

Nonextensive statistics in stringy space-time foam models and entangled meson states

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The possibility of generation of nonextensive statistics, in the sense of Tsallis, due to space-time foam is discussed within the context of a particular kind of foam in string/brane theory, the D-particle foam model. The latter involves pointlike brane defects (D-particles), which provide the topologically nontrivial foamy structures of space-time. A stochastic Langevin equation for the velocity recoil of D-particles can be derived from the pinched approximation for a sum over genera in the calculation of the partition function of a bosonic string in the presence of heavy D-particles. The string coupling in standard perturbation theory is related to the exponential of the expectation of the dilaton. Inclusion of fluctuations of the dilaton itself and uncertainties in the string background will then necessitate fluctuations in g_s . The fluctuation in the string coupling in the sum over genera typically leads to a generic structure of the Langevin equation where the coefficient of the noise term fluctuates owing to dependence on the string coupling g_s . The positivity of g_s leads naturally to a stochastic modeling of its distribution with a χ distribution. This then rigorously implies a Tsallis-type nonextensive or, more generally, a superstatistics distribution for the recoil velocity of D-particles. As a concrete and physically interesting application, we provide a rigorous estimate of an ω -like effect, pertinent to CPT violating modifications of the Einstein-Podolsky-Rosen correlators in entangled states of neutral kaons. In the case of D-particle foam fluctuations, which respect the Lorentz symmetry of the vacuum on average, we find that the ω effect may be within the range of sensitivity of future meson factories.

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I. INTRODUCTION

The standard model of particle physics is considered to be successful even though it has many undetermined parameters which need to be fitted to data. One reason for the great interest in physics beyond the standard model (SM) is the absence, within its framework, of quantum gravity (QG). A full picture of QG still remains elusive. One interesting approach to it, based on string theory (ST) [1,2], has the advantage of also unifying QG with the nuclear and electromagnetic forces. Moreover flavor mixing phenomena are not understood in the sense that phenomenological mixing matrices are not derived from a fundamental point of view. There may be some relationship between these two deficiencies, and QG may play a role in that. For instance, (a small) part of the observed *mass differences* between neutrinos might be quantum gravitational in origin as argued recently [3] within the context of a space-time foam model in string theory with space-time defects [4] in the form of pointlike D(irichlet)-branes (D-particles) [2,5,6]. The interaction of the defects with string matter in such a model induces flavor mixing and nontrivial contributions to the cosmological constant (or better vacuum energy) proportional to the mixing angle and the mass differences among the neutrino states.

Because of its foundations based on local relativistic unitary quantum field theory, SM has CPT symmetry [7]. Hence the detection of possible violations of CPT clearly necessitates physics beyond the standard model. CPT violation might be an important feature of QG, which may be

exhibited by some stringy models of space-time foam with defects, of the type discussed in [4]. Recently, an interesting signature of CPT violation due to decoherence of matter as a result of QG effects in stochastic space-time foam models, has been suggested in Ref. [8], implying new physics beyond SM in the entanglement properties of neutral meson pairs. The proposed signature is the so-called omega effect where modifications to the standard Einstein-Podolsky-Rosen (EPR) correlations appear for entangled states of neutral flavored mesons in meson factories [8,9].

Neutral mesons, such as the K mesons, have in the past been pivotal in the study of discrete symmetries [10]. The decay of a (generic) meson (such as the ϕ meson produced in collisions of e^+ and e^- with quantum numbers $J^{PC} = 1^{--}$ [11]), leads to a pair state $|i\rangle$ of neutral mesons (M) which has the form of the entangled state

$$|i\rangle = \frac{1}{\sqrt{2}}(|\bar{M}_0(\vec{k})\rangle|M_0(-\vec{k})\rangle - |M_0(\vec{k})\rangle|\bar{M}_0(-\vec{k})\rangle). \quad (1.1)$$

This state has $CP = +$. If CPT is not well defined (actually *perturbatively*, i.e. although the concept of the anti-particle still exists, it is slightly modified), then M_0 and \bar{M}_0 may not be identified and the requirement of $CP = +$ can be relaxed [8,9]. Consequently the state of the meson pair can be parametrized to have the form

$$|i\rangle = (|\bar{M}_0(\vec{k})\rangle|M_0(-\vec{k})\rangle - |M_0(\vec{k})\rangle|\bar{M}_0(-\vec{k})\rangle) + \omega(|\bar{M}_0(\vec{k})\rangle|M_0(-\vec{k})\rangle + |M_0(\vec{k})\rangle|\bar{M}_0(-\vec{k})\rangle), \quad (1.2)$$

where $\omega = |\omega|e^{i\Omega}$ is a complex *CPT* violating (CPTV) parameter [8]. It turns out to be difficult to generate this ω term since reasonable attempts based on local effective Lagrangians with explicit *CPT* violating terms do not lead to it. Recent approaches have concentrated on properties of space-time foam and related decoherence, which by their nature go beyond the concept of local effective field theory. From extrapolations of black hole physics and quantum uncertainty relations to the smallest scales it seems unavoidable that space and time should fluctuate on a space scale l_P , the Planck length, and a time scale t_P , the Planck time. Such quantum fluctuations lead to a background state which will be generically called space-time foam [12]. The full understanding of quantum states of matter in such a medium foam requires a deeper understanding of QG. Consequently the approaches to it have ranged from the purely phenomenological to those based on a more fundamental theory such as strings. Even in the latter, phenomenological assumptions will have to be made since we do not have a developed string field theory and formulations have typically been based on fixed backgrounds. Indeed a plausible phenomenological approach based on thermal baths, at least in its natural and most straightforward form, cannot generate an ω term [13]. On the other hand a stochastic approach based and inspired by theoretical considerations of D-particle recoil from stringy matter in D-particle foam models [4] has predicted and quantified this omega effect [9]. However, the quantification was based on somewhat phenomenological and naive estimates. In this work we shall provide more rigorous estimates of this effect within D-particle foam models.

In the treatments so far of D-particle foam the stochastic modeling has concentrated on the randomness of the recoil-velocity vector of the D-particle defect during its topologically nontrivial interaction with the matter string state (i.e. the capture and subsequent reemission of the matter state). In this work we want to incorporate the effects of the (target space) quantum fluctuations of the D-particle which, within the bosonic string model, can be calculated using perturbation theory. As we shall discuss, the renormalization of a certain subleading divergence to all orders in string perturbation theory leads to fluctuations superposed on a drift velocity of the D-particle obtained after the initial recoil on interacting with stringy matter. Furthermore, at the phenomenological level uncertainty in the string vacua can lead to a fluctuating string coupling g_s which is related to vacuum expectation value of the dilaton field. Incorporation of such fluctuations lead to superstatistics [14] of the velocity recoil distribution, nonextensive statistics of Tsallis [15] being one possibility. The recoil fluctuations of the D-particle lead to effective stochastic backreaction on space-time that cannot be neglected. Hence the space-time metric will have induced stochastic contributions from stochasticity in the recoil [16].

In this work we shall concentrate our discussion on the bosonic string. Although admittedly, this is not relevant

phenomenologically, it is the only case where we manage to sum over world sheet genera and express our basic results on stochastic fluctuations of space-time and fuzziness of string coupling in a closed form. We shall make some comments on the robustness of our results and thus their extension to supersymmetric cases at the end of our article. This constitutes an active part of our research at present.

Within the bosonic σ -model framework of D-particle foam, we shall calculate the stochastic fluctuations of the space-time induced by D-particle recoil, and then we shall use it to estimate the resulting ω -effect in the initial state of entangled mesons in this concrete microscopic model. Contrary to previous naive estimates [9], based on dimensional analysis, in this particular example of (string inspired) foam, the effect is calculated rather rigorously, following conformal field theory methods in the world sheet of the string. In particular, for the magnitude of the ω -effect in the initial entangled meson states (1.2), we find the following result for the square of the amplitude of the parameter ω :

$$|\omega|^2 \sim \frac{m_1^2 + m_2^2}{(m_1 - m_2)^2} \frac{k^2}{M_P^2}, \quad (1.3)$$

for nonrelativistic entangled states of mesons, with (near degenerate) masses m_i , $i = 1, 2$ and M_P the quantum-gravity scale, assumed to be the four-dimensional Planck scale, 10^{19} GeV. For comparison, we remind the reader that the naive phenomenological estimates of [9] lead to effects of order $|\omega|_{\text{naive}}^2 \sim \zeta^2 \frac{k^4}{M_P^2(m_1 - m_2)^2}$, with ζ a typical momentum transfer variable, during the interaction of the meson state with the D-particle, $\zeta \sim \frac{\Delta \vec{k}}{k}$, which in [9] has been assumed not very small. What the detailed string calculation in this paper shows, in which the stochastic space-time fluctuations are due to quantum fluctuations of recoil velocities about a zero (Lorentz invariant) average, is that the parameter ζ of [9] depends on details of the meson system, such as the mass and momenta of the mass eigenstates, and is effectively of order $\zeta^2 \sim (m_1^2 + m_2^2)/k^2$. For the case of neutral kaons in ϕ -factories, with $k \sim \mathcal{O}(1 \text{ GeV})$, this is already at the order of magnitude required for detectability in an upgrade in DAΦNE [9]. Hence these models of D-particle foam may be falsifiable in such future meson factories. Of course, the estimate assumes the neutral kaon as interacting with the space-time foam as an elementary entity, ignoring details of strongly interacting constituents in the kaon substructure. Taking into account such strong interaction effects might change the above estimates.

The structure of the article is as follows: in the next section we will discuss general features of the D-particle foam model, on which our work is based. In Sec. III we review the basic mathematical properties of the deformations of the bosonic sigma model, describing the interac-

tion of string matter with D-particle defects. In Sec. IV we discuss resummation of higher world sheet topologies and their implication on inducing stochastic fluctuations in the recoil velocity of the D-particle. The latter are expressed through a Langevin-type stochastic equation, which replaces the tree-level world sheet renormalization-group equation for the recoil velocity, interpreted as a sigma-model coupling. In Sec. V, we solve this Langevin equation but also discuss “fuzzy” aspects of the string coupling in a full quantum-gravity setting of this string model of foam. For D-particle foam we thus arrive, in the sense of Tsallis, at a superstatistics description. We discuss the application of our results to a precise estimation of decoherence-induced CPT-violating effects in entangled states of mesons (the ω effect) in Sec. VI, where we estimate the effect in this model of (D-particle) space-time and compare it to previous estimates. Our conclusions and outlook are presented in Sec. VII. Some technical aspects are reviewed in two appendices.

II. GENERIC FEATURES OF D-PARTICLE FOAM MODEL

In this section we shall briefly review the basic features of the D-particle foam model discussed in [4]. We will use some established results and constructs from string/brane theory [2,5], which we shall discuss briefly for the benefit of the nonexpert reader. In particular, zero-dimensional D-branes [6] occur (in bosonic and some supersymmetric string theories) and are also known as D-particles. Interactions in string theory are, as yet, not treated as systematically as in ordinary quantum field theory where a second quantized formalism is defined. The latter leads to the standard formulations by Schwinger and Feynman of perturbation series. When we consider stringy matter interacting with other matter or D-particles, the world lines traced out by point particles are replaced by two-dimensional world sheets. World sheets are the parameter space of the first quantized operators (fermionic or bosonic) representing strings. In this way the first quantized string is actually represented by a two-dimensional (world sheet) quantum field theory. An important consistency requirement of this first quantized string theory is conformal invariance which determines the space-time dimension and/or structure. This symmetry permits the representation of interactions through the construction of measures on inequivalent Riemann surfaces [1]. In and out states of stringy matter are represented by vertex insertions at the boundaries. The D-particles as solitonic states [5] in string theory do fluctuate themselves quantum mechanically; this is described by stringy excitations, corresponding to open strings with their ends attached to the D-particles and higher dimensional D-branes. In a first quantized (world sheet) language, such fluctuations are also described by Riemann surfaces of higher topology with appropriate Dirichlet boundary conditions (c.f. Fig. 1). The plethora

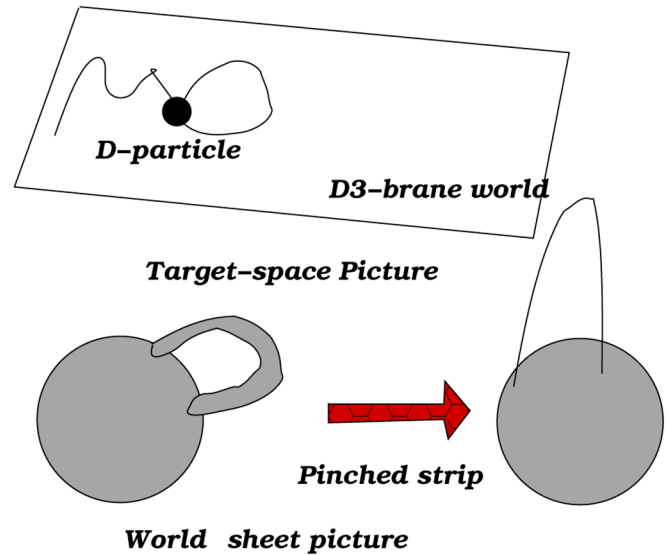


FIG. 1 (color online). *Upper picture:* A fluctuating D-particle is described by open strings attached to it. As a result of conservation of string fluxes [2,5,6] that accompany the D-branes, an isolated D-particle cannot occur, but it has to be connected to a D-brane world through flux strings. *Lower picture:* World sheet diagrams with annulus topologies, describing the fluctuations of D-particles as a result of the open string states ending on them. Conformal invariance implies that pinched surfaces, with infinitely long thin strips, have to be taken into account. In bosonic string theory, such surfaces can be resummed [21].

of Feynman diagrams in higher-order quantum field theory is replaced by a small set of world sheet diagrams classified by moduli which need to be summed or integrated over [17]. The model of space-time foam, used in understanding the omega effect, is based on D-particles populating a bulk geometry between parallel D-brane worlds. The model is termed D-foam [4] (c.f. Fig. 2), and our world is modeled as a three-brane moving in the bulk geometry; as a result, D-particles cross the brane world and appear for an observer on the brane as foamy structures which flash on and off.

