section, by using the multiple closed superstring effective action (2.11) obtained in the previous section, we analyze the closed string interactions. In the IR limit, diagonal components are relevant degrees of freedom. Thus, the theory becomes a free theory in this limit. The leading interaction in the perturbative string theory is the interaction between two closed strings, which is realized as the recombination. The recombination can be investigated by using the Yang-Mills theory in [11] which is introduced as the low energy effective action of D strings.

Let us start from the two-dimensional effective action (2.11). The bosonic part is written as

$$S = -\frac{\theta}{8\pi g^2}\int_{-\infty}^{\infty} d\tau \int_0^{2\pi w} d\sigma \mathrm{tr}\bigg(\frac{g^2}{\theta|z|^2}F_{z\bar{z}}^2 + 4D_+\phi_i D_-\phi_i + \frac{\theta|z|^2}{g^2}[\phi_i, \phi_j][\phi_i, \phi_j]\bigg). \tag{3.1}$$

In [2], we identify the overall coefficient $\frac{\theta}{8\pi g^2}$ with $\frac{1}{8\pi\alpha'}$ in the free string limit. Off-diagonal fields are regarded as local fields since we focus on the region where two strings are close to each other.

The string coupling is very weak in the IR region. On the analogy of matrix string theory [3], the string coupling will behave $g_s \propto \frac{1}{|z|}$. We can interpret this relation as representing the equivalence between the IR limit and the weak coupling limit.

The supergravity solution of fundamental strings which is dual to our effective theory is described by [4]

$$ds^2 = \frac{U^6}{g_{\mathrm{YM}}^4 2^7\pi^4 N}dx^2 + \frac{1}{2\pi g_{\mathrm{YM}}^2}dU^2 + \frac{U^2}{2\pi g_{\mathrm{YM}}^2}d\Omega^2,$$
$$e^{\phi} = \left(\frac{g_{\mathrm{YM}}^6 2^8\pi^5 N}{U^6}\right)^{-1/2}. \tag{3.2}$$

We can read the scaling behavior of x as $x \sim 1/U^3$ from the metric. In the IR limit, the string coupling vanishes $e^{\phi} \sim U^3 \sim 1/x \to 0$, which is consistent with our picture.

It seems that this kind of running coupling behavior makes it difficult to treat the interaction. However, the recombination happens at a definite scale. For simplicity, we fix the scale of z as $|z| = z_r \equiv 1$ and treat this process in the real time.[1] We map the coordinates from $z = e^{\tau + i\sigma}$ into $z = e^{i(t+\sigma)}$ by the analytic continuation $\tau \to it$. Then, the complex plane is mapped into the cylinder with the radius $|z| = 1$. The bosonic part of the action becomes

$$S = -\frac{\theta}{8\pi g^2}\int_{-\infty}^{\infty} dt \int_0^{2\pi w} d\sigma \mathrm{tr}\bigg(\frac{g^2}{\theta}F_{z\bar{z}}^2 + 4D_+\phi_i D_-\phi_i + \frac{\theta}{g^2}[\phi_i, \phi_j][\phi_i, \phi_j]\bigg). \tag{3.4}$$

This action is the two-dimensional SU(2) Yang-Mills theory in the Lorentzian metric. Since we are considering the

[1]For a generic value of z_r, the coupling constant is changed as $g_s^2 \to g_s^2/z_r^2$. By taking the following rescaling,

$$q \to \frac{q}{z_r^2}, \qquad \varphi \to \frac{\varphi}{z_r}, \qquad \sigma \to \sigma z_r, \tag{3.3}$$

we can absorb this factor. It is consistent with our claim (3.26) $q \sim g_s^2$.

large winding number w, the world-sheet length of the string is very large. Thus, the solution of a single closed string might be represented locally as

$$\phi_2 = \sqrt{\frac{g^2}{\theta}}\sigma, \qquad A_\pm = \psi = 0, \qquad \phi_i = 0 \qquad (i = 3, \ldots, 9), \tag{3.5}$$

in $U(1)$ gauge theory.

