Higher charmonia and *X, Y, Z* states with screened potential

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We incorporate the color-screening effect due to light quark pair creation into the heavy quark-antiquark potential, and investigate the effects of screened potential on the spectrum of higher charmonium. We calculate the masses, electromagnetic decays, and E1 transitions of charmonium states in the screened potential model, and propose possible assignments for the newly discovered charmonium or charmoniumlike "*X, Y, Z*" states. We find the masses of higher charmonia with screened potential are considerably lower than those with unscreened potential. The $\chi_{c2}(2P)$ mass agrees well with that of the $Z(3930)$, and the mass of $\psi(4415)$ is compatible with $\psi(5S)$ rather than $\psi(4S)$. In particular, the discovered four *Y* states in the initial state radiation process, i.e., $Y(4008)$, $Y(4260)$, $Y(4320/4360)$, $Y(4660)$ may be assigned as the $\psi(3S)$, $\psi(4S)$, $\psi(3D)$, $\psi(6S)$ states, respectively. The $X(3940)$ and $X(4160)$ found in the double charmonium production in $e^+ e^-$ annihilation may be assigned as the $\eta_c(3S)$ and $\chi_{c0}(3P)$ states. Based on the calculated E1 transition widths for $\chi_{c1}(2P) \rightarrow \gamma J/\psi$ and $\chi_{c1}(2P) \rightarrow \gamma \psi(2S)$ and other results, we argue that the $X(3872)$ may be a $\chi_{c1}(2P)$ dominated charmonium state with some admixture of the $D^0 \bar{D}^{*0}$ component. Possible problems encountered in these assignments and comparisons with other interpretations for these *X, Y, Z* states are discussed in detail. We emphasize that more theoretical and experimental investigations are urgently needed to clarify these assignments and other interpretations.

I. INTRODUCTION

In recent years, a number of charmoniumlike states, the so-called "*X, Y, Z*" mesons, have been found at *B* factories and other experiments (for reviews see e.g. [1–3]). These states all lie above the open charm (e.g. $D^{(*)}\bar{D}^{(*)}$) thresholds, and the observed masses and decays make the identifications of these new states very puzzling. Apart from the conventional charmonium, many exotic candidates, such as molecules, tetra-quarks and charmonium-hybrids, are suggested. However, despite many exciting hints, none of the exotic states has been firmly established so far, and more theoretical studies are needed to explain the existing data and to confront new experimental tests. In the domain of conventional charmonium, one of the main difficulties to assign the new mesons as excited charmonia is that the observed masses do not fit the predictions of potential models with linear confinement potential (the quenched potential). However, the situation can be more complicated than the simple potential model calculations by including the coupled channel effects, or the string breaking effects. These effects could make the masses of higher charmonia lower than the potential model predictions. One distinct example is the state $Z(3930)$ observed in the two-photon process [4], which is now identified with the 2*P* charmonium χ'_{c2}, but its mass is about 40 MeV lower than the prediction given by the quenched potential model [5]. Another example is the $X(3940)$ observed in double charmonium production in $e^+ e^-$ annihilation [6], which is likely (though not necessarily) to be the 3*S* spin-singlet

charmonium state η''_c [7], but this assignment will imply that the mass of the 3*S* pseudoscalar charmonium is smaller than the prediction given by the quenched potential model by about 100 MeV [5]. Therefore, it is possible that the quenched potential model may overestimate the masses of charmonia in the energy region well above the open-charm thresholds.

Although very successful in the prediction of the charmonium spectrum below the open-charm threshold, it is well known that the quenched potential model, which incorporates a Coulomb term at short distances and the linear confining interaction at large distances [8,9], will not be reliable in the domain beyond the open-charm threshold. This is because the linear potential, which is expected to be dominant in this mass region, will be screened or softened by the vacuum polarization effects of dynamical fermions [10]. Unfortunately, this screening effect has not been directly detected with the standard Wilson loop technique in the unquenched lattice calculations. The reason might be that the Wilson loop operator almost decouples from the physical ground state at a large lattice distance that consists of two disjoint strings [11]. In other words, to simulate the screening effect out with the Wilson loop technique needs a very long lattice time, which might be too far beyond the ability of the simulation at present. On the other hand, the screening or the string breaking effect has been demonstrated indirectly by the investigation of the mixing of a static quark-antiquark string with a static light meson-antimeson system in $n_f = 2$ lattice QCD [12]. This effect is also confirmed by the calculations within

some holographic QCD models [13] recently. However, since the simulations of lattice QCD still have large uncertainties and difficulties in handling higher excited states, in order to investigate the screening effects on the charmonium spectrum it should be useful to improve the potential model itself to incorporate the screening effect and compare the model predictions with experimental data. Such screened potential models [14–17] were proposed many years ago in the study of heavy quarkonium and heavy flavor mesons, as well as light hadrons [18]. In recent years the screened potential models have again been used to investigate the heavy quarkonium spectrum and leptonic decay widths [19]. In addition to heavy hadrons, the spectra of light hadrons have also been investigated with the screened potential [20], and it is argued that the large degeneracy observed in the excited meson spectrum by the Crystal Barrel Collaboration in proton-antiproton annihilation in the range 1.9–2.4 GeV [21] may indicate the flattening of the confinement potential due to the color-screening effects [20]. On this point, it is important to further examine experimentally whether or not the linear Regge trajectory of the meson spectrum can hold for even higher excited mesons. More complete data are needed in the future to clarify the issue of the light meson spectrum regarding the color-screening effect. Although no definite conclusion can be drawn at present, it is certainly useful to study the color-screening effects on the mass spectra for both heavy and light hadrons, especially for the newly discovered higher excited states.