Even at low energies E , such a foam may have observable consequences e.g. decoherence effects which may be of magnitude $O([\frac{E}{M_p}]^n)$ with $n = 1, 2$, where M_p is the Planck mass or change in the usual Lorentz invariant dispersion relations. The study of D-brane dynamics has been made possible by Polchinski’s realization [5] that such solitonic string backgrounds can be described in a conformally invariant way in terms of world sheets with boundaries [5]. On these boundaries Dirichlet boundary conditions for the collective target-space coordinates of the soliton are imposed [18]. When low-energy matter given by a closed string propagating in a $(d + 1)$ -dimensional space-time collides with a very massive D-particle (0-brane) embedded in this space-time, the D-particle recoils as a result [19] in a nonrelativistic manner. We shall con-

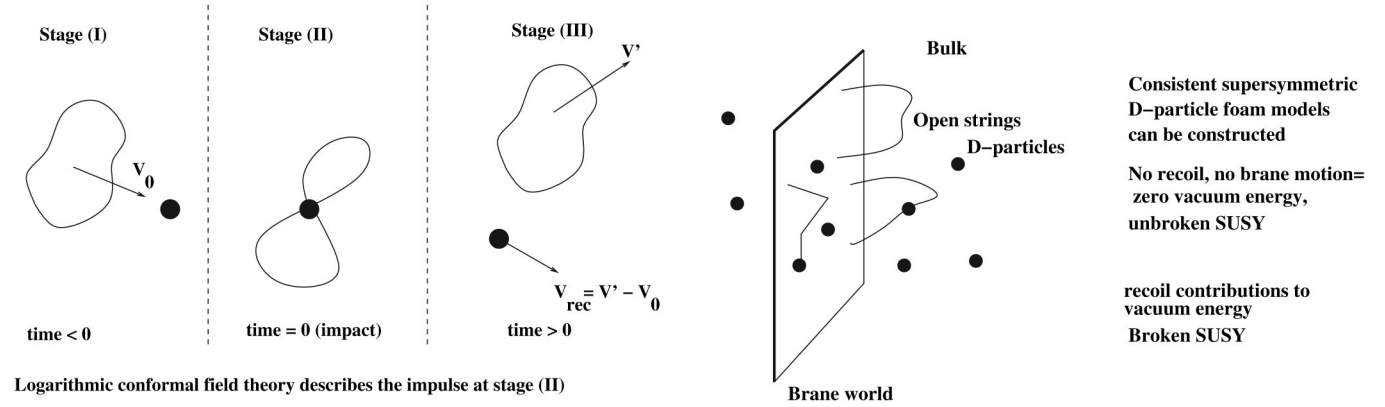


FIG. 2. Schematic representation of a D-foam. The figure also indicates the capture/recoil process of a string state by a D-particle defect for closed (left panel) and open (right panel) string states, in the presence of D-brane world. The presence of a D-brane is essential due to gauge flux conservation, since an isolated D-particle cannot exist. The intermediate composite state at $t = 0$, which has a lifetime within the stringy uncertainty time interval δt , of the order of the string length, and is described by world sheet logarithmic conformal field theory, is responsible for the distortion of the surrounding space-time during the scattering, and subsequently leads to induced metrics depending on both coordinates and momenta of the string state. This results on modified dispersion relations for the open string propagation in such a situation [4], leading to nontrivial “optics” for this space-time.

sider the simple case of bosonic stringy matter coupling to D-particles. Hence we can only discuss matters of principle and ignore issues of stability due to tachyons. However we should note that an open string model needs to incorporate for completeness, higher dimensional D-branes such as the D3-brane. This is due to the vectorial charge carried by the string owing to the Kalb-Ramond field. Higher dimensional D-branes (unlike D-particles) can carry the charge from the endpoints of open strings that are attached to them. For a closed bosonic string model the inclusion of such D-branes is not imperative (see Fig. 2) although D-particle fluctuations would generally require them. The details of the higher dimensional branes are not essential for our analysis however. The current state of phenomenological modeling of the interactions of D-particle foam with stringy matter will be briefly summarized now. Since there are no rigid bodies in general relativity the recoil fluctuations of the brane and the effective stochastic backreaction on space-time cannot be neglected. As we will discuss, D-particle recoil in the “tree approximation” i.e. in lowest order in the string coupling g_s , is required to cancel infrared singularities in a higher-order disc or Riemann sphere amplitude in open or closed string theory, respectively; the recoil induces a nontrivial space-time metric. For ε a positive infinitesimal, $\Theta_\varepsilon(t)$, the regularized step function is introduced, in terms of a contour integral

$$\Theta_\varepsilon(t) = \frac{1}{2\pi i} \int_{-\infty}^{\infty} \frac{d\omega}{\omega - i\varepsilon} e^{i\omega t}. \quad (2.1)$$

For closed strings colliding with a heavy (i.e. nonrelativistic) D-particle, the metric has the form [20]

$$g_{ij} = \delta_{ij}, \quad g_{00} = -1, \\ g_{0i} = \varepsilon(\varepsilon y_i + u_i t) \Theta_\varepsilon(t), \quad i = 1, \dots, d, \quad (2.2)$$

where the suffix 0 denotes temporal (Liouville) components; $u_i = (k_1 - k_2)_i$ is small; $k_1(k_2)$ is the momentum of the propagating closed-string state before (after) the recoil; y_i are the spatial collective coordinates of the D-particle; and ε^{-2} is identified with the target Minkowski time t for $t \gg 0$ after the collision. The latter requirement is consistent with ε being infinitesimal. For our purposes the Liouville and Minkowski times can be identified. Now for large t , to leading order,

$$g_{0i} \simeq \bar{u}_i \equiv \frac{u_i}{\varepsilon} \propto g_s \frac{\Delta p_i}{M_s}, \quad (2.3)$$

where Δp_i is the momentum transfer during a collision and M_s is the string mass scale; $g_s < 1$ is the string coupling, assumed weak; and the combination M_s/g_s is the D-particle mass, playing the role of the quantum gravity scale in this problem, i.e. the Planck mass. This formalism was used to establish a phenomenological model where the couplings u_i were taken to be stochastic and modeled by a Gaussian process. The latter assumption was not based on analysis of the underlying string theory. The Gaussian process represents a large universality class for stochastic processes and in this sense is an understandable assumption. Our purpose is to determine the justification of using Gaussian (or other) distributions for the velocity recoil in the context of D-particle foam.

III. MODULI AND VERTEX OPERATORS FOR D-PARTICLES: A COMPREHENSIVE REVIEW

In order to understand the dynamics of D-particle foam it is imperative to consider D-particle quantization and, in particular, D-particle recoil as a result of scattering off stringy matter. The issue of recoil is not fully understood

and it is not our purpose here to delve into the subtleties of recoil and operators describing recoil. We will rather proceed on the basis of a proposal which has had some success in the past [19]. The energy of a D-particle is independent of its position. Consequently in bosonic string theory there are 25 zero modes, the Dirichlet directions. Zero modes lead to infrared divergences in loops in a field theory setting where collective coordinates are used to isolate these infrared divergences. This is essentially due to the naivety of the perturbation series that is used and can be addressed using a coherent state formalism. The situation is similar but, in some ways, worse for string theory since the second quantized formalism for strings is more rudimentary. In string theory the use of first quantization requires a sum over Riemann sheets with different moduli parameters. The formal transition from one surface to another of lower genus (within string perturbation theory) has singularities associated with infrared divergences in the integration over moduli parameters. In the case of a disc D an incipient annulus Σ can be found by making two punctures and attaching a long thin strip (somewhat like the strap in a handbag and dubbed wormholes). In the limit of vanishing width, this wormhole is represented by a region in moduli space, integration over which leads to an infrared divergence. A similar argument would apply to a Riemann sphere where two punctures would be connected by a long thin tube. For the disc the divergence will be due to propagation of zero mode open string state while for the Riemann sphere it would be an analogous closed string state along the wormhole. Let us examine this divergence explicitly. Consider the correlation function $\langle V_1 V_2 V_3 \dots V_n \rangle_\Sigma$ for vertex operators [1] $\{V_i\}_{i=1, \dots, n}$, where Σ is the Riemann surface of an annulus; it can be deformed into a disc with a wormhole attached [18,21]. The set of vertex operators include necessarily any higher dimensional D-branes necessary to conserve string charge. However such vertex insertions clearly do not affect the infrared divergence caused by wormholes. The correlation function can be expressed as

$$\langle V_1 V_2 V_3 \dots V_n \rangle_\Sigma = \sum_a \int ds_1 \int ds_2 \int \frac{dq}{q} \times q^{h_a-1} \langle \phi_a(s_1) \phi_a(s_2) V_1 V_2 V_3 \dots V_n \rangle_D, \tag{3.1}$$

where s_1 and s_2 are the positions of the punctures on ∂D . The $\{\phi_a\}$ are a complete set of eigenstates of the Virasoro operator L_0 with conformal weights h_a [22] and q is a Teichmuller parameter associated with the added thin strip. Clearly there is a potential divergence associated with its disappearance $q \rightarrow 0$ and this corresponds to a long thin strip attached to the disc. For a static D-particle the string coordinates $\vec{X}_D = \{X^1, X^2, X^3, \dots, X^{25}\}$ have Dirichlet boundary conditions while X^0 has Neumann boundary conditions (in the static gauge)

$$\frac{\partial}{\partial \sigma} X^0(\tau, \sigma)|_{\sigma=0} = 0 = \frac{\partial}{\partial \sigma} X^0(\tau, \sigma)|_{\sigma=\pi}, \tag{3.2}$$

where (τ, σ) is a coordinisation of the world sheet. The associated translational zero mode is given by

$$\phi^i(X, \omega) = \frac{\sqrt{g_s}}{4} \partial_n X_D^i e^{i\omega X^0}, \tag{3.3}$$

where ∂_n denotes a derivative in the X^i Dirichlet direction, and is an element of the set $\{\phi_a\}$. The conformal weight $h_i = 1 + \alpha' \omega^2$. The relevant part of the integral in (3.1) is

$$\int_0^1 dq \int_{-\infty}^{\infty} d\omega q^{-1+\alpha' \omega^2} = \int_0^1 dq \frac{1}{q(-\log q)^{1/2}} \tag{3.4}$$

and is divergent because of the behavior of the integrand near $q = 0$. This can be regularized by putting a lower cutoff $q > \delta \rightarrow 0$ in the integral. The correlation function on Σ can be computed to be [18]

$$\langle V_1 V_2 V_3 \dots V_n \rangle_\Sigma = -\frac{g_s}{16T} \log \delta \langle \partial_n \vec{X}_D(s_1) \cdot \partial_n \vec{X}_D(s_2) \times V_1 V_2 V_3 \dots V_n \rangle_D, \tag{3.5}$$

where T is a cutoff for large (target) time. Division by it removes divergencies due to the integration over the world sheet zero modes of the target time. These should not be confused with divergencies associated with pinched world sheet surfaces, proportional to $\log \delta$, that we are interested in here. These latter divergences cause conventional conformal invariance to fail.

However, in one approach, it was argued sometime ago, that these divergences can be canceled if D-particles are allowed to recoil [19,21], as a result of momentum conservation during their scattering with string states. In fact, in our D-particle foam model [4], this is not a simple scattering process, as it involves capture and reemission of the string state by the D-particle defect. In simple terms, this process involves splitting of strings by the defect. In a world sheet (first quantization) framework, such processes are described by appropriate vertex operators, whose operator product expansion close on a (local) logarithmic algebra [19,21]. The translational zero modes are associated with infinitesimal translations in the Dirichlet directions. Given that D-particles do not have any internal or rotational degrees of freedom, these modes should give us valuable information concerning recoil. Moreover (for $\omega = 0$) there is a degeneracy in the conformal weights between ϕ^i , $\partial_\omega \phi^i$ and the identity operator and also the conformal blocks in the corresponding algebra have logarithmic terms.

