Pion superfluidity beyond mean field approximation in the Nambu–Jona-Lasinio model

Chengfu Mu and Pengfei Zhuang

Physics Department, Tsinghua University, Beijing 100084, China

(Received 13 March 2009; published 7 May 2009)

We investigate pion superfluidity in the frame of a two flavor Nambu–Jona-Lasinio model beyond mean field approximation. We calculate the thermodynamics to the next to leading order in an expansion in the inverse number of colors, including both quark and meson contributions at finite temperature and baryon and isospin density. Because of the meson fluctuations, the Sarma phase which exists at the mean field level is washed away, and the Bose-Einstein condensation region at low isospin density is highly suppressed.

DOI: 10.1103/PhysRevD.79.094006 PACS numbers: 12.39.−x, 03.75.Nt, 21.65.Qr

I. INTRODUCTION

The study on quantum chromodynamics (QCD) phase structure is recently extended to finite isospin density [1]. The physical motivation to study QCD at finite isospin density and the corresponding pion superfluidity is related to the investigation of compact stars, isospin asymmetric nuclear matter, and heavy ion collisions at intermediate energies.

While the perturbation theory of QCD can well describe the properties of new QCD phases at extremely high temperature and density, the study on the phase structure at moderate temperature and density depends on lattice QCD calculation and effective models with QCD symmetries. The lattice simulation at finite isospin chemical potential [2] shows that there is a phase transition from normal phase to pion superfluidity phase at a critical isospin chemical potential which is about the pion mass in the vacuum. The QCD phase structure at finite isospin density is also investigated in low energy effective models, such as the Nambu–Jona-Lasinio (NJL) model [3] applied to quarks [4–8] which is simple but enables us to see directly how the dynamic mechanism of isospin symmetry breaking operates. Near the phase transition point, the chiral and pion condensates calculated in this model are in good agreement with the lattice simulation [2].

In a pion superfluid at zero baryon chemical potential, the quark and antiquark of a condensed pair have the same isospin chemical potential and in turn the same Fermi surface. When a nonzero baryon chemical potential is turned on, it can be regarded as a Fermi surface mismatch between the quark and antiquark. The pion superfluidity in baryonic matter is recently discussed at mean field level in the NJL model in chiral limit [9] and in the real case with finite current quark mass [10]. The pion superfluid can exist when the baryon density is not very high, otherwise the system will be in normal phase without pion condensation because of too strong a mismatch. Inside the pion superfluid, the condensed state is separated into two phases. At small isospin chemical potential μ_I, the homogeneous and isotropic Sarma phase [11] is free from the Sarma instability [11] and magnetic instability [12] due to the strong coupling and large enough effective quark mass. It is therefore the stable ground state. At large μ_I, while the Sarma instability can be cured via fixing baryon density n_B to be nonzero, its magnetic instability implies that the inhomogeneous and anisotropic Larkin-Ovchinnikov-Fudde-Ferrell (LOFF) phase [13] is favored more than the Sarma phase. In the intermediate μ_I region, the stable ground state is the Sarma phase at higher n_B and LOFF phase at lower n_B.

The Bose-Einstein condensation—Bardeen-Cooper-Shriffer (BEC-BCS) crossover at finite baryon and isospin chemical potentials—is investigated in the NJL model [14]. The pion condensation undergoes a BEC-BCS crossover when the isospin chemical potential increases. The point here is that the crossover is not triggered by increasing the strength of attractive interaction among quarks but driven by changing the isospin density. It is found that the chiral symmetry restoration at finite temperature and density plays an important role in the BEC-BCS crossover.

Most of the work in the NJL model is mainly based on the mean field approximation to the quark mass and on the random phase approximation (RPA) for the Bethe-Salpeter equation for the meson masses [15]. If one examines the thermodynamic potential in the mean field approximation, one sees immediately the deficit of this approach, viz., that only the quarks contribute to the thermodynamic potential with mesons playing no role whatsoever. This is clearly inadequate and unphysical, since one expects at least that the pionic degrees of freedom should dominate the system at low temperature, while the quark degrees of freedom should be relevant only in the chiral symmetry restoration phase. As such, this indicates that calculations in the NJL model must be performed beyond the mean field approximation. In Ref. [16], the thermodynamics of a quark-meson plasma is calculated to order $1/N_c$ in an expansion in the inverse number of colors, and pions as Goldstone particles corresponding to spontaneous chiral symmetry restoration do control the thermodynamic functions at low temperature and density.

