PHYSICAL REVIEW D **79,** 103528 (2009)

Gauss-Bonnet braneworld cosmological effect on relic density of dark matter

Nobuchika Okada and Satomi Okada

Institute of Particle and Nuclear Studies, High Energy Accelerator Research Organization (KEK), 1-1 Oho, Tsukuba, Ibaraki 305-0801, Japan

(Received 29 March 2009; published 27 May 2009)

In Gauss-Bonnet braneworld cosmology, the Friedmann equation of our four-dimensional Universe on 3-brane is modified in a high energy regime (Gauss-Bonnet regime), while the standard expansion law is reproduced in low energies (standard regime). We investigate the Gauss-Bonnet braneworld cosmological effect on the thermal relic density of cold dark matter when the freeze-out of the dark matter occurs in the Gauss-Bonnet regime. We find that the resultant relic density is considerably reduced when the transition temperature, which connects the Gauss-Bonnet regime with the standard regime, is low enough. This result is in sharp contrast with the result previously obtained in the Randall-Sundrum braneworld cosmology, where the relic density is enhanced.

DOI: 10.1103/PhysRevD.79.103528 PACS numbers: 95.35.+d, 98.80.Cq

I. INTRODUCTION

Recent various cosmological observations, in particular, the WMAP satellite [1], have established the Λ cold dark matter (ΛCDM) cosmological model with a great accuracy, and the relic abundance of the cold dark matter is estimated as (68% C.L. uncertainties)

$$\Omega_{\rm CDM} h^2 = 0.1131 \pm 0.0034. \tag{1}$$

However, clarifying the identity of the dark matter particle is still a prime open problem in particle physics and cosmology. Since the standard model (SM) has no suitable candidate for the cold dark matter, the observation of the dark matter suggests new physics beyond the SM in which the dark matter particle is naturally provided. Many candidates for dark matter have been proposed in various new physics models, for example, the neutralino in supersymmetric models is one of the promising candidates [2].

Among several possibilities, the dark matter as the thermal relic may be the most plausible scenario, since in this case, the relic abundance of the dark matter is insensitive to the history of the early universe before the freeze-out time of the dark matter, such as the mechanism of reheating after inflation etc. This scenario allows us to estimate the dark matter relic density by solving the Boltzmann equation [3],

$$\frac{dY}{dx} = -\frac{s\langle\sigma v\rangle}{xH}(Y^2 - Y_{\rm EQ}^2), \tag{2}$$

where $Y = n/s$ is the yield defined by the ratio of the dark matter number density (n) to the entropy density of the universe $s = 0.439 g_* m^3/x^3$, $g_* \sim 100$ is the effective total number of relativistic degrees of freedom, and $x = m/T$ is the ratio between the dark matter mass (m) and the temperature of the universe (T). The yield in equilibrium $Y_{\rm EQ}$ is written as $Y_{\rm EQ} = 0.145(g/g_*)x^{3/2}e^{-x}$ for $x \gtrsim 3$ with g being the degrees of freedom of the dark matter. In the standard cosmology, the Hubble parameter is described by the total energy density of the universe [$\rho = (\pi^2/30)g_*(m/x)^4$] through the Friedmann equation of the form,

$$H^2 = \frac{8\pi}{3M_{\rm Pl}^2}\rho, \tag{3}$$

where $M_{\rm Pl} = 1.22 \times 10^{19}$ GeV is the Planck mass. Using explicit formulas presented above, the Boltzmann equation can be rewritten into the form

$$\frac{dY}{dx} = -\frac{\lambda}{x^2}\langle\sigma v\rangle(Y^2 - Y_{\rm EQ}^2), \tag{4}$$

with an x-independent constant $\lambda = xs/H = 0.26\sqrt{g_*} m M_{\rm Pl}$.

