PHYSICAL REVIEW D **79,** 094008 (2009)

Very high energy proton-proton cross section

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(Received 16 December 2008; revised manuscript received 20 February 2009; published 8 May 2009)

The recent Pierre Auger Observatory result suggesting a coincidence of extensive air showers arrival directions with ''nearby'' active galactic nuclei and HiRes discovery of the Greisen-Zatsepin-Kuzmin cutoff indicates protons to be only or at least the strongly dominant component of primary extra galactic cosmic ray flux. However, showers initiated by these ultrahigh energy particles developed faster than predicted by the simulation calculations with conventional interaction models. This could be evidence of the substantial increase of the p-air cross section. The progress in understanding the proton-proton cross section description allows us to examine this possibility, and eventually reject it as an explanation of the ultrahigh energy cosmic ray ''pure proton'' controversy.

DOI: 10.1103/PhysRevD.79.094008 PACS numbers: 13.60.Hb, 13.85.Hd

I. INTRODUCTION

Recently announced discoveries by the Pierre Auger Observatory [1] and HiRes Fly's Eye (HiRes) experiment [2] suggest the cosmic ray (CR) flux above the energy of about 10^{18} eV consists mainly of protons. In this paper we examine the question of whether giant extensive air showers (EAS), as we see them, can be initiated by CR protons of such ultrahigh energies with the interaction cross section changed appropriately.

For very high energies only some characteristics of the shower can be measured with good enough accuracy. The situation is different than the one in ''the knee'' energies ($\sqrt{s}$ of the order of few TeV), where the experimental information on each individual shower could be ''multicomponent'', very detailed, and accurate. The very steep CR energy spectrum makes the 10^{18} eV events 1×10^9 times less frequent than those of 10^{16} eV. The most important and well-measured property of the giant EAS is the shower longitudinal development profile, given experimentally, e.g., as a distribution, along the shower axis, of the intensity of fluorescent light emitted when the charged shower particles excite air molecules. This distribution could be measured to some, not very great, accuracy. The normalization (total number of particles), which is related to the total primary particle energy, and first moment (given usually as $x_{\max}$—the position of the maximum number of the particles in the cascade) are the parameters available for further study. The next moment, the width of $x_{\max}$ distribution, is also used for the estimation of the primary cosmic ray mass spectrum (see, e.g., [3]). The recent measurements of the Pierre Auger Observatory [4] and HiRes [2] together with the older ones from Fly's Eye [5] and Yakutsk [6] give the most complete information we can get about the average value of $x_{\max}$ as a function of estimated primary particle energy, and all this data, when confronted with the calculations, seem to contradict the ''pure proton'' ultrahigh energy cosmic ray flux composition hypothesis.

The EAS data have been analyzed for more then 20 years by the widely available simulation programs. The one called CORSIKA (cosmic ray simulations for Kascade) [7,8] is the best known example. Different high energy interaction models incorporated into CORSIKA are EPOS, DPMJET, NEXUS, QGSJET, SIBYLL, and VENUS [9]. In spite of the continuous progress in the modeling, the correct description of EAS (with the lack of contradictions from the accelerator measured characteristics) is still far off. For a recent example, see, e.g., Ref. [10] by the KASCADE Collaboration.

The development of EAS is determined by the probabilities of particles to interact (or decay) and inclusive momentum distributions of produced particles. The decay constants and branching ratios are very well known. The interaction probabilities are defined by the cross section values. We are not going to discuss here the effects of geometry and atmospheric profiles, as they are not relevant with respect to the accuracy of giant EAS measurements.

The inclusive energy distributions are known experimentally up to the SPS (and the Tevatron) energies and this knowledge is, of course, not as good as we wish it to be. Lower energy experiments (around $\sqrt{s} \approx 20$ GeV) with stationary targets deliver more precise data and, what is more important, they cover the very forward region, which in fact controls a development of the cascade. The information we have is limited and we need to follow some, more or less elaborate, models to extrapolate it by 4–5 orders of magnitude of incoming particle energy. More details can be found, e.g., in Ref. [11].