The solution which represents the intersecting strings is written as

$$(\phi_2)_{\text{b.g.}} = \begin{pmatrix} q\sqrt{\frac{g^2}{\theta}}\sigma & 0 \\ 0 & -q\sqrt{\frac{g^2}{\theta}}\sigma \end{pmatrix} = q\sqrt{\frac{g^2}{\theta}}\sigma\sigma^3, \qquad A_\pm = \psi = 0, \qquad \phi_i = 0 \qquad (i = 3, \ldots, 9). \tag{3.6}$$

Two strings intersect at $\sigma = 0$. Each string is connected with each other at a point far from the origin $\sigma = 0$, which cannot be seen in this local solution near the intersection point. Thus, if the recombination occurs at this point, we obtain the single closed string. q is a parameter which is related to the intersection angle ϕ as

$$q \equiv \tan\frac{\phi}{2}. \tag{3.7}$$

q is the dimensionful quantity in the world-sheet sense and, as we will see later, we relate it to the string coupling constant g_s. If we take $q \ll 1$, two strings are very close to each other in the target space, especially around the intersection point. Thus, our effective theory is valid.

We consider the fluctuations around this background. We turn on the fluctuations $a_\pm$ and φ,

$$\phi_2 = q\sqrt{\frac{g^2}{\theta}}\sigma\sigma^3 + \sqrt{\frac{g^2}{\theta}}\varphi\sigma^1, \qquad A_+ = a_+\sigma^2, \qquad A_- = a_-\sigma^2, \tag{3.8}$$

since other fluctuations decouple from these fields at the quadratic level. σ^i ($i = 1, 2, 3$) are the Pauli matrices.

The quadratic Lagrangian is obtained as

$$L = \frac{g^2}{\theta}[-(\partial_+ a_- - \partial_- a_+)^2 - 2(\partial_+\varphi - iqa_+\sigma)(\partial_-\varphi - iqa_-\sigma) - \sqrt{2}qa_-\varphi + \sqrt{2}qa_+\varphi]. \tag{3.9}$$

Note that these fluctuation terms do not come from $[\phi_i, \phi_j][\phi_i, \phi_j]$ terms. By choosing the gauge condition

$$a_+ = -a_- \equiv \frac{a}{\sqrt{2}}, \tag{3.10}$$

the quadratic Lagrangian is written by the parameter τ and σ as

$$L = \frac{g^2}{\theta}[(\partial_t a)^2 + (\partial_t\varphi)^2 - ((\partial_\sigma\varphi)^2 + 2q\sigma a\partial_\sigma\varphi + (qa\sigma)^2 - 2qa\varphi)]. \tag{3.11}$$

In order to solve the equation of motion for the fluctuations, we expand the fluctuations by the mass eigenfunctions

$$a(t, \sigma) = \sum_{n\geq 0}\tilde{a}(\sigma)C_n(t), \qquad \varphi(t, \sigma) = \sum_{n\geq 0}\tilde{\varphi}(\sigma)C_n(t), \tag{3.12}$$

where $C_n(t)$ satisfy the equations

$$(\partial_t^2 + m_n^2)C_n(t) = 0. \tag{3.13}$$

The differential equations are solved in Appendix B 1. For the lowest mode $n = 0$, the eigenfunctions are calculated as

$$C_0(t) = C_0 \cdot \exp(\sqrt{q}t), \qquad \tilde{a}_0(\sigma) = \tilde{\varphi}_0(\sigma) = \exp\left(-\frac{q}{2}\sigma^2\right), \tag{3.14}$$

with the eigenvalue

$$m_0^2 = -q. \tag{3.15}$$

These eigenfunctions are localized near the intersection point $\sigma \sim 0$. This mode is tachyonic and it causes the recombination [11].

B. The recombination probability

Since the transition probability from two strings into the single string depends on the string coupling constant g_s, we can identify g_s by our investigation. We estimate the recombination probability per unit time. Since we put two strings with the relative center of mass velocity $v = 0$ as the initial condition, the recombination always occurs if we wait long enough.

Before calculating the probability, we confirm that the lowest mode C_0 (from now on, we denote it as C) indeed causes the recombination. During and after the recombination, we investigate the time evolution of the tachyonic mode. Small fluctuations trigger the recombination and, if we wait long enough, the recombination is over.