The effect of vacuum polarization due to dynamical quark pair creation may also be compensated by that of the hadron loop induced by virtual D meson pairs in the so-called coupled-channel model [8,22–24], for charmonium system. In Ref. [25], we compared the coupled-channel model with the screened potential model in charmonium spectrum in the mass region below 4 GeV. With the same quenched limit, the two models are found to have similar global features. It is not surprising since, in the quark-meson duality picture, the two models may describe roughly the same effects. In practice, calculations with the screened potential model are simpler to deal with, though detailed predictions for the spectrum can be somewhat different from the coupled-channel model [25].

In this paper, we calculate the mass spectrum of the charmonium especially the higher charmonium using a nonrelativistic Schrödinger equation with the Coulomb potential plus a screened linear potential which is nearly the same as that in [14,16] but with slightly different parameters (see Sec. II for details). Spin-dependent interactions are considered perturbatively. With one additional screening parameter μ, the model predicts that the masses of higher charmonium are lowered, compared with the quenched linear potential, and this mass suppression tends to be strengthened when the charmonium states vary from lower levels to higher ones. For instance, we find that the

calculated mass of χ'_{c2} is 3937 MeV, fitting well the experimental value of $Z(3930)$ [4], and the mass of $\psi(5S)$ is close to the observed $\psi(4415)$. Consequently, this mass spectrum will leave more room for new assignments for some of the observed X, Y, Z states in the charmonium family. These possible assignments will be suggested and discussed in detail in this paper.

In the following, we first introduce the screened potential model in Sec. II, and then study some decay and transition processes of charmonia in Sec. III. In Sec. IV we will discuss possible assignments for the observed charmonium(-like) states. A summary will be given in Sec. V.

II. THE SCREENED POTENTIAL MODEL

As a minimal model describing the charmonium spectrum we use a nonrelativistic potential model with screening effect being considered [14,16]. We use a potential as

$$V_{\mathrm{scr}}(r) = V_V(r) + V_S(r), \tag{1}$$

$$V_V(r) = -\frac{4}{3}\frac{\alpha_C}{r}, \tag{2}$$

$$V_S(r) = \lambda\left(\frac{1 - e^{-\mu r}}{\mu}\right), \tag{3}$$

where μ is the screening factor which makes the long-range scalar part of $V_{\mathrm{scr}}(r)$ flat when $r \gg \frac{1}{\mu}$ and still linearly rising when $r \ll \frac{1}{\mu}$, λ is the linear potential slope, and α_C is the coefficient of the Coulomb potential. Figure 1. shows the screened potential departure from linear in large length with the parameters given in (10).

For hyperfine interactions, we only consider the spin-dependent interactions which include three parts as follows.
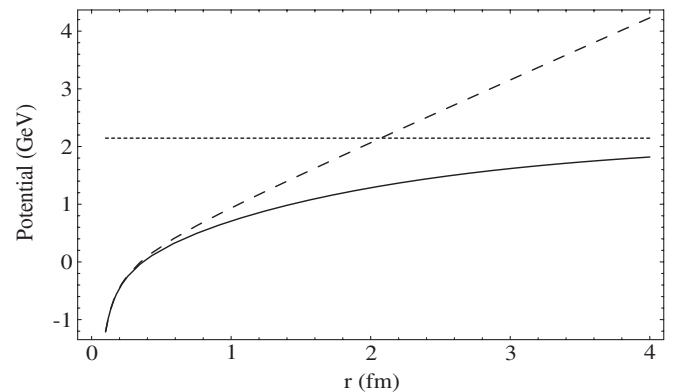

FIG. 1. Comparison of the unscreened potential $V(r)$ (dashed line) and the screened potential V_{scr} (solid line) for $r = 0.1$–4 fm with parameters taken from (10). The asymptotic limit of V_{scr} is shown by the dotted line.

The spin-spin contact hyperfine interaction is

$$H_{SS} = \frac{32\pi\alpha_C}{9m_c^2}\tilde{\delta}_\sigma(r)\vec{S}_c \cdot \vec{S}_{\bar{c}}, \tag{4}$$

where $\tilde{\delta}_\sigma(r)$ is usually taken to be $\delta(\vec{r})$ in nonrelativistic potential models, but here we take $\tilde{\delta}_\sigma(r) = (\sigma/\sqrt{\pi})^3 e^{-\sigma^2 r^2}$ as in Ref. [5] since it is an artifact of an $O(v_c^2/c^2)$ expansion of the T-matrix [26] in a range comparable to $1/m_c$.

The spin-orbit term and the tensor term take the common forms:

$$H_{LS} = \frac{1}{2m_c^2 r}(3V_V'(r) - V_S'(r))\vec{L} \cdot \vec{S}, \tag{5}$$

and

$$H_T = \frac{1}{12m_c^2}\left(\frac{1}{r}V_V'(r) - V_V''(r)\right)T. \tag{6}$$

These spin-dependent interactions are dealt with perturbatively. They are diagonal in a $|J, L, S\rangle$ basis with the matrix elements

$$\langle\vec{S}_c \cdot \vec{S}_{\bar{c}}\rangle = \tfrac{1}{2}S^2 - \tfrac{3}{4}, \tag{7}$$

TABLE I. Experimental and theoretical mass spectrum of charmonium states. The experimental masses are PDG [27] averages. The masses are in units of MeV, while the averaged radii are in units of fm. The results of our screened potential model are shown in comparison with that of Ref. [5] including the NR and GI models [5].

