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Conformal windows of $Sp(2N)$ and $SO(N)$ gauge theories

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We study the nonperturbative dynamics of nonsupersymmetric asymptotically free gauge theories with fermionic matter in distinct representations of the $SO(N)$ and $Sp(2N)$ gauge groups. We use different analytic methods to unveil the associated conformal windows for the relevant matter representations. We propose a direct test for confronting and establishing the validity of the analytic methods used to constrain the conformal windows. By comparing the resulting windows for SU, Sp, and SO a pleasing universal picture emerges.

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I. INTRODUCTION

Models of dynamical breaking of the electroweak symmetry are theoretically appealing and constitute one of the best motivated natural extensions of the standard model (SM). These are also among the most challenging models to work with since they require deep knowledge of gauge dynamics in a regime where perturbation theory fails. In particular, it is of utmost importance to gain information on the nonperturbative dynamics of non-Abelian four-dimensional gauge theories.

Recent studies of the dynamics of gauge theories featuring fermions transforming according to higher dimensional representations of the new gauge group led to several interesting phenomenological possibilities [1–3] such as minimal walking technicolor [4] and ultra minimal walking technicolor [5]. Higher dimensional representations have been used earlier in particle physics phenomenology. Time honored examples are grand unified models. Theories with fermions transforming according to higher dimensional representations develop an infrared fixed point (IRFP) for a very small number of flavors and colors [1,3,6]. This was considered unlikely to occur for nonsupersymmetric gauge theories with fermionic matter [7]. This discovery is important since it allows the construction of several explicit UV-complete models able to break the electroweak symmetry dynamically while naturally featuring small contributions to the electroweak precision parameters [4,8,9]. Simultaneously it also helps alleviating the flavor changing neutral currents, while the models also feature explicit candidates of asymmetric dark matter [4,5]. These models are economical since they require the introduction of a very small number of underlying elementary fields and can feature a light composite Higgs [2,3,10]. Recent analyses lend further support to the latter observation [11,12].

At large distances theories developing an IRFP are conformal. One can envision several ways to depart from conformality. For example, one can add a relevant operator such as an explicit fermion-mass term or decrease the number of flavors. If the departure from conformality is *soft*, meaning that the IRFP is quasi reached the gauge coupling constant runs slowly over a long range of energies and the theory is said to *walk* [13–16]. This is, however, not the best way to define a walking theory since the coupling constant is not a physical quantity. In fact one should look at two and higher point correlators and determine the associated scaling exponent. In a (quasi)-conformal theory the scaling will have a characteristic power law behavior. Gauge theories developing an IRFP are natural ultraviolet completions of unparticle [17] models [18,19]. The effects of the instantons and their interplay with the fermion-mass operator on the conformal window have been evaluated in [20]. Within the SD approach these effects were investigated in [21].

Non-Abelian gauge theories exist in a number of distinct phases, which can be classified according to the characteristic dependence of the potential energy on the distance between two well separated static sources. The collection of all of these different behaviors, when represented, for example, in the flavor-color space, constitutes the *phase diagram* of the given gauge theory. The phase diagram of $SU(N)$ gauge theories as functions of number of flavors, colors, and matter representation has been investigated in [1,3,6,19,22]. Interesting applications have been envisioned not only for the LHC phenomenology [1,4,23–26] but also for cosmology [5,27–38]. The nonperturbative dynamics of these models is being investigated via first principles lattice computations by several groups [39–48]. In the literature the reader can also find various attempts to gain information on the nonperturbative gauge dynamics using gauge-gravity type duality, and we cite here only a few recent efforts [49–51].

Here, we extend the analysis of the zero temperature and matter density phase diagram to $SO(N)$ and $Sp(2N)$ gauge theories. Our results will lead to a deeper understanding of the (conformal) gauge dynamics of nonsupersymmetric gauge theories, while it will enlarge the number of nonsupersymmetric gauge theories that can be used for extending the SM.

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The analytical tools we will use for such an exploration are i) The conjectured all-orders beta function (BF) for nonsupersymmetric gauge theories with fermionic matter in arbitrary representations of the gauge group [6]; ii) The truncated Schwinger-Dyson (SD) equation [52–54] (referred also as the ladder approximation in the literature); The Appelquist-Cohen-Schmaltz (ACS) conjecture [55], which makes use of the counting of the thermal degrees of freedom at high and low temperature.

We will show that relevant constraints can be deduced for any gauge theory and any representation only via the all-orders beta function and the SD results. The ACS conjecture is, unfortunately, not sufficiently constraining when studying theories with matter in higher dimensional representations of SO and Sp gauge theories. This is in complete agreement with our earlier results for SU gauge theories [56]. We will rediscuss the phase diagram of the $SU(2)$ gauge theory with fundamental fermions. The results, here, seem to disagree with the ones in [55]. We suggest that by investigating the dynamics of the $SU(2)$ gauge theory with five Dirac flavors in the fundamental representation of the underlying gauge theory via first principles lattice simulations one will be able to test the ACS conjecture as well as the all-orders beta function one.

The paper is organized as follows: In Sec. II, we will introduce the all-orders beta function, in Sec. III, we will summarize the basic points about the SD approximation, and in Sec. IV, we will briefly summarize the thermal degrees of freedom method to bound the conformal window. The phase diagram of $Sp(2N)$ gauge theories with matter in the vector and two-index representation will be investigated in Sec. V, while in Sec. VI, we will investigate the one for $SO(N)$ gauge theories. We will conclude in Sec. VII.

II. ALL-ORDERS BETA FUNCTION CONJECTURE

Recently, we have conjectured an all-orders beta function, which allows for a bound of the conformal window [6] of $SU(N)$ gauge theories for any matter representation. It is written in a form useful for constraining the phase diagram of strongly coupled theories. It is inspired by the Novikov-Shifman-Vainshtein-Zakharov beta function for supersymmetric theories [57,58], and the renormalization scheme coincides with the Novikov-Shifman-Vainshtein-Zakharov one. The predictions of the conformal window coming from the above beta function are nontrivially supported by all the recent lattice results [39–42,46,47,59].

It reproduces the exact supersymmetric results when reducing the matter content to the one of supersymmetric gauge theories. In particular, we compared our prediction for the running of the coupling constant for the pure Yang-Mills theories with the one studied via the Schroedinger functional [59–61] and found an impressive agreement. We have also predicted that the IRFP for $SU(3)$ gauge theories could not extend below 8.25 number of flavors. Subsequent numerical analysis [46,47,62] confirmed our prediction.

Here, we further assume the form of the beta function to hold for $SO(N)$ and $Sp(2N)$ gauge groups. Consider a generic gauge group with $N_f(r_i)$ Dirac flavors belonging to the representation r_i, $i = 1, \ldots, k$ of the gauge group. The conjectured beta function reads

$$\beta(g) = -\frac{g^3}{(4\pi)^2}\frac{\beta_0 - \frac{2}{3}\sum_{i=1}^{k} T(r_i)N_f(r_i)\gamma_i(g^2)}{1 - \frac{g^2}{8\pi^2}C_2(G)(1 + \frac{2\beta_0'}{\beta_0})}, \qquad (1)$$

with

$$\beta_0 = \frac{11}{3}C_2(G) - \frac{4}{3}\sum_{i=1}^{k} T(r_i)N_f(r_i) \quad \text{and} \qquad (2)$$
$$\beta_0' = C_2(G) - \sum_{i=1}^{k} T(r_i)N_f(r_i).$$

The generators T_r^a, $a = 1 \ldots N^2 - 1$ of the gauge group in the representation r are normalized according to $\mathrm{Tr}[T_r^a T_r^b] = T(r)\delta^{ab}$, while the quadratic Casimir $C_2(r)$ is given by $T_r^a T_r^a = C_2(r)I$. The trace normalization factor $T(r)$ and the quadratic Casimir are connected via $C_2(r)d(r) = T(r)d(G)$, where $d(r)$ is the dimension of the representation r. The adjoint representation is denoted by G.

