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Does bulk viscosity create a viable unified dark matter model?

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We investigate in detail the possibility that a single imperfect fluid with bulk viscosity can replace the need for separate dark matter and dark energy in cosmological models. With suitable choices of model parameters, we show that the background cosmology in this model can mimic that of a ΛCDM universe to high precision. However, as the cosmic expansion goes through the decelerating-accelerating transition, the density perturbations in this fluid are rapidly damped out. We show that, although this does not significantly affect structure formation in baryonic matter, it makes the gravitational potential decay rapidly at late times, leading to modifications in predictions of cosmological observables such as the CMB power spectrum and weak lensing. This model of unified dark matter is thus difficult to reconcile with astronomical observations. We also clarify the differences with respect to other unified dark matter models where the fluid is barotropic, i.e., $p = p(\rho)$, such as the (generalized) Chaplygin gas model, and point out their observational difficulties. We also summarize the status of dark sector models with no new dynamical degrees of freedom introduced and discuss the problems with them.

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I. INTRODUCTION

In recent years the cosmological picture that over 95% of the energy in our universe is contributed by a dark sector has been supported independently by a number of observations, notably those of type Ia supernovae (SN) luminosity distances, cosmic microwave background (CMB) anisotropies, the power spectrum of clustered matter, and weak lensing [1–4]. This dark sector can be further subdivided into dark matter and dark energy according to their different gravitational properties. The concordance ΛCDM paradigm, in which dark matter is assumed to be weakly (or just gravitationally) interacting massive particles (the cold dark matter), and dark energy is a positive cosmological constant (Λ) or slowly varying scalar field, has been successful in confronting all these observational data sets. However, the smallness of the cosmological constant, and the fact that it only becomes dominant recently make this model conceptually unattractive and stimulates the examination of new models (for a review see [5]).

Since both cold dark matter (CDM) and dark energy are invisible and have as yet unknown origins, it is natural to consider the possibility that they are actually not two exotic matter species but just different aspects of a single fluid. These scenarios are frequently dubbed unified dark matter (UDM) models. In the context of general relativity they assume an equation of state $p = p(\rho)$ or $p = p(H)$, where H is the Hubble rate; in universes with zero spatial curvature these prescriptions are identical. A particular class of cosmology with this equation of state was investigated in the context of studies of cosmological bulk viscosity [6–11] in which the viscosity coefficient is $\eta(\rho) = \alpha\rho^m$ and the effective pressure in the Friedmann equation is $p' = (\gamma - 1)\rho - 3H\eta(\rho)$, and of string production effects [12] which mimic the effects of bulk viscosity of this form with $m = 3/2$ [7].

The flat bulk viscous cosmologies also include as subcases the so-called Chaplygin gas model ($p = -A\rho^{-\alpha}$) [13–15], which is just a bulk viscosity for dust ($\gamma = 1$) with $m + 1/2 \equiv -\alpha$, and its generalizations to $\rho + p = B\rho^\lambda$ [13], which is just $\gamma = 0$ and $m + 1/2 \equiv \lambda$, or other functional forms $p(\rho)$ [5]. The Chaplygin gas models are simple, with no new dynamical degrees of freedom, and yet produce an interesting time evolution for the dark energy. However, it was shown subsequently that the density perturbation of this fluid will either blow up or experience rapid oscillatory damping at late times, so the models can be stringently constrained by the matter power spectrum [16]. This distinctive behavior arises because there is a minimum possible total density of the universe at late times.

There have been some explicit investigations in the bulk viscous cosmology in connection with the dark energy problem. Fabris *et al.* [17,18] investigated the case in which the viscosity coefficient has the form $\eta(\rho) = \alpha\rho^m$ and considered both background and perturbed evolutions of a universe dominated by viscous matter. However, these studies were limited by certain simplifications. In Ref. [17], for example, the authors obtained an analytical solution for ρ_v (where the subscript v denotes viscous matter) as a function of scale factor a, under the assumption that no other matter species exist in the universe (the same assumption as [7]). In Ref. [18] the authors added a baryonic matter species and estimated the cosmological parameters in detail using Bayesian statistics; but their calculation was largely confined to the special case $m = 0$ and used only supernovae data to derive the constraints.

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We believe that such an analysis needs to be generalized. First, as will be seen below, $m = 0$ is not necessarily the best fit for these models, even for the background cosmology. Second, no radiation component was included in previous analysis. This is a reasonable simplification as long as we are only concerned with the late-time cosmic expansion and only using the supernova data. But when observables are also related to the high-redshift features, such as the CMB shift parameter, the radiation must be taken into account. In fact, the properties of the viscosity could significantly modify the early-time evolution of the universe if the late-time evolution is consistent with the supernova data, and so the CMB shift parameter could be effective in constraining the model. Third, a detailed study of the linear perturbations in the model is needed as a critical check of the model's feasibility. As we will see below, the UDM model based on viscous matter has several distinctive predictions regarding the perturbations when compared to the ΛCDM paradigm or the Chaplygin gas model. Such a feature is very general, making the model barely compatible with observations. This indicates that a UDM model without new dynamical degrees of freedom is unlikely to be observationally acceptable.