State		Expt.	Theor. of ours		Theor. of Ref. [5]	
			Mass	$\langle r^2\rangle^{1/2}$	NR	GI
1S	$J/\psi(1^3S_1)$	3096.916 ± 0.011	3097	0.41	3090	3098
	$\eta_c(1^1S_0)$	2980.3 ± 1.2	2979		2982	2975
2S	$\psi'(2^3S_1)$	3686.093 ± 0.034	3673	0.91	3672	3676
	$\eta_c'(2^1S_0)$	3637 ± 4	3623		3630	3623
3S	$\psi(3^3S_1)$	4039 ± 1	4022	1.38	4072	4100
	$\eta_c(3^1S_0)$		3991		4043	4064
4S	$\psi(4^3S_1)$	4263^{+8}_{-9}	4273	1.87	4406	4450
	$\eta_c(4^1S_0)$		4250		4384	4425
5S	$\psi(5^3S_1)$	4421 ± 4	4463	2.39		
	$\eta_c(5^1S_0)$		4446			
6S	$\psi(6^3S_1)$		4608	2.98		
	$\eta_c(6^1S_0)$		4595			
1P	$\chi_2(1^3P_2)$	3556.20 ± 0.09	3554	0.71	3556	3550
	$\chi_1(1^3P_1)$	3510.66 ± 0.07	3510		3505	3510
	$\chi_0(1^3P_0)$	3414.75 ± 0.31	3433		3424	3445
	$h_c(1^1P_1)$	3525.93 ± 0.27	3519		3516	3517
2P	$\chi_2(2^3P_2)$	$3929 \pm 5 \pm 2$	3937	1.19	3972	3979
	$\chi_1(2^3P_1)$		3901		3925	3953
	$\chi_0(2^3P_0)$		3842		3852	3916
	$h_c(2^1P_1)$		3908		3934	3956
3P	$\chi_2(3^3P_2)$		4208	1.67	4317	4337
	$\chi_1(3^3P_1)$		4178		4271	4317
	$\chi_0(3^3P_0)$		4131		4202	4292
	$h_c(3^1P_1)$		4184		4279	4318
1D	$\psi_3(1^3D_3)$		3799	0.96	3806	3849
	$\psi_2(1^3D_2)$		3798		3800	3838
	$\psi(1^3D_1)$	3775.2 ± 1.7	3787		3785	3819
	$\eta_{c2}(1^1D_2)$		3796		3799	3837
2D	$\psi_3(2^3D_3)$		4103	1.44	4167	4217
	$\psi_2(2^3D_2)$		4100		4158	4208
	$\psi(2^3D_1)$	4153 ± 3	4089		4142	4194
	$\eta_{c2}(2^1D_2)$		4099		4158	4208
3D	$\psi_3(3^3D_3)$		4331	1.94		
	$\psi_2(3^3D_2)$		4327			
	$\psi(3^3D_1)$		4317			
	$\eta_{c2}(3^1D_2)$		4326			

$$\langle \vec{L} \cdot \vec{S} \rangle = [J(J+1) - L(L+1) - S(S+1)]/2 \quad (8)$$

and the tensor operator T has nonvanishing diagonal matrix elements only between $L > 0$ spin-triplet states, which are

$$\langle^3 L_J | T |^3 L_J \rangle = \begin{cases} -\frac{L}{6(2L+3)}, & J = L+1 \\ \frac{1}{6}, & J = L \\ -\frac{(L+1)}{6(2L-1)}, & J = L-1. \end{cases} \quad (9)$$

For the model parameters, we take

$$\alpha_C = 0.5007, \qquad \lambda = 0.21 \text{ GeV}^2,$$
$$\mu = 0.0979 \text{ GeV}, \qquad \sigma = 1.362 \text{ GeV}, \quad (10)$$
$$m_c = 1.4045 \text{ GeV}, \qquad \alpha_S = 0.26,$$

where $\alpha_C \approx \alpha_s(m_c v_c)$ and $\alpha_S \approx \alpha_s(2m_c)$ are essentially the strong coupling constants but at different scales. The former is for large distances and used to determine the spectrum while the latter is for short distances and used for QCD radiative corrections in charmonium decays (see below in the next section). Here μ is the characteristic scale for color screening, and $1/\mu$ is about 2 fm, implying that at distances larger than $1/\mu$ the static color source in the $c\bar{c}$ system gradually becomes neutralized by the produced light quark pair, and string breaking emerges. With these values of the parameters for the potential, we can calculate the spectrum of the charmonium system. The results are shown in Table I. For comparison, we also list the experimental values [27] and those predicted by the quenched potential model [5] in Table I.

III. SOME DECAY PROCESSES

A. Leptonic decays

The electronic decay width of the vector meson is given by the Van Royen-Weisskopf formula [28] with QCD radiative corrections taken into account [29].

TABLE II. Leptonic widths (in units of KeV) for charmonium states without S-D mixing in the screened potential model. The widths calculated with and without QCD corrections are marked by Γ_{ee} and Γ_{ee}^0, respectively. The experimental values are taken from PDG [27].