The cross sections are, of course, not measured for the energies of our present interest, but significant progress in understanding extrapolation procedures has been made in

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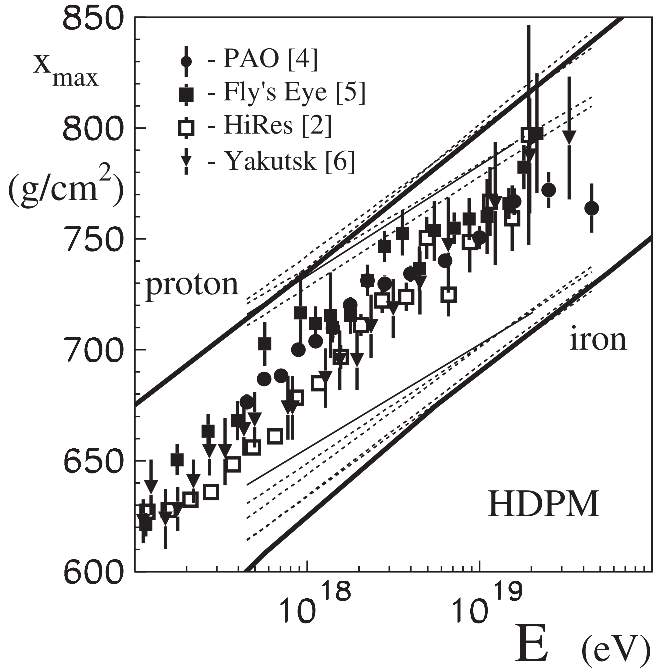

FIG. 1. The prediction of $x_{\rm max}$ depths calculated with HDPM (thick solid line) in comparison with data and other model results. Thin dashed lines show results of other CORSIKA models (EPOS, SIBYLL, and two versions of the QGSJET model from top to bottom around 10^{19} eV for the proton case, respectively, [4]. Thin solid lines represent the well-known Fly's Eye group calculations [5].

recent years. This could be used for the analysis to the ultrahigh energy cosmic ray data.

II. $X_{\rm max}$ RESULTS

In Fig. 1 the situation of the first moment of the shower development is shown in a conventional way. The depth in the atmosphere of the position of the maximum number of particles in the extensive air shower $x_{\rm max}$ is presented as a function of primary particle energy. The energy is related closely to the total number of particles in the cascade while the $x_{\rm max}$ is related rather to the particle energy per nucleon.

The HDPM interaction code [12] introduced in a first version of CORSIKA has been used here in the simplified, averaged, version. The calculations were made by numerical integration of the set of particle transport equations. The results of our calculations compared with contemporary CORSIKA (version 6.735) output produced with different interaction models, as seen in Fig. 1, are of the quality required to provide a conclusion from the data shown (including possible systematics).

III. RECENT PROGRESS IN CROSS SECTION DESCRIPTION

The role of (inelastic) cross section values for the cascade development is obvious. The cross sections involved in the EAS development are at least hadron-nucleus (in general nucleus-nucleus) cross sections. Before we discuss this complex case we would like to look closely at the interaction of single nucleons (protons).

A. Hadron-proton scattering cross section

Recent years brought significant progress to the theoretical description of the cross sections (total, inelastic, and elastic). It is based, still, on the optical picture

$$\sigma_{\rm tot} = 2\int[1 - {\rm Re}(e^{i\chi(b)})]d^2\mathbf{b}, \qquad \sigma_{\rm inel} = \int 1 - |e^{i\chi(b)}|^2 d^2\mathbf{b}. \tag{1}$$

Using the optical analogy one can interpret the $(1 - e^{i\chi(\mathbf{b})})$ term as a transmission coefficient for a given impact parameter $\mathbf{b}$. Considering two colliding objects we can assume (for pure absorptive potential)

$$\chi(b) = i\omega(b) = iK_{ab}\int d^2\mathbf{b}'\rho_a(\mathbf{b})\rho_b(\mathbf{b}+\mathbf{b}'), \tag{2}$$

where ρ_h is a particle's "opaqueness" (the matter density integrated along the collision axis).

In the series of papers by Block and co-workers [13], the approximation of χ of the form inspired by QCD was examined and gave a very good description of pp and $p\bar{p}$ data. Assuming the vector meson dominance and the additive quark model, it could be used with the same parameters also for photon-proton and photo-photon scattering cross section calculations.

Another parametrization was proposed by Pérez-Peraza and collaborators in Ref. [14]. The fit to differential elastic scattering data has been made. The consistency with measured ISR, SPS, and Tevatron [15] cross sections within the framework of an adopted model makes the eventual confidence band narrow.