Solving the Boltzmann equation, we can obtain the thermal relic abundance of the dark matter at the present universe. As is well known, an approximate formula of the solution to the Boltzmann equation (for the S-wave annihilation process) can be described as

$$Y(\infty) \simeq \frac{x_d}{\lambda\langle\sigma v\rangle}, \tag{5}$$

where $x_d = m/T_d$ with the decoupling temperature T_d. It is useful to express the relic density in terms of the ratio of the dark matter density to the critical density, $\Omega h^2 = m s_0 Y(\infty) h^2/\rho_c$, where $\rho_c = 1.1 \times 10^{-5} h^2\ {\rm cm}^{-3}$, $h = 0.71^{+0.04}_{-0.03}$, and $s_0 = 2900\ {\rm cm}^{-3}$ [3]:

$$\Omega h^2 \simeq \frac{1.07 \times 10^9 x_d\ {\rm GeV}^{-1}}{\sqrt{g_*} M_{\rm Pl}\langle\sigma {\rm v}\rangle}. \tag{6}$$

In a typical dark matter scenario such as the weakly interacting massive particle scenario, $x_d \sim 23$ and thus the dark matter relic density is controlled only by its annihilation cross section $\langle\sigma v\rangle$.

Recently, the braneworld models have attracted a great deal of attention as a novel higher dimensional theory. In these models, it is assumed that the standard model particles are confined on a "3-brane," while gravity resides in the whole higher dimensional spacetime. The braneworld cosmology based on the model first proposed by Randall and Sundrum (RS) [4], the so-called RS II model, has been intensively investigated [5]. It has been found that [6] the Friedmann equation in the RS cosmology leads to a nonstandard expansion law in high energies, while the standard cosmology is reproduced in low energies.

Since the Hubble parameter is involved in the Boltzmann equation, the dark matter relic abundance depends on the expansion law of the early universe at freeze-out time. Therefore, if our Universe is higher dimensional and obeys the nonstandard expansion law at the freeze-out time, the resultant relic abundance of the dark matter will be altered. The RS braneworld cosmological effect on the dark matter physics has been investigated in detail [7–10], in particular, it has been shown that the resultant dark matter relic abundance is considerably enhanced. In the same context of the RS braneworld cosmology, other cosmological issues such as leptogenesis [11] and the cosmological gravitino problem in supersymmetric models [12] have also been examined.

In this paper, we investigate the nonstandard cosmological effect on the dark matter relic abundance in the context of the Gauss-Bonnet (GB) braneworld cosmology. Once the Gauss-Bonnet term is added in the RS braneworld model, the Friedmann equation in high energies is modified from the one in the RS braneworld cosmology. We will show that the GB braneworld cosmological effect works to reduce the thermal relic abundance of the dark matter. This is in sharp contrast with the RS braneworld cosmological effect.

II. RELIC DENSITY UNDER THE MODIFIED FRIEDMANN EQUATION

Let us begin with parametrizing a modified Friedmann equation in a certain class of braneworld cosmological models such as

$$H = H_{\rm st} F(x_t/x), \tag{7}$$

where $H_{\rm st}$ is the Hubble parameter in the standard cosmology, and the function F denotes the modification of the Friedmann equation in braneworld models. Phenomenologically viable models should reproduce the standard cosmology in low energies, so that we impose a condition: $F(x_t/x) = 1$ for $x_t/x \leq 1$. Here, $x_t = T_t/m$ and we call T_t "transition temperature" at which the modified expansion law shifts into the standard cosmological one. The model-independent constraints on T_t is given by the success of the big bang nucleosynthesis (BBN) in the standard cosmology, and the transition should complete before the BBN era, $T_t \gtrsim 1$ MeV.[1] For concreteness, we assume

$$F(x_t/x) = \left(\frac{x_t}{x}\right)^{\gamma}, \tag{8}$$

for $x_t/x > 1$ with a constant γ. We can see that this parameterization is in fact a good approximation for Friedmann equations obtained in known braneworld cosmological models. For example, $\gamma = 2$ corresponds to the RS braneworld cosmology, while $\gamma = -2/3$ to the GB braneworld cosmology as we will see later. Here we keep γ as a free parameter to make our discussion applicable to general braneworld models (if such models exist).

