Mass spectrum of the scalar hidden charm and bottom tetraquark states

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In this article, we study the mass spectrum of the scalar hidden charm and bottom tetraquark states with the QCD sum rules. The numerical results are compared with the corresponding ones from a relativistic quark model based on a quasipotential approach in QCD. The relevant values from the constituent diquark model based on the constituent diquark masses and the spin-spin interactions are also discussed.

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I. INTRODUCTION

The Babar, Belle, CLEO, D0, CDF, and FOCUS collaborations have discovered (or confirmed) a large number of charmoniumlike states $X(3940)$, $X(3872)$, $Y(4260)$, $Y(4008)$, $Y(3940)$, $Y(4325)$, $Y(4360)$, $Y(4660)$, etc, and revitalized the interest in the spectroscopy of the charmonium states [1–4]. For a concise review of the experimental situation of the new charmoniumlike states, one can consult Ref. [5]. Many possible assignments for those states have been suggested, such as multiquark states (irrespective of the molecule type and the diquark-antidiquark type), hybrid states, charmonium states modified by nearby thresholds, threshold cusps, etc [1–4]. The observed decay channels are $J/\psi\pi^+\pi^-$ or $\psi'\pi^+\pi^-$; an essential ingredient for understanding the structures of those mesons is whether or not the $\pi\pi$ comes from a resonance state.

The $Z^+(4430)$ observed in the decay mode $\psi'\pi^+$ by the Belle collaboration is the most interesting subject [6]. We can distinguish the multiquark states from the hybrids or charmonia with the criterion of nonzero charge. The $Z^+(4430)$ cannot be a pure $c\bar{c}$ state due to the positive charge, and may be a $c\bar{c}u\bar{d}$ tetraquark state. The *BABAR* collaboration did not confirm this resonance [7], i.e. they observed no significant evidence for a $Z(4430)$ signal for any of the processes investigated, neither in the total $J/\psi\pi$ or $\psi'\pi$ mass distribution nor in the corresponding distributions for the regions of $K\pi$ mass for which observation of the $Z(4430)$ signal was reported.

In 2008, the Belle collaboration reported the first observation of two resonancelike structures [thereafter we will denote them as $Z(4050)$ and $Z(4250)$ respectively] in the $\pi^+\chi_{c1}$ invariant mass distribution near 4.1 GeV in the exclusive decays $\bar{B}^0 \to K^-\pi^+\chi_{c1}$ [8]. Their quark contents must be some special combinations of the $c\bar{c}u\bar{d}$, just like the $Z^+(4430)$, they cannot be the conventional mesons. They may be the tetraquark states [9,10] or the molecular states [11–14]. The $Z(4050)$ and $Z(4250)$ lie about (0.5–0.6) GeV above the $\pi^+\chi_{c1}$ threshold, the decay $Z \to \pi^+\chi_{c1}$ can take place with the "fall-apart" mechanism and it is Okubo-Zweig-Iizuka superallowed, which can take into account the large total width naturally.

The spins of the $Z(4050)$ and $Z(4250)$ are not determined yet, they can be scalar or vector mesons. In Refs. [9,10], we assume that the hidden charm mesons $Z(4050)$ and $Z(4250)$ are vector (and scalar) tetraquark states, and study their masses with the QCD sum rules. The numerical results indicate that the mass of the vector hidden charm tetraquark state is about $M_Z = (5.12 \pm 0.15)$ GeV or $M_Z = (5.16 \pm 0.16)$ GeV, while the mass of the scalar hidden charm tetraquark state is about $M_Z = (4.36 \pm 0.18)$ GeV. The scalar hidden charm tetraquark states may have smaller masses than the corresponding vector states.

The mass is a fundamental parameter in describing a hadron, whether or not there exist those hidden charm tetraquark configurations is of great importance itself, because it provides a new opportunity for a deeper understanding of the low energy QCD.

In this article, we study the mass spectrum of the scalar hidden charm and bottom tetraquark states using the QCD sum rules [15,16]. In the QCD sum rules, the operator product expansion is used to expand the time-ordered currents into a series of quark and gluon condensates which parametrize the long distance properties of the QCD vacuum. Based on the quark-hadron duality, we can obtain copious information about the hadronic parameters at the phenomenological side [15,16].

The $Z(4050)$ and $Z(4250)$ can be tentatively identified as the scalar hidden charm ($c\bar{c}$) tetraquark states, while the scalar hidden bottom ($b\bar{b}$) tetraquark states may be observed at the LHCb, where the $b\bar{b}$ pairs will be copiously produced. The hidden charm and bottom tetraquark states (Z) have the symbolic quark structures:

$$\begin{aligned} &Z^+ = Q\bar{Q}u\bar{d}; \qquad Z^0 = \frac{1}{\sqrt{2}}Q\bar{Q}(u\bar{u} - d\bar{d}); \\ &Z^- = Q\bar{Q}d\bar{u}; \qquad Z_s^+ = Q\bar{Q}u\bar{s}; \qquad Z_s^- = Q\bar{Q}s\bar{u}; \\ &Z_s^0 = Q\bar{Q}d\bar{s}; \qquad \bar{Z}_s^0 = Q\bar{Q}s\bar{d}; \\ &Z_\varphi = \frac{1}{\sqrt{2}}Q\bar{Q}(u\bar{u} + d\bar{d}); \qquad Z_\phi = Q\bar{Q}s\bar{s}, \end{aligned} \tag{1}$$

where the Q denotes the heavy quarks c and b.

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The colored objects (diquarks) in a confining potential can result in a copious spectrum, there maybe exist a series of orbital angular momentum excitations. In the heavy quark limit, the c (and b) quark can be taken as a static well potential, which binds the light quark q to form a diquark in the color antitriplet channel. We take the diquarks as the basic constituents following Jaffe and Wilczek [17,18]. The heavy tetraquark system could be described by a double-well potential with two light quarks $q'\bar{q}$ lying in the two wells, respectively.

The diquarks have five Dirac tensor structures, scalar $C\gamma_5$, pseudoscalar C, vector $C\gamma_\mu\gamma_5$, axial vector $C\gamma_\mu$, and tensor $C\sigma_{\mu\nu}$. The structures $C\gamma_\mu$ and $C\sigma_{\mu\nu}$ are symmetric, the structures $C\gamma_5$, C and $C\gamma_\mu\gamma_5$ are antisymmetric. The attractive interactions of one-gluon exchange favor formation of the diquarks in color antitriplet $\bar{3}_c$, flavor antitriplet $\bar{3}_f$, and spin singlet 1_s [19,20]. In this article, we assume the scalar hidden charm and bottom mesons Z consist of the $C\gamma_5 - C\gamma_5$ type diquark structures rather than the $C - C$ type diquark structures, and construct the interpolating currents:

$$
\begin{aligned}
J_{Z^+}(x) &= \epsilon^{ijk}\epsilon^{imn} u_j^T(x) C\gamma_5 Q_k(x)\bar{Q}_m(x)\gamma_5 C\bar{d}_n^T(x),\\
J_{Z^0}(x) &= \frac{\epsilon^{ijk}\epsilon^{imn}}{\sqrt{2}}[u_j^T(x)C\gamma_5 Q_k(x)\bar{Q}_m(x)\gamma_5 C\bar{u}_n^T(x)\\
&\quad - (u\to d)],\\
J_{Z_s^+}(x) &= \epsilon^{ijk}\epsilon^{imn} u_j^T(x) C\gamma_5 Q_k(x)\bar{Q}_m(x)\gamma_5 C\bar{s}_n^T(x),\\
J_{Z_s^0}(x) &= \epsilon^{ijk}\epsilon^{imn} d_j^T(x) C\gamma_5 Q_k(x)\bar{Q}_m(x)\gamma_5 C\bar{s}_n^T(x),\\
J_{Z_\varphi}(x) &= \frac{\epsilon^{ijk}\epsilon^{imn}}{\sqrt{2}}[u_j^T(x)C\gamma_5 Q_k(x)\bar{Q}_m(x)\gamma_5 C\bar{u}_n^T(x)\\
&\quad + (u\to d)],\\
J_{Z_\phi}(x) &= \epsilon^{ijk}\epsilon^{imn} s_j^T(x) C\gamma_5 Q_k(x)\bar{Q}_m(x)\gamma_5 C\bar{s}_n^T(x),
\end{aligned}
\tag{2}
$$

where the $i, j, k, \ldots$ are color indexes. In the isospin limit, the interpolating currents result in three distinct expressions for the correlation functions $\Pi(p)$, which are characterized by the number of the s quarks they contain.

The article is arranged as follows: we derive the QCD sum rules for the scalar hidden charm and bottom tetraquark states Z in Sec. II; in Sec. III, we provide numerical results and discussions; Sec. IV is reserved for conclusion.