In the paper by Ishida and Igi [16], $K^{\pm}p$, $\pi^{\pm}p$, $p\bar{p}$, and pp, the forward scattering amplitude was parametrized in the form which, for high energies, leads to the saturation of the $\log^2 s$ Froissart bound. The universality of the asymptotic behavior [pp, $(p\bar{p})$, $\pi^{\pm}p$, and $K^{\pm}p$] gives additional evidence that the proposed picture is correct and extrapolations are thus, again, justified stronger.

The $\log^2 s$ character of the cross section rise was pointed out also by the COMPETE Collaboration [17].

The similar and, again, self-consistent description of data on charged pion and proton (antiproton) projectiles has been used by Block and Halzen in Ref. [18]. Their "best fit" performed with some new statistical tools supports the necessity of the $\log^2$ component.

To be fair we should cite here also the work which does not favor the $\log^2 s$ cross section rise. In Ref. [19] the standard Regge and (soft) pomeron exchange contributions are supplemented by a hard pomeron and two-pomeron exchange terms making possible a very fast rise of the cross section already in the Tevatron energies. However, the predictions made are quite uncertain due to the very crude treatment of double pomeron exchanges.

We would like to remind one here also about the older result from Ref. [20] where the conclusion was similar. The cross sections for interactions of $K^{\pm}$, $\pi^{\pm}$, $p\bar{p}$, and pp, and a wide collection of elastic data were a parametrized assumed mechanism of geometrical scaling with a $\log^2 s$ asymptotic rise of cross sections.

At very high energies where the proton-proton cross section is big, the "proton central opaqueness" in the optical models reaches the black limit and thus it cannot get bigger; thus the further rise of the cross sections can be realized by the rise of the proton transverse dimensions. The relevance of the geometrical scaling seems to be increasing when one moves to extremely high cosmic ray energies.

The resulting cross sections are shown in Fig. 2 together with approximations by Durand and Pi [21], the result of Ref. [14], and Block parametrization [22], in comparison with the data from stationary target pp and $p\bar{p}$ accelerator experiments, colliders ISR, SPS, and Tevatron, and points inferred from the cosmic ray experimental data. The last will be discussed further below.

It is seen that the spread between different models and extrapolations even at the highest energies is very small. It is much smaller than the error bars shown by the cosmic ray points.

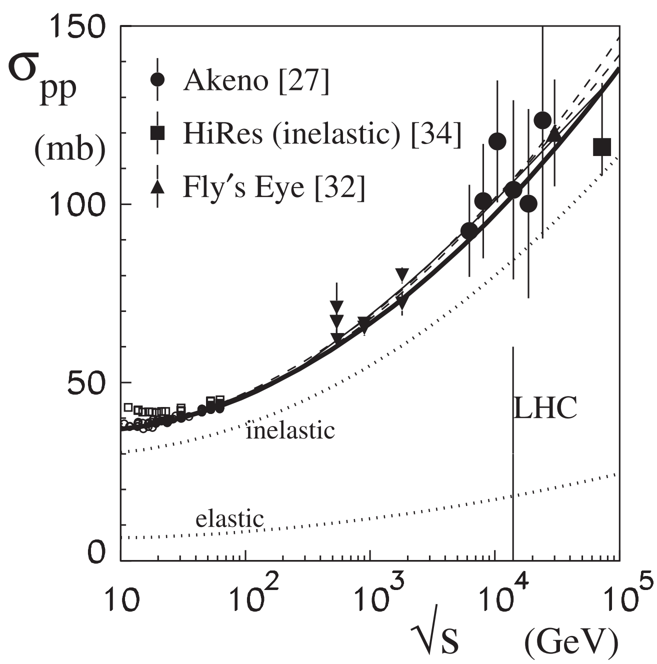

FIG. 2. Proton-proton total cross section (inelastic and elastic contributions are given by dotted lines) calculated with geometrical scaling [20] compared with the data from accelerator measurements [35] and cosmic ray data points recalculated as described in Sec. III C. Dashed thin lines are the results of [22] (the upper line) and [14] (the lower). The thin solid line is the phenomenological fit of [21].

B. Proton-nucleus and nucleus-nucleus interaction cross sections

The cosmic ray data on the proton-proton cross section at very high energies are based on EAS observations. The conversion procedure from p-air to pp is necessarily involved here.