To understand the formal structure of the world sheet deformation operators pertinent to the recoil/capture process, we first notice that the world sheet boundary operator \mathcal{V}_D describing the excitations of a moving heavy D0-brane is given in the tree approximation by

$$\mathcal{V}_D = \int_{\partial D} (y_i \partial_n X^i + u_i X^0 \partial_n X^i) \equiv \int_{\partial D} Y_i(X^0) \partial_n X^i, \quad (3.6)$$

where u_i and y_i are the velocity and position of the D-particle, respectively, and $Y_i(X^0) \equiv y_i + u_i X^0$. To describe the capture/recoil we need an operator which has nonzero matrix elements between different states of the D-particle and is turned on ‘‘abruptly’’ in target time. One way of doing this is to put [19] a $\Theta(X^0)$, the Heaviside function, in front of \mathcal{V}_D which models an impulse whereby the D-particle starts moving at $X^0 = 0$. Using Gauss’s theorem this impulsive \mathcal{V}_D , denoted by $\mathcal{V}_D^{\text{imp}}$, can be represented as

$$\begin{aligned} \mathcal{V}_D^{\text{imp}} &= \sum_{i=1}^{25} \int_D d^2z \partial_\alpha ([u_i X^0] \Theta(X^0) \partial^\alpha X^i) \\ &= \sum_{i=1}^{25} \int_{\partial D} d\tau u_i X^0 \Theta(X^0) \partial_n X^i. \end{aligned} \quad (3.7)$$

Since X^0 is an operator it will be necessary to define $\Theta(X^0)$ as an operator using the contour integral

$$\Theta_\varepsilon(X^0) = -\frac{i}{2\pi} \int_{-\infty}^{\infty} \frac{d\omega}{\omega - i\varepsilon} e^{i\omega X^0} \quad \text{with } \varepsilon \rightarrow 0+. \quad (3.8)$$

Hence we can consider

$$\begin{aligned} D_\varepsilon(X^0) &\equiv D(X^0; \varepsilon) = X^0 \Theta_\varepsilon(X^0) \\ &= -\int_{-\infty}^{\infty} \frac{d\omega}{(\omega - i\varepsilon)^2} e^{i\omega X^0}. \end{aligned} \quad (3.9)$$

The introduction of the feature of impulse in the operator breaks conventional conformal symmetry, but a modified logarithmic conformal algebra holds. A generic logarithmic algebra in terms of operators \mathcal{C} and \mathcal{D} and the stress tensor $T(z)$ (in complex tensor notation) satisfies the operator product expansion

$$T(z)\mathcal{C}(w, \bar{w}) \sim \frac{\Delta}{(z-w)^2} \mathcal{C}(w, \bar{w}) + \frac{\partial \mathcal{C}(w, \bar{w})}{(z-w)} + \dots, \quad (3.10)$$

$$\begin{aligned} T(z)\mathcal{D}(w, \bar{w}) &\sim \frac{\Delta}{(z-w)^2} \mathcal{D}(w, \bar{w}) + \frac{1}{(z-w)^2} \mathcal{C}(w) \\ &+ \frac{\partial \mathcal{D}(w)}{(z-w)} + \dots, \end{aligned} \quad (3.11)$$

and

$$\langle \mathcal{C}(z, \bar{z}) \mathcal{C}(0, 0) \rangle \sim 0, \quad (3.12)$$

$$\langle \mathcal{C}(z, \bar{z}) \mathcal{D}(0, 0) \rangle \sim \frac{c}{|z|^{2\Delta}}, \quad (3.13)$$

$$\langle \mathcal{D}(z, \bar{z}) \mathcal{D}(0, 0) \rangle \sim \frac{c}{|z|^{2\Delta}} (\log|z| + \mathfrak{d}), \quad (3.14)$$

where \mathfrak{d} is a constant. Since the conformal dimension of e^{iqX^0} is $\frac{q^2}{2}$ we find that

$$\begin{aligned} T(w)D_\varepsilon(z) &\sim -\frac{\varepsilon^2}{2(w-z)^2} D_\varepsilon(z) + \frac{1}{(w-z)^2} \varepsilon \Theta_\varepsilon(X^0) \\ &+ \dots \end{aligned} \quad (3.15)$$

and so a logarithmic conformal algebra structure arises if we define

$$C_\varepsilon(X^0) \equiv C(X^0; \varepsilon) = \varepsilon \Theta_\varepsilon(X^0), \quad (3.16)$$

suppressing, for simplicity, the nonholomorphic piece. The above logarithmic conformal field theory structure is found with this identification. Similarly we find

$$T(w)C_\varepsilon(z) \sim -\frac{\varepsilon^2}{2(w-z)^2} C_\varepsilon(z) + \dots$$

Consequently Δ for $C_\varepsilon(z)$ and $D_\varepsilon(z)$ is $-\frac{\varepsilon^2}{2}$. A calculation (in a Euclidean metric) for a disc of size L with a short-distance world sheet cutoff a reveals that as $\varepsilon \rightarrow 0$

$$\langle C_\varepsilon(z) C_\varepsilon(0) \rangle \sim O(\varepsilon^2), \quad (3.17)$$

$$\langle C_\varepsilon(z, \bar{z}) D_\varepsilon(0) \rangle \sim \frac{\pi}{2} \sqrt{\frac{\pi}{\varepsilon^2 \alpha}} \left(1 - 2\varepsilon^2 \log \left| \frac{z}{a} \right|^2 \right), \quad (3.18)$$

$$\langle D_\varepsilon(z, \bar{z}) D_\varepsilon(0) \rangle \sim \frac{\pi}{2} \sqrt{\frac{\pi}{\varepsilon^2 \alpha}} \left(\frac{1}{\varepsilon^2} - 2 \log \left| \frac{z}{a} \right|^2 \right), \quad (3.19)$$

where $\alpha = \log| \frac{L}{a} |^2$. We consider $\varepsilon \rightarrow 0+$ such that

$$\varepsilon^2 \alpha \sim \frac{1}{2\eta} = O(1), \quad (3.20)$$

where η is the time signature and the right-hand side is kept fixed as the cutoff runs; it is then straightforward to see that (3.17), (3.18), and (3.19) are consistent with (3.12), (3.13), and (3.14). It is only under the condition (3.20) that the recoil operators C_ε and D_ε obey a closed logarithmic conformal algebra [19]:

$$\begin{aligned} \langle C_\varepsilon(z) C_\varepsilon(0) \rangle &\sim 0, & \langle C_\varepsilon(z) D_\varepsilon(0) \rangle &\sim 1, \\ \langle D_\varepsilon(z) D_\varepsilon(0) \rangle &\sim -2\eta \log|z/L|^2. \end{aligned} \quad (3.21)$$

The reader should notice that the full recoil operators, involving $\partial_n X^i$ holomorphic pieces with the conformal-dimension-one entering (3.7), obey the full logarithmic algebra (3.12), (3.13), and (3.14) with conformal dimensions $\Delta = 1 - \frac{\varepsilon^2}{2}$. From now on we shall adopt the Euclidean signature $\eta = 1$.

We next remark that, at tree level in the string perturbation sense, the stringy sigma model (inclusive of the D-particle boundary term and other vertex operators) is a two-dimensional renormalizable quantum field theory; hence

for generic couplings g^i it is possible to see how the couplings run in the renormalization group sense with changes in the short-distance cutoff through the beta functions β^i . In the world sheet renormalization group [23], based on expansions in powers of the couplings, β^i has the form (with no summation over the repeated indices)

$$\beta^i = y_i g^i + \dots, \quad (3.22)$$

where y_i is the anomalous dimension, which is related to the conformal dimension Δ_i by $y_i = \Delta_i - \delta$, with δ the engineering dimension (for the holomorphic parts of vertex operators for the open string $\delta = 1$). The ... in (3.22) denote higher orders in g^i . Consequently, in our case, we note that the (renormalized) D-particle recoil velocities u^i constitute such σ -model couplings, and to lowest order in the renormalized coupling u_i the corresponding β function satisfies

$$\frac{du^i}{d \log \Lambda} = -\frac{\varepsilon^2}{2} u^i, \quad (3.23)$$

where Λ is a (covariant) world sheet renormalization-group scale. In our notation, we identify the logarithm of this scale with $\alpha = \log|\frac{L}{a}|^2$, satisfying (3.20).

An important comment is now in order concerning the interpretation of the flow of this world sheet renormalization-group scale as a target-time flow. The target time t is identified through $t = 2 \log \Lambda$. For completeness we recapitulate the arguments of [19] leading to such a conclusion. Let one make a scale transformation on the size of the world sheet

$$L \rightarrow L' = e^{t/4} L \quad (3.24)$$

which is a finite-size scaling (the only one which has physical sense for the open string world sheet). Because of the relation between ε and L (3.20) this transformation will induce a change in ε

$$\varepsilon^2 \rightarrow \varepsilon'^2 = \frac{\varepsilon^2}{1 + \varepsilon^2 t} \quad (3.25)$$

(note that if ε is infinitesimally small, so is ε' for any finite t). From the scale dependence of the correlation functions (3.21) that C_ε and D_ε transform as

$$D_\varepsilon \rightarrow D_{\varepsilon'} = D_\varepsilon + t C_\varepsilon, \quad C_\varepsilon \rightarrow C_{\varepsilon'} = C_\varepsilon. \quad (3.26)$$

From this transformation one can then see that the coupling constants in front of C_ε and D_ε in the recoil operator (3.6), i.e. the velocities u_i and spatial collective coordinates y_i of the brane, must transform like

$$u_i \rightarrow u_i, \quad y_i \rightarrow y_i + u_i t. \quad (3.27)$$

This transformation is nothing other but the Galilean transformation for the heavy D-particles and thus it demonstrates that the finite-size scaling parameter t , entering (3.24), plays the role of target time, on account of (3.20). Notice that (3.27) is derived upon using (3.21), that is in the

limit where $\varepsilon \rightarrow 0$. This will become important later on, where we shall discuss (stochastic) relaxation phenomena in our recoiling D-particle.

Thus, in the presence of recoil a world sheet scale transformation leads to an evolution of the D-brane in target space, and from now on we identify the world sheet renormalization-group scale with the target time t . In this sense, Eq. (3.23) is an evolution equation in target time.

However, this equation does not capture quantum-fluctuation aspects of u^i about its classical trajectory with time $u_i(t)$. Going to higher orders in perturbation theory of the quantum field theory at fixed genus does not qualitatively alter the situation in the sense that the equation remains deterministic. In the next section we shall consider the effect of string perturbation theory where higher-genus surfaces are considered and resummed in some appropriate limits that we shall discuss in detail.

IV. STRING PERTURBATION THEORY AND IMPLICATION FOR RECOIL VELOCITY

It is not possible to exactly sum up higher orders in string perturbation theory. We have seen that infrared singularities in the integration over the moduli of the Riemann surface (representing the world sheet) in the wormhole limit are related to the recoil operators for the D-particle. The wormhole construction [24] is a way of constructing higher-genus surfaces from lower-genus ones. Since it will be relevant to us later, we should note that g_s the string coupling is given by

$$g_s = e^{\langle \Phi \rangle}, \quad (4.1)$$

where Φ is the spin zero dilaton mode which is part of the massless string multiplet. Here $\langle \dots \rangle$ denotes the string path integral $\int DX e^{S_\sigma} \Phi$, where S_σ is the string σ -model action in the presence of string backgrounds such as the dilaton and the Kalb-Ramond modes. In particular the σ -model deformation due to the dilaton has the form

$$\frac{1}{4\pi} \int_\Sigma d\sigma d\tau \sqrt{\gamma} \Phi(X) R^{(2)}(\tau, \sigma) \quad (4.2)$$

on a world sheet Riemann surface Σ , where $\gamma_{\alpha\beta}$ is the induced metric on the world sheet, $\gamma = |\det \gamma_{\alpha\beta}|$ and $R^{(2)}$ is the associated Ricci curvature scalar. Now the Euler characteristic χ of Σ is given by

$$\chi = \frac{1}{4\pi} \int_\Sigma d\sigma d\tau \sqrt{\gamma} R^{(2)} = 2(1 - g), \quad (4.3)$$

where g is the genus and is an integer valued invariant. If we split the dilaton into a classical (world sheet coordinate independent) part $\langle \Phi \rangle$ and a quantum part $\varphi =: \Phi :$, where $: \dots :$ denotes appropriate normal ordering, we can write $\Phi = \langle \Phi \rangle + \varphi$. The σ -model partition function Z can be written as a sum over genera

$$Z = \sum_{\chi} \iint d\gamma_{\alpha\beta} dX e^{-S_{\text{rest}} - \chi(\Phi) - (1/4\pi) \int_{\Sigma} d\sigma d\tau \sqrt{\gamma} \varphi R^{(2)}(\tau, \sigma)}, \tag{4.4}$$

$$= \sum_{\chi} g_s^{-\chi} \iint d\gamma_{\alpha\beta} dX e^{-S_{\text{rest}} - (1/4\pi) \int_{\Sigma} d\sigma d\tau \sqrt{\gamma} \varphi R^{(2)}(\tau, \sigma)}, \tag{4.5}$$

where S_{rest} denotes a σ -model action involving the rest of the background deformations except the dilaton. For the moment we will assume that the theory is such that a potential is generated for Φ which suppresses the fluctuations represented by φ . In general we would have to consider $g_s = e^{\Phi}$ which would then make the string coupling a field.