II. QCD SUM RULES FOR THE SCALAR TETRAQUARK STATES Z

In the following, we write down the two-point correlation functions $\Pi(p)$ in the QCD sum rules

$$\Pi(p) = i\int d^4x e^{ip\cdot x}\langle 0|T\{J(x)J^\dagger(0)\}|0\rangle, \tag{3}$$

where the $J(x)$ denotes the interpolating currents $J_{Z^+}(x)$, $J_{Z^0}(x)$, $J_{Z_s^+}(x)$, etc.

We can insert a complete set of intermediate hadronic states with the same quantum numbers as the current operator $J(x)$ into the correlation functions $\Pi(p)$ to obtain the hadronic representation [15,16]. After isolating the ground state contribution from the pole term of the Z, we get the following result:

$$\Pi(p) = \frac{\lambda_Z^2}{M_Z^2 - p^2} + \cdots, \tag{4}$$

where the pole residue (or coupling) λ_Z is defined by

$$\lambda_Z = \langle 0|J(0)|Z(p)\rangle. \tag{5}$$

After performing the standard procedure of the QCD sum rules, we obtain the following six sum rules:

$$\lambda_i^2 e^{-(M_i^2/M^2)} = \int_{\Delta_i}^{s_i^0} ds\rho_i(s)e^{-(s/M^2)}, \tag{6}$$

$$
\begin{aligned}
\rho_{q\bar{q}}(s) &= \frac{1}{512\pi^6}\int_{\alpha_{\min}}^{\alpha_{\max}} d\alpha \int_{\beta_{\min}}^{1-\alpha} d\beta\alpha\beta(1-\alpha-\beta)^3(s-\tilde{m}_Q^2)^2(7s^2 - 6s\tilde{m}_Q^2 + \tilde{m}_Q^4)\\
&\quad + \frac{m_Q\langle\bar{q}q\rangle}{16\pi^4}\int_{\alpha_{\min}}^{\alpha_{\max}} d\alpha \int_{\beta_{\min}}^{1-\alpha} d\beta(1-\alpha-\beta)(\alpha+\beta)(s-\tilde{m}_Q^2)(\tilde{m}_Q^2 - 2s)\\
&\quad + \frac{m_Q\langle\bar{q}g_s\sigma Gq\rangle}{64\pi^4}\int_{\alpha_{\min}}^{\alpha_{\max}} d\alpha \int_{\beta_{\min}}^{1-\alpha} d\beta(\alpha+\beta)(3s - 2\tilde{m}_Q^2) + \frac{m_Q^2\langle\bar{q}q\rangle^2}{12\pi^2}\int_{\alpha_{\rm mix}}^{\alpha_{\max}} d\alpha\\
&\quad + \frac{m_Q^2\langle\bar{q}g_s\sigma Gq\rangle^2}{192\pi^2 M^6}\int_{\alpha_{\rm mix}}^{\alpha_{\max}} d\alpha\tilde{\tilde{m}}_Q^4\delta(s-\tilde{\tilde{m}}_Q^2) - \frac{m_Q^2\langle\bar{q}q\rangle\langle\bar{q}g_s\sigma Gq\rangle}{24\pi^2}\int_{\alpha_{\rm mix}}^{\alpha_{\max}} d\alpha\left[1 + \frac{s}{M^2}\right]\delta(s-\tilde{\tilde{m}}_Q^2),
\end{aligned}
\tag{7}
$$

$$\begin{aligned}
\rho_{q\bar{s}}(s) = {} & \frac{1}{512\pi^6}\int_{\alpha_{\min}}^{\alpha_{\max}} d\alpha \int_{\beta_{\min}}^{1-\alpha} d\beta\alpha\beta(1-\alpha-\beta)^3(s-\tilde{m}_Q^2)^2(7s^2-6s\tilde{m}_Q^2+\tilde{m}_Q^4) \\
& + \frac{m_s m_Q}{256\pi^6}\int_{\alpha_{\min}}^{\alpha_{\max}} d\alpha \int_{\beta_{\min}}^{1-\alpha} d\beta\beta(1-\alpha-\beta)^2(s-\tilde{m}_Q^2)^2(5s-2\tilde{m}_Q^2) \\
& + \frac{m_s\langle\bar{s}s\rangle}{32\pi^4}\int_{\alpha_{\min}}^{\alpha_{\max}} d\alpha \int_{\beta_{\min}}^{1-\alpha} d\beta\alpha\beta(1-\alpha-\beta)(10s^2-12s\tilde{m}_Q^2+3\tilde{m}_Q^4) \\
& + \frac{m_Q\langle\bar{q}q\rangle}{16\pi^4}\int_{\alpha_{\min}}^{\alpha_{\max}} d\alpha \int_{\beta_{\min}}^{1-\alpha} d\beta\alpha(1-\alpha-\beta)(s-\tilde{m}_Q^2)(\tilde{m}_Q^2-2s) \\
& + \frac{m_Q\langle\bar{s}s\rangle}{16\pi^4}\int_{\alpha_{\min}}^{\alpha_{\max}} d\alpha \int_{\beta_{\min}}^{1-\alpha} d\beta\beta(1-\alpha-\beta)(s-\tilde{m}_Q^2)(\tilde{m}_Q^2-2s) \\
& + \frac{m_Q\langle\bar{q}g_s\sigma Gq\rangle}{64\pi^4}\int_{\alpha_{\min}}^{\alpha_{\max}} d\alpha \int_{\beta_{\min}}^{1-\alpha} d\beta\alpha(3s-2\tilde{m}_Q^2) + \frac{m_Q\langle\bar{s}g_s\sigma Gs\rangle}{64\pi^4}\int_{\alpha_{\min}}^{\alpha_{\max}} d\alpha \int_{\beta_{\min}}^{1-\alpha} d\beta\beta(3s-2\tilde{m}_Q^2) \\
& + \frac{m_s\langle\bar{s}g_s\sigma Gs\rangle}{32\pi^4}\int_{\alpha_{\min}}^{\alpha_{\max}} d\alpha \int_{\beta_{\min}}^{1-\alpha} d\beta\alpha\beta\left[\tilde{m}_Q^2-2s-\frac{s^2}{6}\delta(s-\tilde{m}_Q^2)\right] + \frac{m_s m_Q^2\langle\bar{q}q\rangle}{16\pi^4}\int_{\alpha_{\min}}^{\alpha_{\max}} d\alpha \int_{\beta_{\min}}^{1-\alpha} d\beta(\tilde{m}_Q^2-s) \\
& + \frac{m_Q^2\langle\bar{q}q\rangle\langle\bar{s}s\rangle}{12\pi^2}\int_{\alpha_{\mathrm{mix}}}^{\alpha_{\max}} d\alpha + \frac{m_s m_Q^2\langle\bar{q}g_s\sigma Gq\rangle}{64\pi^4}\int_{\alpha_{\mathrm{mix}}}^{\alpha_{\max}} d\alpha - \frac{m_s m_Q\langle\bar{q}q\rangle\langle\bar{s}s\rangle}{24\pi^2}\int_{\alpha_{\mathrm{mix}}}^{\alpha_{\max}} d\alpha\alpha[2+s\delta(s-\tilde{\tilde{m}}_Q^2)] \\
& - \frac{m_Q^2[\langle\bar{q}q\rangle\langle\bar{s}g_s\sigma Gs\rangle+\langle\bar{s}s\rangle\langle\bar{q}g_s\sigma Gq\rangle]}{48\pi^2}\int_{\alpha_{\mathrm{mix}}}^{\alpha_{\max}} d\alpha\left[1+\frac{s}{M^2}\right]\delta(s-\tilde{\tilde{m}}_Q^2) \\
& + \frac{m_s m_Q[2\langle\bar{q}q\rangle\langle\bar{s}g_s\sigma Gs\rangle+3\langle\bar{s}s\rangle\langle\bar{q}g_s\sigma Gq\rangle]}{144\pi^2}\int_{\alpha_{\mathrm{mix}}}^{\alpha_{\max}} d\alpha\alpha\left[1+\frac{s}{M^2}+\frac{s^2}{2M^4}\right]\delta(s-\tilde{\tilde{m}}_Q^2) \\
& + \frac{m_Q^2\langle\bar{q}g_s\sigma Gq\rangle\langle\bar{s}g_s\sigma Gs\rangle}{192\pi^2 M^6}\int_{\alpha_{\mathrm{mix}}}^{\alpha_{\max}} d\alpha\tilde{\tilde{m}}_Q^4\delta(s-\tilde{\tilde{m}}_Q^2), \qquad (8)
\end{aligned}$$