The theory of scattering of nuclei has been known for years [23]. However, we think that the applicability of some simplifications used at low energies can be questioned when the individual nucleon transverse sizes rise substantially. To explain the point some brief introduction is needed.

The scattering of particles on the close many-particle system (nucleus) can be treated as a superposition of individual interactions each with a specific phase shift. The overall phase shift for the incoming wave is a sum of all of the two-particle phase shifts,

$$\chi_A(b, \{\mathbf{d}\}) = \sum_{j=1}^{A} \chi_j(\mathbf{b} - \mathbf{d}_j). \tag{3}$$

The set of vectors in the impact parameter plane $\{\mathbf{d}\} = \{d_1, d_2, \ldots, d_A\}$ describes the particular positions of all A nucleons within the nucleus. Equation (3) is the essence of Ref. [23] and defines the Glauber approximation.

On the other hand, the scattering process can be treated as the single collision process with its own nuclear phase shift $\chi_{\rm opt}(b)$. To get the consistency with Eq. (3) it is required that

$$e^{i\chi_{\rm opt}(b)} = \int |\psi(\{\mathbf{d}\})|^2 e^{i\sum_{j=1}^{A}\chi_j(\mathbf{b}-\mathbf{d}_j)} \prod_{j=1}^{A} d^2\mathbf{d}_j = \langle e^{i\chi(b,\{\mathbf{d}\})}\rangle, \tag{4}$$

in what defines the relation of the individual projectile-nucleon and overall projectile-nucleus opacities. The averaging is over all configurations of the nucleons $\{\mathbf{d}\}$.

To go further with the calculations of $\chi_{\rm opt}$ a commonly used assumption has to be made. If we assume that the number of scattering centers (A) is large and the transparency of the nucleus as a whole remains constant then

$$\chi_{\rm opt}(b) = i \int d^2\mathbf{d}\rho_A(\mathbf{d})[1 - e^{i\chi(\mathbf{b}-\mathbf{d})}], \tag{5}$$

where ρ_A is the distribution of scattering center (nucleon) positions in the nucleus ($\sum \varrho_j$).

And finally, assuming that the individual nucleon opacity $|1 - e^{i\chi(b)}|$ is very sharply peaked compared with ρ_A then with the help of the optical theorem the simple formula can be found

$$\sigma_{pA}^{\rm inel} = \int d^2\mathbf{b}[1 - e^{-\sigma_{pp}^{\rm tot}\rho_A(b)}] = \int d^2\mathbf{b}\left\{1 - \left[1 - \sigma_{pp}^{\rm tot}\frac{\rho_A}{A}\right]^A\right\}, \quad (6)$$

where the last equality holds in the large A limit [certainly Eq. (6) cannot be used for $A = 1$]. This result is often but not quite correctly called the "Glauber approximation." As has been shown, the original Glauber assumption given by Eq. (3) is here supported by a small nucleon size and a large value of A.

The inelastic interaction, by definition, contains also two-prong quasielastic scattering and some diffractive interactions with a single particle on the projectile/target hemisphere. The cross section responsible for the energy dissipation processes involved in the cosmic ray passage through the atmosphere is not the inelastic one defined above but rather the one called the "production" (absorption) cross section.

One of the ways to take it into account is the multiple scattering approach which is used to make p-air to pp conversion [24].

The $\sigma_{\rm prod}$ can be interpreted in the probabilistic way by identifying the $[1 - |e^{i\chi(\mathbf{b})}|^2]$ term as the probability of inelastic scattering (particle production) at impact parameter $\mathbf{b}$. This can be extended to the interaction with nucleus in a straightforward way. If we denote this probability by $P(\mathbf{b})$ and nucleons in a nucleus A are distributed according to ρ_A, then the averaged probability of the inelastic interaction with one of the nucleons is

$$\bar{P}_A(\mathbf{b}) = \int d^2\mathbf{d}\frac{\rho_A(\mathbf{b})}{A}P(\mathbf{b} - \mathbf{d}). \quad (7)$$

The production cross section with the whole nucleus is then

$$\sigma_{pA}^{\rm prod} = \int d^2\mathbf{b}\{1 - [1 - \overline{P_A(\mathbf{b})}]^A\}. \quad (8)$$