The recombination can be seen by the diagonalization of the background with the off-diagonal fluctuations

$$\phi_2(t, \sigma) = \frac{1}{2}\sqrt{\frac{g^2}{\theta}}\begin{pmatrix} q\sigma & C(t)\tilde{\varphi}_0(\sigma) \\ C(t)\tilde{\varphi}_0(\sigma) & -q\sigma \end{pmatrix} \xrightarrow{\text{diag}} \frac{1}{2}\sqrt{\frac{g^2}{\theta}}\begin{pmatrix} \sqrt{(q\sigma)^2 + C^2(t)e^{-q\sigma^2}} & 0 \\ 0 & -\sqrt{(q\sigma)^2 + C^2(t)e^{-q\sigma^2}} \end{pmatrix}. \tag{3.16}$$

Note that although this phenomenon locally represents the recombination of 2 strings → 2 strings, the final state is connected globally. Thus, this recombination represents the transition from two closed strings into the single closed string. We need to clarify the validity of the approximation since this geometrical picture comes from the fluctuation analysis. Our analysis is valid if the terms quadratic in the fluctuations are much smaller than the terms in the background. This condition is

$$(q\sigma)^2 \gg C^2 e^{-q\sigma^2}. \tag{3.17}$$

Since $q\sigma^2 \sim O(1)$ as can be seen in (3.19), we obtain

$$q \gg C^2. \tag{3.18}$$

Our analysis is valid under this condition.

The action quadratic in the C mode is obtained as

$$S = \frac{1}{8\pi}\int_{-\infty}^{\infty} dt \int_{-\pi w}^{\pi w} d\sigma((\partial_t C(t))^2 + qC^2(t)) \times \exp\left(-\frac{q}{2}\sigma^2\right) \sim \frac{1}{8\pi}\sqrt{\frac{\pi}{2q}}\int_{-\infty}^{\infty} dt((\partial_t C(t))^2 + qC^2(t)). \tag{3.19}$$

After integrating the σ direction, this action can be regarded as a quantum mechanics of a particle moving in the inverse harmonic oscillator [12]. As seen in (3.16), $C(t)$ is related to the separation of recombined strings. If C takes a large value, then we can regard it as the signal for recombination. Once C obtains the large value, the recombination is over. The separation of strings grows and finally we obtain a single closed string. In the inverse harmonic oscillator potential, the wave functions of the particle will keep spreading to the larger $|C|$. Thus, the recombination probability approaches 1 at $t \to \infty$.

The parameters in (3.19) can be interpreted in terms of the quantum mechanics of the particle moving in the inverse harmonic potential as

$$m \equiv \frac{1}{8\pi}\sqrt{\frac{\pi}{2q}}, \qquad \omega \equiv \sqrt{q}, \tag{3.20}$$

where m is the mass of the particle and ω is the frequency.

The Schrödinger equation is given by

$$i\frac{\partial\psi}{\partial t} = -\frac{1}{4m}\frac{\partial^2\psi}{\partial C^2} - m\omega^2 C^2\psi. \tag{3.21}$$

As an initial condition at $t = 0$, we consider the wave function to be a Gaussian which is labeled by the parameter ϕ. The derivation of the solution of the equation (3.21) is discussed in Appendix B 2. For large t, the wave function behaves

$$\psi(C, t) \sim (2/\pi)^{1/4} b^{-1/2}\exp\left(-\frac{1}{2}(\omega t + i\phi)\right) \times \exp\left(-e^{-2\omega t}\frac{C^2}{b^2} + im\omega C^2\right). \tag{3.22}$$

As discussed previously, once recombination happens, we obtain the single closed string as a final state. Since the fluctuation analysis is valid in the region $\sqrt{q} \gg C$, the geometric picture is also reliable in this region. We judge that the recombination has happened if the value of $C(t)$ grows beyond $\sqrt{q} = \omega$. Thus, the recombination probability at a time t is estimated as

$$P(t) = 2\int_{\omega}^{\infty} dC|\psi(C, t)|^2 = 1 - \mathrm{Erf}(\sqrt{2}b^{-1}e^{-t\omega}\omega) = 1 - \mathrm{Erf}(\sqrt{4m\omega^3 \sin 2\phi}\, e^{-t\omega}). \tag{3.23}$$