$$\begin{aligned}
\rho_{s\bar{s}}(s) = {} & \frac{1}{512\pi^6}\int_{\alpha_{\min}}^{\alpha_{\max}} d\alpha \int_{\beta_{\min}}^{1-\alpha} d\beta\alpha\beta(1-\alpha-\beta)^3(s-\tilde{m}_Q^2)^2(7s^2-6s\tilde{m}_Q^2+\tilde{m}_Q^4) \\
& + \frac{m_s m_Q}{256\pi^6}\int_{\alpha_{\min}}^{\alpha_{\max}} d\alpha \int_{\beta_{\min}}^{1-\alpha} d\beta(\alpha+\beta)(1-\alpha-\beta)^2(s-\tilde{m}_Q^2)^2(5s-2\tilde{m}_Q^2) \\
& + \frac{m_s\langle\bar{s}s\rangle}{16\pi^4}\int_{\alpha_{\min}}^{\alpha_{\max}} d\alpha \int_{\beta_{\min}}^{1-\alpha} d\beta\alpha\beta(1-\alpha-\beta)(10s^2-12s\tilde{m}_Q^2+3\tilde{m}_Q^4) \\
& + \frac{m_Q\langle\bar{s}s\rangle}{16\pi^4}\int_{\alpha_{\min}}^{\alpha_{\max}} d\alpha \int_{\beta_{\min}}^{1-\alpha} d\beta(\alpha+\beta)(1-\alpha-\beta)(s-\tilde{m}_Q^2)(\tilde{m}_Q^2-2s) \\
& + \frac{m_Q\langle\bar{s}g_s\sigma Gs\rangle}{64\pi^4}\int_{\alpha_{\min}}^{\alpha_{\max}} d\alpha \int_{\beta_{\min}}^{1-\alpha} d\beta(\alpha+\beta)(3s-2\tilde{m}_Q^2) \\
& + \frac{m_s\langle\bar{s}g_s\sigma Gs\rangle}{16\pi^4}\int_{\alpha_{\min}}^{\alpha_{\max}} d\alpha \int_{\beta_{\min}}^{1-\alpha} d\beta\alpha\beta\left[\tilde{m}_Q^2-2s-\frac{s^2}{6}\delta(s-\tilde{m}_Q^2)\right] + \frac{m_s m_Q^2\langle\bar{s}s\rangle}{8\pi^4}\int_{\alpha_{\min}}^{\alpha_{\max}} d\alpha \int_{\beta_{\min}}^{1-\alpha} d\beta(\tilde{m}_Q^2-s) \\
& + \frac{m_Q^2\langle\bar{s}s\rangle^2}{12\pi^2}\int_{\alpha_{\mathrm{mix}}}^{\alpha_{\max}} d\alpha + \frac{m_s m_Q^2\langle\bar{s}g_s\sigma Gs\rangle}{32\pi^4}\int_{\alpha_{\mathrm{mix}}}^{\alpha_{\max}} d\alpha - \frac{m_s m_Q\langle\bar{s}s\rangle^2}{12\pi^2}\int_{\alpha_{\mathrm{mix}}}^{\alpha_{\max}} d\alpha\alpha[2+s\delta(s-\tilde{\tilde{m}}_Q^2)] \\
& - \frac{m_Q^2\langle\bar{s}s\rangle\langle\bar{s}g_s\sigma Gs\rangle}{24\pi^2}\int_{\alpha_{\mathrm{mix}}}^{\alpha_{\max}} d\alpha\left[1+\frac{s}{M^2}\right]\delta(s-\tilde{\tilde{m}}_Q^2) + \frac{5m_s m_Q\langle\bar{s}s\rangle\langle\bar{s}g_s\sigma Gs\rangle}{144\pi^2}\int_{\alpha_{\mathrm{mix}}}^{\alpha_{\max}} d\alpha\left[1+\frac{s}{M^2}+\frac{s^2}{2M^4}\right] \\
& \times\delta(s-\tilde{\tilde{m}}_Q^2) + \frac{m_Q^2\langle\bar{s}g_s\sigma Gs\rangle^2}{192\pi^2 M^6}\int_{\alpha_{\mathrm{mix}}}^{\alpha_{\max}} d\alpha\tilde{\tilde{m}}_Q^4\delta(s-\tilde{\tilde{m}}_Q^2), \qquad (9)
\end{aligned}$$

where the i denotes the $c\bar{c}q\bar{q}$, $c\bar{c}q\bar{s}$, $c\bar{c}s\bar{s}$, $b\bar{b}q\bar{q}$, $b\bar{b}q\bar{s}$, and $b\bar{b}s\bar{s}$ channels, respectively; the s_i^0 are the corresponding continuum threshold parameters and the M^2 is the Borel parameter; $\alpha_{\max} = \frac{1+\sqrt{1-\frac{4m_Q^2}{s}}}{2}$, $\alpha_{\min} = \frac{1-\sqrt{1-\frac{4m_Q^2}{s}}}{2}$, $\beta_{\min} = \frac{\alpha m_Q^2}{\alpha s - m_Q^2}$, $\tilde{m}_Q^2 = \frac{(\alpha+\beta)m_Q^2}{\alpha\beta}$, $\tilde{\tilde{m}}_Q^2 = \frac{m_Q^2}{\alpha(1-\alpha)}$. The thresholds Δ_i can be sorted into three sets, we introduce the $q\bar{q}$, $q\bar{s}$, and $s\bar{s}$ to denote the light quark constituents in the scalar tetraquark states to simplify the notations, $\Delta_{q\bar{q}} = 4m_Q^2$, $\Delta_{q\bar{s}} = (2m_Q + m_s)^2$, $\Delta_{s\bar{s}} = 4(m_Q + m_s)^2$.

We carry out the operator product expansion to the vacuum condensates adding up to dimension 10. In calculation, we take the assumption of vacuum saturation for high dimension vacuum condensates; they are always factorized to lower condensates with vacuum saturation in the QCD sum rules, factorization works well in the large N_c limit. In this article, we take into account the contributions from the quark condensates, mixed condensates, and neglect the contributions from the gluon condensate. The contributions from the gluon condensates are suppressed by large denominators and would not play any significant roles for the light tetraquark states [21,22], the heavy tetraquark state [10], and the heavy molecular state [23]. There are many terms involving the gluon condensate for the heavy tetraquark states and heavy molecular states in the operator product expansion (one can consult Refs. [10,23] for example), we neglect the gluon condensates for simplicity. Furthermore, we neglect the terms proportional to the m_u and m_d, their contributions are of minor importance.

Differentiate the Eq. (6) with respect to $\frac{1}{M^2}$, then eliminate the pole residues λ_i, we can obtain the six sum rules for the masses of the Z,

$$M_i^2 = \frac{\int_{\Delta_i}^{s_i^0} ds \frac{d}{d(-1/M^2)} \rho_i(s) e^{-(s/M^2)}}{\int_{\Delta_i}^{s_i^0} ds \rho_i(s) e^{-(s/M^2)}}. \quad (10)$$

III. NUMERICAL RESULTS AND DISCUSSIONS

The input parameters are taken to be the standard values $\langle\bar{q}q\rangle = -(0.24 \pm 0.01\ \text{GeV})^3$, $\langle\bar{s}s\rangle = (0.8 \pm 0.2)\langle\bar{q}q\rangle$, $\langle\bar{q}g_s\sigma Gq\rangle = m_0^2\langle\bar{q}q\rangle$, $\langle\bar{s}g_s\sigma Gs\rangle = m_0^2\langle\bar{s}s\rangle$, $m_0^2 = (0.8 \pm 0.2)\ \text{GeV}^2$, $m_s = (0.14 \pm 0.01)\ \text{GeV}$, $m_c = (1.35 \pm 0.10)\ \text{GeV}$, and $m_b = (4.8 \pm 0.1)\ \text{GeV}$ at the energy scale $\mu = 1\ \text{GeV}$ [15,16,24].

In the conventional QCD sum rules [15,16], there are two criteria (pole dominance and convergence of the operator product expansion) for choosing the Borel parameter M^2 and threshold parameter s_0. The light tetraquark states cannot satisfy the two criteria, although it is not an indication of nonexistence of the light tetraquark states (For detailed discussions about this subject, one can consult Refs. [10,25]). We impose the two criteria on the heavy tetraquark states to choose the Borel parameter M^2 and threshold parameter s_0.

If the resonancelike structures $Z(4050)$ and $Z(4250)$ observed by the Belle collaboration in the $\pi^+\chi_{c1}$ invariant mass distribution near 4.1 GeV in the exclusive decays $\bar{B}^0 \to K^-\pi^+\chi_{c1}$ are scalar tetraquark states [8], the threshold parameter can be tentatively taken as $s_{q\bar{q}}^0 = (4.248 + 0.5)^2\ \text{GeV}^2 \approx 23\ \text{GeV}^2$ to take into account all possible contributions from the ground states, where we choose the energy gap between the ground states and the first radial excited states to be 0.5 GeV. Taking into account the $SU(3)$ symmetry of the light flavor quarks, we expect the threshold parameters $s_{q\bar{s}}^0$ and $s_{s\bar{s}}^0$ are slightly larger than the $s_{q\bar{q}}^0$. Furthermore, we take into account the mass difference between the c and b quarks, the threshold parameters in the hidden bottom channels are tentatively taken as $s_{q\bar{q}}^0 = 138\ \text{GeV}^2$, $s_{q\bar{s}}^0 = 140\ \text{GeV}^2$, and $s_{s\bar{s}}^0 = 142\ \text{GeV}^2$.