In the multiple scattering picture the point-nucleon approximation can be also introduced simplifying the cross section formula. If one puts $P(\mathbf{b}) = \delta^2(\mathbf{b})\sigma_{pp}^{\rm prod}$, then

$$\bar{P}_A(\mathbf{b}) = \frac{\rho_A(\mathbf{b})}{A}\sigma_{pp}^{\rm prod}, \quad (9)$$

in what leads to

$$\sigma_{pA}^{\rm prod} = \int d^2\mathbf{b}\left\{1 - \left[1 - \sigma_{pp}^{\rm prod}\frac{\rho_A(\mathbf{b})}{A}\right]^A\right\}. \quad (10)$$

The above equation has a very similar form to Eq. (6) but the difference is also quite clear, but it is sometimes called the Glauber approximation too (see, e.g., Ref. [24].

This "multiple scattering" approach gives the pp total (using $\sigma^{\rm prod}$, after respective quasielastic corrections) cross section points higher than "the Glauber" even with the point-nucleon approximation. Summation of probabilities instead of phase shifts cuts the interference parts.

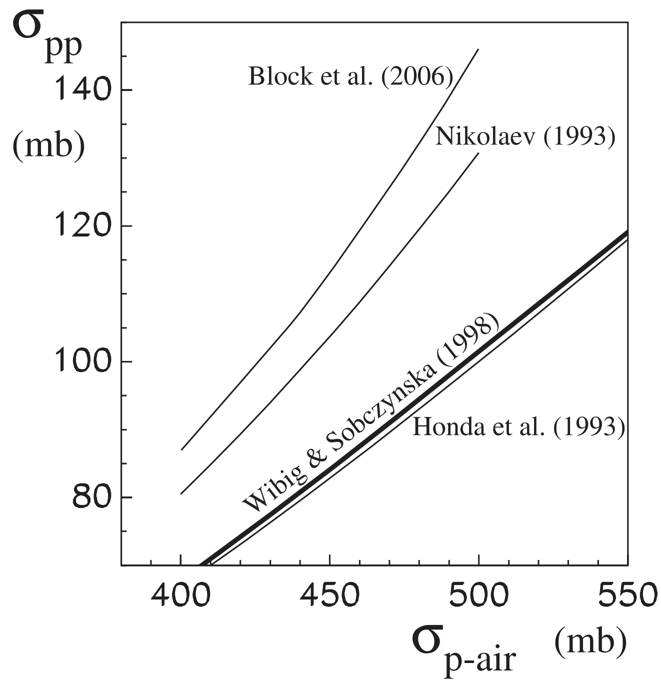

FIG. 3. The relation between pp and p-air cross sections calculated with the exact Glauber formula and other propositions used to convert cosmic ray EAS attenuation data to pp total cross section. The difference related partially to the meaning of presented σ's is discussed in the text.

We have performed respective calculations based on exact Glauber formalism. The results are shown in Fig. 3 as a relation between total pp and p-air interaction cross sections. It is shown there with other propositions. The difference between the Refs. [25,26] result and Honda and ours is in the meaning of $\sigma_{p\text{-air}}$ which is taken once as measured and onceas corrected to the inelastic cross section as described in Ref. [27].

The importance of this relation is that it allows one to get the pp cross section from the cosmic ray data on the EAS attenuation length measured experimentally which is related to the p-air interaction cross section.

C. Cosmic ray data

The conversion p-air to pp cross section is not the only problem with utilizing the EAS data. The cross sections are available mostly (with some exceptions) as a rate of the attenuation of the shower and this process involves also other properties of the proton interaction with air nuclei. The relation

$$\Lambda_{\rm att} = k\frac{14.5m_p}{\sigma_{p\text{-air}}^{\rm prod}}, \quad (11)$$

where $\sigma_{pp}^{\rm prod}$ is the production cross section, and

$$\sigma^{\text{prod}}_{p\text{-air}} = \sigma^{\text{tot}}_{p\text{-air}} - \sigma^{\text{el}}_{p\text{-air}} - \sigma^{\text{q-el}}_{p\text{-air}} \quad (12)$$

makes the problem. The reason of some additional uncertainty from the experimental point of view is in the quasielastic contribution.

The quasielastic cross section $\sigma^{\text{q-el}}_{p\text{-air}}$ can be defined as two-prongs inelastic [25,26], but also can be increased by a diffractive contribution [28] or by interactions with the inelasticity, defined as a fraction of interaction energy realized to secondary particle production, smaller than, e.g., 0.05 [29]. All of these different approaches have to affect the factor k in Eq. (11).