The rest of this paper is organized as follows. In Sec. II, we give the basic field equations which will be used in the subsequent analysis. In Sec. III we consider the background evolution of a universe dominated by a viscous matter (in the presence of normal matter species such as baryons, photons, and neutrinos). Using the SN data and CMB shift parameter, we find the best-fit model parameters and show that these give a background cosmology very similar to the prediction of the ΛCDM paradigm. Section IV is devoted to the perturbed evolution of general bulk viscosity models, which we find to behave very differently from both ΛCDM and other UDM models. Finally, we discuss our results and conclude in Sec. V. We will frequently call the UDM fluid with $p = p(\rho, H)$ "the viscous dark matter."

II. THE FIELD EQUATIONS

In this section we list the general field equations that govern the evolution of the cosmological background and its first-order perturbations in general relativity, which will be used in later sections. The perturbation equations will be given in the covariant and gauge invariant formalism, using the method of 3 + 1 decomposition.

The main idea of 3 + 1 decomposition is to make spacetime splits of physical quantities with respect to the 4-velocity u^a of an observer. The projection tensor h_{ab} is defined as $h_{ab} = g_{ab} - u_a u_b$ and can be used to obtain covariant tensors perpendicular to u. For example, the covariant spatial derivative $\hat{\nabla}$ of a tensor field $T^{b\cdots c}_{d\cdots e}$ is defined as

$$\hat{\nabla}^a T^{b\cdots c}_{d\cdots e} \equiv h^a_i h^b_j \cdots h^c_k h^r_d \cdots h^s_e \nabla^i T^{j\cdots k}_{r\cdots s}. \tag{1}$$

The energy-momentum tensor and covariant derivative of the 4-velocity are decomposed, respectively, as

$$T_{ab} = \pi_{ab} + 2q_{(a}u_{b)} + \rho u_a u_b - p h_{ab}, \tag{2}$$

$$\nabla_a u_b = \sigma_{ab} + \varpi_{ab} + \tfrac{1}{3}\theta h_{ab} + u_a A_b. \tag{3}$$

In the above, π_{ab} is the projected symmetric trace-free (PSTF) anisotropic stress, q_a the vector heat flux vector, p the isotropic pressure, σ_{ab} the PSTF shear tensor, $\varpi_{ab} = \hat{\nabla}_{[a}u_{b]}$, the vorticity, $\theta = \nabla^c u_c \equiv 3\dot{a}/a$ (a is the mean expansion scale factor) the expansion scalar, and $A_b = \dot{u}_b$ the acceleration; the overdot denotes time derivative expressed as $\dot{\phi} = u^a \nabla_a \phi$, brackets mean antisymmetrization, and parentheses symmetrization. The 4-velocity normalization is chosen to be $u^a u_a = 1$. The quantities π_{ab}, q_a, ρ, p are referred to as *dynamical* quantities and σ_{ab}, ϖ_{ab}, θ, A_a as *kinematical* quantities. Note that the dynamical quantities can be obtained from the energy-momentum tensor T_{ab} through the relations

$$\begin{aligned} \rho &= T_{ab}u^a u^b, & p &= -\tfrac{1}{3}h^{ab}T_{ab}, \\ q_a &= h^d_a u^c T_{cd}, & \pi_{ab} &= h^c_a h^d_b T_{cd} + p h_{ab}. \end{aligned} \tag{4}$$

Decomposing the Riemann tensor and making use of the Einstein equations, we obtain, after linearization, a set of propagation and constraint equations governing the evolution of perturbed physical quantities. Here we shall only list the equations that will be used in later sections, and for more details we refer the reader to [19].

The first equation we will use is the Raychaudhuri equation:

$$\dot{\theta} + \frac{1}{3}\theta^2 - \hat{\nabla}^a A_a + \frac{\kappa}{2}(\rho + 3p) = 0. \tag{5}$$

The second equation to be used involves the projected Ricci scalar $\hat{R}$ into the hypersurfaces orthogonal to u^a, which can be expressed as

$$\hat{R} \doteq 2\kappa\rho - \tfrac{2}{3}\theta^2. \tag{6}$$

Since we are considering a spatially flat universe, the spatial curvature vanishes (at background level) on large scales and so $\hat{R} = 0$. Thus, from Eq. (6), we obtain

$$\tfrac{1}{3}\theta^2 = \kappa\rho, \tag{7}$$

which is the Friedmann equation that governs the expansion of the universe in standard general relativity.

Furthermore, we will need the conservation equations for the energy-momentum tensor,

$$\dot{\rho} + (\rho + p)\theta + \hat{\nabla}^a q_a = 0, \tag{8}$$

$$\dot{q}_a + \tfrac{4}{3}\theta q_a + (\rho + p)A_a - \hat{\nabla}_a p + \hat{\nabla}^b \pi_{ab} = 0. \tag{9}$$

Equations (5), (8), and (9) involve both background and first-order perturbed quantities (such as A_a, q_a). To obtain equations for the background cosmology it is sufficient to neglect all first-order terms, while to obtain corresponding equations for the perturbation evolution we need to take the spatial derivatives $\hat{\nabla}_a$ [cf. Equation (1)] of these equations. For the perturbation analysis it is then more convenient to work in the k space because we shall confine ourselves to the linear regime where different k modes decouple. Following [19], we make the following harmonic expansions of our perturbation variables:

$$\hat{\nabla}_a\rho = \sum_k \frac{k}{a}\mathcal{X}Q_a^k \qquad \hat{\nabla}_a p = \sum_k \frac{k}{a}\mathcal{X}^p Q_a^k$$
$$q_a = \sum_k qQ_a^k \qquad \pi_{ab} = \sum_k \Pi Q_{ab}^k$$
$$\hat{\nabla}_a\theta = \sum_k \frac{k^2}{a^2}\mathcal{Z}Q_a^k \qquad A_a = \sum_k \frac{k}{a}AQ_a^k$$

in which Q^k is the eigenfunction of the comoving spatial Laplacian $a^2\hat{\nabla}^2$ satisfying

$$\hat{\nabla}^2 Q^k = \frac{k^2}{a^2}Q^k, \qquad (10)$$

and Q_a^k, Q_{ab}^k are given by $Q_a^k = \frac{a}{k}\hat{\nabla}_a Q^k$, $Q_{ab}^k = \frac{a}{k}\hat{\nabla}_{\langle a}Q_{b\rangle}^k$.