Here we take it for granted that the energy gap between the ground states and the first radial excited states is about 0.5 GeV, and use those values as a guide to determine the threshold parameters s_0 with the QCD sum rules.

The contributions from the high dimension vacuum condensates in the operator product expansion are shown in Figs. 1 and 2, where (and thereafter) we use the $\langle\bar{q}q\rangle$ to denote the quark condensates $\langle\bar{q}q\rangle$, $\langle\bar{s}s\rangle$ and the $\langle\bar{q}g_s\sigma Gq\rangle$ to denote the mixed condensates $\langle\bar{q}g_s\sigma Gq\rangle$, $\langle\bar{s}g_s\sigma Gs\rangle$. From the figures, we can see that the contributions from the high dimension condensates change quickly with variation of the Borel parameter at the values $M^2 \leq 2.6\ \text{GeV}^2$ and $M^2 \leq 7.0\ \text{GeV}^2$ for the $c\bar{c}$ channels and $b\bar{b}$ channels, respectively. Such an unstable behavior cannot lead to stable sum rules; our numerical results confirm this conjecture. At the values $M^2 \geq 2.6\ \text{GeV}^2$ and $s_0 \geq 23\ \text{GeV}^2$, the contributions from the $\langle\bar{q}q\rangle^2 + \langle\bar{q}q\rangle\langle\bar{q}g_s\sigma Gq\rangle$ term are less than (or equal to) 10% for the $c\bar{c}q\bar{q}$ channel, the corresponding contributions are smaller for the $c\bar{c}q\bar{s}$ and $c\bar{c}s\bar{s}$ channels; the contributions from the vacuum condensate of the highest dimension $\langle\bar{q}g_s\sigma Gq\rangle^2$ are less than (or equal to) 2% for all the $c\bar{c}$ channels, we expect the operator product expansion is convergent in the $c\bar{c}$ channels. At the values $M^2 \geq 7.0\ \text{GeV}^2$ and $s_0 \geq 136\ \text{GeV}^2$, the contributions from the $\langle\bar{q}q\rangle^2 + \langle\bar{q}q\rangle\langle\bar{q}g_s\sigma Gq\rangle$ term are less than 10% for the $b\bar{b}q\bar{q}$ channel, the corresponding contributions are smaller for the $b\bar{b}q\bar{s}$ and $b\bar{b}s\bar{s}$ channels; the contributions from the vacuum condensate of the highest dimension $\langle\bar{q}g_s\sigma Gq\rangle^2$ are less than (or equal) 6% for all the $b\bar{b}$ channels. We expect the operator product expansion is convergent in the $b\bar{b}$ channels.

In Fig. 3, we plot the contributions from different terms in the operator product expansion. From the figures, we can see that the main contributions come from the perturbative term and the $\langle\bar{q}q\rangle + \langle\bar{q}g_s\sigma Gq\rangle$ term; the operator product expansion is convergent, and the interpolating currents containing more s quarks have better convergent behavior.

In this article, we take the uniform Borel parameter $M^2_{\rm min}$, i.e. $M^2_{\rm min} \geq 2.6\ {\rm GeV}^2$ and $M^2_{\rm min} \geq 7.0\ {\rm GeV}^2$ for the $c\bar{c}$ channels and $b\bar{b}$ channels, respectively.

In Fig. 4, we show the contributions from the pole terms with variation of the Borel parameter and the threshold parameter. The pole contributions are larger than (or equal to) 50% at the value $M^2 \leq 3.2\ {\rm GeV}^2$ and $s_0 \geq 23\ {\rm GeV}^2$, $23\ {\rm GeV}^2$, $24\ {\rm GeV}^2$ for the $c\bar{c}q\bar{q}$, $c\bar{c}q\bar{s}$, $c\bar{c}s\bar{s}$ channels, respectively, and larger than (or equal to) 50% at the value $M^2 \leq 8.0\ {\rm GeV}^2$ and $s_0 \geq 136\ {\rm GeV}^2$, $138\ {\rm GeV}^2$, $138\ {\rm GeV}^2$ for the $b\bar{b}q\bar{q}$, $b\bar{b}q\bar{s}$, and $b\bar{b}s\bar{s}$ channels, respectively. Again we take the uniform Borel parameter $M^2_{\rm max}$, i.e. $M^2_{\rm max} \leq 3.2\ {\rm GeV}^2$ and $M^2_{\rm max} \leq 8.0\ {\rm GeV}^2$ for the $c\bar{c}$ channels and $b\bar{b}$ channels, respectively.

In this article, the threshold parameters are taken as $s_0 = (24 \pm 1)\ {\rm GeV}^2$, $(24 \pm 1)\ {\rm GeV}^2$, $(25 \pm 1)\ {\rm GeV}^2$, $(138 \pm 2)\ {\rm GeV}^2$, $(140 \pm 2)\ {\rm GeV}^2$, and $(140 \pm 2)\ {\rm GeV}^2$ for the $c\bar{c}q\bar{q}$, $c\bar{c}q\bar{s}$, $c\bar{c}s\bar{s}$, $b\bar{b}q\bar{q}$, $b\bar{b}q\bar{s}$, and $b\bar{b}s\bar{s}$ channels, respectively; the Borel parameters are taken as $M^2 = (2.6\text{–}3.2)\ {\rm GeV}^2$ and $(7.0\text{–}8.0)\ {\rm GeV}^2$ for the $c\bar{c}$ channels and $b\bar{b}$ channels, respectively. In those regions, the two criteria of the QCD sum rules are fully satisfied [15,16].

Taking into account all uncertainties of the input parameters, finally we obtain the values of the masses and pole resides of the Z, which are shown in Figs. 5 and 6 and Tables I and II.

From Table I, we can see that the $SU(3)$ breaking effects for the masses of the hidden charm and bottom tetraquark states are buried in the uncertainties. The central value of the scalar tetraquark state $c\bar{c}q\bar{q}$ is slightly larger than the one $M_Z = (4.36 \pm 0.18)$ GeV obtained in Ref. [10], where the contributions from the terms involving the gluon condensate are taken into account. We can draw the conclusion that the gluon condensate plays a tiny important role and can be safely neglected.

The meson $Z(4250)$ may be a scalar tetraquark state $(c\bar{c}u\bar{d})$, the decay $Z(4250) \to \pi^+\chi_{c1}$ can take place with the Okubo-Zweig-Iizuka superallowed "fall-apart" mechanism, which can take into account the large total width naturally. Other possibilities, such as a hadro-charmonium resonance and a $D_1^+\bar{D}^0 + D^+\bar{D}_1^0$ molecular state are not excluded; more experimental data are still needed to identify it. It is difficult to identify the $Z(4050)$ as the scalar tetraquark state $(c\bar{c}u\bar{d})$ considering its small mass. There is still a lack of experiential candidates to identify the scalar tetraquark states $c\bar{c}q\bar{s}$, $c\bar{c}s\bar{s}$, $b\bar{b}q\bar{q}$, $b\bar{b}q\bar{s}$, and $b\bar{b}s\bar{s}$.

In Table I, we also present the results from a relativistic quark model based on a quasipotential approach in QCD [26,27], the central values of our predictions are larger than the corresponding ones from the quasipotential model, about (0.4–0.7) GeV. In Refs. [26,27], Ebert *et al.* take the diquarks as bound states of the light and heavy quarks in the color antitriplet channel, and calculate their mass spectrum using a Schrodinger type equation, then take the masses of the diquarks as the basic input parameters, and study the mass spectrum of the heavy tetraquark states as bound states of the diquark-antidiquark system. In the conventional quark models, the constituent quark masses are taken as the basic input parameters, and fitted to reproduce the mass spectra of the well-known mesons and baryons. However, the present experimental knowledge about the phenomenological hadronic spectral densities of the tetraquark states is rather vague, whether or not there exist tetraquark states is not confirmed with confidence, and no knowledge about the high resonances exists. The predicted constituent diquark masses cannot be confronted with the experimental data.