Additionally the factor k has to describe the role in the cascade development of the secondaries and their subsequent interactions. This of course is the source of additional uncertainties of a systematic type because of

(i) The multiparticle production process, especially at very high energies, is unknown, and even more, the contemporary Monte Carlo programs are not able to simulate some shower characteristics consistently (see, e.g., [10]) even in much lower energies.

(ii) k should also take into account the particular experimental setup, the trigger, and possible biases due to the steep CR energy spectrum.

The value of k should be adjusted with the help of self-consistent extensive Monte Carlo simulations (as, e.g., shown in Ref. [29]) taking into account properly the quasielastic contributions, however they are particularly defined.

The reanalysis of k done recently in Ref. [22] gave an interesting observation. It seems that the value of k is changing with time. Starting from the value of 1.6 in the 1990s it seems to decrease continuously. In the paper by Block [22] one value of k was proposed which could describe all the experiments in the best way with the additional assumption that the predictions for σ_{inel} obtained in Ref. [18] are the correct ones. The confirmation of the result obtained in Ref. [22] is that the "model independent" estimation of k by the HiRes group gives the result in agreement with the value of $k = 1.26$.

Table I (from Ref. [22]) illustrates the situation.

The proposed change of the value of k is the reason of the position of the dot-dashed line in Fig. 4 which looks to be in obvious contradiction with the data points. But the points there are as they were originally published by listed experimental groups with their own k values. The Akeno points, for example, should go down when k changes from 1.5 to 1.26 according to Ref. [22].

We propose to leave the original $\sigma_{p\text{-air}}$ points as they were published but change the procedure of the conversion of $\sigma_{p\text{-air}}$ to σ_{pp}. The thick line in Fig. 4 is obtained using our geometrical scaling fit of accelerator data and the transformation coefficient shown in Fig. 3 and pp cross section shown in Fig. 2.

TABLE I. Values of k used in different experiments, and the "best" universal value from Ref. [22] compared with values obtained from simulation calculations.

Experiment	k
Fly's Eye	1.6
Akeno	1.5
Yakutsk	1.4
HiRes	1.21
EASTOP	1.15
"'Best value" from Ref. [22] 1.264 ± 0.0330 ± 0.013	
CORSIKA simulations:	
With SIBYLL	1.15
With QGSJET	1.3

There are also shown for comparison other cross sections adopted for various high energy interaction models in the CORSIKA program.

Concluding, we show in Table II some predictions of various authors concerning the pp total cross section for the LHC energy ($\sqrt{s} = 14$ TeV).

As it is seen, all the estimated values are very close and, with a high degree of confidence, it can be stated that the pp cross section predicted for the LHC energies is expected to be equal to about 108 mb (within a few millibarns

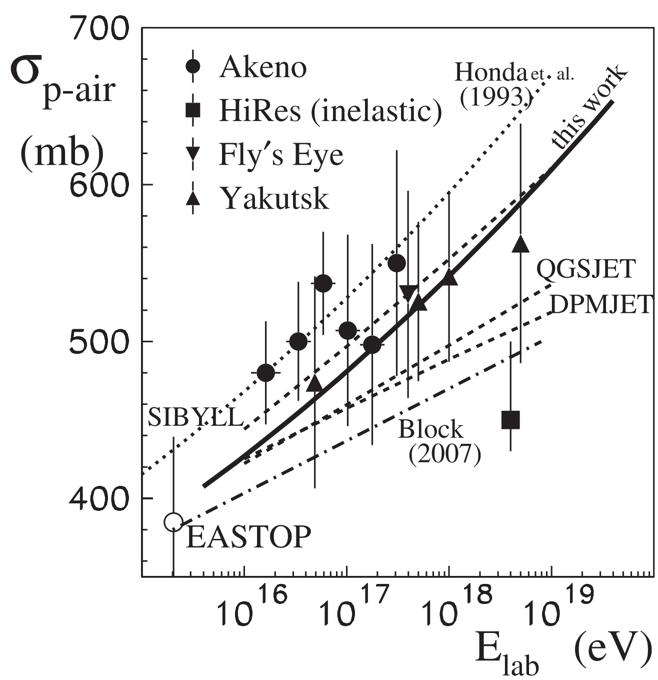

FIG. 4. Proton-air interaction cross section calculated according to exact Glauber formula (thick line) and the values used by the DPMJET, QGSJET, and SIBYLL models of the CORSIKA program (dashed lines). The fit to the original Akeno data is shown by a dotted line and the recent result of Block by a dot-dashed line. The points are Akeno [27], Fly's Eye [32], Hi-Res [34], Yakutsk [33], and the recent result from the EASTOP experiment [36].