The perturbed version of Eq. (8) is

$$\dot{\mathcal{X}}_a + (\rho + p)(\mathcal{Z}_a - a\theta A_a) + (\mathcal{X}_a + \mathcal{X}_a^p)\theta + a\hat{\nabla}_a\hat{\nabla}^b q_b = 0. \qquad (11)$$

In Sec. IV we shall use the harmonic expansion coefficients of Eqs. (5), (9), and (11) to derive the evolution equation for the density perturbation of the viscous dark matter.

III. THE BACKGROUND EVOLUTION

The field equations governing the background cosmic expansion are the Friedmann equation, the Raychaudhuri equation, and the conservation equations for energy densities of the different matter species (baryons, radiation, and viscous dark matter). Not all these equations are independent, and we choose the Friedmann equation (here $H = \dot{a}/a = \theta/3$),

$$3H^2 = 8\pi G(\rho_D + \rho_B + \rho_R), \qquad (12)$$

and the conservation equations,

$$\dot{\rho}_B + 3H\rho_B = 0, \qquad (13)$$

$$\dot{\rho}_R + 4H\rho_R = 0, \qquad (14)$$

$$\dot{\rho}_D + 3H(\rho_D + p_D) = 0, \qquad (15)$$

as our starting point. In these equations ρ_B, ρ_R, and ρ_D are, respectively, the energy densities of baryonic matter, radiation, and the (viscous) dark matter. The pressure $p_D = (\gamma - 1)\rho_D - 3\alpha H\rho_D^m$ is the effective pressure of the dark matter, with γ, α, m being our free model parameters.

We shall not follow [18] by dividing ρ_D into different components with different equation of states (EoS), although it can be useful mathematically, since that might hide the fact that there is only a single fluid with varying EoS. When it comes to the perturbative evolution of this fluid, it will be misleading if one thinks that part of the fluid behaves as a cosmological constant which has no perturbation, while another part simply clusters as CDM, there being *no* interactions between them. We also note that the Hubble expansion rate H appears in the expression of p_D, which means that the EoS of the viscous dark matter depends also on the existence and properties of other matter species, and so we cannot neglect the effects of baryonic matter and radiation (at early times).

Next, we estimate the free model parameters γ, α, and m. We will set $\gamma = 1$ in all the calculations below because we want the viscous dark matter to behave like CDM at (early) times when the correction term is not important. For m, earlier studies [7] showed that when $m > 1/2$ the universe will start from a de Sitter phase and finally evolve towards power-law perfect-fluid dominated expansion, while if $m < 1/2$ it is just the opposite, with late-time approach to de Sitter evolution; the special $m = 1/2$ case corresponds to power-law evolution throughout. In fact, from the expression $p_D = -3\alpha H\rho_D^m$ we can see that if ρ_D dominates over other matter species (so that $H \propto \rho_D^{1/2}$), then p_D will be a constant if $m = -1/2$. We can estimate that the best-fit value of m is around $-1/2$. In what follows, m will be taken as a free parameter to be constrained by data. Finally, α is obviously a dimensional constant. To determine its value, we note that to explain the SNe data we need the candidate for dark energy to contribute an effective energy density and pressure of the same order as ρ_Λ in the ΛCDM model. Thus, we have

$$\rho_{D0} \simeq \rho_{CDM0} + \rho_\Lambda, \qquad 3\alpha H_0\rho_{D0}^m \simeq \rho_\Lambda$$

in which the quantities on the left-hand sides are for our model while those on the right-hand sides are for the ΛCDM model. A subscript 0 here denotes the present value of a quantity. Using the results $\Omega_{CDM} \simeq 0.20$ and $\Omega_\Lambda \simeq 0.76$, we calculate from the above two equations that $\beta \equiv \alpha H_0\rho_{DM0}^{m-1} \simeq 0.26$. This gives us a sense about the magnitude of the dimensionless quantity β, which is chosen as another model parameter instead of α and will be constrained below.

With the above preliminaries, it is now straightforward to rewrite the conservation equation of the viscous dark matter, Eq. (15), as

$$\varrho^* + 3\gamma\varrho = 9\beta\varrho^m\left[\frac{\varrho + r_{bd}e^{-3N} + r_{rd}e^{-4N}}{1 + r_{bd} + r_{rd}}\right]^{1/2}, \qquad (16)$$

where we have defined $\varrho \equiv \rho_{DM}/\rho_{DM0}$, $r_{bd} \equiv \rho_{B0}/\rho_{DM0}$, $r_{rd} \equiv \rho_{R0}/\rho_{DM0}$, and also used the Friedmann equation (12) to substitute for the Hubble parameter, H. The star denotes a derivative with respect to $N = \log a$. Because there are in general no closed-form solutions to these cosmological equations, we will use Eq. (16), together with Eqs. (12)–(14), in our subsequent numerical calculation. Note that r_{bd} and r_{rd} are constants which are fixed once Ω_B and Ω_R are known (using the fact that the universe is spatially flat, so that $\Omega_{DM} = 1 - \Omega_B - \Omega_R$): $\Omega_{DM} = 1 - \Omega_B - \Omega_R$):

$$r_{bd} = \frac{\Omega_B}{1 - \Omega_B - \Omega_R}, \qquad r_{rd} = \frac{\Omega_R}{1 - \Omega_B - \Omega_R}. \tag{17}$$

A natural choice of the initial (final) condition of Eq. (16) is $\varrho(N = 0) = 1$.