In Refs. [28–30], Maiani *et al.* take the diquarks as the basic constituents, examine the rich spectrum of the diquark-antidiquark states with the constituent diquark masses and the spin-spin interactions, and try to accommodate some of the newly observed charmoniumlike resonances not fitting a pure $c\bar{c}$ assignment. The predictions depend heavily on the assumption that the light scalar mesons $a_0(980)$ and $f_0(980)$ are tetraquark states, the basic parameters (constituent diquark masses) are estimated thereafter. The predications $M_{c\bar{c}q\bar{q}} = 3723$ MeV [28] and $M_{c\bar{c}s\bar{s}} = 3834$ MeV [30] (for the tetraquark states $c\bar{c}q\bar{q}$ and $c\bar{c}s\bar{s}$, respectively) are about 0.6 GeV smaller than the corresponding ones in the present work.

In Ref. [31], Zouzou *et al.* solve the four-body $(\bar{Q}\,\bar{Q}\,qq)$ problem by three different variational methods with a nonrelativistic potential considering explicitly virtual meson-meson components in the wave functions, search for possible bound states below the threshold for the spontaneous dissociation into two mesons, and observe that the exotic bound states $\bar{Q}\,\bar{Q}\,qq$ maybe exist for unequal quark masses (the ratio m_Q/m_q is large enough). The studies using a potential derived from the MIT bag model in the Born-Oppenheimer approximation support this observation [32,33]. In Ref. [34], Manohar and Wise study systems of two heavy-light mesons interacting through a one-pion exchange potential determined by the heavy meson chiral perturbation theory and observe that the long range potential maybe sufficiently attractive to produce a weakly bound two-meson state in the case $Q = b$. In Ref. [35], the $L = 0$ tetraquark states $QQ\overline{QQ}$ (Q denotes both Q and q) are analyzed in a chromo-magnetic model where only a constant hyperfine potential is retained.

If there exist scalar tetraquark states $\bar{Q}\,\bar{Q}\,qq$, we can construct the $C\gamma_\mu - C\gamma^\mu$ type interpolating currents to study them with the QCD sum rules, as the $\bar{Q}\,\bar{Q}$ and qq favor forming diquarks in the symmetric sextet 6_f with the spin-parity $J^P = 1^+$ due to Fermi statistics. The attractive interactions of one-gluon exchange favor formation of the diquarks in color antitriplet $\bar{3}_c$, flavor antitriplet $\bar{3}_f$, and spin singlet 1_s [19,20]. We expect the scalar $C\gamma_\mu - C\gamma^\mu$ type tetraquark states $\bar{Q}\,\bar{Q}\,qq$ are heavier than the corresponding $C\gamma_5 - C\gamma_5$ type tetraquark states $\bar{Q}Q\bar{q}q$, our

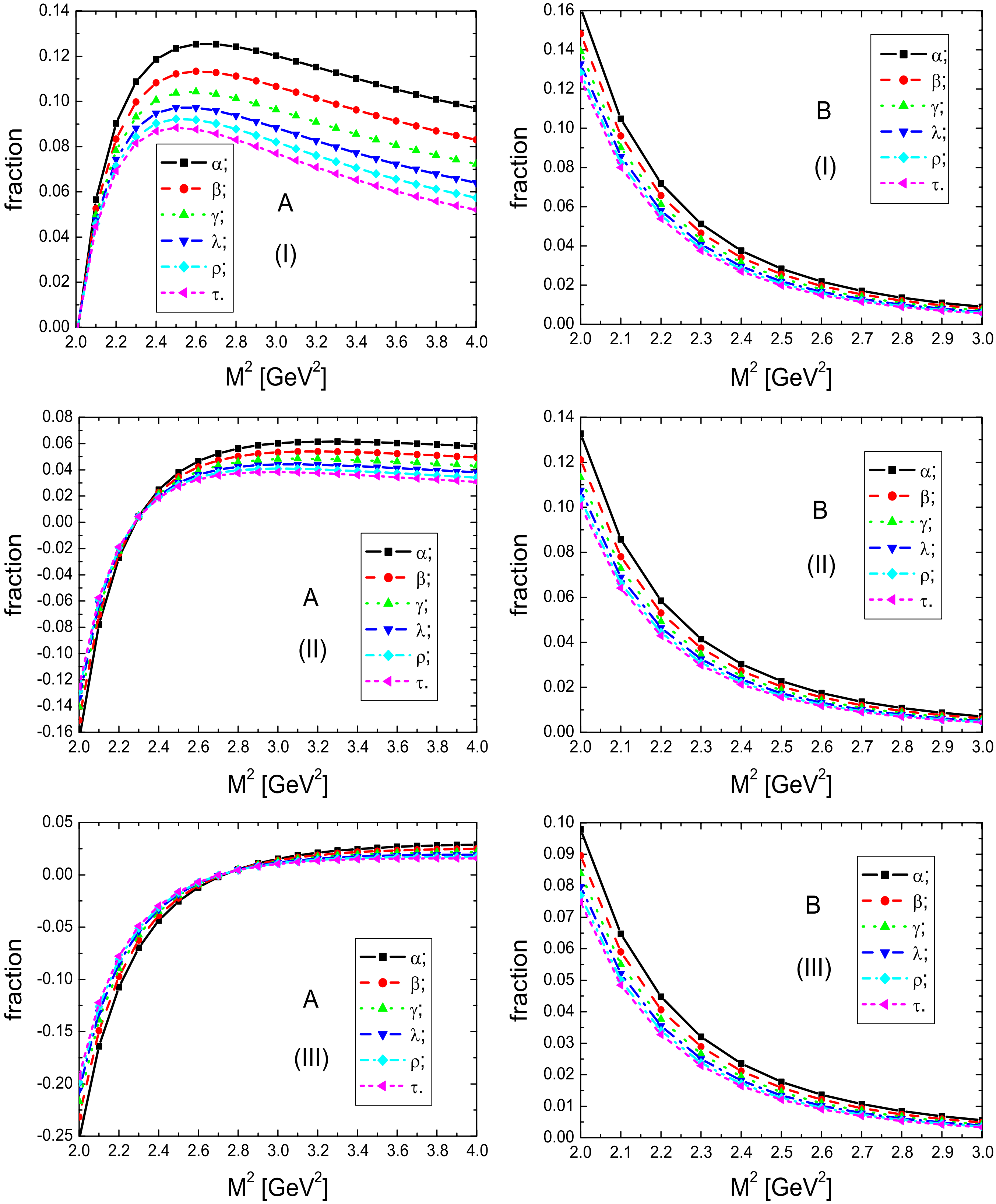

FIG. 1 (color online). The contributions from different terms with a variation of the Borel parameter M^2 in the operator product expansion. The A and B denote the contributions from the $\langle\bar{q}q\rangle^2 + \langle\bar{q}q\rangle\langle\bar{q}g_s\sigma Gq\rangle$ term and the $\langle\bar{q}g_s\sigma Gq\rangle^2$ term, respectively. The (I), (II), and (III) denote the $c\bar{c}q\bar{q}$, $c\bar{c}q\bar{s}$, and $c\bar{c}s\bar{s}$ channels, respectively. The notations α, β, γ, λ, ρ, and τ correspond to the threshold parameters $s_0 = 21\ \text{GeV}^2$, $22\ \text{GeV}^2$, $23\ \text{GeV}^2$, $24\ \text{GeV}^2$, $25\ \text{GeV}^2$, and $26\ \text{GeV}^2$, respectively.