First we approximately solve the Boltzmann equation with the modified Friedmann equation in the nonstandard cosmology. Equation (4) is modified into

$$\frac{dY}{dx} = -\frac{\lambda}{x^2}\left(\frac{\langle\sigma v\rangle}{F(x_t/x)}\right)(Y^2 - Y_{\rm EQ}^2), \tag{9}$$

from which we can understand that the effect of the modified Friedmann equation is equivalent to modify the annihilation cross section in the standard cosmology;

$$\langle\sigma v\rangle \to \left(\frac{\langle\sigma v\rangle}{F(x_t/x)}\right) = \langle\sigma v\rangle\left(\frac{x}{x_t}\right)^{\gamma} \tag{10}$$

in the era $x_t/x > 1$ which we are interested in. Equation (6) implies that the braneworld cosmological effect enhances (reduces) the thermal relic abundance of dark matter for $\gamma > 0$ ($\gamma < 0$).

For simplicity, we parameterize the thermal average of the annihilation cross section times the relative velocity as $\langle\sigma v\rangle = \sigma_n x^{-n}$ with a (mass dimension 2) constant σ_n and an integer n ($n = 0$ and 1 corresponds to S-wave and P-wave processes, respectively). At the early time, the dark matter particle is in the thermal equilibrium, and Y tracks $Y_{\rm EQ}$ closely. To begin, consider the small deviation from the thermal distribution $\Delta = Y - Y_{\rm EQ} \ll Y_{\rm EQ}$. The Boltzmann equation leads to

$$\Delta \simeq -\frac{x^2(x_t/x)^{\gamma}\frac{dY_{\rm EQ}}{dx}}{\lambda\sigma_n x^{-n}(2Y_{\rm EQ} + \Delta)} \simeq \frac{x_t^{\gamma}}{2\lambda\sigma_n}x^{2+n-\gamma}, \tag{11}$$

where we have used an approximation formula $dY_{\rm EQ}/dx \simeq -Y_{\rm EQ}$. As the temperature decreases or equivalently x becomes large, the deviation grows since $Y_{\rm EQ}$ is exponentially dumping. Eventually the decoupling occurs at x_d roughly evaluated as $\Delta(x_d) \simeq Y(x_d) \simeq Y_{\rm EQ}(x_d)$. At a fur-

[1]The precision measurements of the gravitational law in the submillimeter range lead to much more stringent constraint, for example, $T_t \gtrsim 1$ TeV for the RS braneworld model [4]. However, this constraint is, in general, quite model dependent and can be moderated in some extended models [13]. In this paper, we consider only the model-independent BBN constraint on the transition temperature.

ther low temperature, $\Delta \simeq Y \gg Y_{\rm EQ}$ is satisfied and the $Y_{\rm EQ}^2$ term in the Boltzmann equation can be neglected so that

$$\frac{d\Delta}{dx} = -\frac{\lambda\sigma_n}{x_t^\gamma} x^{\gamma-n-2}\Delta^2, \tag{12}$$

and the solution is formally given by

$$\frac{1}{\Delta(x)} = \frac{1}{\Delta(x_d)} + \frac{\lambda\sigma_n}{(\gamma-n-1)x_t^\gamma}(x^{\gamma-n-1} - x_d^{\gamma-n-1}). \tag{13}$$

For the S-wave process ($n = 0$), for example, the well-known result in the standard cosmology (corresponding to $\gamma = 0$), $1/Y(\infty) \simeq \lambda\sigma_0/x_d$, is obtained. When we take $\gamma = 2$, our analysis here is the same as the one in [7] for the RS braneworld cosmology. For $n = 0$, for example, $\Delta(x)^{-1}$ is continuously growing, and this growth stops at $x = x_t$, so the resultant relic abundance has been found to be $1/Y(\infty) \simeq \lambda\sigma_0 x_t$. Then, we obtain the ratio of the energy density of the dark matter in the RS braneworld cosmology ($\Omega_{\rm (RS)}$) to the one in the standard cosmology ($\Omega_{(s)}$) such that

$$\frac{\Omega_{\rm (RS)}}{\Omega_{(s)}} \simeq \left(\frac{x_t}{x_{d(s)}}\right), \tag{14}$$

where $x_{d(s)}$ is the decoupling temperature in the standard cosmology. The relic abundance is enhanced by the RS braneworld effect for $x_t > x_{d(s)}$, while it should saturate to the standard result for $x_t < x_{d(s)}$.