We have used the supernovae luminosity distance data [2] and CMB shift parameter [20] to constrain the model parameters m and β, analytically marginalizing the current Hubble expansion rate H_0 and assuming the baryon density today is fixed by big bang nucleosynthesis and measurements of the light element abundances. The result is shown in Fig. 1 and the best-fit parameters we find are $(m, \beta) = (-0.4, 0.236)$ which lie close to our estimate above ($m = -0.5$, $\beta = 0.264$). Figure 2 shows the cosmic evolution of the fractional energy densities in the bulk viscous model with the above best-fit parameters. It can be seen there that the viscous model mimics the concordance ΛCDM paradigm extremely well all through its cosmic history. The model therefore appears to work well as a description of the background cosmology.

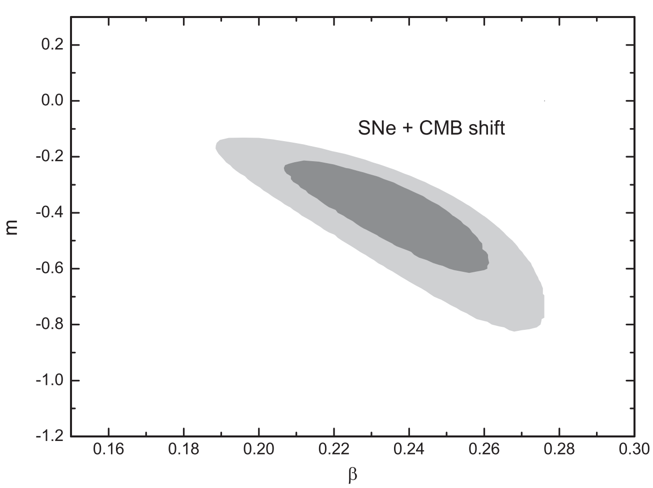

FIG. 1. The joint constraints on m and β from supernovae and CMB shift-parameter data. The dark grey and light grey areas denote the 68% and 95% confidence regions, respectively. The current Hubble expansion rate H_0 is marginalized over analytically and the other parameters used are $\Omega_R = 8.475 \times 10^{-5}$ and $\Omega_R + \Omega_B = 0.04$ evaluated at the present time.

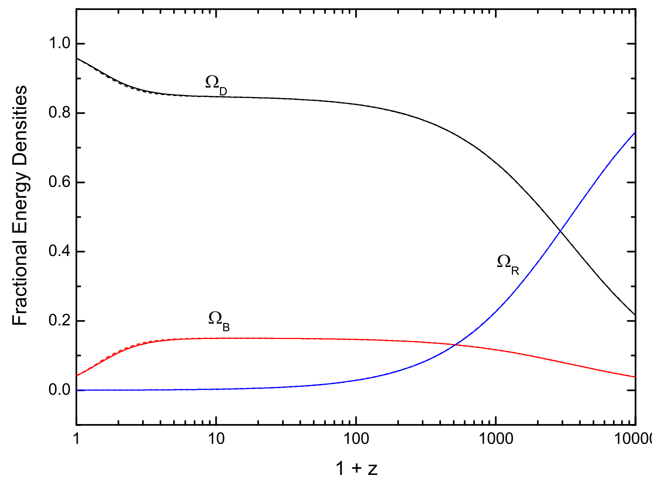

FIG. 2 (color online). Solid lines: The evolution of the fractional energy densities of viscous dark matter Ω_D (black), baryons Ω_B (red), and radiation (including photons and massless neutrinos) Ω_R (blue), versus $1 + z$ where z is the redshift. The model parameters here are chosen as $m = -0.4$ and $\beta = 0.236$. Dashed lines: The same evolutions for the concordance ΛCDM model. Here, Ω_D denotes the fractional energy density of the dark sector, i.e., dark energy (a cosmological constant) plus cold dark matter. The other parameters used (for both models) are $\Omega_R = 8.475 \times 10^{-5}$ and $\Omega_R + \Omega_B = 0.04$, evaluated at the present time.

IV. THE EVOLUTION OF FIRST-ORDER DENSITY PERTURBATIONS

Despite the excellent coincidence between the viscous dark matter and ΛCDM models for the background evolution found in the last section, we should also investigate the formation of large-scale structure to test whether the viscous model is also a feasible model for dark energy. In this section we show that, generally, a bulk viscosity depending on the energy density, i.e., $p_D = -\eta(\rho_D)\nabla^a u_a$, will significantly influence the formation and evolution of large-scale cosmological structure. This very different prediction from that of the ΛCDM model indicates that stringent constraints can be placed on the viability of the bulk viscosity models using observational data on the CMB spectrum, matter power spectrum, and weak gravitational lensing.