A characteristic feature of the Sarma phase is the intermediate temperature superfluidity [17]: the superfluidity happens at finite temperature but disappears at zero temperature. Since the mean field treatment is a good approximation only at zero temperature [18], a careful study on the Sarma phase needs to go beyond the mean field. As for the BEC-BCS crossover induced by the change in density, the description on the BEC phase at low density should be closely related to whether the meson fluctuations are included or not. In this paper, we investigate the pion superfluidity in the frame of the NJL model beyond mean field approximation. We will focus on the effect of meson fluctuations on the Sarma phase and the BEC-BCS crossover at finite temperature and baryon and isospin density.

The paper is organized as follows. In Sec. II we present the thermodynamics of the pion superfluidity and the gap equations for the chiral and pion condensates in the NJL model in and beyond mean field approximation. In Sec. III we calculate the phase diagram and see the meson effect on the Sarma phase and BEC-BCS crossover. We summarize and conclude in Sec. IV.

II. THERMODYNAMICS OF THE PION SUPERFLUIDITY

The two flavor $SU(2)$ NJL Lagrangian density is defined as

$$\mathcal{L} = \bar{\psi}(i\gamma^\mu\partial_\mu - m_0 + \mu\gamma_0)\psi + G[(\bar{\psi}\psi)^2 + (\bar{\psi}i\gamma_5\tau\psi)^2] \tag{1}$$

with scalar and pseudoscalar interactions corresponding to σ and π excitations, where ψ is the quark field, m_0 the current quark mass, G the coupling constant with dimension $(\mathrm{GeV})^{-2}$, and μ the quark chemical potential matrix in flavor space $\mu = \mathrm{diag}(\mu_u, \mu_d) = \mathrm{diag}(\mu_B/3 + \mu_I/2, \mu_B/3 - \mu_I/2)$ with μ_B and μ_I being baryon and isospin chemical potential. The Lagrangian density has the symmetry $U_B(1) \otimes SU_I(2) \otimes SU_A(2)$ corresponding to baryon number symmetry, isospin symmetry, and chiral symmetry, respectively. However, at nonzero isospin chemical potential, the isospin symmetry $SU_I(2)$ breaks down to $U_I(1)$ global symmetry with the generator I_3 which is related to the condensation of charged pions. At zero baryon chemical potential, the Fermi surfaces of $u(d)$ and anti-$d(u)$ quarks coincide and hence the condensate of u and anti-d quarks is favored at sufficiently high $\mu_I > 0$ and the condensate of d and anti-u quarks is favored at sufficiently high $\mu_I < 0$. We introduce the chiral condensate

$$\sigma = \langle\bar{\psi}\psi\rangle \tag{2}$$

and the pion condensate

$$\pi = \sqrt{2}\langle\bar{\psi}i\gamma_5\tau_+\psi\rangle = \sqrt{2}\langle\bar{\psi}i\gamma_5\tau_-\psi\rangle \tag{3}$$

with $\tau_\pm = (\tau_1 \pm i\tau_2)/\sqrt{2}$. A nonzero condensate σ means spontaneous chiral symmetry breaking, and a nonzero condensate π means spontaneous isospin symmetry breaking.

In mean field approximation the thermodynamic potential includes the condensation part and the quark part,

$$\Omega_{\mathrm{mf}} = G(\sigma^2 + \pi^2) + \Omega_q, \tag{4}$$

and the quark part can be evaluated as a summation of four quasiparticle contributions [10],

$$\Omega_q = -6\sum_{i=1}^{4}\int\frac{d^3\mathbf{p}}{(2\pi)^3}g(\omega_i), \tag{5}$$

where ω_i are the dispersions of the quasiparticles,

$$\omega_1 = E_- + \mu_B/3, \qquad \omega_2 = E_- - \mu_B/3, \qquad \omega_3 = E_+ + \mu_B/3, \qquad \omega_4 = E_+ - \mu_B/3 \tag{6}$$

with the definitions

$$E_\pm = \sqrt{(E_p \pm \mu_I/2)^2 + 4G^2\pi^2}, \qquad E_p = \sqrt{p^2 + m^2}, \tag{7}$$

and the function $g(x)$ is defined as $g(x) = x/2 + T\ln(1 + e^{-x/T})$. The effective quark mass m is controlled by the chiral condensate, $m = m_0 - 2G\sigma$. The gap equations to determine the condensates σ (or quark mass m) and π can be obtained by the minimum of the thermodynamic potential $\Omega_{\mathrm{mf}}(T, \mu_B, \mu_I, m, \pi)$,

$$\frac{\partial\Omega_{\mathrm{mf}}}{\partial m} = 0, \qquad \frac{\partial\Omega_{\mathrm{mf}}}{\partial\pi} = 0, \qquad \frac{\partial^2\Omega_{\mathrm{mf}}}{\partial m^2} > 0, \qquad \frac{\partial^2\Omega_{\mathrm{mf}}}{\partial\pi^2} > 0. \tag{8}$$