TABLE II. Predicted values of pp cross section at LHC energies.

Author	Year	Ref.	$\sigma_{\rm tot}$
Honda *et al.*	1993	[27]	110.4
Wibig and Sobczynska	1998	[20]	102.5
Cudell *et al.*	2002	[17]	111.5 ± 1.2
Velasco *et al.*	1999	[30]	104.17 ± 4.4
Pérez-Peraza *et al.*	2005	[14]	108.27 + 4.4 − 3.17
Block *et al.*	1999	[13]	108 ± 3.4
Block and Halzen	2006	[18]	107.3 ± 1.2
Ishida and Igi	2007	[16]	109.5 ± 2.8

"error box"). The result 102.5 mb is excluded by recent fits but the confidence level is not very tight. The result of Ref. [19] of 125 ± 35 mb was not included in Table II. Its error is rather of a systemetic nature, so it should be read as the cross section could be anywhere between 100 and 160 mb. The predictive power of this statement is not very strong. In the near future it is expected that the LHC will produce cross section results which put a new constrain on the validity of geometrical scaling at very high energies.

IV. "PROTONS ONLY" AND HDPM RESULTS ON $x_{\rm max}$

As shown in Fig. 1 the pure proton flux around the ankle is excluded when HDPM with default parameters is used. The analytic shower development allows us to test if the correction factor, e.g., of the form proposed in Ref. [29]

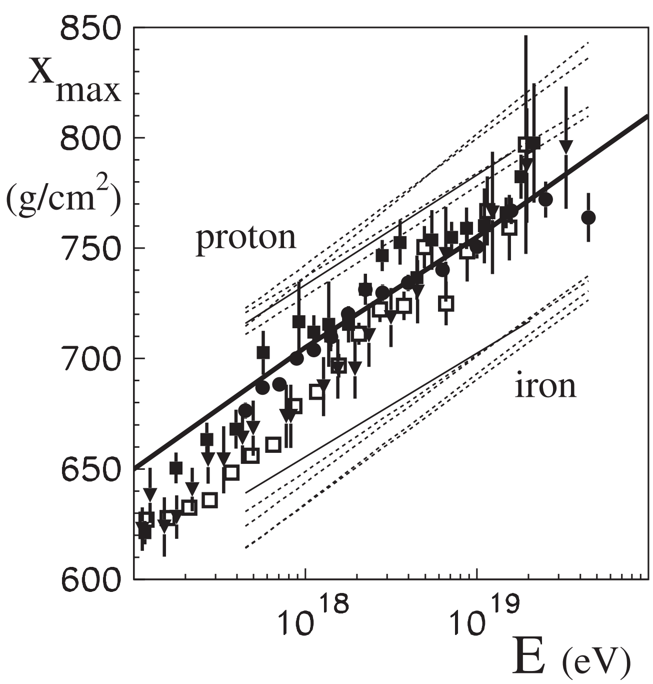

FIG. 5. HDPM the best result on $x_{\rm max}$ average value for pure protons with the cross section increased.