Let us first concentrate on the special case considered above, with $p_D = -\alpha\rho_D^m\nabla^a u_a = -3\alpha H\rho_D^m$. For an observer comoving with dark matter particles (with 4-velocity u_a), the energy-momentum tensor could be written as

$$T_{ab} = \rho_D u_a u_b - p_D(g_{ab} - u_a u_b). \tag{18}$$

In the ΛCDM paradigm, if one chooses the observer to be comoving with the dark matter particles as above, then obviously the peculiar velocity is zero, $v_D = 0$, which implies, by the conservation of energy-momentum tensor, that $A_a = 0$ for this observer, and there is no acceleration.

In the bulk viscous model, however, $v_D = 0$ and $A_a = 0$ are different choices of frame which can be used in numerical calculations. Here, for convenience, we will choose the $v_D = 0$ frame, in which, again from the conservation of energy, $A_a \neq 0$, which is easy to understand because the dark matter particles themselves have interactions (the pressure p_D) and cannot be acceleration free.

Taking the spatial derivative of p_D and picking the harmonic coefficients, we obtain

$$\mathcal{X}^p_D = m p_D \Delta_D - \alpha \rho_D^m \frac{k}{a} \mathcal{Z} = p_D \left(m\Delta_D + \frac{k\mathcal{Z}}{3\mathcal{H}} \right), \quad (19)$$

where $\mathcal{H} = a'/a$ with $' \equiv d/d\tau$ (τ is the conformal time defined by $ad\tau = dt$) and $\Delta_D = \mathcal{X}_D/\rho_D$ is the density contrast of the viscous dark matter.

As the anisotropic stress vanishes up to first order in perturbations, from Eq. (9) we obtain

$$(\rho_D + p_D)A = \mathcal{X}^p_D. \quad (20)$$

Similarly, from Eq. (11) we have

$$\Delta'_D + (1 + w)k\mathcal{Z} - 3w\mathcal{H}\Delta_D = 0, \quad (21)$$

where we have defined the zero-order EoS parameter $w \equiv w(a) = p_D/\rho_D$ for the viscous dark matter so that

$$w = -3\alpha H \rho_D^{m-1}. \quad (22)$$

Finally, taking the spatial derivative of the Raychaudhuri equation (5), we get

$$k\mathcal{Z}' + k\mathcal{H}\mathcal{Z} - k^2 A + 3(\mathcal{H}' - \mathcal{H}^2)A = -\frac{\kappa}{2}(\mathcal{X} + 3\mathcal{X}^p)a^2. \quad (23)$$

We shall assume here that the universe is dominated by the viscous dark matter, so that on the right-hand side of Eq. (23) $\mathcal{X}$ and $\mathcal{X}^p$ can be replaced by $\mathcal{X}_D$ and $\mathcal{X}^p_D$, respectively. In more general cases it is straightforward to include contributions from other matter species. Then, from Eqs. (19)–(21) and (23), we can eliminate $\mathcal{Z}$ and A to obtain

$$\Delta''_D + [C_1(a) + k^2 C_2(a)]\Delta'_D + [C_3(a) + k^2 C_4(a)]\Delta_D = 0, \quad (24)$$

where we have defined

$$C_1(a) = \mathcal{H} + \frac{\kappa \rho \mathrm{D} a^2}{2\mathcal{H}} w + \left(\frac{\mathcal{H}'}{\mathcal{H}} - \mathcal{H} \right) \frac{w}{1 + w} - \frac{w'}{1 + w} - 3w\mathcal{H}; \quad (25)$$

$$C_2(a) = -\frac{w}{3(1 + w)} \frac{1}{\mathcal{H}}; \quad (26)$$

$$C_3(a) = -\frac{\kappa \rho_D a^2}{2}[(1 + w)(1 + 3mw) + 3w^2] - 3\left(\frac{w'}{1 + w}\mathcal{H} + w\mathcal{H}' \right) - 3w\mathcal{H}^2 - 3w(\mathcal{H}' - \mathcal{H}^2)\left(m + \frac{w}{1 + w} \right); \quad (27)$$

$$C_4(a) = w\left(m + \frac{w}{1 + w} \right). \quad (28)$$

It is straightforward to check that when $w = 0$ (whether or not $m = 0$), this reduces to the evolution equation for the CDM density contrast, which does not depend on k, indicating that the evolution of the CDM density contrast is scale independent. For the viscous dark matter, however, we see that the equation depends on k in the coefficients of both Δ' and Δ. Equation (24) is effectively an equation for a damped simple oscillator, with time-varying and scale-dependent frequency and damping force, and no driving force. On small scales ($k \gg \mathcal{H}$) and at late times [when $w \sim \mathcal{O}(-1)$], the $C_2(a)$ and $C_4(a)$ terms dominate the coefficients of Δ' and Δ, respectively; it is then easy to show that the oscillator is overdamped so that its amplitude decays to zero rapidly with no oscillations. This qualitative feature can be seen in the upper panel of Fig. 3, where we plot the evolution of the dark matter density contrast, Δ_D, for four different length scales, $k = 0.005$, 0.01, 0.05, 0.1 Mpc^{-1}, for our best-fit model parameters ($m = -0.4$, $\beta = 0.236$). We see that the evolution of Δ_D deviates from that predicted by the ΛCDM model only at late times, but nonetheless significantly. The scale-dependent evolution of density contrast is actually a general feature in the UDM models without new dynamical degrees of freedom [16,21,22]

The lower panel of Fig. 3 displays the evolution of the baryon density contrast Δ_B. Because viscous dark matter clusters more weakly than cold dark matter, the gravitational potential that the baryons lie in is also weaker, making the baryons less clustered. However, since this effect is indirect, the deviation of Δ_B from the ΛCDM prediction is considerably smaller than that of Δ_D. Note that the galaxy surveys actually measure the clustering of luminous (baryonic) matter, rather than that of dark matter. Consequently, these measurements can only be applied to Δ_B, which is just weakly dependent on the model parameters [23]. Nevertheless, as in the bulk viscous model, both the amplitude and the shape of baryonic matter power spectrum are distinct from those predicted by the ΛCDM paradigm, and therefore we expect that stringent constraints can be obtained from it.