From the first order derivatives, we have

$$m\left(\frac{1}{4G} + \frac{\partial\Omega_q}{\partial m^2}\right) = \frac{m_0}{4G}, \qquad \pi\left(G + \frac{\partial\Omega_q}{\partial\pi^2}\right) = 0. \tag{9}$$

Considering the relations between μ_I, μ_B and μ_u, $\mu_{\bar{d}}$, $\mu_u = \mu_B/3 + \mu_I/2$, and $\mu_{\bar{d}} = -\mu_B/3 + \mu_I/2$, the baryon and isospin density $n_B = -\partial\Omega_{\mathrm{mf}}/\partial\mu_B$ and $n_I = -\partial\Omega_{\mathrm{mf}}/\partial\mu_I$ can be expressed in terms of the u and $\bar{d}$ quark density $n_u = -\partial\Omega_{\mathrm{mf}}/\partial\mu_u$ and $n_{\bar{d}} = -\partial\Omega_{\mathrm{mf}}/\partial\mu_{\bar{d}}$,

$$n_I = \tfrac{1}{2}(n_u + n_{\bar{d}}), \qquad n_B = \tfrac{1}{3}(n_u - n_{\bar{d}}). \tag{10}$$

It is easy to see that n_B plays the role of density asymmetry for pion condensation. For isospin symmetric matter with $n_B = 0$ the only possible homogeneous and isotropic pion condensed state is the BCS state. The Sarma state appears only in isospin asymmetric matter with $n_B \neq 0$. The two gap equations (9) and two number equations (10) determine self-consistently m, π, μ_I, and μ_B as functions of T, n_I, and n_B at the mean field level.

We now consider the meson contribution to the thermodynamics of the system. The meson modes are regarded as quantum fluctuations above the mean field in the NJL model and can be calculated in the frame of RPA [15]. For the mean field quark propagator with off-diagonal elements in flavor space,

$$\mathcal{S}^{-1}(p) = \begin{pmatrix} \gamma^\mu p_\mu + \mu_u\gamma_0 - m & 2iG\pi\gamma_5 \\ 2iG\pi\gamma_5 & \gamma^\mu p_\mu + \mu_d\gamma_0 - m \end{pmatrix}, \tag{11}$$

we must consider all possible channels in the bubble summation in RPA. In the pion superfluidity region, σ and charged pions are coupled to each other and the uncharged pion is decoupled from them. Using matrix notation for the meson polarization function $1 - 2G\Pi(q)$ [7],

$$1 - 2G\Pi = \begin{pmatrix} 1-2G\Pi_{\sigma\sigma} & -2G\Pi_{\sigma\pi_+} & -2G\Pi_{\sigma\pi_-} & 0 \\ -2G\Pi_{\pi_+\sigma} & 1-2G\Pi_{\pi_+\pi_+} & -2G\Pi_{\pi_+\pi_-} & 0 \\ -2G\Pi_{\pi_-\sigma} & -2G\Pi_{\pi_-\pi_+} & 1-2G\Pi_{\pi_-\pi_-} & 0 \\ 0 & 0 & 0 & 1-2G\Pi_{\pi_0\pi_0} \end{pmatrix} \tag{12}$$

with the quark bubbles

$$\Pi_{jk}(q) = i\int\frac{d^4p}{(2\pi)^4}\mathrm{Tr}[\Gamma_j^*\mathcal{S}(p+q)\Gamma_k\mathcal{S}(p)], \quad j,k = \sigma, \pi_+, \pi_-, \pi_0, \tag{13}$$

where the trace $\mathrm{Tr} = \mathrm{Tr}_C\,\mathrm{Tr}\,\mathrm{Tr}_D$ is taken in color, flavor, and Dirac spaces and the meson vertices are defined as

$$\Gamma_j = \begin{cases} 1 & j=\sigma \\ i\tau_+\gamma_5 & j=\pi_+ \\ i\tau_-\gamma_5 & j=\pi_- \\ i\tau_3\gamma_5 & j=\pi_0, \end{cases} \qquad \Gamma_j^* = \begin{cases} 1 & j=\sigma \\ i\tau_-\gamma_5 & j=\pi_+ \\ i\tau_+\gamma_5 & j=\pi_- \\ i\tau_3\gamma_5 & j=\pi_0, \end{cases} \tag{14}$$

and the meson masses M_j are determined by

$$\det[1 - 2G\Pi(q_0 + \mu_j = M_j, \mathbf{q} = 0)] = 0 \tag{15}$$

with meson chemical potentials $\mu_\sigma = 0$, $\mu_{\pi_+} = \mu_I$, $\mu_{\pi_-} = -\mu_I$, $\mu_{\pi_0} = 0$.