The beta function is given in terms of the anomalous dimension of the fermion mass $\gamma = -d\ln m/d\ln\mu$, where m is the renormalized mass, similar to the supersymmetric case [57,58,63]. The loss of asymptotic freedom is determined by the change of sign in the first coefficient β_0 of the beta function. This occurs when

$$\sum_{i=1}^{k}\frac{4}{11}T(r_i)N_f(r_i) = C_2(G), \qquad \text{Loss of AF.} \qquad (3)$$

At the zero of the beta function we have

$$\sum_{i=1}^{k}\frac{2}{11}T(r_i)N_f(r_i)(2 + \gamma_i) = C_2(G). \qquad (4)$$

Hence, specifying the value of the anomalous dimensions at the IRFP yields the last constraint needed to construct the conformal window. Having reached the zero of the beta function the theory is conformal in the infrared. For a theory to be conformal the dimension of the nontrivial spinless operators must be larger than 1 in order not to contain negative norm states [64–66]. Since the dimension of the chiral condensate is $3 - \gamma_i$ we see that $\gamma_i = 2$, for all representations r_i, yields the maximum possible bound

$$\sum_{i=1}^{k}\frac{8}{11}T(r_i)N_f(r_i) = C_2(G). \qquad (5)$$

In the case of a single representation this constraint yields

$$N_f(r)^{\rm BF} \geq \frac{11}{8}\frac{C_2(G)}{T(r)}. \quad (6)$$

The actual size of the conformal window can be smaller than the one determined by the bound above, Eqs. (3) and (5). It may happen, in fact, that chiral symmetry breaking is triggered for a value of the anomalous dimension less than two. If this occurs, the conformal window shrinks. Within the ladder approximation [52,53] one finds that chiral symmetry breaking occurs when the anomalous dimension is close to 1. Picking $\gamma_i = 1$ we find

$$\sum_{i=1}^{k}\frac{6}{11}T(r_i)N_f(r_i) = C_2(G). \quad (7)$$

When considering two distinct representations the conformal window becomes a three-dimensional volume, i.e. the conformal volume [22]. Of course, we recover the results by Banks and Zaks [67] valid in the perturbative regime of the conformal window.

III. SCHWINGER-DYSON IN THE RAINBOW APPROXIMATION

For nonsupersymmetric theories an old way to get quantitative estimates is to use the *rainbow* approximation to the Schwinger-Dyson equation [68,69], see Fig. 1. Here, the full nonperturbative fermion propagator in momentum space reads

$$iS^{-1}(p) = Z(p)(\not{p} - \Sigma(p)), \quad (8)$$

and the Euclidianized gap equation in Landau gauge is given by

$$\Sigma(p) = 3C_2(r)\int\frac{d^4k}{(2\pi)^4}\frac{\alpha((k-p)^2)}{(k-p)^2}\frac{\Sigma(k^2)}{Z(k^2)k^2 + \Sigma^2(k^2)}, \quad (9)$$

where $Z(k^2) = 1$ in the Landau gauge, and we linearize the equation by neglecting $\Sigma^2(k^2)$ in the denominator. Upon converting it into a differential equation and assuming that the coupling $\alpha(\mu) \approx \alpha_c$ is varying slowly [$\beta(\alpha) \simeq 0$] one gets the approximate (WKB) solutions

$$\Sigma(p) \propto p^{-\gamma(\mu)}, \qquad \Sigma(p) \propto p^{\gamma(\mu)-2}. \quad (10)$$

The critical coupling is given in terms of the quadratic Casimir of the representation of the fermions

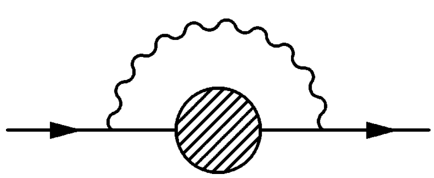

FIG. 1. Rainbow approximation for the fermion self-energy function. The boson is a gluon.

$$\alpha_c \equiv \frac{\pi}{3C_2(r)}. \quad (11)$$

The anomalous dimension of the fermion-mass operator is

$$\gamma(\mu) = 1 - \sqrt{1 - \frac{\alpha(\mu)}{\alpha_c}} \sim \frac{3C_2(r)\alpha(\mu)}{2\pi}. \quad (12)$$

The first solution corresponds to the running of an ordinary mass term (*hard* mass) of nondynamical origin and the second solution to a *soft* mass dynamically generated. In fact, in the second case one observes the $1/p^2$ behavior in the limit of large momentum.

Within this approximation spontaneous symmetry breaking occurs when α reaches the critical coupling α_c given in Eq. (11). From Eq. (12) it is clear that α_c is reached when γ is of order unity [15,52,53]. Hence, the symmetry breaking occurs when the soft and the hard mass terms scale as a function of the energy scale in the same way. In Ref. [52], it was noted that in the lowest (ladder) order, the gap equation leads to the condition $\gamma(2-\gamma) = 1$ for chiral symmetry breaking to occur. To all orders in perturbation theory this condition is gauge invariant and also equivalent nonperturbatively to the condition $\gamma = 1$. However, to any finite order in perturbation theory these conditions are, of course, different. Interestingly, the condition $\gamma(2-\gamma) = 1$ leads again to the critical coupling α_c when using the perturbative leading order expression for the anomalous dimension, which is $\gamma = \frac{3C_2(r)}{2\pi}\alpha$.

To summarize, the idea behind this method is simple. One simply compares the two couplings in the infrared associated to i) an infrared zero in the β function, call it α^* with ii) the critical coupling, denoted with α_c, above which a dynamical mass for the fermions generates nonperturbatively and chiral symmetry breaking occurs. If α^* is less than α_c, chiral symmetry does not occur, and the theory remains conformal in the infrared, vice versa if α^* is larger than α_c, then the fermions acquire a dynamical mass, and the theory cannot be conformal in the infrared. The condition $\alpha^* = \alpha_c$ provides the desired $N_f^{\rm SD}$ as function of N. In practice, to estimate α^* one uses the two-loop beta function while one uses the truncated SD equation to determine α_c as we have done before. This corresponds to when the anomalous dimension of the quark mass operator becomes approximately unity.

The two-loop fixed point value of the coupling constant is

$$\frac{\alpha^*}{4\pi} = -\frac{\beta_0}{\beta_1}, \quad (13)$$

with the following definition of the two-loop beta function:

$$\beta(g) = -\frac{\beta_0}{(4\pi)^2}g^3 - \frac{\beta_1}{(4\pi)^4}g^5, \quad (14)$$

where g is the gauge coupling, and the beta function coefficients are given by

$$\beta_0 = \tfrac{11}{3}C_2(G) - \tfrac{4}{3}T(r)N_f, \tag{15}$$

$$\beta_1 = \tfrac{34}{3}C_2^2(G) - \tfrac{20}{3}C_2(G)T(r)N_f - 4C_2(r)T(r)N_f. \tag{16}$$

To this order the two coefficients are universal, i.e. do not depend on which renormalization group scheme one has used to determine them. The perturbative expression for the anomalous dimension reads

$$\gamma(g^2) = \frac{3}{2}C_2(r)\frac{g^2}{4\pi^2} + O(g^4), \tag{17}$$

with $\gamma = -d\ln m/d\ln\mu$ and m the renormalized fermion-mass.