On the other hand, if $\gamma < 0$, we arrived at the result which is in sharp contrast with the one in the RS braneworld cosmology. For simplicity, we consider only $n = 0$ in the following equation, and the analysis for $n > 0$ is straightforward. In this case, we obtain

$$\frac{1}{Y(\infty)} \simeq \frac{\lambda\sigma_0}{1-\gamma}\left(\frac{x_t}{x_d}\right)^{-\gamma} x_d^{-1}, \tag{15}$$

and the ratio of the energy density of the dark matter in this braneworld cosmology ($\Omega_{(b)}$) to the one in the standard cosmology ($\Omega_{(s)}$) is evaluated as

$$\frac{\Omega_{(b)}}{\Omega_{(s)}} \simeq (1-\gamma)\left(\frac{x_d}{x_t}\right)^{-\gamma}\left(\frac{x_d}{x_{d(s)}}\right). \tag{16}$$

Thus, the resultant relic density is reduced in the case for $\gamma < 0$ and $x_d < x_t$. This is nothing but the case that happens in the GB braneworld cosmology.

III. GAUSS-BONNET BRANEWORLD COSMOLOGICAL EFFECT

Motivated by string theory considerations, it is a natural extension to add higher curvature terms to the bulk gravity action in the RS braneworld model [14]. Among possible terms, the Gauss-Bonnet invariant is of particular interest in five dimensions, since it is a unique nonlinear term in curvature which yields second order gravitational field equations. The five-dimensional gravitational action with the GB invariant is given by

$$\mathcal{S} = \frac{1}{2\kappa_5^2}\int d^5x\sqrt{-g_5}[-2\Lambda_5 + \mathcal{R} + \alpha(\mathcal{R}^2 - 4\mathcal{R}_{ab}\mathcal{R}^{ab} + \mathcal{R}_{abcd}\mathcal{R}^{abcd})] - \int_{\rm brane} d^4x\sqrt{-g_4}(\sigma + \mathcal{L}_{\rm matter}), \tag{17}$$

where $\kappa_5^2 = 8\pi/M_5^3$ with the five-dimensional Planck scale M_5, $\sigma > 0$ is the brane tension, and $\Lambda_5 < 0$ is the bulk cosmological constant. The RS model is recovered in the limit $\alpha \to 0$.

Imposing a Z_2 parity across the brane in an anti-de Sitter bulk and modeling the matters on the brane as a perfect fluid, the modified Friedmann equation on the spatially flat brane has been found as [15,16]

$$\kappa_5^2(\rho + m_\sigma^4) = 2\mu\sqrt{1+\frac{H^2}{\mu^2}}\left(3 - \beta + 2\beta\frac{H^2}{\mu^2}\right), \tag{18}$$

where $\beta = 4\alpha\mu^2 = 1 - \sqrt{1 + 4\alpha\Lambda_5/3}$ and $m_\sigma = \sigma^{1/4}$. There are four free parameters, κ_5, m_σ, μ, and β, corresponding to the original free parameters, κ_5, σ, Λ_5, and α, which are constrained by phenomenological requirements. To reproduce the Friedmann equation of the standard cosmology with zero cosmological constant in the limit $H^2/\mu^2 \ll 1$, we find two relations among the parameters:

$$\kappa_5^2 m_\sigma^4 = 2\mu(3-\beta), \qquad \kappa_4^2 = \frac{8\pi}{M_{\rm Pl}^2} = \frac{\mu}{1+\beta}\kappa_5^2. \tag{19}$$

The modified Friedmann equation can be rewritten in the useful form [17]

$$H^2 = \frac{\mu^2}{\beta}\left[(1-\beta)\cosh\left(\frac{2\chi}{3}\right) - 1\right],$$
$$\rho + m_\sigma^4 = m_\alpha^4 \sinh\chi, \tag{20}$$

where χ is a dimensionless measure of the energy density, and

$$m_\alpha^4 = \sqrt{\frac{8\mu^2(1-\beta)^3}{\beta\kappa_5^4}} = 2\frac{\mu^2}{\kappa_4^2}\sqrt{2\frac{(1-\beta)^3}{\beta(1+\beta)^2}}. \tag{21}$$