The analysis above is generalizable to an arbitrary choice of the viscosity function $\eta(\rho_D)$. To do this, we just need to define

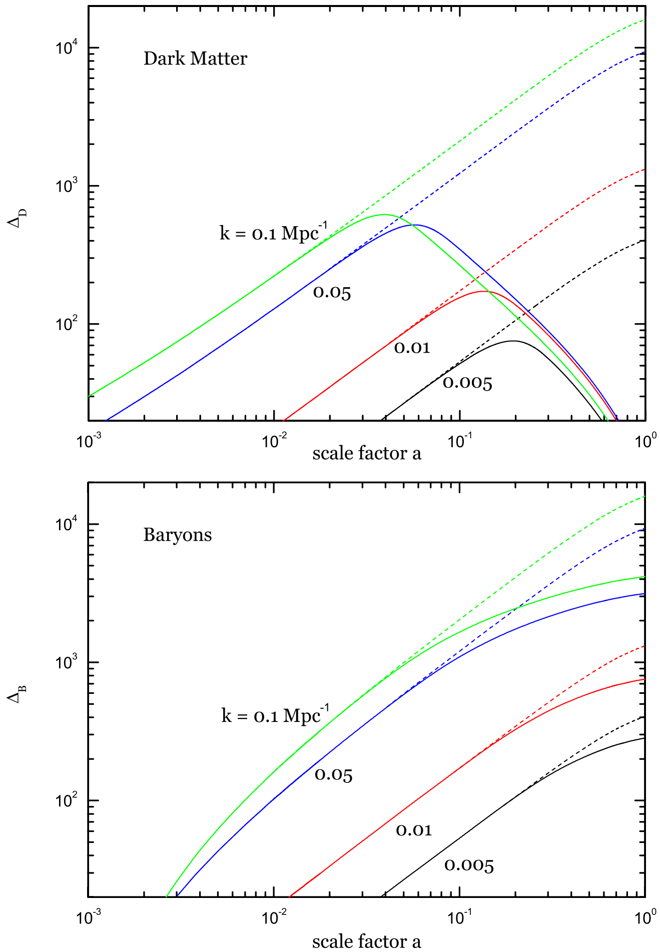

FIG. 3 (color online). Upper panel: The evolution of the density perturbation Δ_D for the viscous dark matter (the solid curves), as compared to the predictions for the cold dark matter model (the dashed curves). The results for four different length scales ($k = 0.005,\ 0.01,\ 0.05,\ 0.1\ \mathrm{Mpc}^{-1}$) are displayed as labeled beside the curves. The model parameters are chosen to be ($m = -0.4$, $\beta = 0.236$). Lower panel: The same but for the baryon density perturbation.

$$\tilde{m}(a) \equiv \frac{\rho_D}{\eta(\rho_D)} \frac{\partial \eta(\rho_D)}{\partial \rho_D} \tag{29}$$

and replace the constants m in Eqs. (27) and (28) with $\tilde{m}(a)$. Then, Eq. (24) describes the evolution of the dark matter density contrast in the general case of $\eta(\rho_D)$.

There are a couple of points to be noted about Eq. (24). First, it is clear that $m = 0$ does not guarantee that C_2 and C_4 are zero, and the EoS of the viscous dark matter w is important. Since $w \neq 0$, if the viscous dark matter is responsible for accelerating the expansion of the universe, our conclusion that the evolution of dark matter density contrast will be sensitively scale dependent and deviate significantly from ΛCDM will hold in general. Note that this is different from the analysis of Ref. [18], which considers the special case of $m = 0$ and concludes that the matter power spectrum in this model behaves well. Second, Eq. (24) is qualitatively different from its counterparts in other models which unify dark matter and dark energy, such as the (generalized) Chaplygin gas model. In the latter we have generally $C_2 = 0$, and so the equation describes an underdamped oscillator rather than an overdamped one. Consequently, the dark matter density contrast oscillates with (rapidly) decaying amplitude (or blows up if C_4 is negative), in contrast to the monotonic decay in our model here. Tracing the derivation of Eq. (24), it is easy to find that the C_2 term comes from the term with $\mathcal{Z}$ in Eq. (19), which is created by the fact that $p_D \propto \nabla^a u_a$. We also note that Eqs. (25) and (26) are independent of m and hence independent of the functional form of $\eta(\rho_D)$. As long as $C_4 > 0$, we will obtain a qualitative picture similar to the one given in Fig. 3.

Although the very different evolution of Δ_D from that in ΛCDM is not directly reflected in the observed galaxy power spectrum, it will modify the gravitational potential and subsequently affect observables such as the CMB, weak lensing, and CMB-galaxy cross correlation. As the viscous dark matter contributes the majority of the total energy in the universe, a decay in its density contrast as in Fig. 3 will also drive the gravitational potential ϕ to decay significantly. This point is verified in Fig. 4, which clearly shows that the decay of ϕ is much faster than that in the ΛCDM model.