For a fixed number of colors the critical number of flavors for which the order of α^* and α_c changes is defined by imposing $\alpha^* = \alpha_c$, and it is given by

$$N_f^{\mathrm{SD}} = \frac{17C_2(G) + 66C_2(r)}{10C_2(G) + 30C_2(r)}\frac{C_2(G)}{T(r)}. \tag{18}$$

Comparing with the previous result obtained using the all-orders beta function, we see that it is the coefficient of $C_2(G)/T(r)$, which is different.

IV. THERMAL COUNTING OF THE DEGREES OF FREEDOM CONJECTURE

The free energy can be seen as a device to count the relevant degrees of freedom. It can be computed, exactly, in two regimes of a generic asymptotically free theory: the very hot and the very cold one.

The zero-temperature theory of interest is characterized using the quantity f_{IR}, related to the free energy by

$$f_{\mathrm{IR}} \equiv -\lim_{T\to 0}\frac{\mathcal{F}(T)}{T^4}\frac{90}{\pi^2}, \tag{19}$$

where T is the temperature and $\mathcal{F}$ is the conventionally defined free energy per unit volume. The limit is well defined if the theory has an IRFP. For the special case of an infrared-free theory

$$f_{\mathrm{IR}} = \#\ \text{Real Bosons} + \frac{7}{4}\#\ \text{Weyl-Fermions}. \tag{20}$$

The corresponding expression in the large T limit is

$$f_{\mathrm{UV}} \equiv -\lim_{T\to\infty}\frac{\mathcal{F}(T)}{T^4}\frac{90}{\pi^2}. \tag{21}$$

This limit is well defined if the theory has an ultraviolet fixed point. For an asymptotically free theory f_{UV} counts the underlying ultraviolet d.o.f. in a similar way.

In terms of these quantities, the conjectured inequality [55] for any asymptotically free theory is

$$f_{\mathrm{IR}} \leq f_{\mathrm{UV}}. \tag{22}$$

This inequality has not been proven but it was shown to be consistent with known results and then used to derive new constraints for several strongly coupled, vectorlike gauge theories. The ACS conjecture has been used also for chiral gauge theories [70]. There it was also found that to make definite predictions a stronger requirement is needed [71].

V. PHASE DIAGRAM OF $Sp(2N)$ GAUGE THEORIES

$Sp(2N)$ is the subgroup of $SU(2N)$, which leaves the tensor $J^{c_1c_2} = (\mathbf{1}_{N\times N}\otimes i\sigma_2)^{c_1c_2}$ invariant. Irreducible tensors of $Sp(2N)$ must be traceless with respect to $J^{c_1c_2}$. Here, we consider $Sp(2N)$ gauge theories with fermions transforming according to a given irreducible representation. Since $\pi^4[Sp(2N)] = Z_2$ there is a Witten topological anomaly [72] whenever the sum of the Dynkin indices of the various matter fields is odd. The adjoint of $Sp(2N)$ is the two-index symmetric tensor.

A. $Sp(2N)$ with vector fields

Consider $2N_f$ Weyl fermions q_c^i with $c = 1,\ldots,2N$ and $i = 1,\ldots,2N_f$ in the fundamental representation of $Sp(2N)$. We have omitted the $SL(2,C)$ spinorial indices. We need an even number of flavors to avoid the Witten anomaly since the Dynkin index of the vector representation is equal to one. In the following Table we summarize the properties of the theory

Fields	$[Sp(2N)]$	$SU(2N_f)$	$T[r_i]$	$d[r_i]$
q	□	□	$\frac{1}{2}$	$2N$
G_μ	Adj = □□	1	$N+1$	$N(2N+1)$

1. Chiral symmetry breaking

The theory is asymptotically free for $N_f \leq 11(N+1)/2$, while the relevant gauge singlet mesonic degree of freedom is

$$M^{[i,j]} = \epsilon^{\alpha\beta}q^{[i}_{\alpha,c_1}q^{j]}_{\beta,c_2}J^{c_1c_2}. \tag{23}$$

If the number of flavors is smaller than the critical number of flavors above which the theory develops an IRFP, we expect this operator to condense and to break $SU(2N_f)$ to the maximal diagonal subgroup, which is $Sp(2N_f)$ leaving behind $2N_f^2 - N_f - 1$ Goldstone bosons. Also, there exist no $Sp(2N)$ stable operators constructed using the invariant tensor $\epsilon^{c_1c_2,\ldots c_{2N}}$ since they will break up into mesons M. This is so since the invariant tensor $\epsilon^{c_1c_2\ldots c_{2N}}$ breaks up into sums of products of $J^{c_1c_2}$.

2. All-orders beta function

A zero in the numerator of the all-orders beta function leads to the following value of the anomalous dimension of the mass operator at the IRFP:

$$\gamma_{\square} = \frac{11(N+1)}{N_f} - 2. \tag{24}$$

Since the (mass) dimension of any scalar gauge singlet operator must be, by unitarity arguments, larger than 1 at the IRFP, this implies that $\gamma_\square \leq 2$. Defining with $\gamma_\square^*$ the maximal anomalous dimension above which the theory loses the IRFP the conformal window is

$$\frac{11}{4}(N+1) \leq \frac{11}{2+\gamma_\square^*}(N+1) \leq N_f \leq \frac{11}{2}(N+1). \tag{25}$$

For the first inequality we have taken the maximal value allowed for the anomalous dimension, i.e. $\gamma_\square^* = 2$.

3. SD

The estimate from the truncated SD analysis yields as critical value of Weyl flavors

$$N_f^{\rm SD} = \frac{2(1+N)(67+100N)}{35+50N}. \tag{26}$$

4. Thermal degrees of freedoms

In the UV we have $2N(2N+1)$ gauge bosons, where the extra factor of 2 comes from taking into account the two helicities of each massless gauge boson, and $4NN_f$ Weyl fermions. In the IR we have $2N_f^2 - N_f - 1$ Goldstones, and hence we have

$$f_{\rm UV} = 2N(2N+1) + 7NN_f, \qquad f_{\rm IR} = 2N_f^2 - N_f - 1. \tag{27}$$

The number of flavors for which $f_{\rm IR} = f_{\rm UV}$ is

$$N_f^{\rm Therm} = \frac{1 + 7N + \sqrt{3(3+10N+27N^2)}}{4}. \tag{28}$$

No information can be obtained about the value of the anomalous dimension of the fermion bilinear at the fixed point. Assuming the conjecture to be valid the critical number of flavors cannot exceed $N_f^{\rm Therm}$. The phase diagram is plotted in Fig. 2.

