

Non-Gaussianity in a three-fluid curvaton modelT. Multamäki,^{*} J. Sainio,[†] and I. Vilja[‡]*Department of Physics and Astronomy, University of Turku, FIN-20014 Turku, Finland*

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The generation of non-Gaussianity is studied in a three-fluid curvaton model. By utilizing second order perturbation theory we derive general formulae for the large scale temperature fluctuation and non-Gaussianity parameter f_{NL} that includes the possibility of a nonadiabatic final state. In the adiabatic limit we recover previously known results. The results are applied to a three-fluid curvaton model where radiation and all of the cold dark matter are produced by the curvaton field decay. We find that the amount of non-Gaussianity in this model decreases as the final state of the system becomes more adiabatic and that the generated non-Gaussianity in the scenario is small, $|f_{\text{NL}}| \sim \mathcal{O}(1)$, in contrast to the two-fluid curvaton model.

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I. INTRODUCTION

The cosmic microwave background (CMB) has proven to be a feast of information for modern cosmology. Ever since its discovery by Penzias and Wilson in 1964 [1], there have been a number of different experiments starting from RELIKT-1 [2] resulting in drastic improvements to the quality of CMB data. A major scientific breakthrough was reached in 1992 when the COBE-satellite was able to detect the presence of the anisotropy in the CMB [3–5].

The most recent data gathered by the WMAP Collaboration [6,7] is consistent with the hypothesis that these perturbations were generated in an era of cosmic inflation. This is generally achieved with a slowly rolling scalar field which leads to an exponential expansion of the Universe and the observed anisotropy is generated by the fluctuations of this inflaton field. Such a minimal scenario leads to adiabatic and Gaussian perturbations in accordance with the current data.

A well-motivated alternative to the simplest inflationary scenario is the curvaton mechanism [8–37] in which the perturbations are generated by a second scalar field dubbed the curvaton which stays subdominant during inflation and the actual expansion of space is still driven by the inflaton. This allows the inflation potential to have more natural properties compared to the single field scenario and can still lead to adiabatic perturbations. However, the extra degrees of freedom in the system now allow for the possibility that the final state is not necessarily purely adiabatic. Instead, the generation of an observable amount of isocurvature perturbations is a possibility that can distinguish the curvaton scenario from the simple single field inflationary model.

Another ingredient that can differentiate the curvaton scenario from the simple inflaton hypothesis is the concept

of non-Gaussianity, which has become more relevant with the data gathered by WMAP and with the advent of the Planck satellite. The current limits from the WMAP 5-year data state that the local non-Gaussianity parameter f_{NL} is limited to the values $-9 < f_{\text{NL}}^{\text{local}} < 111$ [38]. The single field inflationary scenario produces very little of non-Gaussianity, $f_{\text{NL}} \sim \mathcal{O}(1)$, whereas the curvaton scenario might lead to an observable non-Gaussianity [19,39]. Recent studies of the curvaton scenario have largely focused on the generated non-Gaussianity in different models of the curvaton decay [39–42] e.g. with multiple scalar fields [34,43] or with a nonquadratic potential [44]. The presence of non-Gaussianity is therefore of great importance since it can differentiate between different scenarios of the early Universe.

This paper focuses closely on the generation of non-Gaussianity in a three-fluid model of curvaton decay where the curvaton decays into radiation and nonrelativistic cold dark matter. One of the main motivations for this model comes from the fact that the final nonadiabatic state can be calculated accurately compared to the sudden-decay approximation of the two-fluid model. The other properties of this three-fluid model have been discussed previously in several papers: the authors of [45] gave a thorough analysis of the evolution of adiabatic and nonadiabatic components, whereas paper [46] focused on the physically allowed parameter space. The most “realistic” curvaton model can be found in [47] where the freeze-out of dark matter was also included in the analysis. The concept of non-Gaussianity in this three-fluid curvaton scenario was first presented in [45] through first order perturbation theory. The study of non-Gaussianities is however essentially dependent on the second order terms and therefore the use of first order theory is not always justified. Our calculations incorporate the second order perturbations from the start. Since the curvaton scenario might lead to adiabatic and isocurvature perturbations we have derived a generalized non-Gaussianity parameter that includes the adiabatic state as a special case.

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This paper is organized as follows. In Sec. II we present the relevant quantities and equations of motion of the perturbation up to second order. In this section we also derive the generalized non-Gaussianity parameter f_{NL} . In Sec. III we present the equations of motion of the curvaton model and generalize the conserved quantity first presented in [45]. In Sec. IV we present numerical results for f_{NL} in the three-fluid curvaton model both for the case of constant and time-dependent interaction. We end this article with discussion and conclusions in Sec. V.

II. PERTURBATIONS AT FIRST AND SECOND ORDER

The theory of second order cosmological perturbations has been studied rigorously in the recent years. We will follow closely the notation of [48] and use a spatially flat Friedmann-Robertson-Walker background. The metric tensor can be in this case expanded up to second order in the form [48]

$$g_{\mu\nu}dx^\mu dx^\nu = -(1 + 2\phi^{(1)} + \phi^{(2)})dt^2 + a(t)\left(\hat{\omega}_i^{(1)} + \frac{1}{2}\hat{\omega}_i^{(2)}\right) \times dt dx^i + a(t)^2 \left[(1 - 2\psi^{(1)} - \psi^{(2)})\delta_{ij} + \chi_{ij}^{(1)} + \frac{1}{2}\chi_{ij}^{(2)} \right] dx^i dx^j, \quad (1)$$

where $a(t)$ is the scale factor and $\phi^{(r)}$, $\hat{\omega}_i^{(r)}$, $\psi^{(r)}$, and $\chi_{ij}^{(r)}$ are perturbation functions defined in [48] at first ($r = 1$) and second order ($r = 2$). Written in this form, different gauges can be straightforwardly given in terms of the perturbation functions: for example, the Poisson gauge is defined as $\omega^{(r)} = \chi_{ij}^{(r)} = \chi_{ij}^{(r)} = 0$ and the spatially flat one is $\psi^{(r)} = \chi^{(r)} = 0$ [48].