When the contribution from the meson fluctuations is taken into account, the total thermodynamic potential to order $1/N_c$ in an expansion in the inverse number of colors becomes

$$\Omega = \Omega_{\rm mf} + \Omega_{\rm fl}, \tag{16}$$

where the mean field part $\Omega_{\rm mf}(T, \mu_B, \mu_I, m, \pi)$ is shown in (4) and the meson part $\Omega_{\rm fl}(T, \mu_B, \mu_I, m, \pi)$ is expressed in terms of the polarization function [16],

$$\Omega_{\rm fl} = -\frac{i}{2}\int\frac{d^4q}{(2\pi)^4}\ln\det[1 - 2G\Pi(q)]. \tag{17}$$

As is expected physically, the mesonic or collective degrees of freedom play a dominant role at low temperature, while the quark degrees of freedom are most relevant at high temperature [16].

In the chiral symmetry restoration phase at high temperature and/or high density, mesons are not stable bound states, but rather resonant states. They will decay into their quark-antiquark pairs. As a consequence, the determinant in the logarithm of (17) is a complex function in the meson energy plane and the imaginary part can be expressed as a scattering phase shift associated with quark-antiquark scattering. From the calculation in the NJL model with only chiral dynamics [16], the meson width is small around the critical temperature but becomes remarkable when the meson mass is much larger than two times the quark mass, and correspondingly, the contribution from the phase shift to the thermodynamics is negligible at low temperature but significant when the temperature is high enough. For our calculation in the pion superfluidity phase, it can be estimated that the phase shift will be important in the BCS state at high density but its contribution is weakened in the BEC state at low density. Since we focus in this paper on the Sarma phase and the BEC state which exist at low isospin density, we take pole approximation and neglect the scattering phase shift to simplify the numerical calculations. In pole approximation, the meson contribution can be greatly simplified as a summation of four quasiparticles,

$$\Omega_{\rm fl} = \sum_j \Omega_j, \qquad \Omega_j = \int\frac{d^3\mathbf{q}}{(2\pi)^3}\left[\frac{1}{2}(E_j - \mu_j) + T\ln(1 - e^{-(E_j-\mu_j)/T})\right] \tag{18}$$

with meson energies $E_j = \sqrt{M_j^2 + \mathbf{q}^2}$.

While mesons do not change the baryon density of the system, the charged pions modify the isospin density when the meson contribution to the thermodynamics is included,

$$n_I = \tfrac{1}{2}(n_u + n_{\bar d}) + (n_{\pi_+} - n_{\pi_-}), \tag{19}$$

where $n_{\pi_+} = -\partial\Omega_{\pi_+}/\partial\mu_{\pi_+}$ and $n_{\pi_-} = -\partial\Omega_{\pi_-}/\partial\mu_{\pi_-}$ are the π_+ and π_- density.

Up to this point, the order parameters m for chiral phase transition and π for pion superfluidity have been regarded as the values minimizing $\Omega_{\rm mf}$, and $\Omega_{\rm fl}$ has been evaluated at these mean field values, $m = m_{\rm mf}$ and $\pi = \pi_{\rm mf}$. While this is a correct perturbative expansion above the mean field, we may ask the questions: What is the feedback from the mesonic degrees of freedom to the order parameters and could we improve on these mean field values by regarding m and π as variational parameters of the total thermodynamic potential $\Omega = \Omega_{\rm mf} + \Omega_{\rm fl}$? We now perform this procedure and see what the difference between the new and mean field condensates is.

Taking the first order derivatives of the total thermodynamic potential with respect to the unknown quark mass m and pion condensate π, we obtain the following modified gap equations:

$$m\left(\frac{1}{4G} + \frac{\partial\Omega_q}{\partial m^2} + \frac{\partial\Omega_{\rm fl}}{\partial m^2}\right) = \frac{m_0}{4G}, \qquad \pi\left(G + \frac{\partial\Omega_q}{\partial\pi^2} + \frac{\partial\Omega_{\rm fl}}{\partial\pi^2}\right) = 0. \tag{20}$$

In comparison with the mean field gap equations (9), the fluctuation part $\Omega_{\rm fl}$ in the thermodynamic potential leads to a new minimum at $m_{\rm mf+fl}$ and $\pi_{\rm mf+fl}$ that now differs from the mean field one at $m_{\rm mf}$ and $\pi_{\rm mf}$. It is easy to see that the structure of the new gap equations guarantees the two phase transitions. From the second gap equation for pion superfluidity, the trivial solution $\pi = 0$ corresponds to normal quark matter, while the nonzero solution from the zero of the bracket corresponds to the energetically favored pion condensed state. In the chiral limit, there are also two solutions of the first gap equation corresponding, respectively, to the chiral symmetry breaking and restoration phase.