The summation over genera cannot be performed exactly. We will follow an approach using a mechanism due to Fischler and Susskind [21,25] based on a dilute gas of wormholes (proposed originally by Coleman within the context of Euclidean quantum gravity [24]). This results in the structure of recoil (in lowest order) being modified by generating a Gaussian distribution for the recoil velocity u^i .

We present a detailed review of the pertinent formalism in Appendix A. For our purposes in this section we note that, in the case of mixed logarithmic states, the pinched topologies are characterized by divergences of a double logarithmic type [c.f. (A7) in Appendix A] which arise from the form of the string propagator [c.f. (A1) in Appendix A] in the presence of generic logarithmic operators C and D ,

$$\int dq q^{\Delta_\varepsilon - 1} \langle C, D | \begin{pmatrix} 1 & \log q \\ 0 & 1 \end{pmatrix} | C, D \rangle.$$

As shown in [19], the mixing between C and D states along degenerate handles leads formally to divergent string propagators in physical amplitudes, whose integrations have leading divergences of the form $\int \frac{dq}{q} \times \log q \int d^2 z D(z; \varepsilon) \int d^2 z' C(z'^2) \int d^2 z D(z; \varepsilon) \int d^2 z' C(z'; \varepsilon)$. As explained in [21], and reviewed in Appendix A of the current manuscript, these $(\log \delta)^2$ divergences can be canceled by imposing momentum conservation in the scattering process of the light string states off the D-particle background.

We note at this stage that, as mentioned earlier, isolated D-particles do not exist, as a result of their gauge flux conservation requirement. The physically correct way to formulate, therefore, the problem, is to consider groups of N , say, D-particles, which interact among themselves with flux-carrying stretched strings. The above analysis remains intact (in the sense of generalizing straightforwardly) when more than one D-particle is present in a region with typical dimensions smaller than the string length sometimes known as the fat brane [21]. There are, of course, technical

differences in the sense that a non-Abelian structure arises and the \hat{Y} have matrix labels. All qualitative features, however, are preserved concerning the quantization of the D-particle background moduli. For the remainder of this section we shall, therefore, formulate our arguments within this rigorous multi-D-particle picture.

The cancellation of leading divergences of the genus expansion in the non-Abelian case of a group of N D-particles, is demonstrated explicitly in Appendix A. It is shown there that this renormalization requires that the change in (renormalized) velocity of the due to the recoil from the scattering of string states be

$$\bar{U}_i^{ab} = -\frac{1}{M_s} (k_1 + k_2)_i \delta^{ab} = \frac{d\bar{Y}_i^{ab}}{dt}, \quad a, b = 1 \dots N, \tag{4.6}$$

where $k_{1,2}$ are the initial and final momenta in the scattering process and $M_D = 1/\sqrt{\alpha'} g_s$ is the BPS mass of the string soliton [2,5], and $g_s < 1$ is the physical (weak) string coupling. In (4.6), the $k_{1,2}$ are true physical momenta so that M_D represents the actual BPS mass of the D-particles. This means that, to leading order, the constituent D-particles in a group of N of them, say, move parallel to one another with a common velocity and there are no interactions among them. Thus the leading recoil effects imply a commutative structure and the “fat brane” of the group of D-particles behaves as a single D-particle (with a single average collective coordinate of its center of mass). In such a limit one may replace U_i^{ab} by u_i (c.f. previous section), describing the collective recoil velocity of the fat brane. This should be understood throughout this work.

In addition to this divergence, there are subleading $\log \delta$ singularities, corresponding to the diagonal terms $\int d^2 z D(z; \varepsilon) \int d^2 z' D(z'; \varepsilon)$ and $\int d^2 z C(z; \varepsilon) \times \int d^2 z' C(z'; \varepsilon)$. These latter terms are the ones we should concentrate upon for the purposes of deriving the quantum fluctuations of the collective D-particle coordinates. It is these subleading divergences in the genus expansion which lead to interactions between the constituent D-branes and provide the appropriate noncommutative quantum extension of the leading dynamics (4.6). The reader should recall that these (subleading) divergences also showed up in the much simpler case of perpetual Galilean motion of D-branes discussed in [18] [c.f. (3.5)], as a result of the translational symmetries zero mode contributions.

In the weak-coupling case, we can truncate the genus expansion to a sum over pinched annuli (Fig. 3 in Appendix A). This truncation corresponds to a semiclassical approximation to the full quantum string theory in which we treat the D-particles as heavy nonrelativistic objects in target space. Then the dominant contributions to the sum are given by the $\log \delta$ modular divergences described above, and the effects of the dilute gas of wormholes on the disc are to *exponentiate* the bilocal operator (A1) of Appendix A, describing string propagation in a

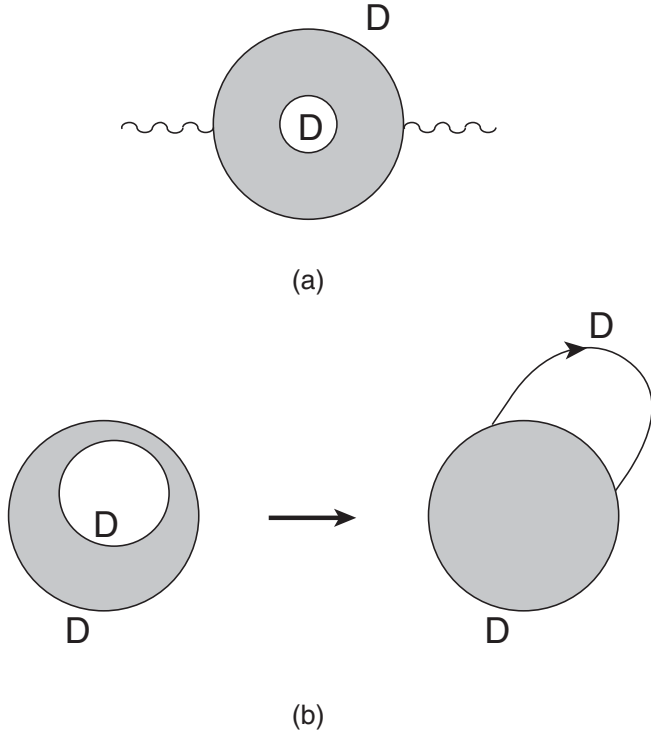


FIG. 3. (a) World sheet annulus diagram for the leading quantum correction to the propagation of a string state V in a D-brane background, and (b) the pinched annulus configuration which is the dominant divergent contribution to the quantum recoil.

pinched annulus. Thus, in the pinched approximation, the genus expansion of the bosonic σ model leads to an effective change in the matrix σ -model action by [21]

$$\Delta S \simeq \frac{g_s^2}{2} \log \delta \sum_{a,b,c,d} \int_{-\infty}^{\infty} d\omega d\omega' \oint_{\partial\Sigma} \oint_{\partial\Sigma'} V_{ab}^i(x; \omega) \times G_{ij}^{ab;cd}(\omega, \omega') V_{cd}^j(x; \omega'), \quad (4.7)$$

where ω, ω' are Fourier variables, defined appropriately in Appendix A, and $G_{ij}, i, j = C, D$ is a metric in the theory space of strings, introduced by Zamolodchikov [26].

The bilocal action (4.7) can be cast into the form of a local world sheet effective action by using standard tricks of wormhole calculus [24] and rewriting it as a functional Gaussian integral [21]

$$e^{\Delta S} = \int [d\check{\rho}] \exp \left[-\frac{1}{2} \sum_{a,b,c,d} \int_{-\infty}^{\infty} d\omega d\omega' \check{\rho}_i^{ab}(\omega) \times \oint_{\partial\Sigma} \oint_{\partial\Sigma'} G_{ab;cd}^{ij}(\omega, \omega') \check{\rho}_j^{cd}(\omega') + g_s \sqrt{\log \delta} \sum_{a,b=1}^N \int_{-\infty}^{\infty} d\omega \check{\rho}_i^{ab}(\omega) \oint_{\partial\Sigma} V_{ab}^i(x; \omega) \right], \quad (4.8)$$

where $\check{\rho}_i^{ab}(\omega)$ are stochastic coupling constants of the

world sheet matrix σ model, which express quantum fluctuations of the corresponding background fields in target space, as a consequence of genus resummation. Thus the effect of the resummation over pinched genera is to induce quantum fluctuations of the collective D-brane background, leading to a set of effective quantum coordinates

$$\check{Y}_i^{ab}(\omega) \rightarrow \hat{Y}_i^{ab}(\omega) = \check{Y}_i^{ab}(\omega) + g_s \sqrt{\log \delta} \check{\rho}_i^{ab}(\omega) \quad (4.9)$$

viewed as position operators in a comoving target-space frame.

Thus we find that the genus expansion in the pinched approximation is [21]

$$\sum_{h^{(p)}} Z_N^{h^{(p)}}[A] \simeq \left\langle \int_{\mathcal{M}} [d\rho] \wp[\rho] W \left[\partial\Sigma; A - \frac{1}{2\pi\alpha'} \rho \right] \right\rangle_0, \quad (4.10)$$

where the sum is over all pinched genera of infinitesimal pinching size, and

$$\wp[\rho] \propto \exp \left[-\frac{1}{2\Gamma^2} \sum_{a,b,c,d} \int_0^1 ds ds' \rho_i^{ab}(X^0(s)) G_{ab;cd}^{ij}(s, s') \times \rho_j^{cd}(X^0(s')) \right] \quad (4.11)$$

is a (appropriately normalized) functional Gaussian distribution on moduli space of width

$$\Gamma = g_s \sqrt{\log \delta}. \quad (4.12)$$

In (4.10) we have normalized the functional Haar integration measure $[d\rho]$ appropriately.

We see therefore that the diagonal subleading logarithmic divergences in the modular cutoff scale δ , associated with degenerate strips in the genus expansion of the matrix σ model, can be treated by absorbing these scaling violations into the width Γ of the probability distribution characterizing the quantum fluctuations of the (classical) D-brane configurations $Y_i^{ab}(X^0(s))$. In this way the interpolation among families of D-brane field theories corresponds to a quantization of the world sheet renormalization-group flows. Note that the world sheet wormhole parameters, being functions on the moduli space of recoil deformations, can be decomposed as

$$\rho_i^{ab}(X^0(s)) = \lim_{\varepsilon \rightarrow 0^+} ([\rho_C]_i^{ab} C(X^0; \varepsilon) + [\rho_D]_i^{ab} D(X^0; \varepsilon)). \quad (4.13)$$

The fields $\rho_{C,D}$ are then renormalized in the same way as the D-brane couplings, so that the corresponding renormalized wormhole parameters generate the same type of (Galilean) β -function equations (3.23).

According to the standard Fischler-Susskind mechanism for canceling string loop divergences [25], modular infinities should be identified with world sheet divergences at lower genera. Thus the strip divergence $\log \delta$ should be associated with a world sheet ultraviolet cutoff scale $\log \lambda$,

which in turn is identified with the target time as described earlier.

We may in effect take δ independent from Λ , in which case we can first let $\varepsilon \rightarrow 0^+$ in the above and then take the limit $\delta \rightarrow 0$. Interpreting $\log \delta$ in this way as a renormalization-group time parameter (interpolating among D-brane field theories), the time dependence of the renormalized width (4.12) expresses the usual properties of the distribution function describing the time evolution of a wave packet in moduli space. The inducing of a statistical Gaussian spread of the D-brane couplings is the essence of the quantization procedure.

A final remark is in order. From the form (3.9) and (3.16) of the recoil operators, it is evident that the dominant contributions in the limit $\varepsilon \rightarrow 0^+$, we consider here, come from the D deformations, pertaining to the recoil velocity u^i of the D-particle (or, better, the center-of-mass velocity of a group of D-particles, as discussed above). From now on, therefore, we restrict our attention to the distribution functions of such recoil velocities:

$$\wp(u) \sim \frac{1}{\Gamma} e^{-(u^2 - \bar{u}^2/\Gamma^2)}, \quad \Gamma = g_s \sqrt{\log \delta}, \quad (4.14)$$

where \bar{u} denotes the classical recoil velocity. Notice that, upon invoking [21] the Fischler-Susskind mechanism [25] for the absorption of the modular infinities to lower-genus (disc) world sheet surfaces, we may identify $\log \delta$ with the target time

$$\log \delta = t, \quad (4.15)$$

where this identification should be understood as being implemented at the end of the computation. To be precise, as explained in [21], the correct form of (4.15) would be $\log \delta = g_s^\chi t$, with $\chi > 0$ an exponent that can only be determined phenomenologically in the approach of [21], by comparing the space-time uncertainty principles, derived in this approach of resumming world sheet genera, with the ones within standard string/brane theory. In fact, in our approach of resumming world sheet pinched surfaces [21], one obtains for the spatial and temporal variances: $\Delta Y^{aa} \Delta t \geq g_s^\chi \sqrt{\alpha'}$, which implies that the standard string-theory result [27], independent of the string coupling, is obtained for $\chi = 0$. This is the case we shall consider here, which leads to the identification (4.15). However, in the modern approach of D-brane theories, one can adjust the uncertainty relations in order to probe minimal distances below the string length, which is achieved by the choice [21], e.g. $\chi = 2/3$, reproducing the characteristic minimal length probed by D-particles [28]. In our case, where, as we shall discuss in the next subsection, the coupling constant of the string may itself fluctuate, it is the mean value of g_s that enters in such relations. This issue is not relevant if we stay within the $\chi = 0$ case, which we do in this article.