Here we have used Eq. (19) to eliminate κ_5 in the last equality. In the same way, we express m_σ as

$$m_\sigma^4 = 2\frac{\mu^2}{\kappa_4^2}\left(\frac{3-\beta}{1+\beta}\right). \tag{22}$$

The evolution of the GB braneworld cosmology is characterized by the two mass scales, m_α and m_σ. Expanding Eq. (20) with respect to χ, we find three regimes for $m_\alpha > m_\sigma$: the GB regime for $\rho \gg m_\alpha^4$,

$$H^2 \simeq \left(\frac{1+\beta}{4\beta}\mu\kappa_4^2\rho\right)^{2/3}, \tag{23}$$

the RS regime for $m_\alpha^4 \gg \rho \gg m_\sigma^4$,

$$H^2 \simeq \frac{\kappa_4^2}{6m_\sigma^4}\rho^2, \tag{24}$$

and the standard regime for $m_\sigma^4 \gg \rho$,

$$H^2 \simeq \frac{\kappa_4^2}{3}\rho. \tag{25}$$

Since we are interested in the GB regime, let us simplify the evolution of the universe by imposing the condition $m_\alpha = m_\sigma$, which leads to

$$3\beta^3 - 12\beta^2 + 15\beta - 2 = 0 \tag{26}$$

and hence, $\beta = 0.151$. In this case, the RS regime is collapsed and there are only two regimes in the evolution of the universe. Applying the parameterization to the nonstandard Friedmann equation in Eqs. (7) and (8),

$$H = H_{\rm st}\left(\frac{\rho_t}{\rho}\right)^{1/6} = H_{\rm st}\left(\frac{x}{x_t}\right)^{2/3}, \tag{27}$$

for $\rho > \rho_t$ or equivalently $x < x_t$, while $H = H_{\rm st}$ for $\rho < \rho_t$, where

$$\rho_t = \frac{27}{16}\left(\frac{1+\beta}{\beta}\right)^2\frac{\mu^2}{\kappa_4^2} \simeq 3.9\mu^2 M_{\rm Pl}^2. \tag{28}$$

In Fig. 1, we show that our approximation for the Friedmann equation is in fact a good approximation to the exact form in Eq. (20).

Now we are ready to see the GB braneworld cosmological effect on the dark matter relic abundance. Equation (27) means that the modified Friedmann equation in the GB braneworld cosmology corresponds to $\gamma = -2/3$, and thus

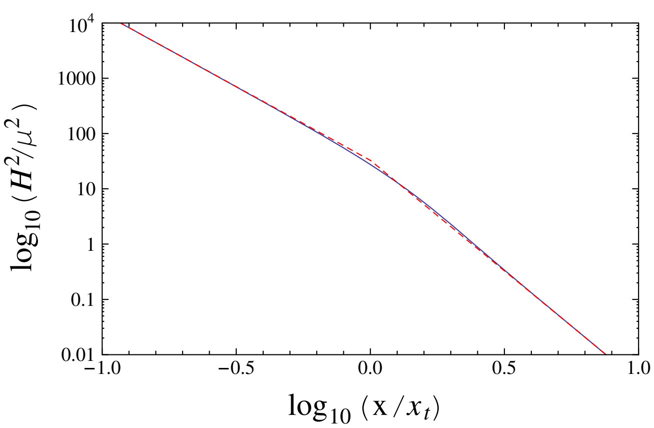

FIG. 1 (color online). The Hubble parameter as a function of ρ in the GB braneworld cosmology. The solid line shows the exact formula of the Friedmann equation, while the dashed line (in red) corresponds to our approximation formula. Here, we have taken a unit $\kappa_4 = 1$.