A useful set of equations can be derived from the continuity equations $T_{i;\mu}^{\mu\nu} = Q_i^\nu$, where $T_i^{\mu\nu}$ is the energy-momentum tensor, Q_i describes the energy transfer between different fluids, and $;$ denotes the covariant derivative. From the continuity equations it follows that the equations determining the background evolution of individual fluids are

$$\dot{\rho}_i = -3H(1 + \omega_i)\rho_i + Q_i, \quad (2)$$

where $\omega_i = P_i/\rho_i$ is the equation of state, P_i the pressure and ρ_i the energy density of the i th fluid, H is the Hubble parameter, and $\dot{} \equiv d/dt$ i.e. derivative with respect to physical time.

At first order one finds the evolution equations of the perturbed energy and pressure densities (on large scales) [49]

$$\delta\dot{\rho}_i^{(1)} + 3H(\delta\rho_i^{(1)} + \delta P_i^{(1)}) - 3(\rho_i + P_i)\dot{\psi}^{(1)} = Q_i\phi^{(1)} + \delta Q_i^{(1)}, \quad (3)$$

and at second order

$$\begin{aligned} \delta\dot{\rho}_i^{(2)} + 3H(\delta\rho_i^{(2)} + \delta P_i^{(2)}) - 3(\rho_i + P_i)\dot{\psi}^{(2)} \\ - 6\dot{\psi}^{(1)}[\delta\rho_i + \delta P_i + 2(\rho_i + P_i)\psi^{(1)}] \\ = Q_i\phi^{(2)} + \delta Q_i^{(2)} - Q_i(\phi^{(1)})^2 + 2\phi^{(1)}\delta Q_i^{(1)}. \end{aligned} \quad (4)$$

In addition to these, we also have the Einstein equations which can be used to give additional limits on the perturbation equations. For example in the Poisson gauge at first order $\psi^{(1)} = \phi^{(1)}$ and on large scales $2\phi^{(1)} + 2\dot{\psi}^{(1)}/H = -\delta\rho/\rho_0$, whereas in the spatially flat gauge $\psi^{(1)} = \chi^{(1)} = 0$ and $2\phi^{(1)} = -\delta\rho/\rho_0$ [48].

At second order the equations get more complex and they are presented in detail in [48]. The equations which we will need are the 00 and ij components of the Einstein equations on large scales which read in the Poisson gauge in a matter dominated universe as

$$\begin{aligned} \phi^{(2)} &= -\frac{1}{2}\frac{\delta\rho^{(2)}}{\rho_0} + 4(\psi^{(1)})^2 \\ \psi^{(2)} - \phi^{(2)} &= -4(\psi^{(1)})^2 - \frac{10}{3}\nabla^{-2}(\partial^i\psi^{(1)}\partial_i\psi^{(1)}) \\ &\quad + 10\nabla^{-4}(\partial^i\partial_j(\partial_i\psi^{(1)}\partial^j\psi^{(1)})). \end{aligned} \quad (5)$$

In the spatially flat gauge the 00 component is

$$\phi^{(2)} = -\frac{1}{2}\frac{\delta\rho^{(2)}}{\rho_0} + 4(\phi^{(1)})^2. \quad (6)$$

A. Curvature perturbations

An elegant way to study the evolution of perturbations is to use gauge-invariant curvature perturbations, which relate to curvature perturbations on homogeneous-density surfaces. At first order they are defined for component i as

$$\zeta_i^{(1)} = -\psi^{(1)} - \frac{\delta\rho_i^{(1)}}{\rho_i'}, \quad (7)$$

where $' \equiv d/d(\ln a)$. At second order the corresponding quantity is defined as

$$\begin{aligned} \zeta_i^{(2)} &= -\psi^{(2)} - \frac{\delta\rho_i^{(2)}}{\rho_i'} + 2\frac{\delta\rho_i^{(1)'}}{\rho_i'}\frac{\delta\rho_i^{(1)}}{\rho_i'} \\ &\quad + 2\frac{\delta\rho_i^{(1)}}{\rho_i'}(\psi^{(1)'} + 2\psi^{(1)}) - \left(\frac{\delta\rho_i^{(1)}}{\rho_i'}\right)^2\left(\frac{\rho_i''}{\rho_i'} - 2\right). \end{aligned} \quad (8)$$

Note that we are here neglecting gradient terms since we are only interested in the large scale behavior of perturbations.

The equation of motion of the first order curvature perturbations can be derived from Eq. (3) and Einstein equations. The result is

$$\zeta_i^{(1)'} = \frac{3\delta P_{\text{int}(i)}}{\rho_i'} - \frac{\delta Q_{\text{int}(i)}}{H\rho_i'} - \frac{H'}{H}\frac{Q_i}{\rho_i'}(\zeta - \zeta_i), \quad (9)$$

where $\delta P_{\text{int}(i)} \equiv \delta P_i - p'_i \delta \rho_i / \rho'_i$ and $\delta Q_{\text{int}(i)} \equiv \delta Q_i - Q'_i \delta \rho_i / \rho'_i$.