We next remark that the nature of the Gaussian correlation is assumed to be delta correlated in time. The Langevin equation [29] implied by (4.9) replaces (3.23) and can be written as

$$\frac{d\bar{u}^i}{dt} = -\frac{1}{4t} \bar{u}^i + \frac{g_s}{\sqrt{2\alpha'}} t^{1/2} \xi(t), \quad (4.16)$$

where $t = \varepsilon^{-2}$ and $\xi(t)$ represents white noise. This equation is valid for large t . From the above analysis it is known [21] (c.f. Appendix A) that to $O(g_s^2)$ the correlation for $\xi(t)$ is \bar{u}^i independent, and for time scales of interest, is correlated like white noise; hence the correlation of $\xi(t)$ has the form

$$\langle \xi(t) \xi(t') \rangle = \delta(t - t'). \quad (4.17)$$

Since the vectorial nature of \bar{u}^i is not crucial for our analysis we will suppress it and consider the single variable \bar{u} .

We should stress that this equation is valid for large t which is required since ε is small. Hence the apparent singularity in Eq. (4.16) at $t = 0$ is not relevant and so we can empirically *regularize* this singularity by changing $\frac{1}{t}$ to $\frac{1}{t+t_0}$ for some $t_0 > 0$; t_0 is the order of the capture time of the ϕ meson by the D-particle. The stochastic Langevin equation (4.16), describes relaxation aspects of the recoiling D-particle with equilibrium being reached only as $\varepsilon \rightarrow 0$ (or $t \rightarrow \infty$).

The reader should notice that in the limit the system reaches equilibrium with a constant in time velocity. It is only in this limit that the Galilean transformation (3.27) applies, as already discussed there. We now proceed to a solution of this Langevin equation and a discussion on the pertinent physical consequences for a statistical population of quantum-fluctuating D-particles.

V. SOLUTION OF LANGEVIN EQUATIONS AND FLUCTUATING STRING COUPLING

In the recent modeling of the omega effect [9], the recoil velocity of the D-particles u has been taken as a classical stochastic variable. The D-particle fluctuations which are described by (4.16) will be superimposed on this stochasticity. Equation (4.16) is a particularly simple equation in the sense that the drift and diffusion terms are independent of u . By making a change of variable it is easy to eliminate the drift term and the resulting equation can then be interpreted in terms of a Wiener process [29]. Let us consider the auxiliary equation

$$\frac{d}{dt} y = -\frac{1}{4(t+t_0)} y, \quad (5.1)$$

which just deals with the drift part of Eq. (4.16). It has a solution

$$y(t) = y(t_0) Y(t),$$

where

$$Y(t) = \exp\left[-\frac{1}{4} \int_0^t \frac{dt'}{t' + t_0}\right] = \left(\frac{t + t_0}{t_0}\right)^{-(1/4)} \quad (5.2)$$

and t_0 is a time much smaller than t . We now define $U(t) = u(t)Y(t)^{-1}$ and readily find that

$$\frac{dU}{dt} = \frac{g_s}{\sqrt{2\alpha'}} t^{1/2} Y(t)^{-1} \xi(t). \quad (5.3)$$

This describes purely diffusive motion and is thus related to the Wiener process; equivalently we can consider the associated probability distribution $p(U, t)$ which satisfies the Fokker-Planck equation

$$\frac{\partial}{\partial t} p(U, t) = \frac{1}{4\alpha'} g_s^2 t \left(\frac{t + t_0}{t_0}\right)^{1/2} \frac{\partial^2}{\partial U^2} p(U, t). \quad (5.4)$$

If at $t = 0$ consider a D-particle velocity recoil u_0 so that

$$p(U, 0) = \delta(U - u_0). \quad (5.5)$$

The Eq. (5.4) can be solved to give

$$p(U, t) = \sqrt{\frac{15\alpha'}{2\pi\eta(t)} \frac{1}{g_s}} \exp\left(-\frac{15\alpha'(U - u_0)^2}{2g_s^2\eta(t)}\right), \quad (5.6)$$

where

$$\eta(t) = 2t_0^2 + 3(t + t_0)^2 \sqrt{1 + \frac{t}{t_0}} - 5t_0^{1/2}(t + t_0)^{3/2}. \quad (5.7)$$

If the D-particle is typically interacting with matter on time scales of t_0 , then the effect of a large number of such collisions can be calculated by performing an ensemble average over a distribution of u_0 . A distribution for u_0 that has been used in modeling is a Gaussian with zero mean and variance σ . This is readily seen to lead to an averaged distribution D-particle velocity recoil distribution $\ll p(u|g_s) \gg$, where

$$\ll p(u|g_s) \gg = \sqrt{\frac{15\alpha'}{2\pi(g_s^2\eta(t) + 15\alpha'\sigma^2)}} \times \exp\left[-\frac{15\alpha'}{2(g_s^2\eta(t) + 15\alpha'\sigma^2)} u^2\right]. \quad (5.8)$$

We have used a notation for p which emphasizes that it is conditional on g_s having a fixed value. The interaction time includes *both* the time for capture and reemission of the string by the D-particle, as well as the time interval until the next capture, during string propagation. In a generic situation, this time could be much larger than the capture time, especially in dilute gases of D-particles, which include less than one D-particle per string ($\alpha^{3/2}$) volume. Indeed, as discussed in detail in [30], using generic properties of strings consistent with the space-time uncertainties [27], the capture and reemission time t_0 , involves the growth of a stretched string between the string state and the D-brane world (c.f. Fig. 2) and is found proportional to the

incident string energy p^0 :

$$t_0 \sim \alpha' p^0 \ll \sqrt{\alpha'}. \quad (5.9)$$

We shall use this result in Sec. VI, where we estimate the strength of the ω effect in the initial entangled state of two mesons (1.2), after the ϕ -meson decay in the presence of D-particles. In such a situation, the interaction time is essentially the capture time t_0 .

We will now examine how the above results are modified when g_s fluctuates. Such issues cannot currently be treated with any level of rigor since they embody issues of string vacua and backgrounds. Hence we will take a somewhat phenomenological stance and consider the effect of a class of stochastic fluctuations for $g_s^{-2} (= e^{-2\langle\Phi\rangle})$ which are varying on a time scale which is slow compared to the drift time scale. This is the arena of superstatistics [14]. Following the analysis of Beck [31] (c.f. Appendix B) we can make the following fairly general classical ansatz for $\frac{1}{g_s^2}$ compatible with its positivity viz.

$$\frac{1}{g_s^2} = \sum_{i=1}^n x_i^2, \quad (5.10)$$

where the x_i are n independent Gaussian variables of zero mean and variance σ_0^2 . The probability distribution p for $\frac{1}{g_s^2}$ is χ^2 with n degrees of freedom, i.e.

$$p\left(\frac{1}{g_s^2}\right) = \frac{1}{\Gamma(\frac{n}{2})} \left[\frac{ng_0^2}{2}\right]^{n/2} \left(\frac{1}{g_s^2}\right)^{(n/2)-1} \exp\left(-\frac{ng_0^2}{2g_s^2}\right). \quad (5.11)$$

Here $\langle\frac{1}{g_s^2}\rangle = \frac{1}{g_0^2}$ and the variance $\text{var}(\frac{1}{g_s^2}) = \frac{2}{ng_0^4}$. If g_0 is held constant the variance can be made small for large n . This is a particular form of the gamma distribution. However other choices for p , such as lognormal and F distributions have also been considered [14] in contexts such as turbulence. We note that

$$\int_0^\infty e^{-wu^2\beta} \beta^{[(n+m)/2]-1} \exp\left(-\frac{n\beta}{2\beta_0}\right) d\beta = \left(u^2w + \frac{n}{2\beta_0}\right)^{-(m+n)/2} \Gamma\left(\frac{m+n}{2}\right). \quad (5.12)$$

We now calculate the probability p is for u as

$$p(u) \equiv \int_0^\infty d\left(\frac{1}{g_s^2}\right) p\left(\frac{1}{g_s^2}\right) \ll p(u|g_s) \gg. \quad (5.13)$$

The choice of p in (5.11) leads to canonical form of distribution for nonextensive statistics [15,31]. Other choices mentioned above give different superstatistics. These wider classes of statistics present opportunities in interpreting the data in neutrino physics (see e.g. [32]). For large n it is straightforward to show that

$$\begin{aligned}
 p(u) &= \sqrt{\frac{15\alpha'}{\pi(g_0^2\eta(t) + 15\alpha'\sigma^2)}} \\
 &\times \frac{\Gamma(\frac{n+1}{2})(ng_0^2)^{n/2}}{\Gamma(\frac{n}{2})\left(\frac{15\alpha'g_0^2}{\eta(t)g_0^2 + 15\alpha'\sigma^2}u^2 + ng_0^2\right)^{(n+1)/2}}. \quad (5.14)
 \end{aligned}$$

On writing $q = 1 + \frac{2}{n+1}$ and $\tilde{g}(t)^2 = (3 - q)(\sigma^2 + \frac{\eta(t)g_0^2}{15\alpha'})$ we find the canonical form for Tsallis statistics [15]

$$p(u) \sim \sqrt{\frac{3-q}{15\tilde{g}(t)^2}} \left(\left[1 + \frac{(q-1)u^2}{\tilde{g}(t)^2} \right]^{-[1/(q-1)]} \right), \quad (5.15)$$

with q the nonextensivity parameter. Hence the stochasticity in the recoil velocity, when stringy matter is captured by the D-particle, leads to a deviation from nonextensive Tsallis statistics. The deviation is suppressed as the interaction time increases. For large n we have weak nonextensivity and the fluctuations for $\frac{1}{g_s^2}$ are small.

The quantity of interest which is an important input for our estimate of the omega effect is the variance of $p(u)$:

$$\text{var}(u)_{\text{non-ext}} = \frac{\eta(t)g_0^2 + 15\alpha'\sigma^2}{15\alpha'(1 - \frac{2}{n})}. \quad (5.16)$$

For reasons mentioned above, we can make the plausible assumption that the initial state of the neutral K meson pairs is governed by the variance at $t \sim t_0 \sim \alpha'p^0$. In such a case, from (5.7) we obtain that $\eta(t \sim t_0) \simeq 2(1 + \sqrt{2})t_0^2 \sim 2(1 + \sqrt{2})(\alpha'p^0)^2$, and hence from (5.16) the variance over such time scales becomes of order

$$\begin{aligned}
 \text{var}(u)_{\text{non-ext}}(t \sim t_0) &\simeq \frac{2(1 + \sqrt{2})[g_0^2(\sqrt{\alpha'}p^0)^2 + 15\sigma^2]}{15(1 - \frac{2}{n})} \\
 &\sim g_0^2\alpha'(p^0)^2 \left(1 + \frac{2}{n} + \dots \right) \\
 &+ \mathcal{O}(\sigma^2), \quad (5.17)
 \end{aligned}$$

where on the right-hand side of the above equation we only gave an order of magnitude estimate, assuming that n is large. The terms of order σ^2 have not been written explicitly. Indeed, in most models of quantum gravity, a natural assumption would be that $\sigma^2 \leq g_0^2\alpha'(p^0)^2$, which is a natural assumption to make for a dispersion due to (quantum) fluctuations of the recoil velocity of heavy D-particles of average mass $M_s/g_0 = 1/(g_0\sqrt{\alpha'})$, where $\langle 1/g_s^2 \rangle = 1/g_0^2$. As we shall discuss in the next section, therefore, such dispersion terms do not lead to dominant contributions to the ω -effect estimates which are of primary interest to us here. These considerations lead to natural estimates for the parameter ζ in the magnitude of the omega effect that we have discussed in our earlier work [9], and we now proceed to examine.

VI. DECOHERENCE AND ENTANGLED STATES: ω EFFECT REVISITED

We shall use a low-energy quantum-mechanical approach for the dynamics of the neutral mesons, which is sufficient for a discussion of the ω effect in nonrelativistic systems of entangled mesons, such as kaons in a ϕ factory [9]. For a description of the stochastically fluctuating space-time effects, we shall make use of the above-derived string-theory effects, in particular, the recoil-velocity dispersion (5.17), including the effects of nonextensive statistics (5.15), due to fluctuations of the string coupling g_s .