state	Γ_{ee}^0	Γ_{ee}	$\Gamma_{ee}^{\text{expt}}$
$1^3S_1(3097)$	11.8	6.60	$5.55 \pm 0.14 \pm 0.02$
$2^3S_1(3686)$	4.29	2.40	2.33 ± 0.07
$3^3S_1(4039)$	2.53	1.42	0.86 ± 0.07
$4^3S_1(4263)$	1.73	0.97	
$5^3S_1(4421)$	1.25	0.70	0.58 ± 0.07
$6^3S_1(4664)$	0.88	0.49	
$1^3D_1(3775)$	0.055	0.031	0.259 ± 0.016
$2^3D_1(4153)$	0.066	0.037	0.83 ± 0.07
$3^3D_1(4361)$	0.079	0.044	

$$\Gamma_{ee}(nS) = \frac{4\alpha^2 e_c^2}{M_{nS}^2} |R_{nS}(0)|^2 \left(1 - \frac{16}{3}\frac{\alpha_S}{\pi}\right), \quad (11)$$

$$\Gamma_{ee}(nD) = \frac{25\alpha^2 e_c^2}{2M_{nD}^2 m_Q^4} |R_{nD}''(0)|^2 \left(1 - \frac{16}{3}\frac{\alpha_S}{\pi}\right), \quad (12)$$

where $M_{nS}(M_{nD})$ is the mass for $nS(nD)$, $e_c = \frac{2}{3}$ is the c quark charge in unit of electron charge, α is the fine structure constant, $R_{nS}(0)$ is the radial S wave function at the origin, and $R_{nD}''(0)$ is the second derivative of the radial D wave function at the origin.

Combined with the parameters (10), we get the results listed in Table II.

B. Two-photon decays

In the nonrelativistic limit, the two-photon decay widths of $^1S_0$, $^3P_0$, and $^3P_2$ can be written as [30]

$$\Gamma^{\text{NR}}(^1S_0 \to \gamma\gamma) = \frac{3\alpha^2 e_c^4 |R_{nS}(0)|^2}{m_c^2}, \quad (13)$$

$$\Gamma^{\text{NR}}(^3P_0 \to \gamma\gamma) = \frac{27\alpha^2 e_c^4 |R_{nP}'(0)|^2}{m_c^4}, \quad (14)$$

$$\Gamma^{\text{NR}}(^3P_2 \to \gamma\gamma) = \frac{36\alpha^2 e_c^4 |R_{nP}'(0)|^2}{5m_c^4}. \quad (15)$$

The first-order QCD radiative corrections to the two-photon decay rates can be accounted for as [30]

$$\Gamma(^1S_0 \to \gamma\gamma) = \Gamma^{\text{NR}}(^1S_0 \to \gamma\gamma)\left[1 + \frac{\alpha_S}{\pi}\left(\frac{\pi^2}{3} - \frac{20}{3}\right)\right], \quad (16)$$

$$\Gamma(^3P_0 \to \gamma\gamma) = \Gamma^{\text{NR}}(^3P_0 \to \gamma\gamma)\left[1 + \frac{\alpha_S}{\pi}\left(\frac{\pi^2}{3} - \frac{28}{9}\right)\right], \quad (17)$$

$$\Gamma(^3P_2 \to \gamma\gamma) = \Gamma^{\text{NR}}(^3P_2 \to \gamma\gamma)\left[1 - \frac{16}{3}\frac{\alpha_S}{\pi}\right]. \quad (18)$$

We can see that $\Gamma(^1S_0 \to \gamma\gamma) \propto |R_{nS}(0)|^2$, which are sensitive to the details of potential near the origin. So we take

$$\Gamma(^1S_0 \to \gamma\gamma) \to \frac{\Gamma(^1S_0 \to \gamma\gamma)}{\Gamma_{ee}(nS)}\Gamma_{ee}^{\text{expt}}(nS) \quad (19)$$

to eliminate this uncertainty.

In the nonrelativistic limit, we can also replace m_c by $M/2$, where M is the mass of the corresponding charmonium state. The results are listed in Table III.

TABLE III.   Two-photon decay widths (in units of KeV) of pseudoscalar ($^1S_0$), scalar ($^3P_0$), and tensor ($^3P_2$) charmonium states. Charmonium masses are in units of MeV.

state	mass	Ref. [31]	Ref. [32]	Ref. [33]	Theory Ref. [34]	Ref. [35]	Ref. [36]	Ours	Experiment PDG [27]
$\eta_c(1^1S_0)$	2980	5.5	3.5	10.94	7.8	5.5	4.8	8.5	$6.7^{+0.9}_{-0.8}$
$\eta_c'(2^1S_0)$	3637	1.8	1.38		3.5	2.1	3.7	2.4	
$\eta_c'(3^1S_0)$	3991		0.94					0.88	
$\chi_{c0}(1^3P_0)$	3415	2.9	1.39	6.38	2.5	5.32		2.5	2.40 ± 0.29
$\chi_{c0}'(2^3P_0)$	3842	1.9	1.11					1.7	
$\chi_{c0}'(3^3P_0)$	4156		0.91					1.2	
$\chi_{c2}(1^3P_2)$	3556	0.50	0.44	0.57	0.28	0.44		0.31	0.49 ± 0.05
$\chi_{c2}'(2^3P_2)$	3929	0.52	0.48					0.23	
$\chi_{c2}'(3^3P_2)$	4208		0.014					0.17	

C. E1 transitions

For the E1 transitions within the charmonium system, we use the formula of Ref. [37]:

$$\Gamma_{E1}(n^{2S+1}L_J \rightarrow n'^{2S'+1}L'_{J'} + \gamma) = \frac{4}{3}C_{fi}\delta_{SS'}e_c^2\alpha|\langle f|r|i\rangle|^2 E_\gamma^3 \tag{20}$$

where E_γ is the emitted photon energy.