We now expand the fluctuation part of the thermodynamic potential around the mean field minimum,

$$\Omega_{\rm fl}(T, \mu_B, \mu_I, m^2, \pi^2) = \sum_{i,j=0}^{\infty} \frac{1}{i!j!} \frac{\partial^i}{\partial(m^2)^i} \frac{\partial^j}{\partial(\pi^2)^j} \times \Omega_{\rm fl}(T, \mu_B, \mu_I, m^2, \pi^2)|_{\rm mf} \times (m^2 - m^2_{\rm mf})^i(\pi^2 - \pi^2_{\rm mf})^j, \tag{21}$$

and, to further simplify the calculation, we consider the expansion only to the first order derivatives. Inserting the expansion into the new gap equations yields the following gap equations:

$$m\left(\frac{1}{4G_\sigma} + \frac{\partial\Omega_q}{\partial m^2}\right) = \frac{m_0}{4G}, \qquad \pi\left(G_\pi + \frac{\partial\Omega_q}{\partial\pi^2}\right) = 0 \tag{22}$$

with two effective coupling constants G_σ and G_π defined by

$$\frac{1}{4G_\sigma} = \frac{1}{4G} + \frac{\partial}{\partial m^2_{\rm mf}}\Omega_{\rm fl}(T, \mu_B, \mu_I, m^2_{\rm mf}, \pi^2_{\rm mf}), \qquad G_\pi = G + \frac{\partial}{\partial\pi^2_{\rm mf}}\Omega_{\rm fl}(T, \mu_B, \mu_I, m^2_{\rm mf}, \pi^2_{\rm mf}). \tag{23}$$

In comparing this group of coupled gap equations with the mean field one (9), one observes that, in the chiral limit, the two groups take the same form, differing only in the effective coupling constants. The coupling constants in the scalar and pseudoscalar channels are the same at mean field level, but they become different and depend on temperature and charge densities when one goes beyond the mean field. If we take $G_\sigma = G_\pi = G$, we recover the mean field case. That is, in this approach, the contribution from meson fluctuations is fully included in G_σ and G_π.

The above approach describes the thermodynamics of a quark-meson plasma with both chiral phase transition and pion superfluidity phase transition beyond the mean field at finite temperature and baryon and isospin density. The two new gap equations (22) determine simultaneously the order parameters σ (or m) and π of the two phase transitions. In the chiral limit, the two phase transitions are fully separated from each other [7]: the chiral symmetry is automatically restored in the pion superfluidity phase. That is, the two order parameters do not coexist in the system. In the real word, chiral symmetry is not fully restored at any isospin chemical potential. However, σ is much smaller than π in the pion superfluidity region [7]. Since we focus in this paper on the fluctuation effect on the pion superfluidity, we will, for the purpose of simplification in numerical calculations, neglect the σ fluctuations and keep only the π fluctuations in the gap equations. Namely, we take $G_\sigma = G$ in the following.

III. PHASE DIAGRAMS IN AND BEYOND MEAN FIELD

Since the NJL model is nonrenormalizable, we should employ a regularization scheme to avoid the divergence in the gap equations. The simplest and normally used way is to introduce a hard three momentum cutoff $|\mathbf{p}| < \Lambda$. In the following numerical calculations, we take the current quark mass $m_0 = 5$ MeV, the coupling constant $G = 4.93\ \mathrm{GeV}^{-2}$, and the cutoff $\Lambda = 653$ MeV [16]. This group of parameters ensures the pion mass $m_\pi = 138$ MeV and the pion decay constant $f_\pi = 93$ MeV in the vacuum.

In the treatment above, we considered the meson fluctuations as a perturbation around the mean field and took only the first order derivatives in the effective coupling constants (23). If this treatment is good, the difference between the two pion condensates calculated in and beyond the mean field approximation should be small. To check the validity region of this method, we show in Fig. 1 the two condensates as a function of isospin density at fixed temperature and baryon density. At low isospin density which corresponds to the BEC region, the difference between the two is really small, but it grows with increasing density and becomes large in the BCS region. Therefore, the approximation with only first order derivatives is good for the study of BEC, but the contribution from the higher order derivatives may be important for the BCS state.