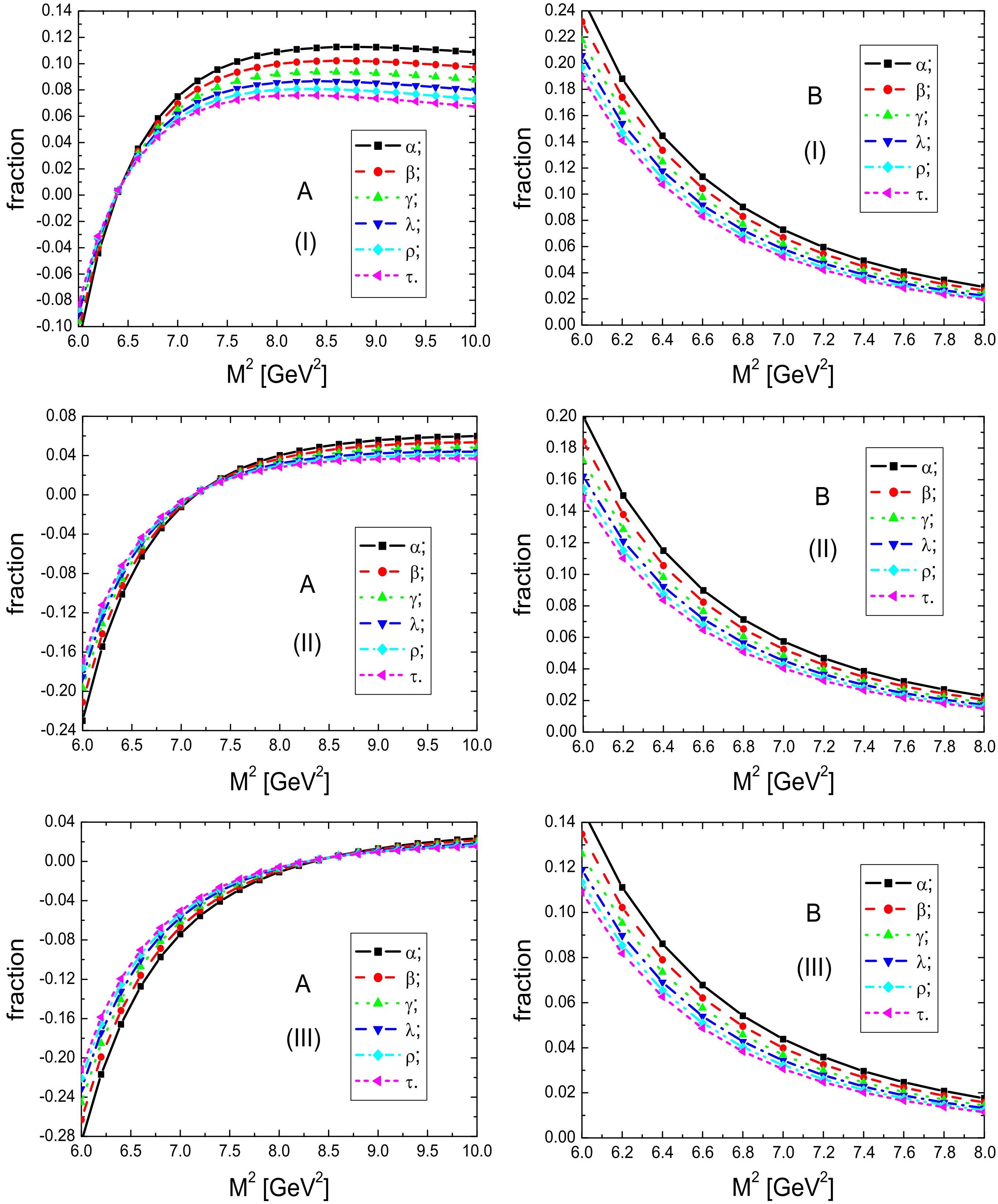

FIG. 2 (color online). The contributions from different terms with a variation of the Borel parameter M^2 in the operator product expansion. The A and B denote the contributions from the $\langle\bar{q}q\rangle^2 + \langle\bar{q}q\rangle\langle\bar{q}g_s\sigma Gq\rangle$ term and the $\langle\bar{q}g_s\sigma Gq\rangle^2$ term, respectively. The (I), (II), and (III) denote the $b\bar{b}q\bar{q}$, $b\bar{b}q\bar{s}$, and $b\bar{b}s\bar{s}$ channels, respectively. The notations α, β, γ, λ, ρ, and τ correspond to the threshold parameters $s_0 = 132\ \mathrm{GeV}^2$, $134\ \mathrm{GeV}^2$, $136\ \mathrm{GeV}^2$, $138\ \mathrm{GeV}^2$, $140\ \mathrm{GeV}^2$, and $142\ \mathrm{GeV}^2$, respectively.

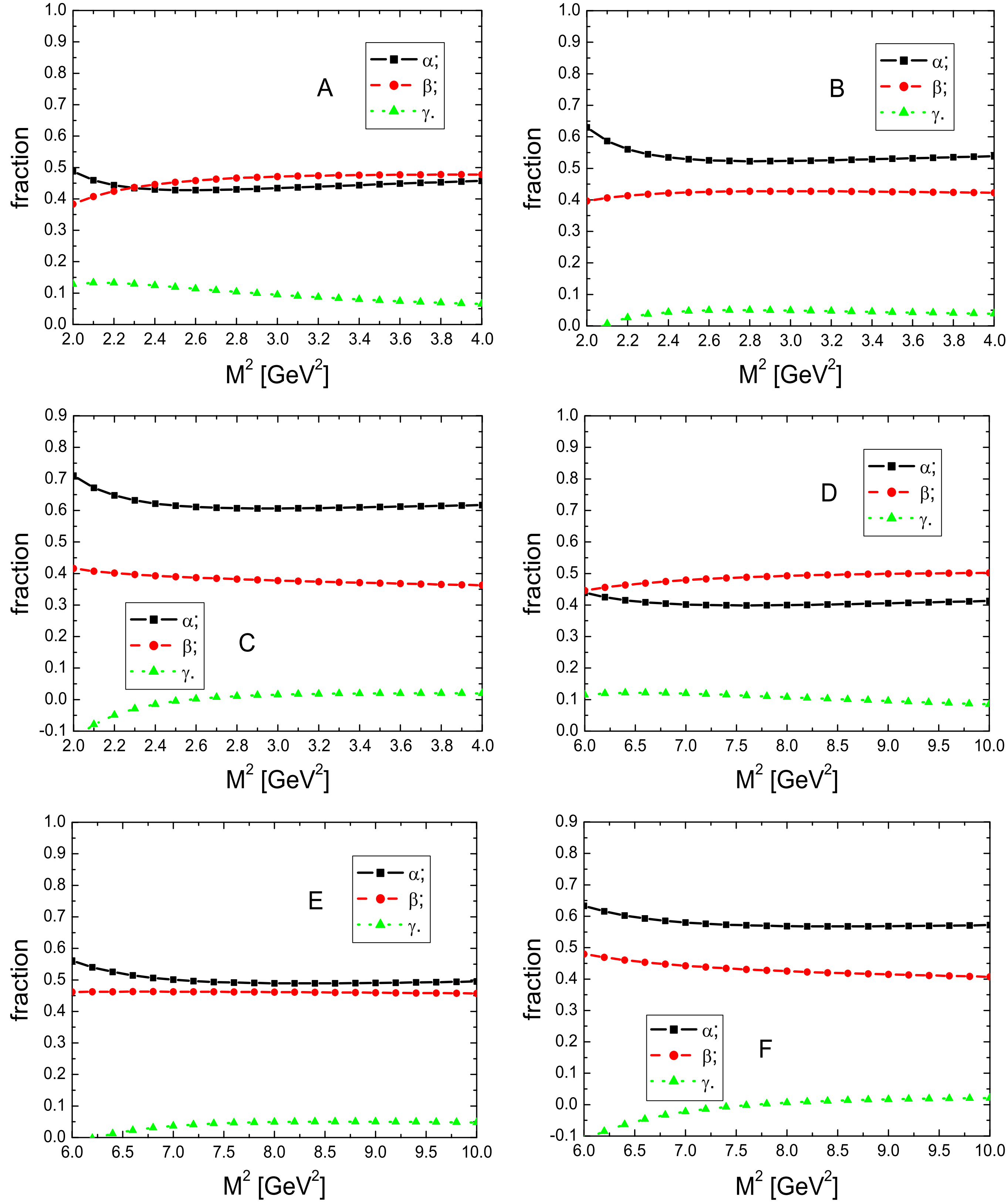

FIG. 3 (color online). The contributions from different terms with a variation of the Borel parameter M^2 in the operator product expansion. The A, B, C, D, E, and F denote the $c\bar{c}q\bar{q}$, $c\bar{c}q\bar{s}$, $c\bar{c}s\bar{s}$, $b\bar{b}q\bar{q}$, $b\bar{b}q\bar{s}$, and $b\bar{b}s\bar{s}$ channels, respectively. The α, β, and γ correspond to the perturbative term, the $\langle\bar{q}q\rangle + \langle\bar{q}g_s\sigma Gq\rangle$ term, and the $\langle\bar{q}q\rangle^2 + \langle\bar{q}q\rangle\langle\bar{q}g_s\sigma Gq\rangle + \langle\bar{q}g_s\sigma Gq\rangle^2$ term, respectively. The threshold parameters are $s_0 = 24$ GeV^2 and 138 GeV^2 for the $c\bar{c}$ channels and $b\bar{b}$ channels, respectively.