Following [9], where we refer the interested reader for details, we consider the following interaction Hamiltonian, which expresses the effective low-energy interaction of the meson states with the D-particle foam space-time background in the model of [4]:

$$\begin{aligned}
 \hat{H} &= g^{01}(g^{00})^{-1}\hat{k} - (g^{00})^{-1} \\
 &\times \sqrt{(g^{01})^2k^2 - g^{00}(g^{11}k^2 + m^2)}, \quad (6.1)
 \end{aligned}$$

where \hat{k} indicates the appropriate momentum operator, with eigenvalue k (along the direction of motion), when acting on momentum eigenstates, i.e. $\hat{k}|\pm k, \uparrow\rangle = \pm k|\pm k, \uparrow\rangle$ together with the corresponding relation for \downarrow . The arrows indicate the appropriate meson ‘‘flavors’’ [9]. The induced metric $g_{\mu\nu}$, on the other hand, is such that

$$\begin{aligned}
 g^{00} &= (-1 + r_4)1, \\
 g^{01} &= g^{10} = r_01 + r_1\sigma_1 + r_2\sigma_2 + r_3\sigma_3, \quad (6.2) \\
 g^{11} &= (1 + r_5)1,
 \end{aligned}$$

where 1, is the identity and σ_i , $i = 1, 2, 3$ are the Pauli matrices.

The target-space metric state, which is close to being flat, can be represented schematically as a density matrix

$$\rho_{\text{grav}} = \int d^5r f(r_\mu) |g(r_\mu)\rangle\langle g(r_\mu)|. \quad (6.3)$$

The parameters r_μ ($\mu = 0, \dots, 5$) are stochastic with a Gaussian distribution $f(r_\mu)$ characterized by the averages

$$\langle r_\mu \rangle = 0, \quad \langle r_\mu r_\nu \rangle = \Delta_\mu \delta_{\mu\nu}. \quad (6.4)$$

The fluctuations experienced by the two entangled neutral mesons will be assumed to be *independent*.

The above parametrization has been taken for simplicity and we will also consider motion to be in the x direction which is natural since the meson pair moves collinearly in the center-of-mass frame.

Space-time deformations of the form (6.2) and the associated Hamiltonians (6.1) have been derived in the context of conformal field theory in earlier works by one of the authors and collaborators [4,33] and details on the relevant derivations will not be given here. We only mention that the variable r_1 in particular, expresses a momentum trans-

fer during the interaction of the (string) matter state with the D-particle defect. In this sense, the off diagonal metric component g_{01} can be represented as

$$g_{01} \sim u_1, \quad (6.5)$$

where $u_1 = g_s \frac{\Delta k}{M_s}$ expresses the momentum transfer along the direction of motion of the matter string (taken here to be the x direction). In the above equation, M_s/g_s is the mass of the D-particle, which for weakly coupled strings with coupling g_s is larger than the string mass scale M_s . In order to address oscillation phenomena, induced by D-particles, the fluctuations of each component of the metric tensor are taken in [9] to have a 2×2 (“flavor”) structure, as in (6.2), and not the simple structure (6.5) considered in earlier works. In the case of neutral kaons, which we concentrate on for concreteness in this section, we use the following notation for the flavors:

$$|K_L\rangle = |\uparrow\rangle, \quad |K_S\rangle = |\downarrow\rangle,$$

which represent the two physical eigenstates, with masses $m_1 \equiv m_L$, $m_2 \equiv m_S$, with

$$\Delta m = m_L - m_S \sim 3.48 \times 10^{-15} \text{ GeV}. \quad (6.6)$$

In this way, the stochastic variables r_μ [(6.4)] in (6.1), are linked with the fluctuations of the D-particle recoil velocity, by representing the latter as

$$u_1 \sim r g_s \frac{k}{M_s}, \quad (6.7)$$

upon the above-mentioned technicality of considering flavor changes in addition to the momentum transfer. Specifically, $|r_i| = \mathcal{O}(g_s r k / M_s)$.

In this sense, the detailed discussion in the previous session on the stochastic fluctuations of the recoil velocity about a zero average value, translates into rewriting (6.4) with variances [c.f. (5.17)]

$$\begin{aligned} \Delta_\mu &\sim g_0^2 \frac{t_0^2}{\alpha'} \left(1 + \frac{2}{n} \dots\right) + \mathcal{O}(\sigma^2) \sim g_0^2 \left(\frac{p^0}{M_s}\right)^2 \left(1 + \frac{2}{n} \dots\right) \\ &\quad + \mathcal{O}(\sigma^2), \\ \mu &= 1, 2, \end{aligned} \quad (6.8)$$

where we considered the capture time $t_0 \sim \alpha' p^0$, with p^0 the energy of the probe, as spanning the essential interaction time with the D-particle of the initial entangled meson state. In this way we extrapolate the result (5.8) to times smaller than $\sqrt{\alpha'}$. This is acceptable, as long as such times are *finite*. From a conformal field theory point of view, this means that we consider the world sheet scaling parameter $1/\varepsilon^2 \sim \ln(L/a)^2 \sim t_0 \ll \sqrt{\alpha'}$ for probe energies $p^0 \ll M_s$.

We next note that the Hamiltonian interaction terms

$$\hat{H}_1 = -(r_1 \sigma_1 + r_2 \sigma_2) \hat{k} \quad (6.9)$$

are the leading-order contribution in the small parameters r_μ in the Hamiltonian H (6.1), since the corresponding variances $\sqrt{\Delta_\mu}$ are small. The term (6.9), has been used in [9] as a *perturbation* in the framework of nondegenerate perturbation theory, in order to derive the “gravitationally dressed” initial entangled meson states, immediately after the ϕ decay. The result is

$$\begin{aligned} |k, \uparrow\rangle_{QG}^{(1)} | -k, \downarrow\rangle_{QG}^{(2)} - |k, \downarrow\rangle_{QG}^{(1)} | -k, \uparrow\rangle_{QG}^{(2)} &= |k, \uparrow\rangle^{(1)} | -k, \downarrow\rangle^{(2)} \\ &\quad - |k, \downarrow\rangle^{(1)} | -k, \uparrow\rangle^{(2)} + |k, \downarrow\rangle^{(1)} | -k, \downarrow\rangle^{(2)} (\beta^{(1)} - \beta^{(2)}) \\ &\quad + |k, \uparrow\rangle^{(1)} | -k, \uparrow\rangle^{(2)} (\alpha^{(2)} - \alpha^{(1)}) \\ &\quad + \beta^{(1)} \alpha^{(2)} |k, \downarrow\rangle^{(1)} | -k, \uparrow\rangle^{(2)} - \alpha^{(1)} \beta^{(2)} |k, \uparrow\rangle^{(1)} | -k, \downarrow\rangle^{(2)}, \end{aligned} \quad (6.10)$$

where

$$\begin{aligned} \alpha^{(i)} &= \frac{{}^{(i)}\langle \uparrow, k^{(i)} | \hat{H}_1 | k^{(i)}, \downarrow \rangle^{(i)}}{E_2 - E_1}, \\ \beta^{(i)} &= \frac{{}^{(i)}\langle \downarrow, k^{(i)} | \hat{H}_1 | k^{(i)}, \uparrow \rangle^{(i)}}{E_1 - E_2}, \quad i = 1, 2, \end{aligned} \quad (6.11)$$

where the index (i) runs over meson species (flavors) ($1 \rightarrow K_L$, $2 \rightarrow K_S$). The reader should notice that the terms proportional to $(\alpha^{(2)} - \alpha^{(1)})$ and $(\beta^{(1)} - \beta^{(2)})$ in (6.10) generate ω -like effects. We concentrate here for brevity and concreteness in the strangeness conserving case of the ω effect in the initial decay of the ϕ meson [8], which corresponds to $r_i \propto \delta_{i2}$. We should mention, however, that in general quantum gravity does not have to conserve this quantum number, and in fact strangeness-violating ω -like terms are generated in this problem through time evolution [9].

We next remark that on averaging the density matrix [c.f. (6.3)] over the random variables r_i , which are treated as independent variables between the two meson particles of the initial state (6.10), we observe that only terms of order $|\omega|^2$ will survive, with the order of $|\omega|^2$ being

$$\begin{aligned} |\omega|^2 &= \tilde{\sum}_{(1),(2)} \left(\mathcal{O} \left(\frac{1}{(E_1 - E_2)^2} \langle \langle \downarrow, k | H_1 | k, \uparrow \rangle \rangle^2 \right) \right) \\ &= \tilde{\sum}_{(1),(2)} \left(\mathcal{O} \left(\frac{\Delta_2 k^2}{(E_1 - E_2)^2} \right) \right) \sim \tilde{\sum}_{(1),(2)} \left(\frac{\Delta_2 k^2}{(m_1 - m_2)^2} \right) \end{aligned} \quad (6.12)$$

for the physically interesting case of nonrelativistic kaons in ϕ factories, in which the momenta are of order of the rest energies. The notation $\tilde{\sum}_{(1),(2)}(\dots)$ above indicates that one considers the sum of the variances Δ_2 over the two meson states 1, 2 as defined above.

The variances in our model of D-foam, which are due to quantum fluctuations of the recoil-velocity variables about the zero average (dictated by the imposed requirement on Lorentz invariance of the string vacuum) are given by (6.8),

with $p^0 \sim m_i$ the energy of the corresponding individual (nonrelativistic) meson state (i), $i = 1, 2$, in the initial entangled state (6.10). It is important to notice that, on taking the sum of the variance Δ_2 over the mesons (1) and (2), the terms proportional to the dispersion σ^2 in the initial recoil velocity u_0 Gaussian distribution in (5.17) give a contribution of order $30\sigma^2$, since σ^2 is assumed universal among particle species. This is a parameter that depends on the details of the foam. As already mentioned in the previous section, one may assume models in which $\sigma^2 \ll (g_0\sqrt{\alpha'}p^0)^2$. In this sense, one is left with the contributions from the first term of the right-hand side of (5.17), and thus we obtain the following estimate for the square of the amplitude of the (complex) ω parameter:

$$|\omega|^2 \sim g_0^2 \frac{(m_1^2 + m_2^2)}{M_s^2} \frac{k^2}{(m_1 - m_2)^2} \left(1 + \frac{2}{n} \dots\right),$$

$$M_s/g_0 \equiv M_P, \quad (6.13)$$

where $M_P = M_s/g_0$ is the (average) D-particle mass, as already mentioned, representing the (average) quantum-gravity scale, which may be taken to be the four-dimensional Planck scale. In the modern version of string theory, M_s is arbitrary and can be as low as a few TeV, but in order to have phenomenologically correct string models with large extra dimensions one also has to have in such cases very weak string couplings g_0 , such that even in such cases of low M_s , the D-particle mass M_s/g_0 is always close to the Planck scale 10^{19} GeV. But of course one has to keep an open mind about ways out of this pattern, especially in view of the string landscape.

The result (6.13), implies, for neutral kaons in a ϕ factory, for which (6.6) is valid, the estimate $|\omega| = \mathcal{O}(10^{-5})$, which, in the sensitive η^{+-} bi-pion decay channel, leads to effects enhanced by 3 orders of magnitude, as a result of the fact that the $|\omega|$ effect always appears in the corresponding observables [8] in the form $|\omega|/|\eta^{+-}|$, and the CP -violating parameter $|\eta^{+-}| \sim 10^{-3}$. At present, this value is still some 2 orders of magnitude away from current bounds of the ω effect by the KLOE collaboration at DAΦNE [34], giving $|\omega| < 10^{-3}$ but it is within the projected sensitivity of the proposed upgrades.

The above estimate is valid for nonrelativistic meson states, where each meson has been treated as a structureless entity when considering its interaction with the D-foam. The inclusion of details of its strongly interacting substructure may affect the results. To consider relativistic meson states, the nonrelativistic quantum mechanics formalism leading to (6.13) in the kaon systems should strictly be replaced by an appropriate relativistic treatment. The major difference in this case is the form of the Hamiltonian, which stems from the expansion of the Dirac Hamiltonian for momenta $k \gg m_i$, m_i the masses. The quantities $E_i \sim k + \frac{m_i^2}{2k}$, $i = 1, 2$ (due to momentum conservation, assumed on average), and the capture times $t_c \sim \alpha' E_i$. Nevertheless,

if one extrapolates naively the above results for the $|\omega|^2$ in such a case, one arrives at the conclusion that for the validity of leading-order perturbation theory for the interaction with the D-foam, one should consider momenta of order $k \sim 30$ GeV. Otherwise higher-order corrections become important. This completes our discussion on the estimates of the ω effect in the initial entangled state of two mesons in a meson factory. As discussed in [9], ω -like terms can also be generated due to the time evolution. One can apply similar estimates for this case too. We shall not do so in this work.

VII. CONCLUSIONS AND OUTLOOK

This work examines the role of quantum string fluctuations (which can give rise to a noncommutative space-time geometry at string scales) on the velocity distribution of D-particles, within a specific kind of foam in string theory. Our Gaussian modeling of the recoil velocity is found to be robust to these fluctuations.