The phase diagrams of pion superfluidity in the $T - n_I$ plane at fixed baryon density and in the $n_B - n_I$ plane at fixed temperature are shown in Fig. 2. The thin and thick solid lines are, respectively, phase transition lines in and

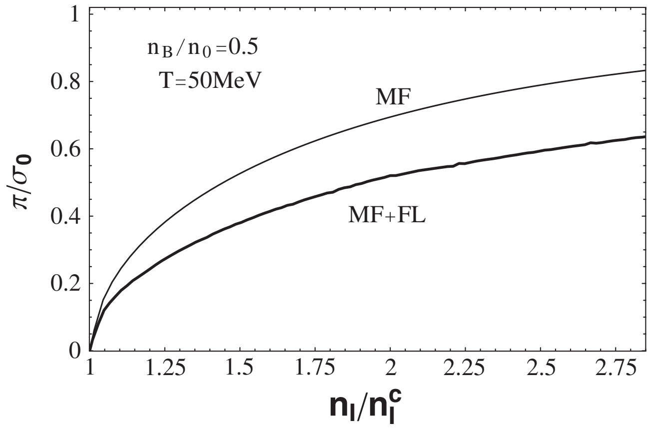

FIG. 1. The pion condensates in the mean field approximation (thin line) and including meson fluctuations (thick line) as functions of isospin density at fixed temperature and baryon density. σ_0 is the chiral condensate in vacuum and n_I^c is the critical isospin density of pion superfluidity.

beyond mean field approximation which separate the normal quark matter at high temperature or high baryon density from the pion superfluidity matter at high isospin density. For pion superfluidity, the averaged Fermi surface of the paired quarks is controlled by isospin chemical potential and the mismatch is served by baryon chemical potential. Therefore, the Sarma phase which is induced by the Fermi surface mismatch may enter the pion superfluidity at nonzero baryon density. In mean field approximation, by analyzing the four quasiparticle dispersions ω_i, the possible types of the Sarma state and their thermodynamic and dynamic instabilities are discussed in detail in Ref. [7]. It is found that the Sarma phase is the ground state of the pion superfluidity at low isospin chemical potential. Very different from the BCS phase structure where the temperature of the pairing state is always lower than the temperature of the normal state, the Sarma phase appears in an intermediate temperature region and the normal state exists in lower and higher temperature regions; see the mean field phase transition line in the $T - n_I$ plane at low isospin density in Fig. 2. However, the phase structure in the $T - n_I$ plane is significantly modified when the meson fluctuations are included. From Fig. 2, the meson effect reduces greatly the pion superfluidity region, and the critical temperature is suppressed from about 150 MeV in the mean field treatment to about 80 MeV in the case beyond the mean field. A qualitative change resulted from the meson fluctuations is that the intermediate temperature superfluidity or the Sarma state in mean field calculations is totally washed away, and the normal quark matter is always above the BCS pairing state. In the $n_B - n_I$ plane, the phase diagram in the mean field approximation is similar to the one in the $n_B - \mu_I$ plane obtained in Ref. [10], and again the meson effect reduces remarkably the pion superfluidity region.

It has been argued both in effective theory and lattice simulation that at finite but not very large isospin density and zero baryon density, the QCD matter is a pure meson matter, i.e., a Bose-Einstein condensate of charged pions. At ultrahigh isospin density, the matter turns to be a Fermi liquid with quark-antiquark cooper pairing [1]. Therefore, there should be a BEC to BCS crossover when the isospin chemical potential increases. There are some equivalent quantities to describe the BEC-BCS crossover induced by changing charge density [19]. Among them are the root-mean-square radius of the Cooper pair which is small in BEC and large in BCS, the s-wave scattering length which is positive in BEC and negative in BCS, the condensate scaled by the Fermi energy which is large in BEC and small in BCS, and the fermion chemical potential which is negative in BEC and positive in BCS. In the following we take the chemical potential to characterize the BEC-BCS