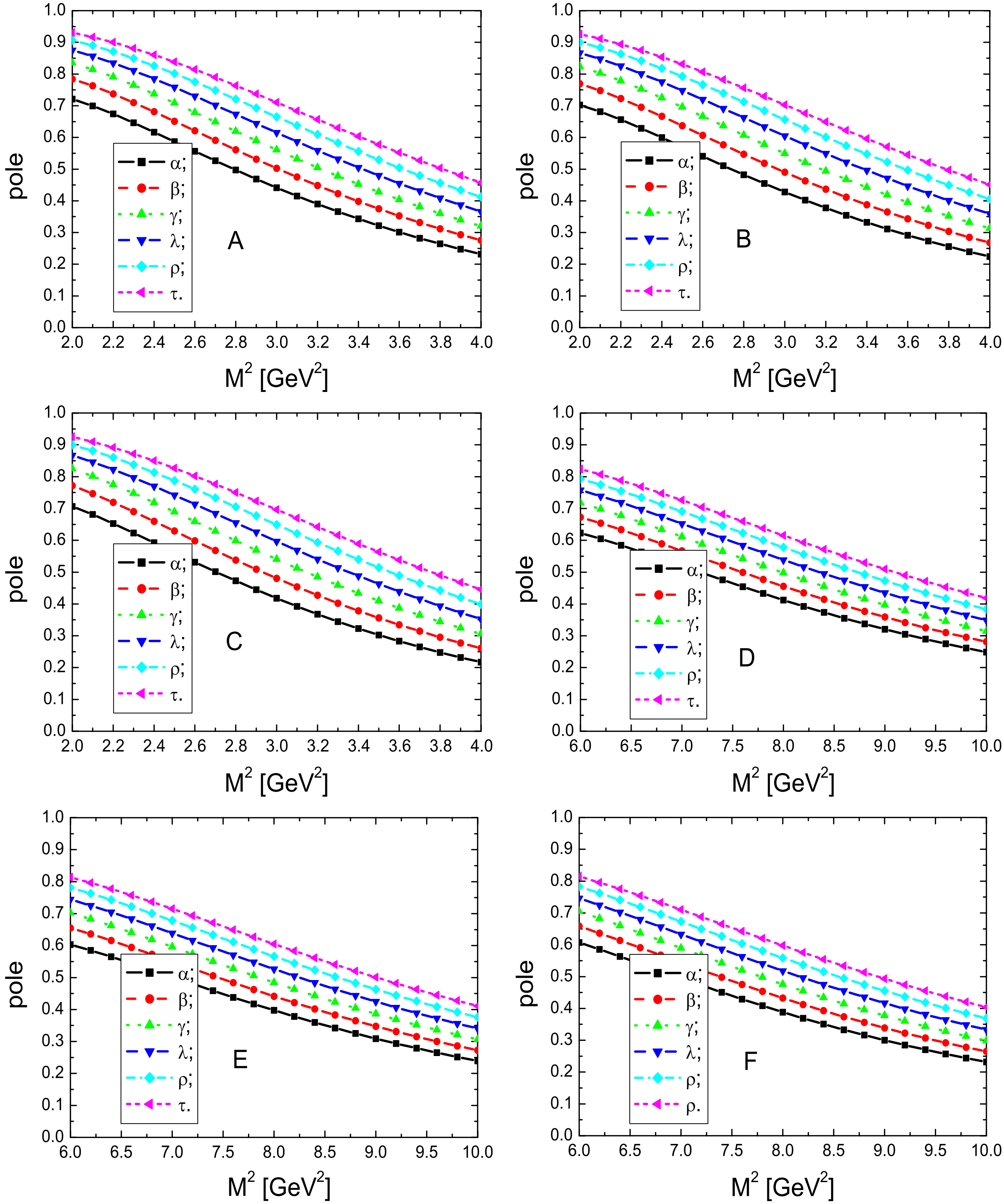

FIG. 4 (color online). The contributions from the pole terms with a variation of the Borel parameter M^2. The A, B, C, D, E, and F denote the $c\bar{c}q\bar{q}$, $c\bar{c}q\bar{s}$, $c\bar{c}s\bar{s}$, $b\bar{b}q\bar{q}$, $b\bar{b}q\bar{s}$, and $b\bar{b}s\bar{s}$ channels, respectively. In the $c\bar{c}$ channels, the notations α, β, γ, λ, ρ, and τ correspond to the threshold parameters $s_0 = 21\ \mathrm{GeV}^2$, $22\ \mathrm{GeV}^2$, $23\ \mathrm{GeV}^2$, $24\ \mathrm{GeV}^2$, $25\ \mathrm{GeV}^2$, and $26\ \mathrm{GeV}^2$ respectively, while in the $b\bar{b}$ channels they correspond to the threshold parameters $s_0 = 132\ \mathrm{GeV}^2$, $134\ \mathrm{GeV}^2$, $136\ \mathrm{GeV}^2$, $138\ \mathrm{GeV}^2$, $140\ \mathrm{GeV}^2$, and $142\ \mathrm{GeV}^2$, respectively.

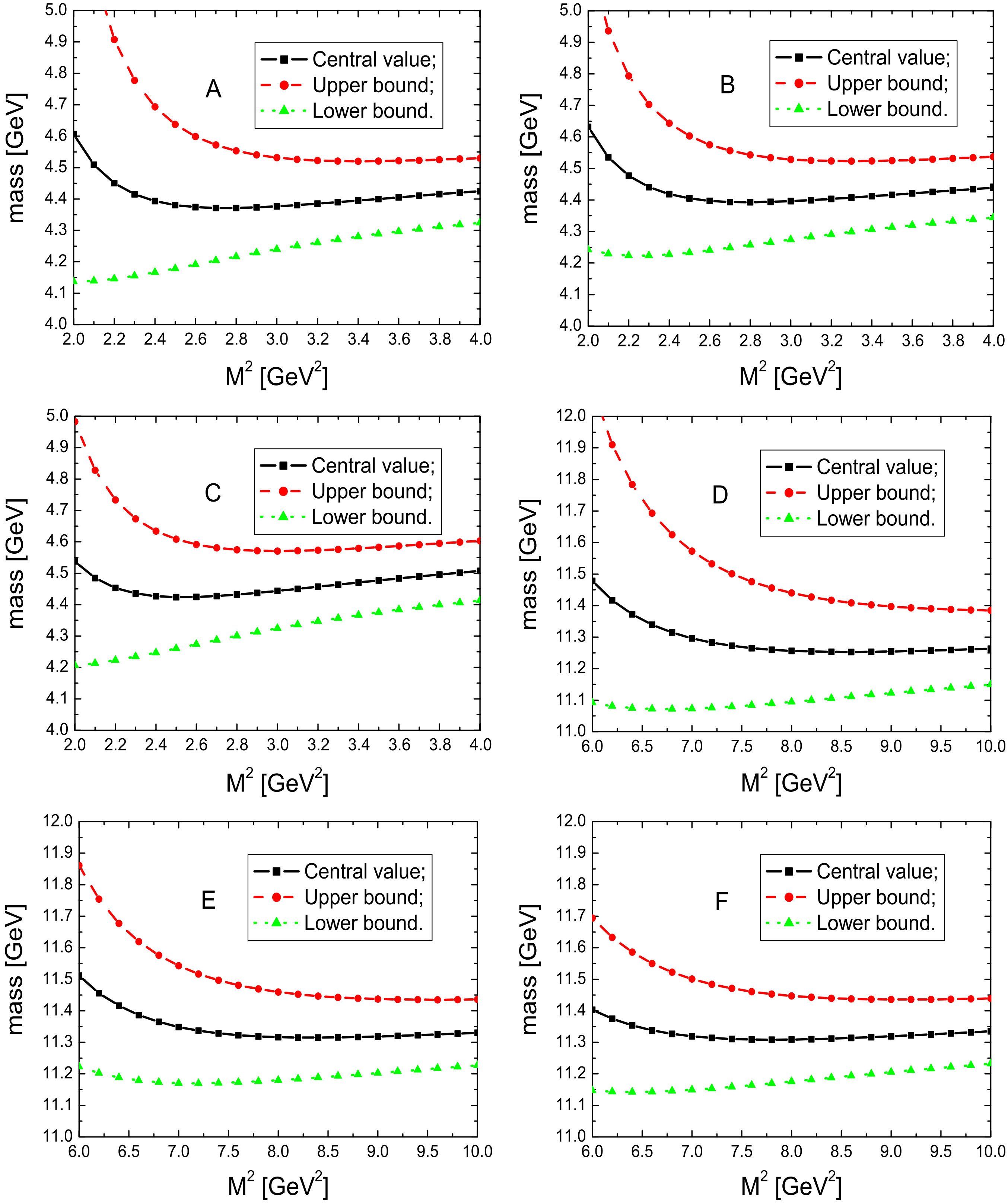

FIG. 5 (color online). The masses of the scalar tetraquark states with a variation of the Borel parameter M^2. The A, B, C, D, E, and F denote the $c\bar{c}q\bar{q}$, $c\bar{c}q\bar{s}$, $c\bar{c}s\bar{s}$, $b\bar{b}q\bar{q}$, $b\bar{b}q\bar{s}$, and $b\bar{b}s\bar{s}$ channels, respectively.