One can confirm that at $t \to \infty$, $P \to 1$. The recombination probability per unit time is calculated as

$$\frac{dP}{dt} = \frac{2}{\sqrt{\pi}}\sqrt{4m\omega^5 \sin 2\phi}\exp(-4m\omega^3 \sin 2\phi e^{-2t\omega} - t\omega) = \frac{2^{1/4}}{\pi^{3/4}} q\sqrt{\sin 2\phi}\exp\left(-\frac{q}{2\sqrt{2\pi}}\sin 2\phi e^{-2\sqrt{q}t} - \sqrt{q}t\right). \tag{3.24}$$

In the small q limit, the probability is proportional to

$$\frac{dP}{dt} \propto q. \tag{3.25}$$

In the perturbative string calculation, this probability is proportional to g_s^2. Thus, we identify

$$q \sim g_s^2. \tag{3.26}$$

The higher order corrections come from the expansion of the exponential function. By expanding this function at a time $t \sim O(\frac{1}{\sqrt{q}})$, and in the small q limit, we obtain

$$\sum_{n=1}^{\infty} P_n q^n, \tag{3.27}$$

which is consistent with the higher order corrections of the perturbative string if we identify q with g_s^2 [13,14].

C. The recombination in matrix string theory

In this section, we estimate the recombination probability of intersecting strings from matrix string theory. The action of matrix string theory is written by the two-dimensional Yang-Mills theory,

$$S = \frac{1}{l_s^2}\int_{-\infty}^{\infty} d\tau \int_0^{2\pi w} d\sigma \mathrm{tr}\Big((l_s^2 g_s^2)F_{z\bar{z}}^2 + 2(D^+\phi_i D_+\phi_i + D^-\phi_i D_-\phi_i) + \frac{1}{l_s^2 g_s^2}[\phi_i, \phi_j][\phi_i, \phi_j] + 2\bar{\psi}(\Gamma^+ D_+ + \Gamma^- D_-)\psi + \frac{2}{g_s l_s}\bar{\psi}\Gamma_i[\phi_i, \psi]\Big), \tag{3.28}$$

where we consider long intersecting strings with the large winding number w. By the identification $\frac{\theta}{g^2} \equiv \frac{1}{l_s^2}$, the action (3.28) is very close to the action (3.4). The different point is the g_s dependence which appears explicitly in (3.28). Thus, we perform the fluctuation analysis for the action (3.28) and investigate the g_s dependence. Since the action is the same apart from the g_s dependence, if we consider the same solution, we obtain the same tachyon mode as in Sec. III A.

By regarding the tachyon effective action as the quantum mechanics of the particle moving in the inverse harmonic oscillator, the parameters g_s and q appear in the mass and frequency of the particle as

$$m \equiv \frac{g_s^2}{8\pi}\sqrt{\frac{\pi}{2q}}, \qquad \omega \equiv \sqrt{q}. \tag{3.29}$$

The only different point with respect to the analysis in Sec. III A is the g_s dependence. The recombination probability at a time t is estimated from the previous calculation (3.24) as

$$P(t) = 1 - \mathrm{Erf}\left(\sqrt{4g_s^2\sqrt{\frac{\pi}{2}}q\sin 2\phi e^{-\sqrt{q}t}}\right). \tag{3.30}$$

Thus, the recombination probability per unit time is

$$\frac{dP}{dt} = \frac{2^{1/4}}{\pi^{3/4}} g_s q\sqrt{\sin 2\phi} \times \exp\left(-\frac{1}{2\sqrt{2\pi}} g_s^2 q \sin 2\phi e^{-2t\sqrt{q}} - \sqrt{q}t\right). \tag{3.31}$$

By the rescaling

$$t \longrightarrow g_s t, \tag{3.32}$$

we obtain

$$\frac{dP}{dt} = \frac{2^{1/4}}{\pi^{3/4}} g_s^2 q\sqrt{\sin 2\phi} \times \exp\left(-\frac{g_s^2 q}{2\sqrt{2\pi}} \sin 2\phi e^{-2g_s t\sqrt{q}} - \sqrt{q}g_s t\right). \tag{3.33}$$

This result is equivalent with the previous result (3.24) if we replace $q \leftrightarrow qg_s^2$. Thus, the identification we have derived in Sec. III B is consistent with matrix string theory.