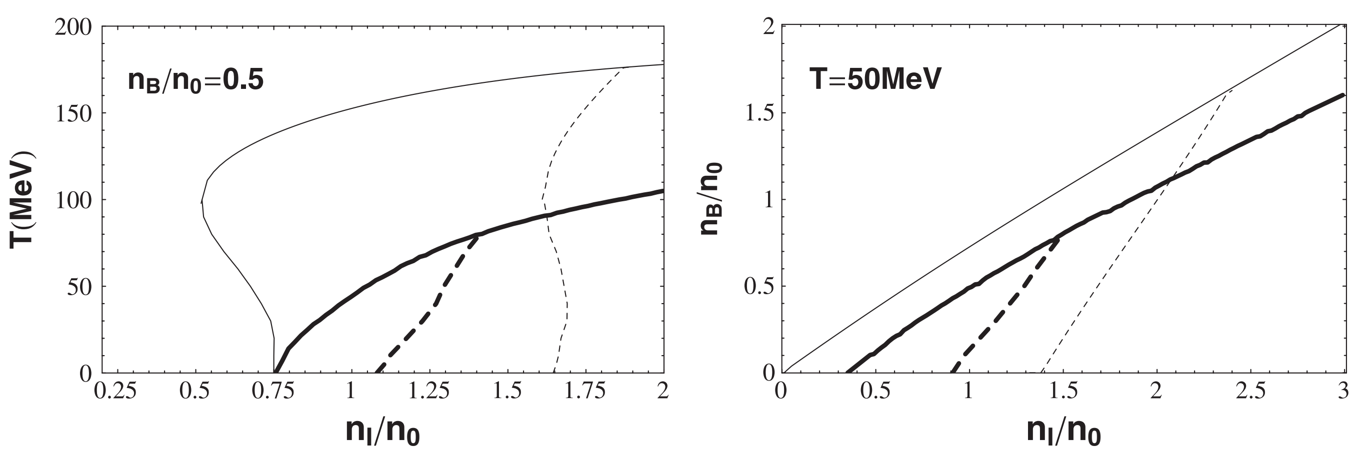

FIG. 2. The phase diagrams of pion superfluidity in the $T - n_I$ plane at fixed baryon density $n_B/n_0 = 0.5$ (left panel) and in the $n_B - n_I$ plane at fixed temperature $T = 50$ MeV (right panel) in the mean field approximation (thin lines) and including meson fluctuations (thick lines). $n_0 = 0.17/\mathrm{fm}^3$ is the normal nuclear density. The solid lines are the phase transition lines and the dashed lines are the BEC-BCS crossover lines.

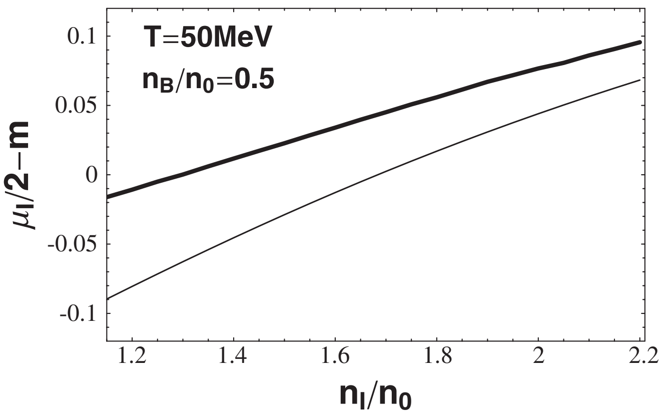

FIG. 3. The effective chemical potential $\mu_I/2 - m$ as a function of isospin density at fixed temperature $T = 50$ MeV and baryon density $n_B/n_0 = 0.5$ in the mean field approximation (thin line) and including mesonic fluctuations (thick line). n_0 is the normal nuclear density.

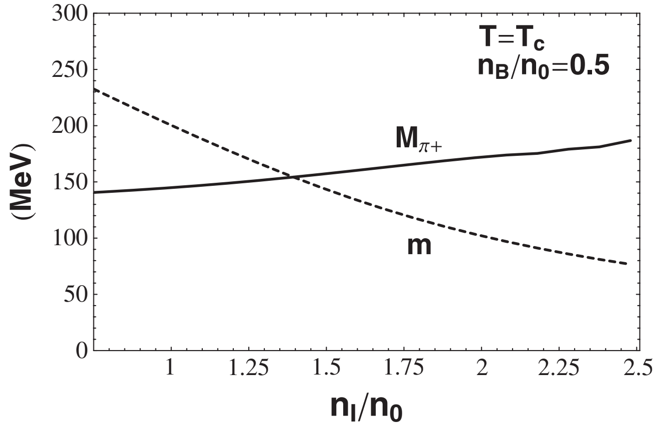

FIG. 4. The meson mass M_{π_+} (solid line) and quark mass m (dashed line) as functions of isospin density at the critical temperature T_c and fixed baryon density $n_B/n_0 = 0.5$. n_0 is the normal nuclear density.

crossover. For relativistic pion superfluidity, the chemical potential which controls the BEC-BCS crossover is $\mu_I/2 - m$ [14] depending on temperature and baryon density through the effective quark mass m, and μ_I can be viewed as the binding energy of the bound state of quark and antiquark in the BEC limit. In Fig. 3 we show $\mu_I/2 - m$ as a function of n_I at fixed temperature and baryon density in and beyond the mean field approximation. In both cases, the effective chemical potential goes up from negative to positive values with increasing isospin density. The zero point, namely, the BEC-BCS crossover point, is located at $n_I/n_0 = 1.68$ in the mean field treatment and $n_I/n_0 = 1.29$ in the case with meson fluctuations. The crossover lines determined by $\mu_I/2 - m = 0$ in $T - n_I$ and $n_B - n_I$ planes are shown in Fig. 2. When the mesonic fluctuations are included, not only the pion superfluidity region is greatly reduced, but also the BEC region is strongly shrunk.