5. A comment on the limit $N = 1$ corresponding to $SU(2)$

In this case, $N_f^{\rm Therm} = 2 + \sqrt{\frac{15}{2}} \simeq 4.74$ and not $4\sqrt{4 - 16/81} \simeq 7.8$ as one deduces from Eq. (11) of [55]. The reason for the discrepancy is due to the fact that the fundamental representation of $SU(2) = Sp(2)$ is pseudoreal and hence the flavor symmetry is enhanced to $SU(2N_f)$. This enhanced symmetry is expected to break spontaneously to $Sp(2N_f)$. This yields $2N_f^2 - N_f - 1$ Goldstone bosons rather than $N_f^2 - 1$ obtained assuming the global symmetry to be $SU(N_f) \times SU(N_f) \times U(1)$ spontaneously broken to $SU(N_f) \times U(1)$. The corrected $N_f^{\rm Therm}$ value for $SU(2)$ is substantially lower than the SD one which is 7.86. The all-orders beta function result is instead 5.5 for the lowest possible value of N_f below which chiral symmetry must break (corresponding to $\gamma_\square = 2$). Imposing $\gamma_\square = 1$ (suggested by the SD approach) the all-orders beta function returns to 7.3, which is closer to the SD prediction. Note that there is some phenomenological interest in the $SU(2)$ gauge theory with fermionic matter in the fundamental representation. For example the case of $N_f = 8$ has been employed in the literature as a possible template for early models of walking technicolor [73].

These results indicate that it is interesting to study the $SU(2)$ gauge theory with $N_f = 5$ Dirac flavors via first principles lattice simulation. This will allow to discrimi-

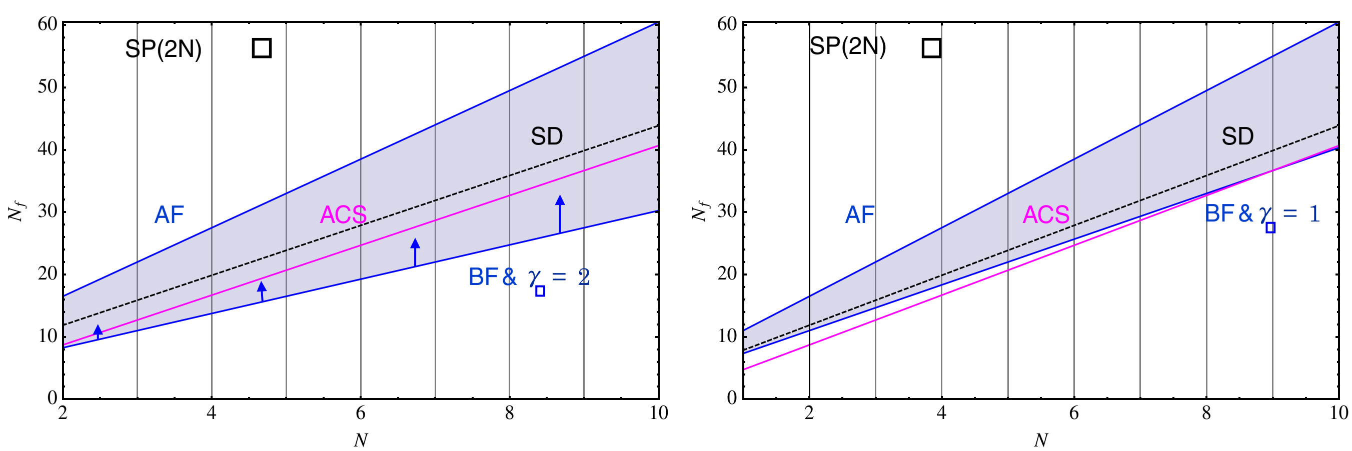

FIG. 2 (color online). Phase diagram of $Sp(2N)$ gauge theories with $2N_f$ fundamental Weyl fermions. *Left panel*: The upper solid (blue) line corresponds to the loss of asymptotic freedom, and it is labeled by AF; the dashed (black) curve corresponds to the SD prediction for the breaking/restoring of chiral symmetry. The solid grey (magenta in color) line corresponds to the ACS bound stating that the conformal region should start above this line. According to the all-orders BF, the conformal window cannot extend below the solid (blue) line, as indicated by the arrows. This line corresponds to the anomalous dimension of the mass reaching the maximum value of 2. *Right panel*: The BF line is plotted assuming the value of the anomalous dimension to be one.

nate between the two distinct predictions, the one from the ACS and the one from the all-orders beta function.

B. $Sp(2N)$ with adjoint matter fields

Consider N_f Weyl fermions $q^i_{\{c_1,c_2\}}$ with c_1 and c_2 ranging from 1 to $2N$ and $i = 1, \ldots, N_f$. This is the adjoint representation of $Sp(2N)$ with Dynkin index $2(N+1)$. Since it is even for any N, there is no Witten anomaly for any N_f. In the following Table we summarize the properties of the theory:

Fields	$[Sp(2N)]$	$SU(N_f)$	$T[r_i]$	$d[r_i]$
q	$\square\square$	$\square$	$N+1$	$N(2N+1)$
G_μ	Adj $= \square\square$	1	$N+1$	$N(2N+1)$

1. Chiral symmetry breaking

The theory is asymptotically free for $N_f \leq 11/2$ (recall that N_f here is the number of Weyl fermions), while the relevant gauge singlet mesonic degree of freedom is

$$M^{\{i,j\}} = \epsilon^{\alpha\beta} q^{\{i}_{\alpha,\{c_1,c_2\}} q^{j\}}_{\beta,\{c_3,c_4\}} J^{c_1c_3} J^{c_2c_4}. \tag{29}$$

If the number of flavors is smaller than the critical number of flavors above which the theory develops an IRFP, we expect this operator to condense and to break $SU(N_f)$ to the maximal diagonal subgroup, which is $SO(N_f)$ leaving behind $(N_f^2 + N_f - 2)/2$ Goldstone bosons.

2. All-orders beta function

Here, the anomalous dimension of the mass operator at the IRFP is

$$\gamma_{\square\square} = \frac{11}{N_f} - 2. \tag{30}$$

Since the dimension of any scalar gauge singlet operator must be larger than 1 at the IRFP, this implies that $\gamma_{\square\square} \leq 2$. Defining with $\gamma^*_{\square\square}$ the maximal anomalous dimension above which the theory loses the IRFP, the conformal window is

$$\frac{11}{4} \leq \frac{11}{2+\gamma^*_{\square\square}} \leq N_f \leq \frac{11}{2}. \tag{31}$$

3. SD

The estimate from the truncated SD analysis yields as critical value of flavors

$$N_f^{\rm SD} = 4.15. \tag{32}$$

4. Thermal degrees of freedoms

In the ultraviolet we have $2N(2N+1)$ gauge bosons and $N(2N+1)N_f$ Weyl fermions. In the IR we have $(N_f^2 + N_f - 2)/2$ Goldstone bosons. Hence,

$$f_{\rm UV} = 2N(2N+1) + \frac{7}{4}N(2N+1)N_f, \qquad f_{\rm IR} = \frac{N_f^2 + N_f - 2}{2}. \tag{33}$$

The number of flavors for which $f_{\rm IR} = f_{\rm UV}$ is

$$N_f^{\rm Therm} = \frac{-2 + 7N + 14N^2 + \sqrt{36 + 36N + 121N^2 + 196N^3 + 196N^4}}{4}. \tag{34}$$

This is a monotonically increasing function of N, which even for a value of N as low as 2 yields $N_f^{\rm Therm} = 35.2$, which is several times higher than the limit set by asymptotic freedom. Although this fact does not contradict the statement that the critical number of flavors is lower than $N_f^{\rm Therm}$, it shows that this conjecture does not lead to useful constraints when looking at higher dimensional representations as we observed in [56] when discussing higher dimensional representations for $SU(N)$ gauge groups The phase diagram is summarized in Fig. 3.