The fast decay of ϕ will enhance the integrated Sachs-Wolfe (ISW) effect, which then contributes a source term $\propto \int^{\tau_0} \phi' j_\ell[k(\tau_0 - \tau)]d\tau$ to the CMB fluctuations, where $j_\ell(k\tau)$ is the spherical Bessel function and ϕ' the (conformal) time derivative of ϕ. This corresponds to more power

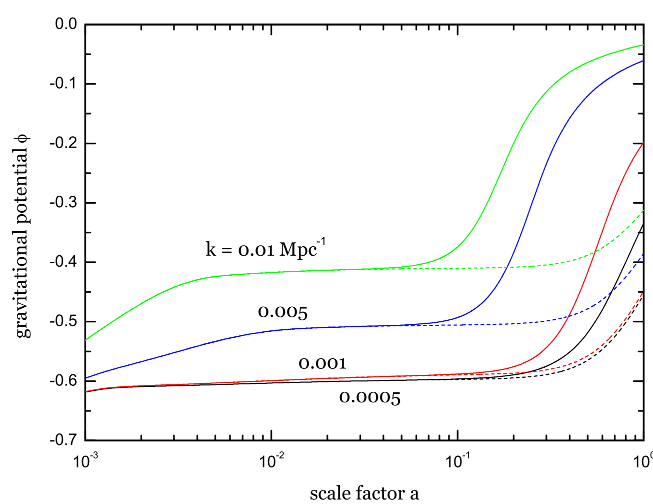

FIG. 4 (color online). The evolution of the gravitational potential ϕ in the bulk viscosity model (solid curve) as compared to the results for the standard ΛCDM paradigm (dashed curves). The results for four different length scales ($k = 0.0005,\ 0.001,\ 0.005$, and $0.01\ \mathrm{Mpc}^{-1}$) are shown. Clearly ϕ decays much faster in the bulk viscosity model.

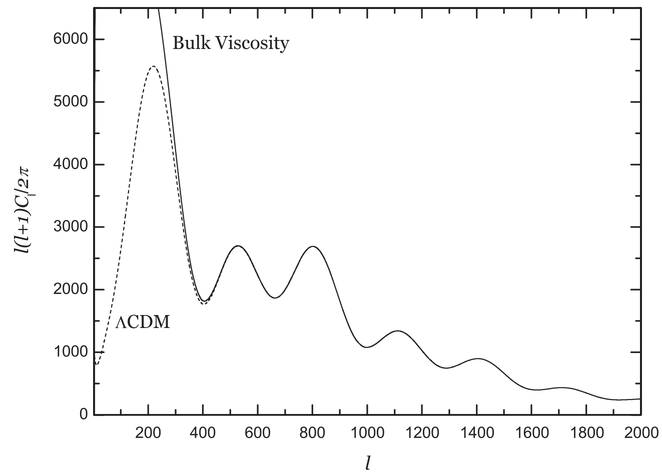

FIG. 5. The CMB temperature fluctuation spectrum of the bulk viscosity model (the solid curve) as compared to the predictions of the ΛCDM paradigm (the dashed curve). The model parameters are chosen to be $m = -0.4$ and $\beta = 0.236$, and all other parameters are the same in the two models.

in the CMB spectrum on large scales [24], as displayed in Fig. 5. Note that the positions of the acoustic peaks for the two models are the same in Fig. 5, because their background evolutions are almost indistinguishable. We can see that comparison with CMB data could provide strong restrictions on the present model as well. Again, because the damping in Δ_D is a general feature in UDM models based on bulk viscosity, so are the enhancements of the ISW effect and the low-ℓ CMB power. In principle, the weak-lensing convergence power spectrum, which reflects the (projected) potential distribution along the line of sight, will also be modified (as compared to the ΛCDM result) significantly by the rapid decay of the potential. This is not considered here because the CMB spectrum itself already places a stringent constraint on the model.

V. DISCUSSION AND CONCLUSIONS

To summarize, in this work we have investigated the background cosmological evolution and large-scale structure formation in the bulk viscous models designed as alternatives to dark energy. A bulk viscosity generates an effective pressure $p = -3\eta(\rho)H$ which, when the function $\eta(\rho)$ is appropriately chosen, could drive the acceleration of the cosmic expansion. Our numerical calculation focuses on a particular choice $\eta(\rho) = \alpha\rho^m$ with α, m being constants. Such a choice has been considered before in the literature [7], in the contexts of both inflationary and late-time accelerating universes. It is interesting to note that, with suitable values of m, the viscous matter behaves as cold dark matter at early times and as a cosmological constant in the future; thus this model naturally unifies dark matter and dark energy, at least at the background level.

We have used the measurements of supernovae luminosity distance and CMB shift parameter to constrain the model parameters and found that for the best-fitting parameters, $m = -0.4$ and $\beta = 0.236$ ($\beta \propto \alpha$), the background evolution of the universe is almost the same as that predicted by the ΛCDM model. From this viewpoint it seems that the model is a feasible alternative to an explicit cosmological constant.