In the BCS limit of the pion superfluidity, the isospin density is high and the paired quark and antiquark is weakly coupled. At the critical temperature, the condensate disappears, the weakly coupled fermions are excited separately, and the system is a Fermi liquid. In the BEC limit, however, the isospin density is low and the paired quark and antiquark is tightly coupled. In this case, above the critical temperature, the system becomes a Bose liquid of tightly bound pions, and the quarks should be too heavy to be excited. This means that, at the critical temperature mesons are lighter than quarks in the BEC limit and quarks are lighter than mesons in the BCS limit. To confirm the BEC-BCS crossover picture obtained above by calculating the effective chemical potential inside the pion superfluidity, we show in Fig. 4 the meson mass M_{π_+} and quark mass m as functions of isospin density at the critical temperature and fixed baryon density. With increasing isospin density, the quark mass drops down but the meson mass goes up monotonously. The two lines cross at about $n_I/n_0 = 1.4$ which is qualitatively in agreement with the BEC-BCS crossover value determined by $\mu_I/2 - m = 0$.

IV. SUMMARY

We have investigated the thermodynamics of a pion superfluid at finite isospin density in the frame of the two flavor NJL model beyond the mean field approximation. Considering the fact that mesons, in particular, pions because of their low mass, dominate the thermodynamics of a quark-hadron system at low temperature, the mesonic fluctuations should be significant for the phase structure of pion superfluidity. By recalculating the minimum of the thermodynamic potential including meson contribution, we derived a new gap equation for the pion condensate which is similar to the mean field form but with a medium dependent coupling constant. From our numerical calculations, the main effects of the meson fluctuations on the phase structure are: (1) the critical temperature of pion superfluidity is highly suppressed and the Sarma phase which exists at low isospin chemical potential in the mean field approximation is fully washed away and (2) the BEC region at low isospin density is significantly shrunk.

ACKNOWLEDGMENTS

The work is supported by the NSFC Grant No. 10735040 and the National Research Program Grants No. 2006CB921404 and No. 2007CB815000.

[1] D. T. Son and M. A. Stephanov, Phys. At. Nucl. **64**, 834 (2001).
[2] J. B. Kogut and D. K. Sinclair, Phys. Rev. D **66**, 034505 (2002); **66**, 014508 (2002); **70**, 094501 (2004).
[3] Nambu and G. Jona-Lasinio, Phys. Rev. **122**, 345 (1961); **124**, 246 (1961).
[4] D. Toublan and J. B. Kogut, Phys. Lett. B **564**, 212 (2003).
[5] M. Frank, M. Buballa, and M. Oertel, Phys. Lett. B **562**, 221 (2003).
[6] A. Barducci, R. Casalbuoni, G. Pettini, and L. Ravagli, Phys. Rev. D **69**, 096004 (2004); **71**, 016011 (2005).
[7] L. He and P. Zhuang, Phys. Lett. B **615**, 93 (2005); L. He, M. Jin, and P. Zhuang, Phys. Rev. D **71**, 116001 (2005).
[8] H. J. Warringa, D. Boer, and J. O. Andersen, Phys. Rev. D **72**, 014015 (2005).
[9] D. Ebert and K. G. Klimenko, J. Phys. G **32**, 599 (2006).
[10] L. He, M. Jin, and P. Zhuang, Phys. Rev. D **74**, 036005 (2006).
[11] G. Sarma, J. Phys. Chem. Solids **24**, 1029 (1963).
[12] S. Wu and S. Yip, Phys. Rev. A **67**, 053603 (2003).
[13] A. I. Larkin and Yu. N. Ovchinnikov, Sov. Phys. JETP **20**, 762 (1965); P. Fulde and R. A. Ferrell, Phys. Rev. **135**, A550 (1964).
[14] G. Sun, L. He, and P. Zhuang, Phys. Rev. D **75**, 096004 (2007).
[15] U. Vogl and Weise, Prog. Part. Nucl. Phys. **27**, 195 (1991); S. P. Klevansky, Rev. Mod. Phys. **64**, 649 (1992); M. K. Volkov, Phys. Part. Nucl. **24**, 35 (1993); T. Hatsuda and T. Kunihiro, Phys. Rep. **247**, 221 (1994); M. Buballa, Phys. Rep. **407**, 205 (2005).
[16] P. Zhuang, J. Hufner, and S. P. Klevansky, Nucl. Phys. **A576**, 525 (1994).
[17] J. Liao and P. Zhuang, Phys. Rev. D **68**, 114016 (2003).
[18] P. Pieri, L. Pisani, and G. C. Strinati, Phys. Rev. B **70**, 094508 (2004).
[19] Sh. Mao, X. Huang, and P. Zhuang, Phys. Rev. C **79**, 034304 (2009).