At second order the corresponding equations read as [50]

$$\begin{aligned} \zeta_i^{(2)'} = & -\frac{1}{\rho'_i H} \left[\left(\delta Q_i^{(2)} - \frac{Q'_i}{\rho'_i} \delta \rho_i^{(2)} \right) + Q_i \frac{\rho'_0}{2\rho_0} \left(\frac{\delta \rho_i}{\rho'_i} \right. \right. \\ & \left. \left. - \frac{\delta \rho^{(2)}}{\rho'_0} \right) \right] - 3 \frac{Q_i}{\rho'_i H} (\phi^{(1)})^2 - 2 \frac{\delta Q_i^{(1)} \phi^{(1)}}{H \rho'_i} + 2 \left[2 \right. \\ & \left. - 3(1 + \omega_i) \right] \zeta_i^{(1)} \zeta_i^{(1)'} - 2 \left[\left(\frac{Q_i \phi^{(1)}}{H \rho'_i} + \frac{\delta Q_i^{(1)}}{H \rho'_i} \right) \zeta_i^{(1)'} \right]' \\ & - \left[\left(\frac{Q'_i}{H \rho'_i} - \frac{1}{2} \frac{Q_i}{H \rho'_i} \frac{\rho'_0}{\rho_0} \right) (\zeta_i^{(1)})^2 \right]'. \end{aligned} \quad (10)$$

There are instances when the definition of different curvature perturbations might fail e.g. when $\rho'_i = 0$. Therefore in our numerical evaluations we have used the spatially flat gauge and evaluated the density perturbations of different components in this gauge at first and second order. The corresponding equations of motion can be easily read from Eqs. (3) and (4) by going to the spatially flat gauge $\psi^{(r)} = \chi^{(r)} = 0$.

B. Non-Gaussianity

The generation of non-Gaussianity in the two-fluid curvaton model at second order has been considered previously in [48,51]. In the two-fluid model the final state is adiabatic whereas in the three-fluid model a significant isocurvature component is a possibility. Here we generalize the results of [48] to include the possibility of a non-adiabatic final state. We follow the standard notation presented in [52] and use the Poisson gauge. Since we are interested only in the large scale non-Gaussianities we can safely ignore some terms from the full expression [52], including the integrated Sachs-Wolfe effect. The temperature fluctuations can in this approximation be written [48] as

$$\begin{aligned} \frac{\Delta T}{T} = & \left[\phi^{(1)} + \tau^{(1)} + \frac{1}{2} (\phi^{(2)} + \tau^{(2)}) - \frac{1}{2} (\phi^{(1)})^2 \right. \\ & \left. + \phi^{(1)} \tau^{(1)} \right]_{\text{Em}}, \end{aligned} \quad (11)$$

where $\phi^{(r)}$ are the metric lapse functions, $\tau = \tau^{(1)} + \frac{1}{2} \tau^{(2)}$ is the intrinsic fractional temperature fluctuation $\tau = \Delta T/T|_{\text{Em}}$, and all the terms are evaluated at the time of emission.

We introduce new variables relating the final values of perturbations to their initial values:

$$\begin{aligned} r_1 = & \frac{\zeta_m^{(1)}|_m}{\zeta_{\sigma,\text{in}}^{(1)}}, & q_1 = & \frac{\zeta_\gamma^{(1)}|_m}{\zeta_{\sigma,\text{in}}^{(1)}}, & r_2 = & \frac{\zeta_m^{(2)}|_m}{(\zeta_{\sigma,\text{in}}^{(1)})^2}, \\ q_2 = & \frac{\zeta_\gamma^{(2)}|_m}{(\zeta_{\sigma,\text{in}}^{(1)})^2}, \end{aligned} \quad (12)$$

where we have assumed that the system consists of three fluids as in the three-fluid curvaton model. We denote the curvaton component by subscript (σ), radiation by (γ), and cold dark matter by (m) and the different numerators are evaluated at the time of decoupling, when the Universe is matter dominated and hence $\zeta^{(i)} \simeq \zeta_m^{(i)}$. The system is adiabatic if $q_1 = r_1$ and $q_2 = r_2$. These variables tell how efficiently the system can convert the initial curvature perturbations into the matter and radiation components. Note that we could have also defined these transfer coefficients in terms of the initial total curvature. However the current choice simplifies the results in the curvaton model under study.

Since the Universe is matter dominated during decoupling we can write $\zeta_m^{(2)}$ in the form

$$\zeta_m^{(2)} = -\psi^{(2)} + \frac{1}{3} \frac{\delta \rho^{(2)}}{\rho_0} + \frac{5}{9} \left(\frac{\delta \rho^{(1)}}{\rho_0} \right)^2, \quad (13)$$

where ρ_0 is the total background energy density. From the definitions of r_1 and r_2 it follows that

$$\zeta_m^{(2)} = r_2 (\zeta_{\sigma,\text{in}}^{(1)})^2 = r_2 \left(\frac{\zeta_m^{(1)}}{r_1} \right)^2 = \frac{25 r_2}{9 r_1^2} (\psi^{(1)})^2, \quad (14)$$

where we have also used the equation $\phi^{(1)} = -3\zeta^{(1)}/5$ which is valid on large scales in a matter dominated universe. By solving $\phi^{(2)}$ from the latter of Einstein equations (5), then eliminating $\delta \rho^{(2)}/\rho_0$ and $\psi^{(2)}$ terms with the first of Einstein equations (5), (13), and (14), and finally using the equality $\phi^{(1)} = -3\zeta^{(1)}/5$ we can write $\phi^{(2)}$ in a matter dominated universe in the form

$$\begin{aligned} \phi^{(2)} = & \left[\frac{16}{3} - \frac{5}{3} \frac{r_2}{r_1^2} \right] (\psi^{(1)})^2 + 2 \nabla^{-2} (\partial^i \psi^{(1)} \partial_i \psi^{(1)}) \\ & - 6 \nabla^{-4} (\partial^i \partial_j (\partial_i \psi^{(1)} \partial_j \psi^{(1)})), \end{aligned} \quad (15)$$

where the inverse Laplacians are to be understood as a formal expression.