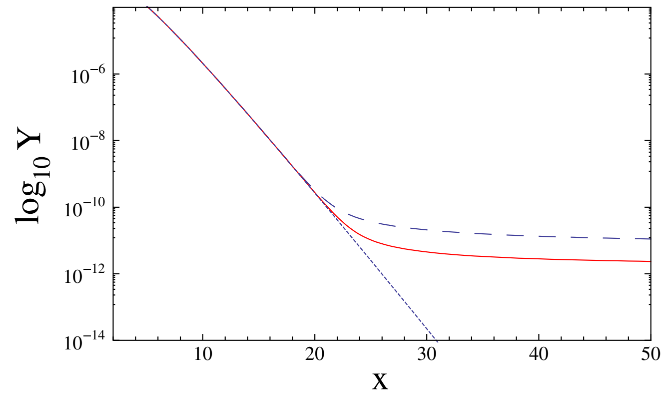

FIG. 2 (color online). Numerical solutions of the Boltzmann equation in the Gauss-Bonnet braneworld cosmology (solid line) and in the standard cosmology (dashed line). The dotted line corresponds to $Y_{\rm EQ}$. The GB braneworld cosmological effect reduces the resultant relic abundance from the one in the standard cosmology.

we find

$$\frac{\Omega_{\rm (GB)}}{\Omega_{(s)}} \simeq \frac{5}{3}\left(\frac{x_t}{x_d}\right)^{2/3}\left(\frac{x_d}{x_{d(s)}}\right) \tag{29}$$

from Eq. (16). Therefore, the GB braneworld cosmological effect reduces the relic density, and the reduction rate is controlled by the transition temperature. This is our main result. For the weakly interacting massive particle dark matter, the typical value of the decoupling temperature is $x_d \sim 23$, and this is not changing so much even under the nonstandard Friedmann equation. Thus, we expect

$$\frac{\Omega_{\rm (GB)}}{\Omega_{(s)}} \simeq \frac{5}{3}\left(\frac{x_t}{23}\right)^{2/3}. \tag{30}$$

For example, $\Omega_{\rm (GB)}/\Omega_{(s)} \simeq 0.25$ for $x_t = 400$.

Finally, let us check that our analytic result given above is a good approximation for the results from the numerical solution of the Boltzmann equation. Fixing the dark matter mass $m = 100$ GeV, $\langle\sigma v\rangle = 10^{-5}/m^2$, and $x_t = 400$, we numerically solve the Boltzmann equation with the Friedmann equations in the standard cosmology and the GB braneworld cosmology, respectively. The numerical results are depicted in Fig. 2. Here, we obtain $\Omega_{\rm (GB)}/\Omega_{(s)} \simeq 0.26$, which is very close to our previous result from analytic formulas.

IV. CONCLUSIONS AND DISCUSSIONS

We have investigated the thermal relic density of the cold dark matter in the braneworld cosmology, in particular, the Gauss-Bonnet braneworld cosmology, which is a natural extension of the Randall-Sundrum braneworld model to include the higher curvature terms. We have modeled the modified Friedmann equation in such a way applicable to general braneworld cosmological models and analytically solved the Boltzmann equation under some approximation. Applying this result to the Gauss-Bonnet braneworld cosmology, we have found that the resultant relic density of the dark matter is considerably reduced from the one in the standard cosmology, when the freeze-out occurs well in the Gauss-Bonnet regime in the evolution of the universe. This conclusion is in sharp contrast with the result in the Randall-Sundrum braneworld cosmology, where the relic density is enhanced by the braneworld cosmological effect.

It is worth applying our results in this paper to concrete dark matter models which have been investigated in the standard cosmology. For supersymmetric models with the neutralino dark matter, the RS braneworld cosmological effect was analyzed [8]. It has been shown that the allowed parameter region for the neutralino dark matter consistent with the observed dark matter density is dramatically modified from the one in the standard cosmology and eventually disappears as the transition temperature is lowered. This is because the RS braneworld cosmological effect enhances the dark matter relic density while supersymmetric models, in particular, the constrained minimal supersymmetric SM, tend to predict an overabundance of neutralino dark matter. On the other hand, the Gauss-Bonnet braneworld cosmological effect reduces the relic density of the dark matter and therefore can enlarge the allowed parameter region in supersymmetric models. A similar effect has been discussed in the scalar-tensor cosmology [18]. This enlargement of the cosmologically allowed parameter region has an impact on the sparticle search at the LHC. As investigated in [11,12] for the RS braneworld cosmology, it would be interesting to consider other cosmological issues also in the GB braneworld cosmology. We leave these subjectsto future works.