The spatial matrix element

$$\langle f|r|i\rangle = \int_0^\infty R_f(r)R_i(r)r^3 dr, \tag{21}$$

involves the initial and final state radial wave functions, and the angular matrix element C_{fi} is

$$C_{fi} = \max(L, L')(2J'+1)\begin{Bmatrix} L' & J' & S \\ J & L & 1 \end{Bmatrix}^2. \tag{22}$$

TABLE IV.   E1 transitions rates of charmonium states in the nonscreened potential model [5] (marked by NR) and our screened potential model (those calculated by the zeroth-order wave functions are marked by SNR_0 and those by the first-order relativistically corrected wave functions are marked by SNR_1).

state	Initial meson	Final meson	E_γ (MeV) NR	$SNR_{0(1)}$	Γ_{thy} (keV) NR	SNR_0	SNR_1	Γ_{expt} (keV)
$2S \rightarrow 1P$	$\psi'(2^3S_1)(3686)$	$\chi_{c2}(1^1P_2)$	128	128	38	43	34	26.3 ± 1.5
		$\chi_{c1}(1^3P_1)$	171	171	54	62	36	27.9 ± 1.5
		$\chi_{c0}(1^3P_0)$	261	261	63	74	25	29.8 ± 1.5
	$\eta_c(2^1S_0)(3637)$	$h_c(2^1P_1)$		109		146	104	
$1P \rightarrow 1S$	$\chi_{c2}(1^3P_2)(3556)$	$J/\psi(1^3S_1)(3097)$	429	429	424	473	309	406 ± 31
	$\chi_{c1}(1^3P_1)(3511)$		390	390	314	354	244	320 ± 25
	$\chi_{c0}(1^3P_0)(3415)$		303	303	152	167	117	131 ± 14
	$h_c(1^1P_1)(3525)$	$\eta_c(1^1S_0)(2980)$	504	504	498	764	323	
$2P \rightarrow 1S$	$\chi_{c2}(2^3P_2)(3929)$	$J/\psi(1^3S_1)$	779	744	81	101	109	
	$\chi_{c1}(2^3P_1)(3872)$		741	697	71	83	45	
	$\chi_{c0}(2^3P_0)(3842)$		681	672	56	74	9.3	
	$h_c(2^1P_1)(3908)$	$\eta_c(1^1S_0)$	839	818	140	134	250	
$2P \rightarrow 2S$	$\chi_{c2}(2^3P_2)$	$J/\psi(2^3S_1)$	276	235	304	225	100	
	$\chi_{c1}(2^3P_1)$		232	182	183	103	60	
	$\chi_{c0}(2^3P_0)$		162	152	64	61	44	
	$h_c(2^1P_1)$	$\eta_c(2^1S_0)$	285	261	280	309	108	
$1D \rightarrow 1P$	$\psi_3(1^3D_3)(3799)$	$\chi_2(1^3P_2)$	242	236	272	284	223	
	$\psi_2(1^3D_2)(3798)$	$\chi_{c2}(1^3P_2)$	236	234	64	70	55	
		$\chi_{c1}(1^3P_1)$	278	276	307	342	208	
	$\psi(1^3D_1)(3775)$	$\chi_{c2}(1^3P_2)$	208	213	4.9	5.8	4.6	<21 [38]
		$\chi_{c1}(1^3P_1)$	250	255	125	150	93	70 ± 17 [38]
		$\chi_{c0}(1^3P_0)$	338	343	403	486	197	172 ± 30 [38]
	$h_{c2}(1^1D_2)(3796)$	$h_c(1^1P_1)$	264	260	339	575	375	

Our results are listed in Table IV. The widths calculated by the zeroth-order wave functions are marked by SNR_0 and those by the first-order relativistically corrected wave functions are marked by SNR_1.

For the first-order relativistic corrections to the wave functions, we include the spin-dependent part of (4)–(6) and the spin-independent part as [39]

$$H_{\text{SI}} = -\frac{\vec{P}^4}{4m_c^3} + \frac{1}{4m_c^2}\nabla^2 V_V(r) - \frac{1}{2m_c^2}\{\{\vec{P}_1 \cdot V_V(r)\Im \cdot \vec{P}_2\}\} + \frac{1}{2m_c^2}\left\{\left\{\vec{P}_1 \cdot \vec{r}\frac{V_V'(r)}{r}\vec{r} \cdot \vec{P}_2\right\}\right\}, \quad (23)$$

where $\vec{P}_1$ and $\vec{P}_2$ are momenta of c and $\bar{c}$ quarks in the rest frame of charmonium, respectively, which satisfy $\vec{P}_1 = -\vec{P}_2 = \vec{P}$, $\Im$ is the unit second-order tensor, and $\{\{\}\}$ is the Gromes's notation

$$\{\{\vec{A} \cdot \Re \cdot \vec{B}\}\} = \tfrac{1}{4}(\vec{A}\vec{B}:\Re + \vec{A}\cdot\Re\vec{B} + \vec{B}\cdot\Re\vec{A} + \Re:\vec{A}\vec{B}), \quad (24)$$

where $\Re$ is any second-order tensor.

Note that we do not include the contributions from the scalar potential in H_{SI} since it is still unclear how to deal with the spin-independent corrections arising from the scalar potential theoretically.

The results of nonrelativistic linear potential model [5] marked by NR are listed for comparison. Our results for the E1 transition widths of SNR_0 are slightly larger than those of the NR's mainly due to the relativistic phase space factor of $E_f^{(c\bar{c})}/M_i^{(c\bar{c})}$ involved in the NR's in [5], where $E_f^{(c\bar{c})}$ is the final state charmonium total energy, and $M_i^{(c\bar{c})}$ is the initial state charmonium mass. Both SNR_0 and NR's results of E1 transitions are larger than experimental values, but we see that in SNR_1 the predicted widths get decreased and fit the experimental values rather well as long as the first-order relativistic corrections to the wave functions are included.