The intrinsic fractional temperature fluctuations $\tau^{(r)}$ can be written at first order as

$$\begin{aligned} \tau^{(1)} = & \frac{1}{4} \frac{\delta \rho_\gamma^{(1)}}{\rho_\gamma} \Big|_{\text{Em}} = - \frac{\delta \rho_\gamma^{(1)}}{\rho'_\gamma} \Big|_{\text{Em}} = \psi^{(1)}|_{\text{Em}} + \zeta_\gamma^{(1)}|_{\text{Em}} \\ = & \left(1 - \frac{5}{3} \frac{q_1}{r_1} \right) \phi^{(1)}|_{\text{Em}}, \end{aligned} \quad (16)$$

where we have used the definitions of $\zeta_\gamma^{(1)}$, $\zeta_m^{(1)}$, q_1 , r_1 , and

$\phi^{(1)} = -3\zeta^{(1)}/5$. At second order the corresponding variable is

$$\tau^{(2)} = \frac{1}{4} \frac{\delta\rho_\gamma^{(2)}}{\rho_\gamma} \Big|_{\text{Em}} - 3(\tau^{(1)})^2. \quad (17)$$

From the definition of $\zeta_\gamma^{(2)}$ we find

$$\frac{1}{4} \frac{\delta\rho_\gamma^{(2)}}{\rho_\gamma} \Big|_{\text{Em}} = \psi^{(2)} + 2(\tau^{(1)}) + 4\tau^{(1)}\phi^{(1)} + \frac{25}{9} \frac{q_2}{r_1^2} (\phi^{(1)})^2, \quad (18)$$

where we have also used the definitions of r_1 and q_2 .

Combining all of the above expressions for perturbations and substituting into the equation for the temperature fluctuations (11) one finally has

$$\begin{aligned} \frac{\Delta T}{T} = & \frac{6r_1 - 5q_1}{3r_1} \left[\phi^{(1)} \right. \\ & + \left[\frac{25(q_2 - q_1^2) - 60q_1r_1 + 96r_1^2 - 30r_2}{6r_1(6r_1 - 5q_1)} \right] (\phi^{(1)})^2 \\ & + \frac{r_1}{6r_1 - 5q_1} \nabla^{-2} (\partial^i \psi^{(1)} \partial_i \psi^{(1)}) - \frac{3r_1}{6r_1 - 5q_1} \\ & \left. \times \nabla^{-4} (\partial^i \partial_j (\partial_i \psi^{(1)} \partial_j \psi^{(1)})) \right], \quad (19) \end{aligned}$$

where we have ignored the momentum dependent terms because we are interested in the large scale non-Gaussianity [48]. To our knowledge this general formula has not been presented before. From Eq. (19) we can read that in the adiabatic limit, i.e. $r_1 = q_1$ and $r_2 = q_2$, we recover at second order an extension of the first order Sachs-Wolfe effect $\Delta T/T = \phi^{(1)}/3$ given in [48]:

$$\begin{aligned} \frac{\Delta T}{T} = & \frac{1}{3} \left[\phi^{(1)} + \frac{1}{2} (\phi^{(2)} - \frac{5}{3} (\phi^{(1)})^2) \right. \\ & + \nabla^{-2} (\partial^i \psi^{(1)} \partial_i \psi^{(1)}) \\ & \left. - 3 \nabla^{-4} (\partial^i \partial_j (\partial_i \psi^{(1)} \partial_j \psi^{(1)})) \right]. \quad (20) \end{aligned}$$

In the opposite case of pure isocurvature perturbation, Eq. (19) is not valid since we have assumed that $r_1 \neq 0$ and for isocurvature perturbations $0 = \zeta^{(1)}|_{\text{Dec}} \simeq \zeta_m^{(1)}|_{\text{Dec}} = r_1 \zeta_{\sigma, \text{in}}^{(1)}$.

We can now define the non-Gaussianity parameter f_{NL} in the general case. Following the notation of [53] we write the temperature fluctuations in the form

$$\frac{\Delta T}{T} = g_T [\phi^{(1)} + f_{\text{NL}} (\phi^{(1)})^2], \quad (21)$$

where the factor g_T depends on the state of the system e.g. for a completely adiabatic one $g_T = 1/3$ and in our calculations $g_T = (6r_1 - 5q_1)/(3r_1)$. We have ignored the gradient terms from our definition of non-Gaussianity but when calculating the bispectrum they need to be included.

Note that this definition leads to a sign difference when compared to the usual approach using the Bardeen potential [48]. Now from Eq. (19) we can easily read the non-Gaussianity parameter

$$f_{\text{NL}} = \frac{25(q_2 - q_1^2) - 60q_1r_1 + 96r_1^2 - 30r_2}{6r_1(6r_1 - 5q_1)}, \quad (22)$$

which is well defined since $r_1 \neq 0$ and $q_1 \leq r_1$.