However, when it comes to the linear perturbations, the viscous dark matter starts to behave very differently from cold dark matter. The pressure of the viscous dark matter tends to resist the growth of the density contrast, and when it becomes significant (at late times) this effect can rapidly damp out the density perturbations, particularly on small scales. Even though this smoothing of the viscous dark matter density perturbation cannot be seen directly, it could reduce the growth rate of density perturbations in the luminous matter (which can be measured by the galaxy power spectrum) and drive the fast decay of the gravitational potential fluctuations, which subsequently modifies the large-scale CMB spectrum, weak lensing, and CMB-galaxy cross correlations.

Note that the model we consider has no explicit ΛCDM limit, i.e., one cannot adjust the model parameters α and m to make the model reduce to ΛCDM exactly, unless a cosmological constant is added and the limit $\alpha \rightarrow 0$ is taken. The latter case, in which we have both an explicit cosmological constant and viscous matter, is not particularly appealing because it will introduce more complexity without solving any of the problems of ΛCDM. Rather, we are interested in whether the viscous dark matter alone could replace ΛCDM completely. This means that the EoS parameter w will necessarily be of order -1 at late times. According to our analysis in Sec. IV [cf. Eqs. (24)–(28)], it is the value of w that determines the evolution of Δ_D, and so we expect that all the qualitative pictures given in Sec. IV will remain in place in any attempts to replace dark matter and dark energy with bulk viscous matter alone. Viscous matter is therefore not a successful contender for the dark sector.

The bulk viscosity model is another candidate to explain the dark energy without introducing dynamical degrees of freedom. In order to explain the accelerating cosmic expansion, one needs a negative (effective) pressure. If no new dynamical degree of freedom is introduced, the negative pressure should be either a constant, or a function of ρ_{DE} (the energy density of the newly added matter), ρ_m (the energy density of the existing matter), or H, or combinations of these variables, which are all the possible variables in the background cosmology [25] (see [26,27] for a counterexample however, where a new degree of freedom is added but is made nondynamical). Hence, we have the following conclusions for each of the allowed prescriptions for a form of newly added matter [28–30]:

(1) The case $p = \text{const}$ is simply a cosmological constant.
(2) An example of the case $p = p(\rho_{DE})$ is the (generalized) Chaplygin gas model [14,15]. The large-

scale structure formation here has been considered in [16,21,22]. In this class of models we have $\mathcal{X}^p \propto \mathcal{X}$ in general, and this leads to a term proportional to $k^2\Delta$ in the evolution equation for the density contrast Δ of the *newly added matter*, and this term dominates on small scales ($k \gg \mathcal{H}$). Depending on the sign of this new term, Δ will either blow up or oscillate and decay rapidly on small scales.

Note that this class of models is frequently considered as either UDM models or dark energy models. In the former case, as shown in [16], the constraints on the models are particularly stringent (note however that, as pointed out in [31,32], in these models the nonlinear corrections can be important, which will make the simple linear treatments inaccurate or even incorrect, and will potentially significantly modify the background evolution as well, depending on the parameters used). In the latter case, density perturbations of CDM and baryons may not be affected significantly, much like the fact that in the above viscous model the baryons are not greatly affected. However, there generally will be other distinct new features of the model [21].

(3) Examples for the case $p = p(\rho_m)$ include the Cardassian model [33], the Palatini $f(R)$ gravity [34], and the $\omega = -3/2$ Brans-Dicke theory with a potential, the linear perturbations of which have been considered in [35–39]. In these models, by a similar argument as in the above, there will be a term proportional to $k^2\Delta_m$ in the evolution equation for the *existing matter* density perturbation Δ_m. As a result, Δ_m will experience blowing up or rapidly decaying oscillation on small scales at late times. Note, however, that in this class of models the averaging over the microscopic structure of the (existing) matter distribution may be a serious issue, rendering the appropriately averaged cosmological behavior very different from naive predictions [40,41] (in a way similar to [32] though with some differences; namely, in [40,41] the averaging is over microscopic scales while in [32] it is over astronomical scales). This is because p for the newly added matter depends algebraically on the density of normal matter particles, and could be very different inside and outside the distribution of the latter. As the normal matter particles only occupy a tiny portion of the total volume of the universe, after averaging, p should be dominated by its value outside the normal matter particles (i.e., in the vacuum), which is likely to be a constant.

(4) The case $p = p(H)$ includes bulk viscosity model with $\eta(\rho) = \text{const}$ considered in [18,42–44], which is a special case for the general model we considered in this work, with $p = p(\rho_{\rm DE}, H)$. Here, the dependence of p on H results in the evolution equation of Δ for the *newly added* matter acquiring both a term proportional to $k^2\Delta$ and one proportional to $k^2\Delta'$. Consequently, on small scales at late times, Δ will decay rapidly without oscillation like an overdamped oscillator. We have shown that the UDM model based on this is generally unable to pass several cosmological tests. Note that, as in the case of $p = p(\rho_{\rm DE})$, the averaging issue again needs to be taken into account if more precise predictions are to be obtained [31,32], though its full effect and significance need careful nonlinear studies such as N-body simulations.

In all such cases (except $p = \text{const}$), we have seen that the evolution of Δ (for either the existing matter or the newly added matter) becomes very irregular. If the matter with irregular perturbation evolution makes a significant contribution to the total energy of the universe, then this will lead to large modifications to the ΛCDM predictions for the matter power spectrum, CMB, weak lensing and similar tests. This indicates that a viable dynamical UDM model is likely to involve extra dynamical degrees of freedom in contrast to that provided by a (generalized) Chaplygin gas or bulk viscosity [45–48].

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