IV. DISCUSSIONS

A. $Z(3930)$

The $Z(3930)$ was discovered by the Belle Collaboration [4] as an enhancement in the $D\bar{D}$ invariant mass near 3.93 GeV/c^2 in the $\gamma\gamma$ collision with the statistical significance of 5.3σ. Results for the mass, width, and product of the two-photon decay width times the branching fraction to $D\bar{D}$ are:

$$M(Z(3930)) = 3929 \pm 5 \pm 2 \text{ MeV}, \quad (25)$$

$$\Gamma(Z(3930)) = 29 \pm 10 \pm 2 \text{ MeV}, \quad (26)$$

$$\Gamma_{\gamma\gamma}\mathcal{B}(Z(3930) \to D\bar{D}) = 0.18 \pm 0.05 \pm 0.03 \text{ KeV}, \quad (27)$$

respectively. The production rate and the angular distribu-

tion in the $\gamma\gamma$ center-of-mass frame suggest that this state is the previously unobserved $\chi_{c2}(2P)$ [4].

The mass of $\chi_{c2}(2P)$ in our model is 3937 MeV, which is consistent with the experimental value in (25), whereas the mass in the quenched potential model [5] is larger than the experimental one by 40–50 MeV (see Table I). As we have mentioned, this is one of our motivations to reexamine the charmonium spectrum in the screened potential model.

The open-charmed decays of $\chi_{c2}(2P)$ were studied in Ref. [1] (the mass was set to be 3931 MeV), and the total width is predicted to be 35 MeV with the branching ratio $\mathcal{B}(Z(3930) \to D\bar{D}) \simeq 74\%$. Together with the data in (27), one can get the two-photon decay width of $Z(3930)$ as about 0.16–0.33 KeV, which is consistent with the predicted value 0.23 KeV for $\chi_{c2}(2P)$ in our model (see Tabel III).

B. $\psi(3770)$, $\psi(4040)$, $\psi(4160)$, $\psi(4415)$

Before the discovery of $X(3872)$, there are only four well-established charmonium states above the $D\bar{D}$ threshold. They are $\psi(3770)$, $\psi(4040)$, $\psi(4160)$, and $\psi(4415)$. They all have quantum number $J^{PC} = 1^{--}$. Conventionally, they were assigned as mainly $\psi(1^3D_1)$, $\psi(3^3S_1)$, $\psi(2^3D_1)$, and $\psi(4^3S_1)$, respectively. These assignments are consistent with the predictions given by the quenched potential models. However, in our screened potential model, the $\psi(4415)$ should be assigned as $\psi(5^3S_1)$ (see Table I).

The experimental di-electron width of $\psi(3770)$ is 0.259 ± 0.016 KeV, larger than that expected for a pure D-wave state, which is probably due to mixing with the 2^3S_1 state induced by the coupled-channel effects as well as by the tensor force. In the following we assume $\psi' \equiv \psi(3686)$ and $\psi'' \equiv \psi(3770)$ to be admixture of $2^3S_1 - 1^3D_1$ charmonium states and

$$|\psi'\rangle = |2^3S_1\rangle\cos\theta + |1^3D_1\rangle\sin\theta, \quad (28)$$

$$|\psi''\rangle = -|2^3S_1S_1\rangle\sin\theta + |1^3D_1\rangle\cos\theta, \quad (29)$$

where θ is the mixing angle. This angle can be estimated by comparing the experimental values of the di-electron widths of $\psi(3770)$ and $\psi(3686)$ with theoretical predictions for pure 1^3D_1 and 2^3S_1 states (see Table II). The result is given by

$$\theta \approx -12° \quad \text{or} \quad \theta \approx 25°. \quad (30)$$

Thereinto, $\theta = 25°$ is not compatible with the width of $\psi' \to \chi_{c0}\gamma$, so we take $\theta = -12°$. Note that $\theta \approx -12°$ is consistent with the results of Refs. [22,40–42].

The S-D mixing may be more serious for $\psi(4160)$ (with $\Gamma_{ee}(\psi(4160)) = 0.83 \pm 0.07$ KeV [27]), if one assigns it as the 2^3D_1 state, since its observed di-electron width is comparable to that of $\psi(4040)$ (with $\Gamma_{ee}(\psi(4040)) = 0.86 \pm 0.07$ KeV [27]). Extracted from the ratio of di-electron widths of $\psi(4160)$ and $\psi(4040)$, the mixing angle

between 2^3D_1 and 3^3S_1 can be as large as $-37°$ in our model and $-35°$ in Refs. [43,44]. Moreover, there could also be mixing between $2D - 4S$ states. This indicates that the observed di-electron widths for higher charmonia (above the open-charm threshold) can be altered by the *S-D* mixing effects due to coupling to the decay channels. Moreover, their masses can also be modified by the *S-D* mixing.

The open-charmed decays of these states are not studied in this paper. However, one may expect that the corresponding calculations and results in our model will be similar to those in the quenched potential model. In Ref. [5], the authors evaluated the open-charmed decay widths of these states using the $^3P_0$ quark pair creation model, and the results are consistent with experimental measurements both for the total widths and the decay patterns. Especially, they predicted that the main decay modes of $\psi(4415)$ [as $\psi(4S)$ in their model] are $D\bar{D}_1$ and $D\bar{D}_2^*$,[1] and the $D\bar{D}_2^*$ mode was confirmed by Belle's measurement [45] recently. The lesson from both the theoretical calculation and the experimental measurement is that the higher excited charmonia tend to decay into excited charm mesons rather than the *S*-wave charm meson pairs. This might be due to relativistic suppression and the node structures of the wave functions of the higher excited states.