In order to compare with observations we need to calculate the bispectrum, which requires that we take into account other effects e.g. the integrated Sachs-Wolfe effect and possible evolution after horizon crossing. This is beyond the scope of this paper and hence left for further work. For details of this procedure we refer to [54,55] where the radiation transfer functions depend now on the adiabicity of the system via g_T .

III. THE CURVATON MODEL

In the three-fluid curvaton model the curvaton fluid decays after inflation into radiation and cold dark matter before big bang nucleosynthesis (BBN). Here we make the same simplifying assumption as in [45] that the cold dark matter fluid does not couple radiation. This system was first studied in [45]. The physically feasible parameter space was explored in [46], where constraints arising from BBN were studied. Temperatures relevant to this system were discussed in [46,47].

A. Evolution equations

As previously mentioned, we denote the curvaton by subscript (σ), radiation by (γ), and cold dark matter by (m). Since the curvaton field is oscillating it can be safely estimated [56] to behave like nonrelativistic matter, i.e. $\omega_\sigma = 0$. For radiation and matter we have $\omega_\gamma = 1/3$ and $\omega_m = 0$ and the interaction terms are [45]

$$\begin{aligned} Q_\sigma &= -\Gamma_\gamma f_\gamma(N) \rho_\sigma - \Gamma_m f_m(N) \rho_\sigma \\ Q_\gamma &= \Gamma_\gamma f_\gamma(N) \rho_\sigma \quad Q_m = \Gamma_m f_m(N) \rho_\sigma, \end{aligned} \quad (23)$$

where Γ_i denote the strength of the interaction and functions $f_i(N)$ allow for time-dependent interactions with $N \equiv \ln(a)$.

Background Eqs. (2) can be written in terms of fractional densities $\Omega_i \equiv \rho_i/\rho$ for which the equations of motion are [45]

$$\begin{aligned} \Omega'_\sigma &= \Omega_\sigma \Omega_\gamma + \frac{Q_\sigma}{H\rho}, & \Omega'_\gamma &= \Omega_\gamma (\Omega_\gamma - 1) + \frac{Q_\gamma}{H\rho}, \\ \Omega'_m &= \Omega_m \Omega_\sigma + \frac{Q_m}{H\rho}, & \left(\frac{1}{H}\right)' &= \left(1 + \frac{1}{3} \Omega_\gamma\right) \left(\frac{1}{H}\right). \end{aligned} \quad (24)$$

From the definition of Ω_i it can be easily seen that $\Omega_\sigma + \Omega_\gamma + \Omega_m = 1$, which means that one equation of motion of Ω_i is redundant.

From Eq. (3) we can read the equations of motion for the first order density perturbations

$$\begin{aligned}\delta\rho_\sigma^{(1)'} &= -3\delta\rho_\sigma^{(1)} - \frac{Q_\sigma}{H} \frac{\delta\rho^{(1)}}{2\rho} + \frac{\delta Q_\sigma^{(1)}}{H}, \\ \delta\rho_\gamma^{(1)'} &= -4\delta\rho_\gamma^{(1)} - \frac{Q_\gamma}{H} \frac{\delta\rho^{(1)}}{2\rho} + \frac{\delta Q_\gamma^{(1)}}{H}, \\ \delta\rho_m^{(1)'} &= -3\delta\rho_m^{(1)} - \frac{Q_m}{H} \frac{\delta\rho^{(1)}}{2\rho} + \frac{\delta Q_m^{(1)}}{H},\end{aligned}\quad (25)$$

in the spatially flat gauge. At second order the corresponding equations in the flat gauge are

$$\begin{aligned}\delta\rho_\sigma^{(2)'} &= -3\delta\rho_\sigma^{(2)} - \frac{Q_\sigma}{2H\rho} \left[\delta\rho^{(2)} - \frac{3}{2} \frac{(\delta\rho^{(1)})^2}{\rho} \right] \\ &\quad + \frac{\delta Q_\sigma^{(2)}}{H} - \frac{\delta\rho^{(1)}\delta Q_\sigma^{(1)}}{H\rho}, \\ \delta\rho_\gamma^{(2)'} &= -4\delta\rho_\gamma^{(2)} - \frac{Q_\gamma}{2H\rho} \left[\delta\rho^{(2)} - \frac{3}{2} \frac{(\delta\rho^{(1)})^2}{\rho} \right] \\ &\quad + \frac{\delta Q_\gamma^{(2)}}{H} - \frac{\delta\rho^{(1)}\delta Q_\gamma^{(1)}}{H\rho}, \\ \delta\rho_m^{(2)'} &= -3\delta\rho_m^{(2)} - \frac{Q_m}{2H\rho} \left[\delta\rho^{(2)} - \frac{3}{2} \frac{(\delta\rho^{(1)})^2}{\rho} \right] \\ &\quad + \frac{\delta Q_m^{(2)}}{H} - \frac{\delta\rho^{(1)}\delta Q_m^{(1)}}{H\rho}.\end{aligned}\quad (26)$$