In this way we have managed to give a rather rigorous estimation (modulo strong interaction effects) of the ω effect in entangled states of mesons, which has been compared to previous naive estimates, based on dimensional analysis. The effect is smaller by roughly 2 orders of magnitude from the current upper bounds set by the KLOE collaboration at DAΦNE. This makes the detectability of the effect in future meson factories a major experimental challenge.

Admittedly, our approach in this paper is based on bosonic string theory which is not the most relevant phenomenologically. World sheet supersymmetric strings do not lead to a closed-form resummation of the leading divergencies of the pinched surfaces, since the latter cancel out [20,33], and the remaining terms are hard to cast in a closed form. However, despite this apparent technical difficulty, the general conclusions drawn from the current work, as far as fuzziness of the string coupling and the target-space time are concerned, are likely to be robust, since they depend on the form of vertex operators for (recoil) zero modes of D-particles (provided of course the theories are restricted to those admitting D-particles). In this sense, our approach here opens up the possibility of extending the analyses of D-particle foam to nonextensive statistics. The latter as a class is ubiquitous but, as far as we are aware, the possibility of nonextensive statistics in D-particle recoil has not hitherto been raised. Moreover, the mechanism makes contact with some nontrivial features of string theory.

The role of such nonextensivity on matter propagation in D-particle foam, matter number distribution functions and (supercritical) string cosmology [35] will be addressed in future publications. This is an important aspect of the formalism discussed here, since it may have a profound influence on the dark matter (and dark energy) distributions in a universe with D-particle foam, which may have phe-

nomenological consequences, as far as constraints on, say, supersymmetric particle physics models are concerned. Indeed, as we have discussed above, the interaction of D-particles with matter leads to local distortions of the neighboring space-time (2.3), which depend on both the string coupling g_s , through the D-particle masses M_s/g_s , and the recoil velocity of the D-particle (i.e. the momentum transfer of the string matter) u_i , which stochastically fluctuates upon summing up higher-genus world sheet topologies. In view of our discussion in this work, both these quantities can be fuzzy, leading to stochastic fluctuations on the space-time metric, on which string matter lives. These fluctuations are up and above any statistical fluctuations in populations of D-particles that characterize the D-particle foam models.

The presence of such fluctuations affect important cosmological quantities that are directly relevant to the (supercritical) universe budget, such as thermal supersymmetric dark matter relic densities, through appropriate modifications of the relevant thermodynamic equations. Hence, the relevant astroparticle physics constraints on supersymmetric models [35] are also modified. As we have noted above, however, the issue as to whether the fuzziness of the string coupling and consequently the non-extensivity of the D-particle foam can lead to observable signatures in Cosmology or astroparticle physics in general, remains to be seen. We hope to be able to report in a more detailed form on these phenomenological issues in the near future.

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APPENDIX A: RECOIL AND LEADING DIVERGENCES IN THE WORLD SHEET GENUS EXPANSION

In this appendix we shall show how the leading $(\log \delta)^2$ modular divergences which appear in (A8) can be removed by invoking an appropriate Ward identity for the fundamental string fields of the matrix σ model. As we shall show, this is equivalent to imposing *momentum conservation* for scattering processes in the matrix D-brane background. We shall be brief in our discussion, restricting ourselves on the main results, of relevance to our discussion in this work. The material in this section is taken from [21], where we refer the interested reader for further details.

As shown in that reference, conformal invariance requires absorbing such singularities into renormalized quantities at lower genera, leading to a generalized version of the Fischler-Susskind mechanism [25]. Such degenerate Riemann surfaces involve a string propagator over thin

long world sheet strips of thickness $\delta \rightarrow 0$ that are attached to a disc. These strips can be thought of as two-dimensional quantum-gravity wormholes. Consider first the resummation of one-loop world sheets, i.e. those with an annular topology, in the pinched approximation (c.f. Fig. 3). String propagation on such a world sheet can be described formally by adding bilocal world sheet operators \mathcal{B} , which in the present case are defined by [21]

$$\mathcal{B}(\omega, \omega') = \sum_{a,b,c,d} \oint_{\partial\Sigma} \oint_{\partial\Sigma'} V_{ab}^i(x; \omega) \times \frac{G_{ij}^{ab;cd}(\omega, \omega')}{L_0 - 1} V_{cd}^j(x; \omega'), \quad (\text{A1})$$

where G_{ij} denotes the Zamolodchikov metric in string-theory space, that is the two-point correlation function of the recoil vertex operators:

$$\oint_{\partial\Sigma_h} V_{ab}^i(x; \omega) \equiv \lim_{\varepsilon \rightarrow 0^+} \frac{ig_s}{2\pi\alpha'} \sum_{k=0}^h \int_0^1 ds_k e^{-i\omega X^0(s_k)} \times \Theta(X^0(s_k); \varepsilon) \bar{\xi}_a(s_k - \varepsilon) \xi_b(s_k) \times \frac{d}{ds_k} X^i(s_k), \quad (\text{A2})$$

and L_0 denotes the usual Virasoro generator. The variable ω is a Fourier variable, appearing in the Fourier transform

$$\check{Y}_i^{ab}(\omega) = \lim_{\varepsilon \rightarrow 0^+} \int_0^\infty dt e^{i\omega t} Y_i^{ab}(X^0; \varepsilon) \quad (\text{A3})$$

of the collective coordinates $Y_i^{ab}(X^0; \varepsilon)$ of the (group of) D-particles [19,21]

$$Y_i^{ab}(X^0; \varepsilon) = \sqrt{\alpha'} Y_{i\varepsilon} + u_i^{ab} X^0, \quad (\text{A4})$$

where Y_i^{ab} , u_i^{ab} denote the collective coordinates and recoil velocities of the group of N D-particles, with the indices $a, b = 1, \dots, N$. The reader should also note that, as explained in [21], the range of the variable $\omega \in (0, \infty)$, as a result of the impulse approximation of recoil, which dictates that the recoil vertex operators are nontrivial only for target times after some definite time [taken to be zero for concreteness in the example discussed here (3.7)]. The operator insertion $(L_0 - 1)^{-1}$ in (A1) represents the string propagator Δ_s on the thin strip of the pinched annulus.

Inserting a complete set of intermediate string states \mathcal{E}_I , we can rewrite (A1) as an integral over the variable $q \equiv e^{2\pi i\tau}$, where τ is the complex modular parameter characterizing the world sheet strip. The string propagator over the strip then reads

$$\Delta_s(z, z') = \sum_I \int dq q^{\Delta_I - 1} \{ \mathcal{E}_I(z) \otimes (\text{ghosts}) \otimes \mathcal{E}_I(z') \}_{\Sigma \# \Sigma'}, \quad (\text{A5})$$

where Δ_I are the conformal dimensions of the states \mathcal{E}_I . The sum in (A5) is over all states which propagate along the long thin strip connecting the discs Σ and Σ' (in the degenerating annulus handle case of interest here, $\Sigma' = \Sigma$). As indicated in (A5), the sum over states must include ghosts, whose central charge cancels that of the world sheet matter fields in any critical string model.

In (A5) we have assumed that the Virasoro operator L_0 can be diagonalized in the basis of string states with eigenvalues their conformal dimensions Δ_I , i.e.

$$L_0|\mathcal{E}_I\rangle = \Delta_I|\mathcal{E}_I\rangle, \quad q^{L_0-1}|\mathcal{E}_I\rangle = q^{\Delta_I-1}|\mathcal{E}_I\rangle. \quad (\text{A6})$$

However, this simple diagonalization fails [19] in the presence of the logarithmic pair of operators (A2), due to the nontrivial mixing between C and D in the Jordan cell of L_0 . Generally, states with $\Delta_I = 0$ may lead to extra logarithmic divergences in (A5), because such states make contributions to the integral of the form $\int dq/q \sim \log \delta$, in the limit $q \sim \delta \rightarrow 0$ representing a long thin strip of thickness δ . We assume that such states are discrete in the space of all string states, i.e. that they are separated from other states by a gap. In that case, there are factorizable logarithmic divergences in (A5) which depend on the background surfaces Σ and Σ' . These are precisely the states corresponding to the logarithmic recoil operators (3.9) and (3.16), with vanishing conformal dimension $\Delta_\varepsilon = -\varepsilon^2/2$ as $\varepsilon \rightarrow 0^+$.

In the case of mixed logarithmic states, the pinched world sheet topologies of Fig. 1 are characterized by divergences of a double logarithmic type which arise from the form of the string propagator in (A1) in the presence of generic logarithmic operators C and D ,

$$\int dq q^{\Delta_\varepsilon-1} \langle C, D | \begin{pmatrix} 1 & \log q \\ 0 & 1 \end{pmatrix} | C, D \rangle. \quad (\text{A7})$$

As shown in [19], the mixing between C and D states along degenerate handles leads formally to divergent string propagators in physical amplitudes, whose integrations have leading divergences of the form

$$\int \frac{dq}{q} \log q \int d^2 z D(z; \varepsilon) \int d^2 z' C(z'; \varepsilon) \simeq (\log \delta)^2 \int d^2 z D(z; \varepsilon) \int d^2 z' C(z'; \varepsilon). \quad (\text{A8})$$

These $(\log \delta)^2$ divergences can be canceled by imposing momentum conservation in the scattering process of the light string states off the D-brane background [21].

We notice at this stage that for brevity we have discussed in this work the simplistic case of a single D-particle interacting with an (open) string state. However, as a result of flux conservation, isolated D-particles do not exist, as we have mentioned above, and hence in realistic situations one deals with groups of N (with N varying) D-particles, interacting among themselves with the exchange of stretched flux-carrying strings. Such groups of N particles can be represented by a non-Abelian Wilson loop operator for the gauge group $SU(N)$. Within the framework of the auxiliary field representation of the Wilson loop operator [21], the effective Abelianization of the matrix σ model leads to a relatively straightforward generalization of the proof of the cancellation of modular infinities by introducing recoil, as we now demonstrate.

The pertinent bilocal term induced by (A8), which exponentiates upon summing over pinched topologies, can be written as a local world sheet effective action using the wormhole parameters $[\rho_{C,D}]_i^{ab}$ to give [21]

$$e^{\Delta S^{CD}} = \lim_{\varepsilon \rightarrow 0^+} \int d\rho_C d\rho_D \exp \left[\sum_{a,b=1}^N \left(-\frac{1}{2g_s^2 (\log \delta)^2} G^{LM} \sum_{c,d=1}^N G^{ij}_{ab;cd} [\rho_L]_i^{ab} [\rho_M]_j^{cd} + \frac{ig_s [\rho_C]_i^{ab}}{2\pi\alpha'} \int_0^1 ds C(X^0(s); \varepsilon) \bar{\xi}_a(s - \varepsilon) \xi_b(s) \frac{d}{ds} x^i(s) + \frac{ig_s [\rho_D]_i^{ab}}{2\pi\alpha'} \int_0^1 ds D(X^0(s); \varepsilon) \bar{\xi}_a(s - \varepsilon) \xi_b(s) \frac{d}{ds} x^i(s) \right) \right]. \quad (\text{A9})$$

Here we have for simplicity considered only the zero frequency modes of the fields involved with respect to the Fourier transformations defined in [21]. They will be sufficient to describe the relevant cancellations. In (A9) the (dimensionless) moduli space metric G^{LM} (where $L, M = C, D$) is an appropriate off diagonal 2×2 matrix, which is required to reproduce the initial bilocal operator with the CD mixing of the logarithmic operators. This off diagonal metric includes all the appropriate normalization factors \mathcal{N}_L for the zero mode states. These factors are essentially the inverse of the CD two-point function, which is finite.

We consider the propagation of two (closed string) matter tachyon states $T_{1,2} = e^{i(k_{1,2})_i x^i}$ in the background of (A9) at tree level. In what follows the effects of the C operator are subleading and can be ignored. Then, we are interested in the amplitude

$$\begin{aligned}
\mathcal{A}_{CD} &\equiv \left\langle\left\langle\left\langle\sum_{c'=1}^N \bar{\xi}_{c'}(0) T_1 T_2 e^{\Delta S^{CD}} \xi_{c'}(1)\right\rangle\right\rangle_0 \\
&= \lim_{\varepsilon \rightarrow 0^+} \sum_{c'=1}^N \int d\rho_C d\rho_D \int Dx D\bar{\xi} D\xi \bar{\xi}_{c'}(0) \exp\left(-N^2 S_0[x] - \sum_{c=1}^N \int_0^1 ds \bar{\xi}_c(s-\varepsilon) \frac{d}{ds} \xi_c(s)\right) T_1[x] T_2[x] \\
&\quad \times \exp\left[\sum_{a,b=1}^N \left(-\frac{1}{2g_s^2(\log\delta)^2} G^{LM} \sum_{c,d=1}^N G_{ab;cd}^{ij} [\rho_L]_i^{ab} [\rho_M]_j^{cd} \right. \right. \\
&\quad \left. \left. + \frac{ig_s[\rho_D]_i^{ab}}{2\pi\alpha'} \int_0^1 ds D(X^0(s); \varepsilon) \bar{\xi}_a(s-\varepsilon) \xi_b(s) \frac{d}{ds} x^i(s)\right)\right] \xi_{c'}(1) + \dots, \tag{A10}
\end{aligned}$$

where ... represent subleading terms. The scaling property (3.27) of the logarithmic operators must be taken into account. Under a scale transformation on the world sheet the C operator emerges from D due to mixing with a scale-dependent coefficient $\sqrt{\alpha'}t$. This will contribute to the scaling infinities we are considering here.