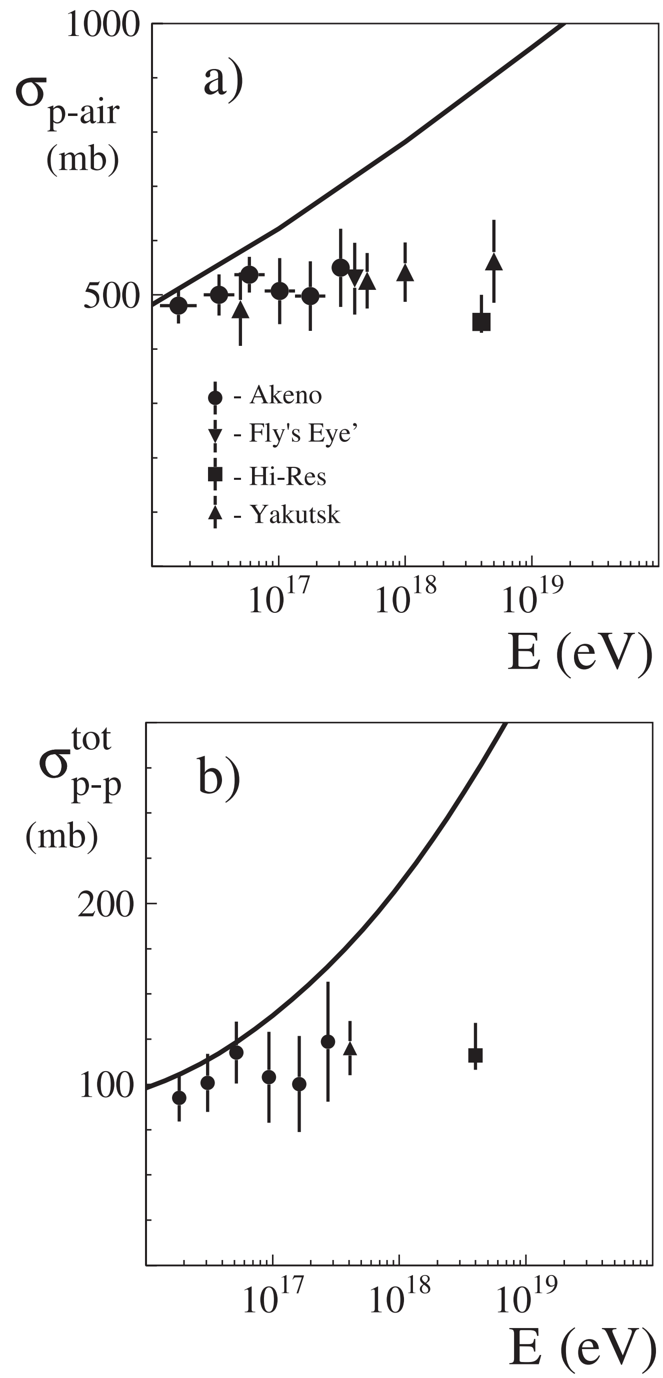

FIG. 6. Proton-air (a) and proton-proton (b) cross sections in the very high energy region. Proton-proton result (b) is obtained using the relation shown in Fig. 3 and the best HDPM fit of the p-air cross section (increased assuming pure protons hypothesis) as shown in (a).

applied to the interaction probabilities (respective cross sections) can make the pure proton hypothesis acceptable.

$$\sigma_{\rm new}(E) = \sigma_{\rm old}(E)\begin{cases} 1 & E \leq 1\ {\rm PeV} \\ 1 + (\delta_{19} - 1)\frac{\lg(E)-15}{(19-15)} & E > 1\ {\rm PeV} \end{cases}. \quad (13)$$

The form of Eq. (13) means that, up to the energies of SPS/Tevatron, where there are measured data, we do not change anything and for higher energies the cross sections are multiplied by a slowly varying factor determined by the value of the correction δ_{19} almost at the end of CR data, at 10^{19} eV.

We found that it is possible to get some nice results as seen in Fig. 5 [31].

But this result needs a cross section as large as the one shown by the line in Fig. 6(a). The points there (as in Fig. 4) show values reported by the four big experimental groups: Akeno [27], Fly's Eye [32], Yakutsk [33], and HiRes [34]. The disagreement is obvious.

We can convert the supposed p-air cross section values to pp total cross sections and compare with the published cosmic ray points. The rise of the pp cross section is even faster than can be expected taking into account the fact that the central part of the nucleus is already almost totally opaque, thus all the p-air cross section rise has to be made by the increase of the size only of the nucleons on the edge of the nucleus. As in Fig. 6(a) the disagreement for pp is unacceptable.

V. CONCLUSION

Presented results show that there is no way to adjust the pp cross section rise such that the EAS calculations with the standard HDPM model (and others, similar, including the CORSIKA code) are able to give the position of the shower maximum in agreement with measurements for pure proton composition at and above 10^{18} eV and preserve simultaneously the observed shower attenuation rate. The actual uncertainties on pp, p-air cross section determination as well as those related to the giant air showers data interpretation demand a drastic change (above a given energy threshold) of the high energy multiparticle production model if one wants to get a pure proton very high energy cosmic ray flux.

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