C. $Sp(2N)$ with two-index antisymmetric representation

Consider N_f Weyl fermions $q^i_{[c_1,c_2]}$ with c_1 and c_2 ranging from 1 to $2N$ and $i = 1, \ldots, N_f$. As for the two-index symmetric case here too the Dynkin index is even and hence we need not to worry about the Witten anomaly. In the following Table we summarize the properties of the theory:

Fields	$[Sp(2N)]$	$SU(N_f)$	$T[r_i]$	$d[r_i]$
q	$\begin{smallmatrix}\square\\\square\end{smallmatrix}$	$\square$	$N-1$	$N(2N-1)-1$
G_μ	Adj $= \square\square$	1	$N+1$	$N(2N+1)$

1. Chiral symmetry breaking

The theory is asymptotically free for $N_f \leq \frac{11(N+1)}{2(N-1)}$ with the relevant gauge singlet mesonic degree of freedom being

$$M^{\{i,j\}} = \epsilon^{\alpha\beta} q^{\{i}_{\alpha,[c_1,c_2]} q^{j\}}_{\beta,[c_3,c_4]} J^{c_1c_3} J^{c_2c_4}. \tag{35}$$

If the number of flavors is smaller than the critical number

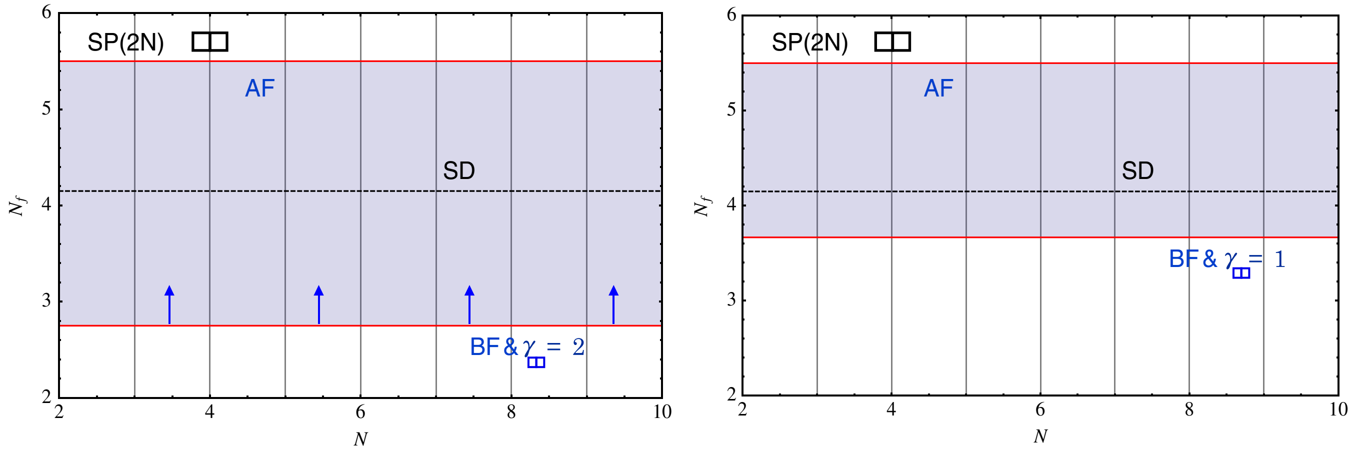

FIG. 3 (color online). Phase diagram of $Sp(2N)$ gauge theories with N_f adjoint Weyl fermions. *Left panel*: The upper solid (red) line corresponds to the loss of asymptotic freedom, and it is labeled by AF; the dashed (black) curve corresponds to the SD prediction for the breaking/restoring of chiral symmetry. According to the all-orders BF, the conformal window cannot extend below the solid (red) line, as indicated by the arrows. This line corresponds to the anomalous dimension of the mass reaching the maximum value of 2. *Right panel*: The BF line is plotted assuming the value of the anomalous dimension to be one.

of flavors above which the theory develops an IRFP, we expect this operator to condense and to break $SU(N_f)$ to the maximal diagonal subgroup, which is $SO(N_f)$ leaving behind $(N_f^2 + N_f - 2)/2$ Goldstone bosons.

2. All-orders beta function

The anomalous dimension of the mass operator at the IRFP is

$$\gamma_{\begin{smallmatrix}\Box\\\Box\end{smallmatrix}} = \frac{11(N+1) - 2N_f(N-1)}{N_f(N-1)}. \qquad (36)$$

Defining with $\gamma^*_{\begin{smallmatrix}\Box\\\Box\end{smallmatrix}}$ the maximal anomalous dimension above which the theory loses the IRFP the conformal window is

$$\frac{11}{4}\frac{N+1}{N-1} \leq \frac{11}{2+\gamma^*_{\begin{smallmatrix}\Box\\\Box\end{smallmatrix}}}\frac{N+1}{N-1} \leq N_f \leq \frac{11}{2}\frac{N+1}{N-1}. \qquad (37)$$

The maximal value allowed for the anomalous dimension is $\gamma^*_{\begin{smallmatrix}\Box\\\Box\end{smallmatrix}} = 2$.

3. SD

The SD analysis yields as critical value of flavors

$$N_f^{\rm SD} = \frac{(1+N)(83N+17)}{5(4N^2 - 3N - 1)}. \qquad (38)$$

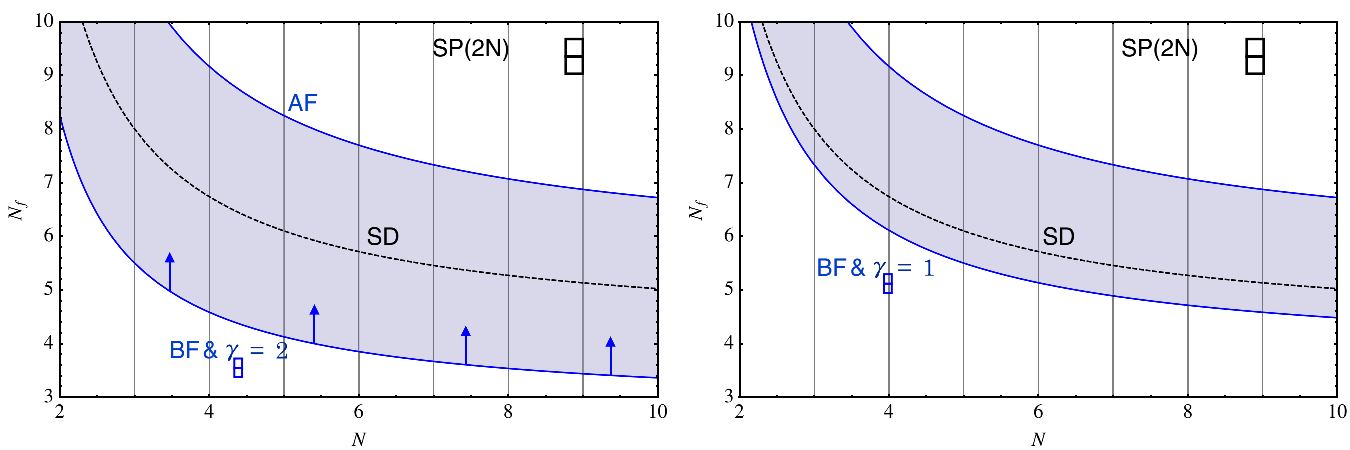

FIG. 4 (color online). Phase diagram of $Sp(2N)$ gauge theories with N_f two-index antisymmetric Weyl fermions. *Left panel*: The upper solid (blue) curve corresponds to the loss of asymptotic freedom, and it is labeled by AF; the dashed (black) curve corresponds to the SD prediction for the breaking/restoring of chiral symmetry. According to the all-orders BF, the conformal window cannot extend below the solid (blue) curve, as indicated by the arrows. This curve corresponds to the anomalous dimension of the mass reaching the maximum value of 2. *Right panel*: The BF curve is plotted assuming the value of the anomalous dimension to be one.