At a large time $t \sim O(\frac{1}{g_s\sqrt{q}})$ in the small q limit, the leading contribution is proportional to $g_s^2 q$. The higher order corrections which come from the expansion of the exponential part are

$$(\text{leading contribution}) \times \sum_{n=1}^{\infty} P_n g_s^{2n}, \tag{3.34}$$

which is consistent with the perturbative string theory.

In [15], the classical BPS solutions that interpolate between the initial and the final string configurations are constructed in matrix string theory. They interpret the amplitude of matrix string theory as the transition amplitude between initial and final configurations, and show that the leading contribution is proportional to $g_s^{-\chi}$, where χ is the Euler characteristic of the interpolating Riemann surface. Thus, they have reproduced the perturbative string amplitude from matrix string theory, which is consistent with our result.

IV. CONCLUSION

We have identified the string coupling g_s in the IIB matrix model. We have constructed the classical solution of the strings in the action which is obtained from the IIB matrix model. Starting from the configuration with the intersection angle $\phi = 2\tan^{-1} q$ and no transverse distance and relative velocity, the recombination happens. This is triggered by the tachyonic fluctuations around the classical solution. After the recombination, we obtain the single closed string. We have estimated the probability of the recombination of two strings per unit time in this situation. The recombination probability is also calculated by the perturbative string theory. The leading contribution is proportional to g_s^2. The higher order corrections are seen as $\sum_{n=2}^{\infty} P_n g_s^{2n}$. Comparing our results with these behaviors, we have identified the string coupling $q \sim g_s^2$.

We have also estimated the recombination probability per unit time in the matrix string theory. The result shows the consistent behavior with that of the perturbative string theory. In the matrix string theory, the Yang-Mills coupling has the dimension $[\text{length}]^{-1}$ in the world sheet and it is inversely proportional to the string coupling g_s. Thus, g_s has the dimension [length] in the world sheet. Since q has the world-sheet dimension $[\text{length}]^{-2}$, the dimensionless parameter is qg_s^2. Since the probability is the dimensionless quantity, q should appear with g_s^2 which is consistent with the result obtained from IIB matrix model.

The parameter q represents the ratio between the target space coordinate ϕ_i and the world-sheet coordinate σ. Another parameter which has this kind of property is the velocity v. Although the recombination probability depends on the relative velocity v, we have put $v = 0$ in this paper. It might be interesting to calculate the probability of the recombination of intersecting strings with nonzero relative velocity.

ACKNOWLEDGMENTS

This work is supported in part by the Grant-in-Aid for Scientific Research from the Ministry of Education, Science and Culture of Japan.

APPENDIX A: NOTATION

1. Light-cone coordinates

$\partial_\pm$ are the derivatives with respect to

$$y_\pm \equiv \frac{1}{\sqrt{2}}(\tau \pm i\sigma), \tag{A1}$$

and they are related to ∂_0 and ∂_1 as

$$\partial_0 = \frac{1}{\sqrt{2}}\left(\frac{\partial_+}{z} + \frac{\partial_-}{\bar{z}}\right), \qquad \partial_1 = \frac{i}{\sqrt{2}}\left(\frac{\partial_+}{z} - \frac{\partial_-}{\bar{z}}\right). \tag{A2}$$

$A_\pm$ are related to A_0 and A_1 as

$$A_0 = \frac{1}{\sqrt{2}}(A_+ + A_-), \qquad A_1 = \frac{i}{\sqrt{2}}(A_+ - A_-). \tag{A3}$$