ACKNOWLEDGMENTS

The authors would like to thank Shinji Komine for collaboration in the early stage of this work. We are grateful to Osamu Seto for carefully reading the manuscript and useful comments. We also wish to thank Andy Okada for his encouragement. This work of N. O. is supported in part by the Grant-in-Aid for Scientific Research from the Ministry of Education, Science and Culture of Japan, Grant No. 18740170.

[1] C.L. Bennett *et al.*, Astrophys. J. Suppl. Ser. **148**, 1 (2003); D.N. Spergel *et al.*, Astrophys. J. Suppl. Ser. **148**, 175 (2003); G. Hinshaw *et al.* (WMAP Collaboration), Astrophys. J. Suppl. Ser. **180**, 225 (2009).
[2] For a review, see, e.g., G. Bertone, D. Hooper, and J. Silk, Phys. Rep. **405**, 279 (2005), and references therein.
[3] See, e.g., E.W. Kolb and M.S. Turner, *The Early Universe* (Addison-Wesley, Reading, MA, 1990).
[4] L. Randall and R. Sundrum, Phys. Rev. Lett. **83**, 4690 (1999).
[5] For a review, see, e.g., D. Langlois, Prog. Theor. Phys. Suppl. **148**, 181 (2002), and references therein.
[6] P. Binetruy, C. Deffayet, and D. Langlois, Nucl. Phys. **B565**, 269 (2000); P. Binetruy, C. Deffayet, U. Ellwanger, and D. Langlois, Phys. Lett. B **477**, 285 (2000); T. Shiromizu, K. i. Maeda, and M. Sasaki, Phys. Rev. D **62**, 024012 (2000); D. Ida, J. High Energy Phys. 09 (2000) 014.
[7] N. Okada and O. Seto, Phys. Rev. D **70**, 083531 (2004).
[8] T. Nihei, N. Okada, and O. Seto, Phys. Rev. D **71**, 063535 (2005); **73**, 063518 (2006).
[9] E. Abou El Dahab and S. Khalil, J. High Energy Phys. 09 (2006) 042.
[10] G. Panotopoulos, J. Cosmol. Astropart. Phys. 05 (2007) 016; N. Okada and O. Seto, Phys. Rev. D **77**, 123505 (2008); J.U. Kang and G. Panotopoulos, J. High Energy Phys. 05 (2008) 036.
[11] N. Okada and O. Seto, Phys. Rev. D **73**, 063505 (2006); M.C. Bento, R. Gonzalez Felipe, and N.M.C. Santos, Phys. Rev. D **73**, 023506 (2006).
[12] N. Okada and O. Seto, Phys. Rev. D **71**, 023517 (2005); E.J. Copeland and O. Seto, Phys. Rev. D **72**, 023506 (2005); N.M.C. Santos, arXiv:hep-ph/0702200.
[13] K. i. Maeda and D. Wands, Phys. Rev. D **62**, 124009 (2000).
[14] J.E. Kim, B. Kyae, and H.M. Lee, Phys. Rev. D **62**, 045013 (2000); Nucl. Phys. **B582**, 296 (2000); **B591**, 587(E) (2000); S. Nojiri, S.D. Odintsov, and S. Ogushi, Int. J. Mod. Phys. A **17**, 4809 (2002); J.E. Lidsey, S. Nojiri, and S.D. Odintsov, J. High Energy Phys. 06 (2002) 026.
[15] C. Charmousis and J.F. Dufaux, Classical Quantum Gravity **19**, 4671 (2002).
[16] K. i. Maeda and T. Torii, Phys. Rev. D **69**, 024002 (2004).
[17] J.E. Lidsey and N.J. Nunes, Phys. Rev. D **67**, 103510 (2003).
[18] R. Catena, N. Fornengo, A. Masiero, M. Pietroni, and M. Schelke, J. High Energy Phys. 10 (2008) 003.