4. Thermal degrees of freedoms

In the ultraviolet we have $2N(2N+1)$ gauge bosons and $(N(2N-1)-1)N_f$ Weyl fermions. In the IR we have $(N_f^2 + N_f - 2)/2$ Goldstone bosons. Hence,

$$f_{\rm UV} = 2N(2N+1) + \frac{7}{4}(N(2N-1)-1)N_f, \qquad f_{\rm IR} = \frac{N_f^2 + N_f - 2}{2}. \tag{39}$$

The number of flavors for which $f_{\rm IR} = f_{\rm UV}$ is

$$N_f^{\rm Therm} = \frac{-9 - 7N + 14N^2 + \sqrt{113 + 190N - 75N^2 - 196N^3 + 196N^4}}{4}. \tag{40}$$

As explained above no useful constraint can be set with this criterion [56]. The phase diagram is summarized in Fig. 6.

5. Summary of the results for $SP(2N)$ gauge theories

In Fig. 5, we summarize the relevant zero temperature and matter density phase diagram as function of the number of colors and Weyl flavors ($N_{\rm Wf}$) for $Sp(2N)$ gauge theories. For the vector representation $N_{\rm Wf} = 2N_f$, while for the two-index theories $N_{\rm Wf} = N_f$. The shape of the various conformal windows are very similar to the ones for $SU(N)$ gauge theories [1,3,6] with the difference that in this case the two-index symmetric representation is the adjoint representation and hence there is one less conformal window.

VI. PHASE DIAGRAM OF $SO(N)$ GAUGE THEORIES

We shall consider $SO(N)$ theories (for $N > 5$) since they do not suffer of a Witten anomaly [72] and, besides, for $N < 7$ can always be reduced to either an SU or an Sp theory.

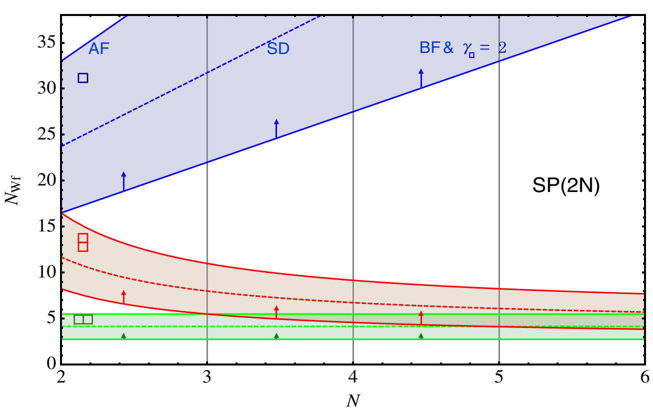

FIG. 5 (color online). Phase diagram, from top to bottom, for $Sp(2N)$ gauge theories with $N_{\rm Wf} = 2N_f$ Weyl fermions in the vector representation (light blue), $N_{\rm Wf} = N_f$ in the two-index antisymmetric representation (light red), and finally in the two-index symmetric (adjoint) [light green]. The arrows indicate that the conformal windows can be smaller, and the associated solid curves correspond to the all-orders beta function prediction for the maximum extension of the conformal windows.

A. $SO(N)$ with vector fields

Consider N_f Weyl fermions q_c^i with $c = 1, \ldots, N$ and $i = 1, \ldots, N_f$ in the vector representation of $SO(N)$. In the following Table we summarize the properties of the theory:

Fields	$[SO(N)]$	$SU(N_f)$	$T[r_i]$	$d[r_i]$
q	$\square$	$\square$	1	N
G_μ	Adj $= \begin{smallmatrix}\square\\\square\end{smallmatrix}$	1	$N-2$	$\frac{N(N-1)}{2}$

1. Chiral symmetry breaking

The theory is asymptotically free for $N_f \le \frac{11(N-2)}{2}$. The relevant gauge singlet mesonic degree of freedom is

$$M^{\{i,j\}} = \epsilon^{\alpha\beta} q_{\alpha,c_1}^{\{i} q_{\beta,c_2}^{j\}} \delta^{c_1 c_2}. \tag{41}$$

If the number of flavors is smaller than the critical number of flavors above which the theory develops an IRFP, we expect this operator to condense and to break $SU(N_f)$ to the maximal diagonal subgroup, which is $SO(N_f)$ leaving behind $(N_f^2 + N_f - 2)/2$ Goldstone bosons.

2. All-orders beta function

The anomalous dimension of the mass operator at the IRFP is

$$\gamma_\square = \frac{11(N-2)}{N_f} - 2. \tag{42}$$

Defining with $\gamma_\square^*$ the maximal anomalous dimension above which the theory loses the IRFP, the conformal window reads

$$\frac{11}{4}N - 2 \le \frac{11}{2+\gamma_\square^*}N - 2 \le N_f \le \frac{11}{2}N - 2. \tag{43}$$

The maximal value allowed for the anomalous dimension is $\gamma_\square^* = 2$.

3. SD

The SD analysis yields as critical value of flavors

$$N_f^{\rm SD} = \frac{2(N-2)(50N-67)}{5(5N-7)}. \tag{44}$$

4. Thermal degrees of freedoms

In the ultraviolet we have $N(N-1)$ gauge bosons and NN_f Weyl fermions. In the IR we have $(N_f^2+N_f-2)/2$ Goldstone bosons. Hence,

$$f_{\rm UV} = N(N-1) + \frac{7}{4}NN_f, \qquad f_{\rm IR} = \frac{N_f^2+N_f-2}{2}. \quad (45)$$

The number of flavors for which $f_{\rm IR} = f_{\rm UV}$ is

$$N_f^{\rm Therm} = \frac{-2+7N+\sqrt{36-60N+81N^2}}{4}. \quad (46)$$

This value is larger than the SD result, and it is larger than the asymptotic freedom constraint for $N < 7$. This is not too surprising since the vector representation of $SO(N)$ for small N becomes a higher representation of other groups for which we have already shown that this method is unconstraining [56].

Note that the ACS line is always above the SD result.