The definition of $F_{z\bar{z}}$ in (2.11) is

$$\begin{aligned} F_{z\bar{z}} &\equiv \bar{z}\partial_+(\bar{z}^{-1}A_-) - z\partial_-(z^{-1}A_+) - [A_+, A_-] \\ &= \partial_+A_- - \partial_-A_+ - [A_+, A_-]. \end{aligned} \tag{A4}$$

2. Pauli matrices

The Pauli matrices in (3.8) satisfy the following relation:

$$[\sigma^i, \sigma^j] = i\epsilon^{ijk}\sigma^k, \qquad \mathrm{tr}(\sigma^i)^2 = \tfrac{1}{2}. \tag{A5}$$

APPENDIX B: CALCULATION

1. The equation of motion for (3.11)

The derivation of the eigenfunctions (3.14) is summarized in this Appendix.

By using the relation

$$\begin{aligned} \partial_+ + \partial_- = -\sqrt{2}i\partial_t, \qquad 2\partial_+\partial_- = -\partial_t^2 + \partial_\sigma^2, \\ \partial_+ - \partial_- = -\sqrt{2}i\partial_\sigma, \end{aligned} \tag{B1}$$

and imposing the gauge fixing condition $a_+ = -a_- \equiv \frac{a}{\sqrt{2}}$, we obtain the Lagrangian (3.11).

The equation of motion for the fluctuation Lagrangian (3.11) is

$$\hat{O}\begin{pmatrix} a(t,\sigma) \\ \varphi(t,\sigma) \end{pmatrix} = 0, \tag{B2}$$

where

$$\hat{O} = \begin{pmatrix} -(q\sigma)^2 - \partial_t^2 & -q\sigma\partial_\sigma + q \\ q\sigma\partial_\sigma + 2q & -\partial_t^2 + \partial_\sigma^2 \end{pmatrix}. \tag{B3}$$

For the mass eigenvalues m_n^2 which satisfy the free field equation

$$(\partial_t^2 + m_n^2)C_n(\tau) = 0, \tag{B4}$$

the equation of motion is given by

$$\begin{pmatrix} -(q\sigma)^2 + m_n^2 & -q\sigma\partial_\sigma + q \\ q\sigma\partial_\sigma + 2q & \partial_\sigma^2 + m_n^2 \end{pmatrix}\begin{pmatrix} \tilde{a}(\sigma) \\ \tilde{\varphi}(\sigma) \end{pmatrix} = 0. \tag{B5}$$

This differential equation is solved with the mass eigenvalue

$$m_n^2 = (2n-1)q. \tag{B6}$$

For the lowest mode $n = 0$, the eigenfunctions are calculated as

$$\begin{aligned} C_0(\tau) &= C_0 \cdot \exp(\sqrt{q}t), \\ \tilde{a}_0(\sigma) &= \tilde{\varphi}_0(\sigma) = \exp\left(-\frac{q}{2}\sigma^2\right). \end{aligned} \tag{B7}$$

For general n, the eigenfunctions are

$$\begin{aligned} \tilde{a}_n(\sigma) &= -e^{-q\sigma^2/2}\sum_{j=0,2,\cdots}^{n}(-1)^{j/2}\frac{4^{j/2}}{j!} \\ &\quad \times \frac{n(n-2)\cdots(n-j+2)}{2n-1}(j-1)\left(\sigma\sqrt{\frac{q}{2}}\right)^j, \\ \tilde{\varphi}_n(\sigma) &= e^{-q\sigma^2/2}\sum_{j=0,2,\cdots}^{n}(-1)^{j/2}\frac{4^{j/2}}{j!} \\ &\quad \times \frac{n(n-2)\cdots(n-j+2)}{2n-1}(2n-j-1)\left(\sigma\sqrt{\frac{q}{2}}\right)^j \end{aligned} \tag{B8}$$