We also need the gauge-invariant curvature perturbations $\zeta_j^{(i)}$ in order to calculate the non-Gaussianity parameter f_{NL} . In the spatially flat gauge these are

$$\begin{aligned}\zeta_i^{(1)} &= -\frac{\delta\rho_i^{(1)}}{\rho_i'}, \\ \zeta_i^{(2)} &= -\frac{\delta\rho_i^{(2)}}{\rho_i'} + [2 - 3(1 + \omega_i)](\zeta_i^{(1)})^2 \\ &\quad - 2\left[\frac{Q_i\phi^{(1)}}{H\rho_i'} + \frac{\delta Q_i^{(1)}}{H\rho_i'}\right]\zeta_i^{(1)} \\ &\quad - \left[\frac{Q_i'}{\rho_i'H} - \frac{1}{2}\frac{Q_i\rho_i'}{\rho_i'H\rho}\right](\zeta_i^{(1)})^2.\end{aligned}\quad (27)$$

The set of equations (24)–(26) can now be evaluated numerically once the initial values have been set. We have chosen the system to be initially radiation dominated and nonadiabatic at first and second order. The energy density of the curvaton field is during oscillations $\rho_\sigma = m^2\sigma^2$ where σ is the amplitude of the field. A small perturbation of the field $\delta\sigma$ leads to [30]

$$\begin{aligned}\tilde{\rho}_\sigma &= \rho_\sigma + \delta\rho_\sigma^{(1)} + \frac{1}{2}\delta\rho_\sigma^{(2)} + \mathcal{O}(\delta\rho_\sigma^{(3)}) \\ &= m^2(\sigma_0^2 + 2\sigma_0\delta\sigma + (\delta\sigma)^2),\end{aligned}\quad (28)$$

which means that at second order

$$\frac{\delta\rho_\sigma^{(2)}}{\rho_\sigma} = \frac{1}{2}\left(\frac{\delta\rho_\sigma^{(1)}}{\rho_\sigma}\right)^2. \quad (29)$$

This forces the second order perturbation of the curvaton to be

$$\begin{aligned}\zeta_{\sigma,\text{in}}^{(2)} &= \left[\frac{1}{2} - \frac{\Gamma_\sigma}{2H}\Omega_\gamma - \frac{2\Gamma_\sigma\Omega_\gamma}{3H + \Gamma_\sigma} + \frac{(\Gamma_\sigma)^2}{2H(3H + \Gamma_\sigma)} \right] \\ &\quad \times (\zeta_\sigma^{(1)})^2|_{\text{in}},\end{aligned}\quad (30)$$

which follows from Eqs. (29) and (8) after short calculations. The first order curvature perturbations are linear equations and we are free to scale them relative to each other and hence set $\zeta_{\sigma,\text{in}}^{(1)} = 1$. Note that their absolute value is ultimately determined by the observed amplitude of the temperature fluctuations, Eq. (21). From Eq. (30) and using the inequality $\Gamma_\sigma \ll H_0$, valid in our calculations, we can therefore safely estimate $\zeta_{\sigma,\text{in}}^{(2)} \approx 1/2$. The other perturbations are initially set to be zero i.e. $\zeta_{\gamma,\text{in}}^{(1)} = \zeta_{m,\text{in}}^{(1)} = \zeta_{\gamma,\text{in}}^{(2)} = \zeta_{m,\text{in}}^{(2)} = 0$ since we have assumed that only the curvaton field has initial perturbations. Based on these initial values and Eqs. (24)–(26) we have calculated the amount of non-Gaussianity in two different situations: (1) the curvaton decays into radiation and cold dark matter with constant couplings, i.e. $f_\gamma(N) = 1$, $f_m(N) = 1$; and (2) $f_\gamma(N) = 1$ and the decay of the curvaton field into the matter component has explicit time dependence $f_m(N)$. We take $f_m(N)$ to be a continuous function with $f_m'(N) = 0$ at $N = 0$.

B. Conserved quantities

The authors of [45] noticed that the curvature perturbation $\zeta_{\text{comp}}^{(1)}$, which is related to a scaled matterlike fluid component

$$\rho_{\text{comp}} = \rho_m + \frac{\Gamma_m}{\Gamma_\gamma + \Gamma_m}\rho_\sigma, \quad (31)$$

is a conserved quantity in the three-fluid model because

$$Q_{\text{comp}} = Q_m + \frac{\Gamma_m}{\Gamma_\gamma + \Gamma_m}Q_\sigma = 0 \quad (32)$$

and $\delta Q_{\text{comp}}^{(1)} = 0$. This result can be generalized to all orders since $\delta Q_{\text{comp}}^{(i)} = 0$ and from Eq. (10) we can easily see that this is equal to $\zeta_{\text{comp}}^{(2)'} = 0$, i.e. $\zeta_{\text{comp}}^{(2)}$ is a conserved quantity. If we now include the possibility of time-dependent interactions as in Eq. (23), $\zeta_{\text{comp}}^{(1)}$ is however no longer conserved because

$$Q_{\text{comp}} = \frac{\Gamma_m\Gamma_\gamma f_m'(N)}{(\Gamma_\gamma + \Gamma_m f_m(N))^2}\rho_\sigma \neq 0. \quad (33)$$