B. $SO(N)$ with adjoint matter fields

Consider N_f Weyl fermions $q^i_{[c_1,c_2]}$ with c_1 and c_2 varying in the range $1,\ldots,N$ and $i = 1,\ldots,N_f$. This is the adjoint representation of $SO(N)$. In the following Table we summarize the properties of the theory:

Fields	$[SO(N)]$	$SU(N_f)$	$T[r_i]$	$d[r_i]$
q	$\begin{smallmatrix}\Box\\\Box\end{smallmatrix}$	$\Box$	$N-2$	$\frac{N(N-1)}{2}$
G_μ	Adj $= \begin{smallmatrix}\Box\\\Box\end{smallmatrix}$	1	$N-2$	$\frac{N(N-1)}{2}$

The analysis leads to a conformal window that is an identical copy of the one for the adjoint matter of the Sp gauge theory, which is also identical to the SU case with adjoint matter. The phase diagram is summarized in Fig. 6.

C. $SO(N)$ with two-index symmetric representation

Consider N_f Weyl fermions $q^i_{\{c_1,c_2\}}$ with c_1 and c_2 varying in the range $1,\ldots,N$ and $i = 1,\ldots,N_f$, i.e. in the two-index symmetric representation of $SO(N)$. In the following Table we summarize the properties of the theory:

Fields	$[SO(N)]$	$SU(N_f)$	$T[r_i]$	$d[r_i]$
q	$\Box\Box$	$\Box$	$N+2$	$\frac{N(N+1)}{2}-1$
G_μ	Adj $= \begin{smallmatrix}\Box\\\Box\end{smallmatrix}$	1	$N-2$	$\frac{N(N-1)}{2}$

1. Chiral symmetry breaking

The theory is asymptotically free for $N_f \le \frac{11(N-2)}{2(N+2)}$. The relevant gauge singlet mesonic degree of freedom is

$$M^{\{i,j\}} = \epsilon^{\alpha\beta} q^{\{i}_{\alpha,\{c_1,c_2\}} q^{j\}}_{\beta,\{c_3,c_4\}} \delta^{c_1c_3}\delta^{c_2,c_4}. \quad (47)$$

If the number of flavors is smaller than the critical number of flavors above which the theory develops an IRFP, we expect this operator to condense and to break $SU(N_f)$ to the maximal diagonal subgroup which is $SO(N_f)$ leaving behind $(N_f^2+N_f-2)/2$ Goldstone bosons.

2. All-orders beta function

The anomalous dimension of the mass operator at the IRFP is

$$\gamma_{\Box\Box} = \frac{11(N-2)}{N_f(N+2)} - 2. \quad (48)$$

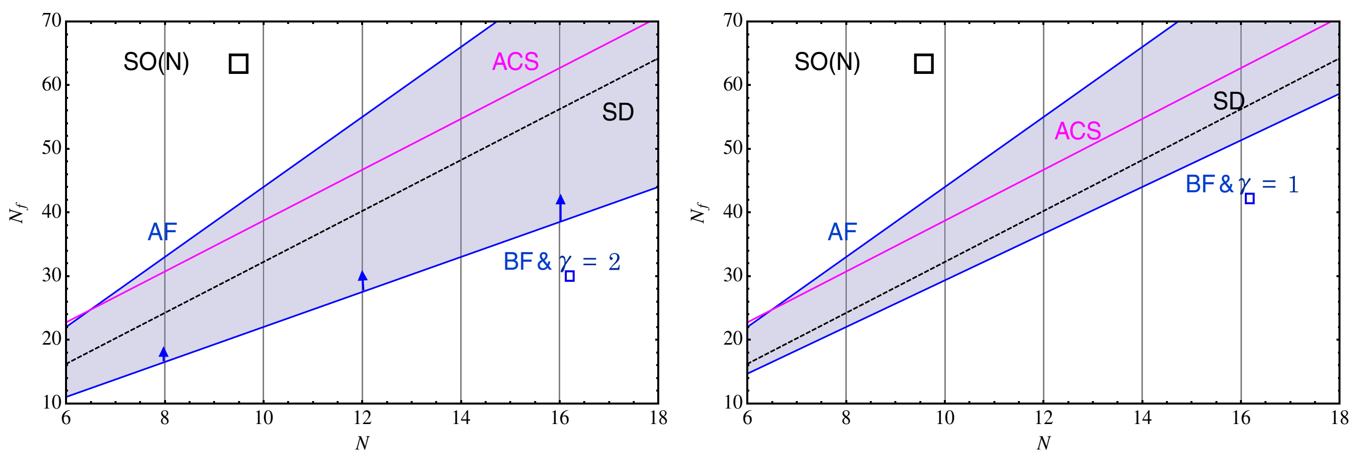

FIG. 6 (color online). Phase diagram of $SO(N)$ gauge theories with N_f fundamental Weyl fermions. *Left panel*: The upper solid (blue) line corresponds to the loss of asymptotic freedom, and it is labeled by AF; the dashed (black) curve corresponds to the SD prediction for the breaking/restoring of chiral symmetry. The solid grey (magenta in color) line corresponds to the ACS bound stating that the conformal region should start above this line. According to the all-orders BF, the conformal window cannot extend below the solid (blue) line, as indicated by the arrows. This line corresponds to the anomalous dimension of the mass reaching the maximum value of 2. *Right panel*: The BF line is plotted assuming the value of the anomalous dimension to be one.

Defining with $\gamma^*_{\Box\Box}$ the maximal anomalous dimension above which the theory loses the IRFP the conformal window reads

$$\frac{11}{4}\frac{N-2}{N+2} \leq \frac{11}{2+\gamma^*_{\Box\Box}}\frac{N-2}{N+2} \leq N_f \leq \frac{11}{2}\frac{N-2}{N+2}. \quad (49)$$

The maximal value allowed for the anomalous dimension is $\gamma^*_{\Box\Box} = 2$.

3. SD

The SD analysis yields as critical value of flavors

$$N_f^{\rm SD} = \frac{(N-2)(83N-34)}{10(2N^2+3N-2)}. \quad (50)$$

4. Thermal degrees of freedoms

In the ultraviolet we have $N(N-1)$ gauge bosons and $(N\frac{(N+1)}{2}-1)N_f$ Weyl fermions. In the IR we have $(N_f^2 + N_f - 2)/2$ Goldstone bosons. Hence,

$$f_{\rm UV} = N(N-1) + \frac{7}{4}\left(N\frac{(N+1)}{2} - 1\right)N_f, \qquad f_{\rm IR} = \frac{N_f^2 + N_f - 2}{2}. \quad (51)$$

The number of flavors for which $f_{\rm IR} = f_{\rm UV}$ is

$$N_f^{\rm Therm} = \frac{-18 + 7N(1+N) + \sqrt{452 + N(-380 + N(-75 + 49N(2+N)))}}{8}. \quad (52)$$

This value is several times larger than the asymptotic freedom result and hence poses no constraint [56]. The phase diagram is summarized in Fig. 7.

5. Summary for $SO(N)$ gauge theories

In Fig. 8, we summarize the relevant zero temperature and matter density phase diagram as a function of the number of colors and Weyl flavors (N_f) for $SO(N)$ gauge theories. The shape of the various conformal windows are very similar to the ones for $SU(N)$ and $Sp(2N)$ gauge with the difference that in this case the two-index antisymmetric representation is the adjoint representation. We have analyzed only the theories with $N \geq 6$ since the remaining smaller N theories can be deduced from Sp and SU using the fact that $SO(6) \sim SU(4)$, $SO(5) \sim Sp(4)$, $SO(4) \sim SU(2) \times SU(2)$, $SO(3) \sim SU(2)$, and $SO(2) \sim U(1)$.