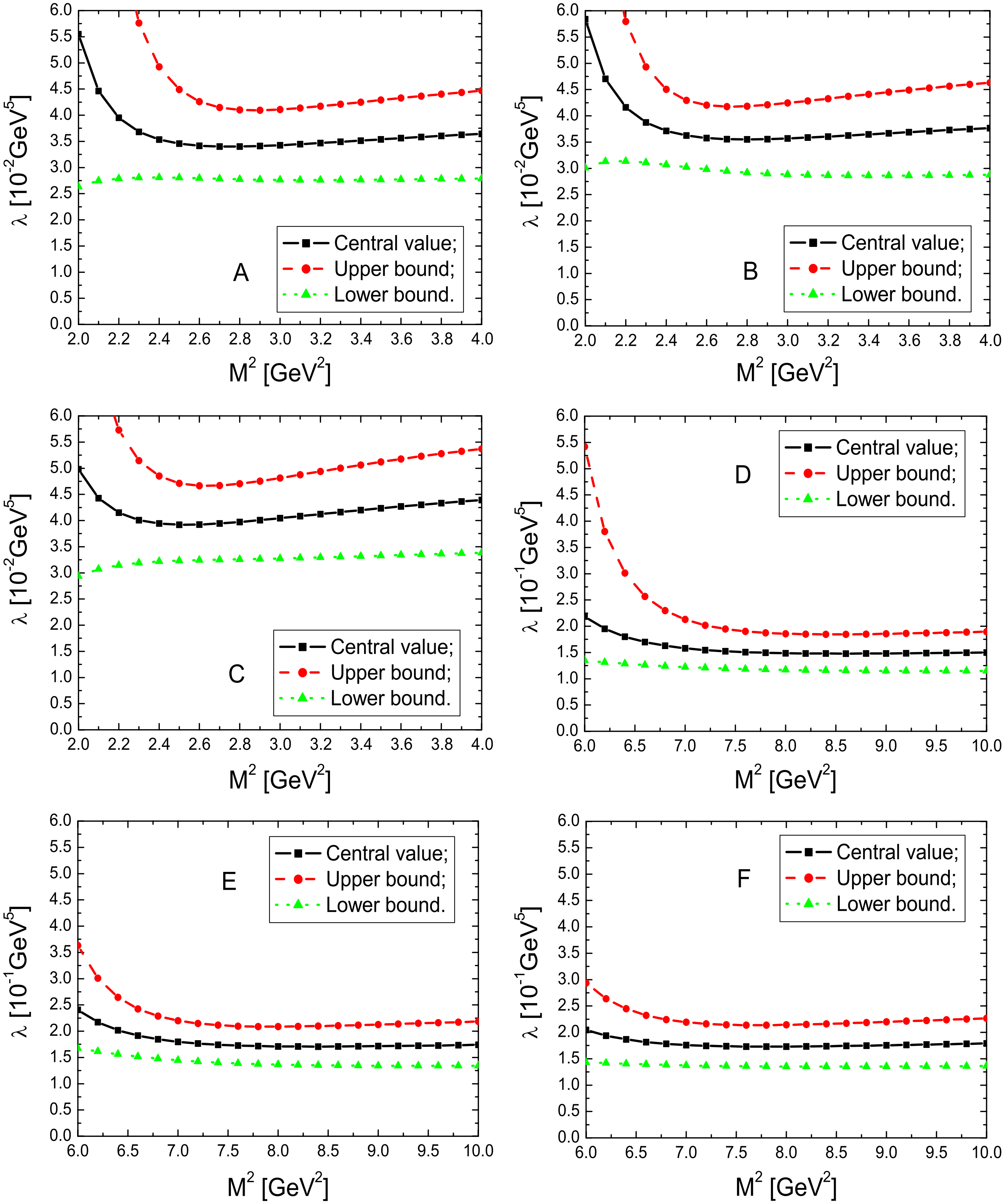

FIG. 6 (color online). The pole residues of the scalar tetraquark states with a variation of the Borel parameter M^2. The A, B, C, D, E, and F denote the $c\bar{c}q\bar{q}$, $c\bar{c}q\bar{s}$, $c\bar{c}s\bar{s}$, $b\bar{b}q\bar{q}$, $b\bar{b}q\bar{s}$, and $b\bar{b}s\bar{s}$ channels, respectively.

TABLE I. The masses (in units of GeV) for the scalar tetraquark states.

Tetraquark states	This work	Refs. [26,27]
$c\bar{c}s\bar{s}$	4.44 ± 0.16	4.051
$c\bar{c}q\bar{s}$	4.39 ± 0.16	3.922
$c\bar{c}q\bar{q}$	4.37 ± 0.18	3.812
$b\bar{b}s\bar{s}$	11.31 ± 0.16	10.662
$b\bar{b}q\bar{s}$	11.33 ± 0.16	10.572
$b\bar{b}q\bar{q}$	11.27 ± 0.20	10.471

TABLE II. The pole residues for the scalar tetraquark states.

Tetraquark states	Pole residues (10^{-2} GeV^5)
$c\bar{c}s\bar{s}$	4.00 ± 0.80
$c\bar{c}q\bar{s}$	3.56 ± 0.70
$c\bar{c}q\bar{q}$	3.41 ± 0.70
$b\bar{b}s\bar{s}$	17.3 ± 4.0
$b\bar{b}q\bar{s}$	17.4 ± 4.0
$b\bar{b}q\bar{q}$	15.2 ± 3.8

numerical results support this conjecture; our works on the $C\gamma_\mu - C\gamma^\mu$ type tetraquark states $\bar{Q}\bar{Q}qq$ will be presented elsewhere.

The nonet scalar mesons below 1 GeV [the $f_0(980)$ and $a_0(980)$ especially] are good candidates for the tetraquark states. However, they cannot satisfy the two criteria of the QCD sum rules, and result in a reasonable Borel window. If the perturbative terms have the main contribution (in the conventional QCD sum rules, the perturbative terms always have the main contribution), we can approximate the spectral density with the perturbative term [25], then take the pole dominance condition, and obtain the approximate relation

$$\frac{s_0}{M^2} \geq 4.7. \tag{11}$$

If we take the Borel parameter that has the typical value $M^2 = 1$ GeV^2, then $s_0 \geq 4.7$ GeV^2, the threshold parameter is too large for the light tetraquark state candidates $f_0(980)$, $a_0(980)$, etc.

On the other hand, the numerous candidates with the same quantum numbers $J^{PC} = 0^{++}$ below 2 GeV cannot be accommodated in one $q\bar{q}$ nonet; some are supposed to be glueballs, molecules, and multiquark states [18,36,37]. Once the main Fock sates of the nonet scalar mesons below 1 GeV^2 are proved to be tetraquark states, we can draw the conclusion that the QCD sum rules are not applicable for the light tetraquark states.

In this article, we calculate the mass spectrum of the scalar hidden charm and bottom tetraquark states by imposing the two criteria of the QCD sum rules. In fact, we usually consult the experimental data in choosing the Borel parameter M^2 and the threshold parameter s_0. There lacks experimental data for the phenomenological hadronic spectral densities of the tetraquark states; the present predictions cannot be confronted with the experimental data.

The LHCb is a dedicated b and c-physics precision experiment at the large hadrom collider (LHC). The LHC will be the world's most copious source of the b hadrons, and a complete spectrum of the b hadrons will be available through gluon fusion. In proton-proton collisions at $\sqrt{s} = 14$ TeV, the $b\bar{b}$ cross section is expected to be $\sim 500\ \mu b$ producing 10^{12} $b\bar{b}$ pairs in a standard year of running at the LHCb operational luminosity of 2×10^{32} cm^{-2} sec^{-1} [38]. The scalar tetraquark states predicted in the present work may be observed at the LHCb, if they indeed exist. We can search for the scalar hidden charm tetraquark states in the $D\bar{D}$, $D^*\bar{D}^*$, $D_s\bar{D}_s$, $D_s^*\bar{D}_s^*$, $J/\psi\rho$, $J/\psi\phi$, $J/\psi\omega$, $\eta_c\pi$, $\eta_c\eta$, ... invariant mass distributions and search for the scalar hidden bottom tetraquark states in the $B\bar{B}$, $B^*\bar{B}^*$, $B_s\bar{B}_s$, $B_s^*\bar{B}_s^*$, $\Upsilon\rho$, $\Upsilon\phi$, $\Upsilon\omega$, $\eta_b\pi$, $\eta_b\eta$, ... invariant mass distributions.

Furthermore, the nonleptonic B decays through $b \to c\bar{c}s$ provide another favorable environment for the production of the scalar hidden charm tetraquark states [39]; we can search for them at the KEK-B or the Fermilab Tevatron.

IV. CONCLUSION

In this article, we study the mass spectrum of the scalar hidden charm and bottom tetraquark states with the QCD sum rules. The numerical results are compared with the corresponding ones from a relativistic quark model based on a quasipotential approach in QCD. The relevant values from the constituent diquark model based on the constituent diquark masses and the spin-spin interactions are also discussed. We can search for the scalar hidden charm and bottom tetraquark states at the LHCb, the KEK-B, or the Fermilab Tevatron.

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