Now in the both situations initially $\rho_m = 0$, $\zeta_{\sigma,\text{in}}^{(1)} = 1$, and $\zeta_{m,\text{in}}^{(1)} = 0$ and we can see from Eq. (27) that

$$\begin{aligned} \zeta_{\text{comp},\text{in}}^{(1)} &= - \frac{\delta\rho_m^{(1)} + \frac{\Gamma_m f_m(N)}{\Gamma_\gamma + \Gamma_m f_m(N)} \delta\rho_\sigma^{(1)}}{-3(\rho_m + \frac{\Gamma_m f_m(N)}{\Gamma_\gamma + \Gamma_m f_m(N)} \rho_\sigma)} \Big|_{N=0} \\ &= - \frac{\frac{\Gamma_m f_m(N)}{\Gamma_\gamma + \Gamma_m f_m(N)} \delta\rho_\sigma^{(1)}}{-3 \frac{\Gamma_m f_m(N)}{\Gamma_\gamma + \Gamma_m f_m(N)} \rho_\sigma} \Big|_{N=0} = - \frac{\rho'_\sigma}{3\rho_\sigma} \zeta_{\sigma,\text{in}}^{(1)} \Big|_{N=0} \\ &= - \frac{\rho'_\sigma}{3\rho_\sigma} \Big|_{N=0} \simeq 1, \end{aligned} \quad (34)$$

where we have used the fact that $\Gamma_\sigma f(N)/H|_{N=0} \ll 1$ in our calculations. Once the curvaton has decayed completely, $\rho_\sigma|_{\text{Dec}} \ll \rho_m|_{\text{Dec}}$ and $\zeta_m^{(i)}|_{\text{Dec}} \simeq \zeta_{\text{comp}}^{(i)}$. If $f_m(N) = 1$, $\zeta_{\text{comp}}^{(i)}$ are conserved and therefore $\zeta_{\sigma,\text{in}}^{(i)} = \zeta_m^{(i)}|_{\text{Dec}}$. In terms of r_1 and r_2 this means that $r_1 = 1$ and $r_2 = 1/2$ which limits the possible values of the resulting non-Gaussianity parameter (22) significantly.

IV. NUMERICAL RESULTS

In [46] we pointed out that the first order perturbation theory was not able to give limits on generated non-Gaussianity. We now apply the second order theory results to calculate the amount of generated non-Gaussianity.

From the reasoning above, we have $\zeta_m^{(1)}|_{\text{Dec}} = \zeta_{\sigma,\text{in}}^{(1)} = 1$ and $\zeta_m^{(2)}|_{\text{Dec}} = \zeta_{\sigma,\text{in}}^{(2)} = 1/2$, or alternatively, $r_1 = 1$ and $r_2 = 1/2$ (the evolution of $\zeta_\gamma^{(1)}$ was explained in detail in [45]). If the curvaton fluid begins to dominate the system before it decays almost all of the radiation originates from the curvaton and hence $\zeta_\gamma^{(1)}|_{\text{Dec}} \simeq \zeta_{\gamma,\text{in}}^{(1)} = 1$. The same reasoning also applies at second order and therefore during decoupling $\zeta_\gamma^{(2)}|_{\text{Dec}} \simeq \zeta_{\gamma,\text{in}}^{(2)} = 1/2$, or $q_1 = 1$, $q_2 = 1/2$. Since $r_1 = q_1$ and $r_2 = q_2$, the system is adiabatic. However, if the curvaton decays before its energy density

begins to dominate the Universe, the curvature perturbations of the radiation fluid remain small and therefore lead to both adiabatic and isocurvature perturbations.

These two differing behaviors can be seen in the Figs. 1(a) and 1(b), where we have plotted the evolution of the second order perturbations $\zeta_i^{(2)}$, $i = \sigma, \gamma, m$. In Fig. 1(a), curvaton dominates universe before its decay, leading to $r_2 \approx 1/2$ and to an adiabatic final state. In the second subfigure, the small initial curvaton density means that the curvaton is subdominant prior to its decay and hence r_2 remains small allowing for nonadiabaticity. From previous work we know that a small initial curvaton density also leads to a larger isocurvature perturbation [46].

In terms of the non-Gaussianity parameter, using the fact that $r_1 = 1$ and $r_2 = 1/2$ means that Eq. (22) can be written in the form

$$f_{\text{NL}} \approx \frac{25(q_2 - q_1^2) - 60q_1 + 81}{6(6 - 5q_1)}. \quad (35)$$

From this we can see that an adiabatic system, i.e. $q_1 = 1$, $q_2 = 1/2$, gives $f_{\text{NL}} = 17/12 \approx 1.42$. Maximizing f_{NL} , we find that $f_{\text{NL}}^{\text{max}} = 2.73$ corresponding to $q_1 = 0.438$, $q_2 = 0.5$ i.e. the system is nonadiabatic.

For different interaction strengths the numerical results are similar and in good agreement with the analytical approximation: the maximum values of non-Gaussianity are close to the expected value and f_{NL} decreases as the system becomes more adiabatic. In a previous paper we studied the same system in detail [46] and found that the system becomes adiabatic as the interaction strengths decrease, hence weaker interaction strengths lead to less non-Gaussianity.

The most interesting observation is that in the three-fluid curvaton model the produced non-Gaussianity is small ($|f_{\text{NL}}| \sim \mathcal{O}(1)$) compared to the one in the two-fluid curvaton model, where much larger non-Gaussianities are easily produced [$|f_{\text{NL}}| \sim \mathcal{O}(10)$ or even $\sim \mathcal{O}(100)$] [6,48]. This result is usually understood in terms of how