At infinite N it is impossible to distinguish theories with matter in the two-index symmetric representation from theories with matter in the two-index antisymmetric. This means that, in this regime, one has an obvious equivalence between theories with these two types of matter. This statement is independent of whether the gauge group is SU, Sp or $SO(N)$. What distinguishes SU from both Sp and SO is the fact that in these two cases one of the two two-index representations is, in fact, the adjoint representation. This simple observation automatically implies that one Weyl flavor in the two-index symmetric (antisymmet-

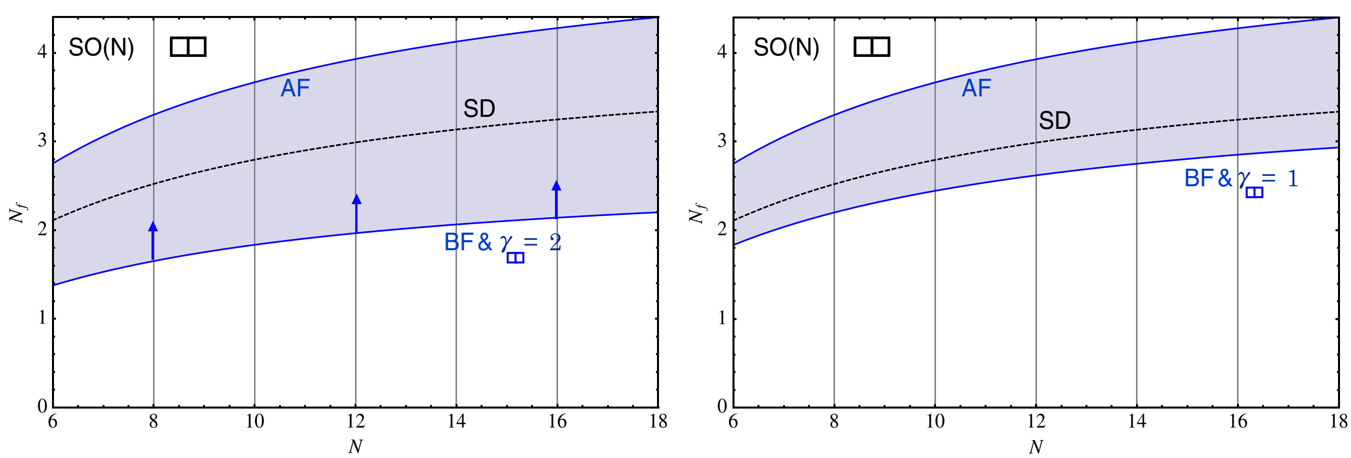

FIG. 7 (color online). Phase diagram of $SO(N)$ gauge theories with N_f Weyl fermions in the two-index symmetric representation. *Left panel*: The upper solid (blue) curve corresponds to the loss of asymptotic freedom, and it is labeled by AF; the dashed (black) curve corresponds to the SD prediction for the breaking/restoring of chiral symmetry. According to the all-orders BF, the conformal window cannot extend below the solid (blue) curve, as indicated by the arrows. This curve corresponds to the anomalous dimension of the mass reaching the maximum value of 2. *Right panel*: The BF curve is plotted assuming the value of the anomalous dimension to be one.

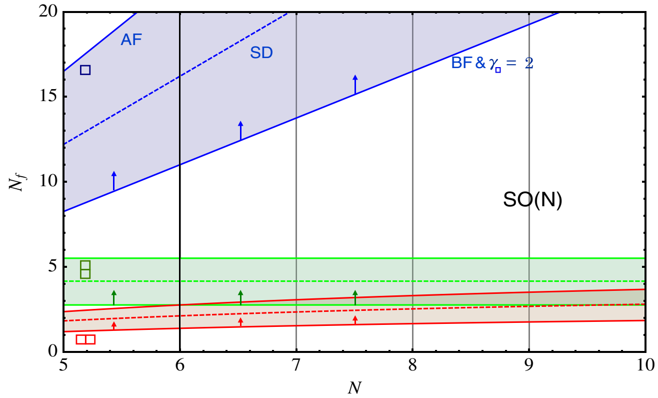

FIG. 8 (color online). Phase diagram of $SO(N)$ gauge theories with N_f Weyl fermions in the vector representation, in the two-index antisymmetric (adjoint) and finally in the two-index symmetric representation. The arrows indicate that the conformal windows can be smaller, and the associated solid curves correspond to the all-orders beta function prediction for the maximum extension of the conformal windows.

ric) representation of $SO(N)(Sp(2N))$ becomes indistinguishable from pure super Yang-Mills at large N. The original observation appeared first within the context of string theory, and it is due to Sugimoto [74] and Uranga [75]. A similar comment was made in [76].

VII. COMPARISON CHART AND CONCLUSIONS

We unveiled the conformal windows for SO and Sp nonsupersymmetric gauge theories with fermions in the vector and two-index representations using three independent analytic methods. In Figs. 5 and 8 we plotted the two phase diagrams as a function of the number of flavors, colors, and matter representation. These phase diagrams are similar to the one for $SU(N)$ gauge theories [1,3,6] summarized in [19]. One observes a universal value, i.e. independent of the representation, of the ratio of the area of the maximum extension of the conformal window, predicted using the all-orders beta function, to the asymptotically free one, as defined in [22]. It is easy to check from our results that this ratio is not only independent on the representation but also on the particular gauge group chosen.

The three different methods we used to unveil the conformal windows are the all-orders beta function (BF), the SD truncated equation, and the thermal degrees of freedom method. In the Table below, we compare directly the various analytical methods. The three plus signs in the second column indicate that the three analytic methods do constrain the conformal window of SU, Sp, and SO gauge theories with fermions in the fundamental representation. Only BF and SD provide useful constraints in the case of the higher dimensional representations as summarized in the third column. When multiple representations participate in the gauge dynamics, the BF constraints can be used directly [5,6] to determine the extension of the conformal (hyper)volumes, while extra dynamical information and approximations are required in the SD approach. Since gauge theories with fermions in several representations of the underlying gauge group must contain higher dimensional representations the ACS is expected to be less efficient in this case [77]. These results are summarized in the fourth column. The all-orders beta function reproduces the supersymmetric exact results when going over the super Yang-Mills case, and the ACS conjecture was proved successful when tested against the supersymmetric conformal window results [55]. However, the SD approximation does not reproduce any supersymmetric results [78]. The results are summarized in the fifth column. Finally, it is of theoretical and phenomenological interest—for example, to construct sensible UV completions of models of dynamical electroweak symmetry breaking and unparticles—to compute the anomalous dimension of the mass of the fermions at the (near) conformal fixed point. Only the all-orders beta function provides a simple closed form expression as it is summarized in the sixth column.

We have also suggested that it is interesting to study the $SU(2)$ gauge theory with $N_f = 5$ Dirac flavors via first principles lattice simulations since it will discriminate between the two distinct predictions, the one from the ACS conjecture and the one from the all-orders beta function.

Our analysis substantially increases the number of asymptotically free gauge theories that can be used to construct SM extensions making use of (near) conformal dynamics. Current lattice simulations can test our predictions and lend further support or even disprove the emergence of a universal picture possibly relating the phase diagrams of gauge theories of fundamental interactions.

TABLE I. Direct comparison among the various analytic methods.

Method	□-Rep.	Higher rep.	Multiple rep.	Susy	γ
BF	+	+	+	+	+
SD	+	+	−	−	−
ACS	+	−	−	+	−

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