The composite D operator insertion in (A10) needs to be normal ordered on the disc. Normal ordering in the present case amounts to subtracting scaling infinities originating from divergent contributions of $D(X^0(s); \varepsilon)$ as $\varepsilon \rightarrow 0^+$. To determine these infinities, we first note that the one-point function of the composite D operators, computed with respect to the free σ model and auxiliary field actions, can be written as

$$\begin{aligned}
&\left\langle\left\langle\left\langle\sum_{c'=1}^N \bar{\xi}_{c'}(0) \exp\left(\sum_{a,b=1}^N \frac{ig_s[\rho_D]_i^{ab}}{2\pi\alpha'} \int_0^1 ds D(X^0(s); \varepsilon) \bar{\xi}_a(s-\varepsilon) \xi_b(s) \frac{d}{ds} x^i(s)\right) \xi_{c'}(1)\right\rangle\right\rangle_0 \\
&= \left\langle\left\langle\sum_{c'=1}^N \bar{\xi}_{c'}(0) \exp\left(-\sum_{a,b,c,d} \frac{g_s^2[\rho_D]_i^{ab} [\rho_D]_j^{cd}}{2(2\pi\alpha')^2} \int_0^1 ds ds' \langle D(X^0(s); \varepsilon) D(X^0(s'); \varepsilon) \rangle_0 \bar{\xi}_a(s-\varepsilon) \xi_b(s) \bar{\xi}_c(s'-\varepsilon) \xi_d(s') \right. \right. \\
&\quad \left. \left. \times \left\langle \frac{d}{ds} x^i(s) \frac{d}{ds'} x^j(s') \right\rangle_0\right)\right\rangle \xi_{c'}(1) \right\rangle \\
&= \exp\left(-\sum_{a,b=1}^N \frac{g_s^2[\rho_D]_i^{ab} [\rho_D]_j^{ba}}{2(2\pi\alpha')^2} \int_0^1 ds ds' \langle D(X^0(s); \varepsilon) D(X^0(s'); \varepsilon) \rangle_0 \left\langle \frac{d}{ds} x^i(s) \frac{d}{ds'} x^j(s') \right\rangle_0\right), \tag{A11}
\end{aligned}$$

where we have used Wick's theorem. The second equality in (A11) follows after removing ambiguous $\Theta(\varepsilon)$ type terms from the Wick expansion in the auxiliary fields using the renormalization scheme described in Appendix B of [21]. One finds that this procedure has the overall effect of replacing the product of auxiliary fields in the first equality in (A11) by the delta functions $\delta_{ad}\delta_{bc}$.

In what follows we shall ignore, for simplicity, the basic divergences that come from the fundamental string propagator in (A11). Such divergences will appear globally in all correlators below and will not affect the final result. As a consequence of the logarithmic algebra and the scale transformation (3.27), there are leading (scaling) divergences in (A11) for $\varepsilon \rightarrow 0^+$ which behave as

$$g_s^2 b \alpha'^{-1/2} t \text{tr}[\rho_D]_i [\rho_D]^i. \tag{A12}$$

Thus, normal ordering of the D operator amounts to adding a term of opposite sign to (A12) into the argument of the exponential in (A10) in order to cancel such divergences.

Let us now introduce a complete set of states $|\mathcal{E}_I\rangle$ into the two-point function of string matter fields on the disc,

$$\langle T_1 T_2 \rangle_0 = \sum_I |\mathcal{N}_I|^2 \langle T_1 | \mathcal{E}_I \rangle_0 \langle \mathcal{E}_I | T_2 \rangle_0, \tag{A13}$$

where \mathcal{N}_I is a normalization factor for the fundamental string states (determined by the Zamolodchikov metric). Taking into account the effects of the C operator included in D under the finite-size scaling (3.24), we see that the leading divergent contributions to (A13) are of the form

$$\langle T_1 T_2 \rangle_0 \simeq -\sqrt{\alpha'} t \langle T_1 | C \rangle_0 \langle C | T_2 \rangle_0 + \dots, \tag{A14}$$

where we have used (3.20), as well as the fact [c.f. (3.17), (3.18), and (3.19)] that the Zamolodchikov metric in the C , D basis behaves as [19]:

$$G_{CC} \sim \varepsilon^2, \quad G_{DD} \sim \varepsilon^{-2}, \quad G_{CD} = G_{DC} \sim \text{const}. \tag{A15}$$

We now notice that the C deformation vertex operator plays the role of the Goldstone mode for the translation symmetry of the fundamental string coordinates x^i , and as such we can apply the corresponding Ward identity in the matrix σ -model path integral to represent the action of the C deformation on physical states by $-i\delta/\delta x^i$ [18,19]. The leading contribution to (A13) can thus be exponentiated to yield

$$\begin{aligned} \langle T_1 T_2 \rangle_0 &\simeq \lim_{\varepsilon \rightarrow 0^+} \sum_{c'=1}^N \int D x D \bar{\xi} D \xi \bar{\xi}_{c'}(0) \exp\left(-N^2 S_0[x] - \sum_{c=1}^N \int_0^1 ds \bar{\xi}_c(s - \varepsilon) \frac{d}{ds} \xi_c(s)\right) T_1[x] \\ &\times \exp\left(-\frac{g_s^2 \sqrt{\alpha'} t}{2} \sum_{a,b=1}^N \int_0^1 ds ds' \bar{\xi}_a(s - \varepsilon) \xi_b(s) \bar{\xi}_b(s' - \varepsilon) \xi_a(s') \frac{\delta}{\delta x_i(s)} \frac{\delta}{\delta x^i(s')}\right) T_2[x] \xi_{c'}(1), \end{aligned} \quad (\text{A16})$$

where we have used the on shell condition $T_j(\frac{\delta}{\delta x_i} \frac{\delta}{\delta x^i}) T_k = 0$ for the tachyon fields. (A16) expresses the non-Abelian version of the Ward identity in the presence of logarithmic deformations.

Using (A12) and (A16) and normalizing the parameters of the logarithmic conformal algebra appropriately, it follows that (A10) can be written as [21]

$$\begin{aligned} \mathcal{A}_{CD} &= \lim_{\varepsilon \rightarrow 0^+} \sum_{c'=1}^N \int d\rho_C d\rho_D \int D x D \bar{\xi} D \xi \bar{\xi}_{c'}(0) \exp\left(-N^2 S_0[x] - \sum_{c=1}^N \int_0^1 ds \bar{\xi}_c(s - \varepsilon) \frac{d}{ds} \xi_c(s)\right) T_1[x] \\ &\times \exp\left[\sum_{a,b=1}^N \left\{ -\frac{1}{2g_s^2 (\log \delta)^2} G^{LM} \sum_{c,d=1}^N G_{ab;cd}^{ij} [\rho_L]_i^{ab} [\rho_M]_j^{cd} \right. \right. \\ &\quad \left. \left. - \frac{g_s^2 \alpha'^{-1/2} t}{2} \eta^{ij} \left([\rho_D]_i^{ab} - \frac{i\sqrt{\alpha'}}{g_s} \int_0^1 ds \bar{\xi}_a(s - \varepsilon) \xi_b(s) \frac{\delta}{\delta x_i(s)} \right) \right. \right. \\ &\quad \left. \left. \times \left([\rho_D]_j^{ba} - \frac{i\sqrt{\alpha'}}{g_s} \int_0^1 ds \bar{\xi}_b(s - \varepsilon) \xi_a(s) \frac{\delta}{\delta x_j(s)} \right) \right\} \right] T_2[x] \xi_{c'}(1) + \dots \end{aligned} \quad (\text{A17})$$

From (A17) it follows that the limit $t \rightarrow \infty$ localizes the world sheet wormhole parameter integrations with delta-function support

$$\prod_{a,b=1}^N \prod_{i=1}^9 \delta\left([\rho_D]_i^{ab} - \frac{\sqrt{\alpha'}}{g_s} (k_1 + k_2)_i \int_0^1 ds \bar{\xi}_a(s - \varepsilon) \xi_b(s)\right), \quad (\text{A18})$$

where $(k_{1,2})_i$ are the momenta of the closed-string matter states. This result shows that the leading modular divergences in the genus expansion are canceled by the scattering of (closed) string states off the matrix D-brane background. Upon rescaling ρ_D by g_s^2 , averaging over the auxiliary boundary fields, and incorporating (A18) as an effective shift in the velocity recoil operator [c.f. (4.9)],

$$\check{Y}_i^{ab}(\omega) \rightarrow \hat{Y}_i^{ab}(\omega) = \check{Y}_i^{ab}(\omega) + g_s \sqrt{\log \delta} \check{\rho}_i^{ab}(\omega) \quad (\text{A19})$$

viewed as position operator in a comoving target-space frame, we can identify this renormalization as fixing the velocity matrix

$$U_i^{ab} = -\sqrt{\alpha'} g_s (k_1 + k_2)_i \delta^{ab} \quad (\text{A20})$$

of the fat brane background. Thus momentum conservation for the D-brane dynamics guarantees conformal invariance of the matrix σ model as far as leading divergences are concerned.

APPENDIX B: NONEXTENSIVE (TSALLIS-TYPE) STATISTICS

In this Appendix we review the analysis of [31] in constructing classes of stochastic differential equations

with fluctuating frictional forces which generate dynamics described by nonextensive statistics in the sense of Tsallis [15].

Our starting point is the linear Langevin equation describing the dynamics of a Brownian particle:

$$\dot{u} = -\gamma u + \sigma L(t), \quad (\text{B1})$$

where $L(t)$ is a Gaussian white noise, $\gamma > 0$ is a friction constant coefficient, and the (real) parameter σ described the strength of the noise.

We can take the probability density of the velocity field u to be Gaussian with average $\langle u \rangle = 0$ and variance $\langle u^2 \rangle = \beta^{-1}$, with $\beta = \frac{\gamma}{\sigma^2}$, which plays the role of an inverse temperature of the Brownian particle, taken for simplicity to be of unit mass. To get Tsallis statistics, we have to allow for the parameter β to fluctuate, which can be achieved by allowing in whole generality the parameters γ and β to fluctuate as well.

The case studied in [31], and used in our work in this article, is the one in which the parameter β is χ^2 distributed with degree n , i.e. the probability density of β is given by

$$p(\beta) = \frac{1}{\Gamma(\frac{n}{2})} \left(\frac{n}{2\beta_0}\right)^{n/2} \beta^{n/2-1} \exp\left(-\frac{n\beta}{2\beta_0}\right), \quad (\text{B2})$$

with β_0 a constant, playing the role of the average of the β fluctuations.

There are many examples which lead to the distribution of the form (B2), for example [31] if β is given by a sum of squares of independent Gaussian random variables X_i , $i = 1, \dots, n$:

$$\beta = \sum_{i=1}^n X_i^2, \quad \text{with} \quad \langle \beta \rangle = \beta_0, \quad \langle \beta^2 \rangle - \beta_0^2 = \frac{2}{n} \beta_0^2. \quad (\text{B3})$$

If the time scale over which β fluctuates is much larger than the typical time scale of order γ^{-1} that the Langevin system (B1) need to reach equilibrium, then the conditional probability $p(u|\beta)$ that the velocity takes on the value u , given a value of β is Gaussian and reads approximately

$$p(u|\beta) = \sqrt{\frac{\beta}{2\pi}} \exp\left(-\frac{1}{2}\beta u^2\right). \quad (\text{B4})$$

The probability to observe *both* a certain value of u and β reads then

$$p(u, \beta) = p(u|\beta)p(\beta) \quad (\text{B5})$$

and therefore the probability of observing a value u regardless of the value of β is

$$p(u) = \int p(u|\beta)p(\beta)d\beta, \quad (\text{B6})$$

which can be evaluated [31], using (B2), to yield

$$p(u) = \frac{\Gamma(\frac{n}{2} + 1)}{\Gamma(\frac{n}{2})} \left(\frac{\beta_0}{\pi n}\right)^{1/2} \frac{1}{(1 + \frac{\beta_0}{n}u^2)^{(n+1)/2}}. \quad (\text{B7})$$

Expression (B7) coincides (up to irrelevant proportionality constants) with the generalized canonical distribution of nonextensive statistical mechanics of Tsallis [15]:

$$p(u) \propto \frac{1}{(1 + \frac{\tilde{\beta}}{2}(q-1)u^2)^{1/(q-1)}}, \quad (\text{B8})$$

provided one makes the identifications $q = 1 + \frac{2}{n+1}$, and $\tilde{\beta} = \frac{2}{3-q}\beta_0$.

In [31] more general cases have been considered, where the Langevin equation (B1) is extended to include arbitrary frictional forces of the form $F(u) = -\frac{\partial}{\partial u} V(u)$.

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