C. $Y(4008)$, $Y(4260)$, $Y(4320/4360)$, $Y(4660)$ and $X(4630)$

The vector state $Y(4260)$ was first discovered by the *BABAR* Collaboration [46] as a relative narrow peak around 4260 MeV in the $J/\psi\pi^+\pi^-$ distribution in the initial state radiation (ISR) process $e^+e^- \to \gamma_{ISR}J/\psi\pi^+\pi^-$. This state was also observed by CLEO [47] and Belle [48], and there is also a relative broad structure near 4.05 GeV, the so-called $Y(4008)$, in the Belle data. Recently, *BABAR* [49] updated their measurement and gave the mass and width of $Y(4260)$ which are consistent with Belle's measurements [48]. However, *BABAR* has not yet seen the structure around 4.05 GeV. The mass and width of the broad structure $Y(4008)$ are given by [48]

$$M(Y(4008)) = 4008 \pm 40^{+114}_{-28} \text{ MeV}, \quad (31)$$

$$\Gamma(Y(4008)) = 226 \pm 44 \pm 87 \text{ MeV}. \quad (32)$$

The averaged mass and width of $Y(4260)$ are given by [27]

$$M(Y(4260)) = 4263^{+8}_{-9} \text{ MeV}, \quad (33)$$

$$\Gamma(Y(4260)) = 95 \pm 14 \text{ MeV}. \quad (34)$$

BABAR also found a structure around 4.32 GeV [50] in the

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[1]In this paper, inclusion of the charge conjugate mode is always implied.

ISR process $e^+e^- \to \gamma_{ISR}\psi'\pi^+\pi^-$ with

$$M(Y(4320)) = 4324 \pm 24 \text{ MeV}, \quad (35)$$

$$\Gamma(Y(4320)) = 172 \pm 33 \text{ MeV}, \quad (36)$$

while in the same process the Belle Collaboration observed two relative narrow peaks around 4.35 GeV with

$$M(Y(4360)) = 4361 \pm 9 \pm 9 \text{ MeV}, \quad (37)$$

$$\Gamma(Y(4360)) = 74 \pm 15 \pm 10 \text{ MeV}, \quad (38)$$

and 4.66 GeV [51] with

$$M(Y(4660)) = 4664 \pm 11 \pm 5 \text{ MeV}, \quad (39)$$

$$\Gamma(Y(4660)) = 48 \pm 15 \pm 3 \text{ MeV}. \quad (40)$$

Aside from the broad structure $Y(4008)$, which might be related to the $\psi(3S)$ state $\psi(4040)$ and even the $\psi(2D)$ state $\psi(4160)$, the other three *Y*-states are considered to be difficult to assign as conventional charmonia since their masses are inconsistent with those predicted by the quenched potential models [5,52], and there are even no enough unassigned states in the charmonium spectrum to accommodate them. As a consequence, these *Y*-states are interpreted totally or partly as exotic states, such as $c\bar{c}g$ hybrid [53], $cq\bar{c}\bar{q}$ tetra-quark state [54–56], bayonium [57], and molecule [58]. As the only exception, in Ref. [59] the $Y(4260)$ was interpreted as the $\psi(4S)$ charmonium. Recently, in Ref. [52] the authors assigned $\psi(3D)$ and $\psi(5S)$ to the $Y(4325/4360)$ and $Y(4660)$ states, although the predicted masses are higher than the experimental values by 50 to 100 MeV.

The situation changes greatly in our screened potential model. In the earlier calculations [16,17], the mass of $\psi(4S)$ was just around 4260 MeV, the mass of $Y(4260)$. Since the predicted higher chamonium spectrum is compressed in the screened potential model, there is enough space to accommodate these *Y*-states. Specifically, in the model of this paper the masses of $\psi(4S)$, $\psi(3D)$ and $\psi(6S)$ (see Table I) are predicted to be 4273, 4317, and 4608 MeV, which are roughly compatible with the observed masses of $Y(4260)$, $Y(4325/4360)$, and $Y(4660)$, respectively. The small mass discrepancies between theoretical predictions and experimental data may be either due to the experimental errors or, more likely, due to the theoretical uncertainties, especially the complicated *S-D* mixing effects, such as the mixing among 4S, 3D, and 5S states.

The di-electron widths of the pure 4^3S_1, 3^3D_1, and 6^3S_1 states in our model are 0.97, 0.044 and 0.49 and KeV, respectively. Experience about the large *S-D* mixing (especially between $\psi(4040)$ and $\psi(4160)$) discussed in the last subsection tells us that the large *S-D* mixing may change these di-electron widths to moderate values. Assuming the di-electron widths of $Y(4260)$,

$Y(4325/4360)$ and $Y(4660)$ to be all about 0.4 to 0.5 KeV, one can extract the branching ratios $\mathcal{B}(Y \rightarrow \psi(\psi')\pi^+\pi^-)$ from the experimental measurements [48,51]. They are all about 1%–2% and the corresponding widths $\Gamma(Y \rightarrow \psi(\psi')\pi^+\pi^-) = 1$–2 MeV. These widths look too large compared with $\Gamma(\psi' \rightarrow J/\psi\pi^+\pi^-) \sim 100$ KeV [27]. However, once the higher charmonium lies well above the open-charm threshold, the di-pion transition rate may be enhanced dramatically by final state interactions between charmed mesons, which are produced in the decay of the charmonium. This is similar to the case in the $\Upsilon(5S)$ di-pion transitions [60], where the rescattering effects between $B^{(*)} - \bar{B}^{(*)}$ mesons are expected to play a crucial role in enhancing the di-pion transition rates of $\Upsilon(5S)$ into $\Upsilon(1S, 2S)$. Furthermore, if the broad structure $Y(4008)$ is indeed due to $\psi(4040)$ as well as $\psi(4160)$, we will have a clear experimental hint for the large di-pion transition rates of $3S$ as well as $2D$ charmonium, which are well above the $D\bar{D}$ threshold.