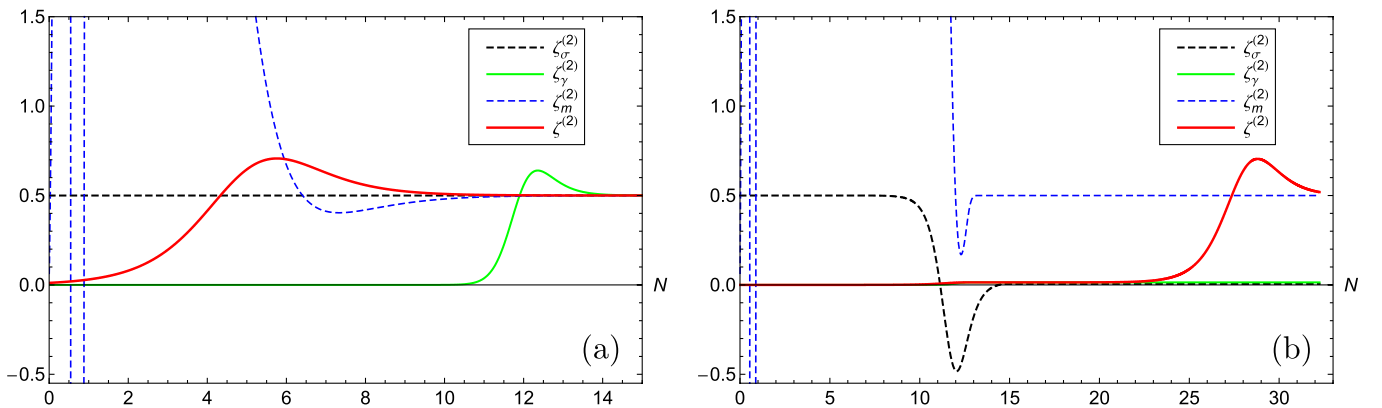


FIG. 1 (color online). Evolution of the second order curvature perturbations in the curvaton-radiation-matter system with $\Gamma_\gamma/H_0 = 10^{-10}$ and $\Gamma_m/H_0 = 10^{-15}$, when (a) $\Omega_{\sigma 0} = 10^{-2}$ and (b) $\Omega_{\sigma 0} = 10^{-7}$. Smaller initial curvaton density leads to a larger isocurvature perturbation. After the curvaton decay, curvature perturbations $\zeta_i^{(2)}$, $i = \sigma, \gamma, m$ remain constant.

efficiently the system can generate matter perturbations [48]: in the two-fluid system we can easily have $r_1 = \zeta_m^{(1)} |_{m} / \zeta_{\sigma, \text{in}}^{(1)} < 1$ and since the observed level of perturbations has to stay constant lower inefficiency generally means a larger level of non-Gaussianity. However in the three-fluid model the system is always perfectly efficient ($r_1 = 1$) and thus we get very little non-Gaussianities. Hence, even a universe with very little observed non-Gaussianity could have had a period dominated by curvaton dynamics.

It should be emphasized that this result is a specific feature of the three-fluid curvaton decay model where the cold dark matter is produced solely by the curvaton decay. By relaxing the underlying assumptions and using e.g. a curvaton model with two noninteracting scalar fields [34] that decay into radiation and dark matter, respectively, a considerably higher level of non-Gaussianity could be reached than in the three-fluid model. This is however beyond the scope of this paper and is left as a further work.

When the decay of the curvaton is modeled by a constant interaction term, the decay begins when the decay width of the curvaton is of the order of the Hubble rate, $\Gamma \sim H$. If the decay process is more involved, such a simple description does not, however, necessarily encompass all of the essential microphysical phenomena. For example, cosmological processes involving different physical scales, such as phase transitions, do not need to conform to the simple fluid description. Instead, multiscale processes are better described by a time-dependent interaction.

The time-dependent scenario was presented and studied in detail in [46], where we found only small differences compared to the continuous case. In order to study the effects of a time-dependent interaction, we choose for the interaction function $f(N) = (\tanh((N - N_*)/\tau) + 1)/2$, where $\tau = 10^{-5}$. The curvature perturbation ζ_{comp} is now no longer conserved because $Q_{\text{comp}} \neq 0$ but since the function $f(N)$ reaches value 1 very quickly, $f'(N) = 0$ is true almost everywhere and ζ_{comp} is almost conserved. Numerical work confirms this conclusion and the value of the non-Gaussianity produced in this scenario is thus very similar to the constant interaction case.

V. DISCUSSION AND CONCLUSIONS

In this paper we have studied the generation of non-Gaussianity in the three-fluid model of curvaton decay by means of second order perturbation theory. In the first part of this paper we introduced general formulae of perturbation theory, different gauge conditions, and concluded it with a derivation of a general formula for the Sachs-Wolfe effect on large scales. It includes the possibility of a non-adiabatic final state and simplifies to the adiabatic formulas presented previously in [48]. In the second part of this paper we applied this formula to the three-fluid model of a curvaton decay and studied the generation of non-Gaussianity in the temperature anisotropy of the CMB.

We find that in general the amount of non-Gaussianity produced in this scenario is small, $f_{\text{NL}} \sim \mathcal{O}(1)$ mainly because the dark matter is also produced by the curvaton, unlike in the two-fluid curvaton models [6,48] which can lead to much larger levels of non-Gaussianity. This conclusion holds both for constant and dynamical interaction strengths between the fluids. This is especially true if the curvaton field fails to dominate the system which leads to large non-Gaussianity in the two-fluid model because then $f_{\text{NL}} \simeq -5/(4r)$ where r is small [6,48]. If the detected non-Gaussianity is small one cannot, however, conclude that the three-fluid model is responsible for it since such an observation is in agreement with the standard inflationary scenario $f_{\text{NL}} = -1/2$ [48,57]. One way to break this degeneracy is to compare the generation of gravity waves in the curvaton scenario [58] with the standard inflation which usually produces a much higher level of gravity waves. Another approach is to use the isocurvature: A nonadiabatic universe with a small amount of non-Gaussianity would indicate the three-fluid curvaton model as a natural candidate.

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