for $n = 0, 2, \ldots,$ and

$$\begin{aligned} \tilde{a}_n(\sigma) &= -e^{-q\sigma^2/2}\sum_{j=1,3,\ldots}^{n}(-1)^{(j-1)/2}\frac{4^{(j-1)/2}}{j!}\left(\frac{j-1}{2}\right) \\ &\quad \times (n-3)\cdots(n-j+2)\left(\sigma\sqrt{\frac{q}{2}}\right)^j, \\ \tilde{\varphi}_n(\sigma) &= e^{-q\sigma^2/2}\sum_{j=1,3,\ldots}^{n}(-1)^{(j-1)/2}\frac{4^{(j-1)/2}}{j!}\left(n-\frac{j+1}{2}\right) \\ &\quad \times (n-3)\cdots(n-j+2)\left(\sigma\sqrt{\frac{q}{2}}\right)^j \end{aligned} \tag{B9}$$

for $n = 3, 5, \ldots.$

2. Solving the Schrödinger equation (3.21)

We summarize the derivation of the solution (3.22) in this Appendix.

We describe the momentum conjugate to the C as Π:

$$\Pi = \frac{\delta L}{\delta \dot{C}} = 2m\dot{C}. \tag{B10}$$

The Hamiltonian is given by

$$H = \Pi\dot{C} - L = \frac{1}{4m}\Pi_0^2 - m\omega^2 C_0^2. \tag{B11}$$

The Schrödinger equation (3.21) is solved by the wave function of the form

$$\psi(C,t) = A(t)\exp(-B(t)C^2), \tag{B12}$$

if A and B satisfy the equation

$$i\dot{A} = \frac{1}{2m}AB, \qquad \frac{i}{m}\dot{B} = \omega^2 + \frac{1}{m^2}B^2. \tag{B13}$$

These are solved in [16] as

$$A = (2\pi)^{-1/4}(b\cos(\phi - i\omega t))^{-1/2},$$
$$B = m\omega\tan(\phi - i\omega t) = m\omega\frac{\sin 2\phi - i\sinh 2\omega t}{\cos 2\phi + \cosh 2\omega t}, \tag{B14}$$

where

$$a^2 \equiv \frac{1}{2m\omega}, \qquad b = a(\sin 2\phi)^{-1/2}. \tag{B15}$$

We put the initial condition of the wave function as

$$\psi(C, t = 0) = (2\pi)^{-1/4}(b\cos\phi)^{-1/2}\exp(-m\omega\tan\phi C^2). \tag{B16}$$

The parameter ϕ controls the initial condition.

[1] N. Ishibashi, H. Kawai, Y. Kitazawa, and A. Tsuchiya, Nucl. Phys. **B498**, 467 (1997).
[2] Y. Kitazawa and S. Nagaoka, Phys. Rev. D **77**, 026009 (2008).
[3] R. Dijkgraaf, E. Verlinde, and H. Verlinde, Nucl. Phys. **B500**, 43 (1997).
[4] N. Itzhaki, J. M. Maldacena, J. Sonnenschein, and S. Yankielowicz, Phys. Rev. D **58**, 046004 (1998).
[5] Y. Kitazawa and S. Nagaoka, Phys. Rev. D **77**, 126016 (2008).
[6] H. Kawai and M. Sato, Phys. Lett. B **659**, 712 (2008).
[7] M. B. Green, J. H. Schwarz, and L. Brink, Nucl. Phys. **B219**, 437 (1983).
[8] A. Connes, M. Douglas, and A. Schwarz, J. High Energy Phys. 02 (1998) 003.
[9] H. Aoki, N. Ishibashi, S. Iso, H. Kawai, Y. Kitazawa, and T. Tada, Nucl. Phys. **B565**, 176 (2000).
[10] M. Li, Nucl. Phys. **B499**, 149 (1997).
[11] K. Hashimoto and S. Nagaoka, J. High Energy Phys. 06 (2003) 034.
[12] A. Hanany and K. Hashimoto, J. High Energy Phys. 06 (2005) 021.
[13] J. Polchinski, Phys. Lett. B **209**, 252 (1988).
[14] M. G. Jackson, N. T. Jones, and J. Polchinski, J. High Energy Phys. 10 (2005) 013.
[15] G. Bonelli, L. Bonora, and F. Nesti, Nucl. Phys. **B538**, 100 (1999).
[16] A. H. Guth and S.-Y. Pi, Phys. Rev. D **32**, 1899 (1985).