It is useful to emphasize that for all the four newly discovered Y states the measurements [46–48,48,49,51]

$$\Gamma_{ee}(Y) \times B(Y \rightarrow \psi\pi^+\pi^-) \approx O(10 \text{ eV}), \qquad (41)$$

where ψ means J/ψ or $\psi(2S)$, and the measured total widths of about $O(100 \text{ MeV})$ altogether may imply that they have similar properties. Therefore, a coherent interpretation for these four Y states is needed. A likely explanation is that they are conventional charmonium states, though other interpretations are also possible and even more interesting. Note that in the hybrid scenario if one of Y states, e.g., the $Y(4260)$ is the 1^{--} hybrid, then one needs to understand why the others, which can no longer be accommodated as 1^{--} hybrids in this mass region, have similar properties to the $Y(4260)$. On the other hand, there are some considered difficulties to assign Y states as conventional charmonia, and the most serious one to assign $Y(4260)$ as the $4S$-dominated charmonium seems to be the observed dip rather than a peak in the R value scanned in e^+e^- annihilation [27] around $Y(4260)$ [this difficulty is common to all resonance interpretations of $Y(4260)$]. A possible explanation for the dip is the destructive interference between the continuum and the resonance. If without any resonance in this region, the continuum contribution should be generally smooth. Another difficulty is the non-observation (not a peak but a dip observed) of the decay modes $D\bar{D}$, $D\bar{D}^*$, $D^*\bar{D}^*$ at the $Y(4260)$ (for a recent report by *BABAR* see Ref. [61]). The dip in the observed charmed meson pairs is probably related to the dip in the R value, since the latter is the measurement of the hadron production cross section in e^+e^- annihilation. The dip in R is just the reflection of the dip in the resonance decays to hadrons (only the charmed hadrons are relevant here). They may all be caused by the interference effects. Moreover, the above-mentioned difficulties are not only for the charmonium assignment but also for other interpretations of $Y(4260)$.

One needs to understand the dip in R around $Y(4260)$ if one tries to interpret $Y(4260)$ as a resonance no matter which kind of resonance it is. Nevertheless, it is instructive to search for decay modes involving the P-wave charm mesons e.g. $D_1\bar{D}$ and other higher charm mesons, apart from the S-wave charm meson pairs, because the higher charmonium may prefer decays to higher charm mesons or multi mesons, due to the form factor suppression with higher momentum released in decays to low charm mesons, and also due to the node structure of higher charmonium state. In this regard, we note that a main decay mode of the $\psi(4415)$ is $D_2\bar{D}$ [45].

Very recently, Belle reported a new vector state $X(4630)$ [62] which was found as a threshold enhancement in the $\Lambda_c^+\Lambda_c^-$ distribution in the ISR process $e^+e^- \rightarrow \gamma_{ISR}\Lambda_c^+\Lambda_c^-$. The mass and width are fitted to be

$$M(X(4630)) = 4634^{+8+5}_{-7-8} \text{ MeV}, \qquad (42)$$

$$\Gamma(X(4630)) = 92^{+40+10}_{-24-21} \text{ MeV}, \qquad (43)$$

which are roughly in agreement with those of $Y(4660)$. Assuming that $X(4630)$ is the same state as the $Y(4660)$ ($\psi(6S)$ in our model) with the di-electron width of about 0.5 KeV, one can extract the partial width

$$\Gamma(X(4630) \rightarrow \Lambda_c^+\Lambda_c^-) \sim 10 \text{ MeV}, \qquad (44)$$

i.e., the branching ratio of about 10%. Such a large baryonic decay width certainly deserves further studying.

To sum up for the discussion in this subsection, our assignments for these newly discovered Y states appear to be consistent with the $4S$, $3D$, $6S$ charmonium mass spectrum predicted by the screened potential model, and other properties may also be understood in these charmonium interpretations. But the issue is far from being conclusive, and many theoretical and experimental investigations are apparently needed to clarify these assignments with other more interesting interpretations such as hybrids and tetraquarks.

D. $X(3872)$

The $X(3872)$ was first observed by Belle [63] in the $J/\psi\pi^+\pi^-$ invariant mass distribution in $B^+ \rightarrow K^+ J/\psi\pi^+\pi^-$ decay as a very narrow peak ($\Gamma_X < 2.3$ MeV) around 3872 MeV. The mass of $X(3872)$ in the $J/\psi\pi^+\pi^-$ mode was recently updated by CDF Collaboration [64] as

$$M(X(3872)) = 3871.61 \pm 0.16 \pm 0.19 \text{ MeV}, \qquad (45)$$

which is very close to the $D^0\bar{D}^{*0}$ threshold $m(D^0\bar{D}^{*0}) = 3871.81 \pm 0.36$ MeV [65]. The spectrum of the di-pion indicates that they come from the ρ resonance and the charge parity of X is even [63]. Moreover, analyses both by Belle [66] and CDF [67] favor the quantum number $J^{PC} = 1^{++}$.

The product branching ratio $\mathcal{B}(B^+ \to XK^+) \cdot \mathcal{B}(X \to J/\psi \rho(\pi^+\pi^-))$ is about $7\text{–}10 \times 10^{-6}$ [68,69]. With the rate of this mode, the relative rates of other decay modes of $X(3872)$ are [70,71]

$$R_{\psi\omega} = \frac{\mathcal{B}(X \to J/\psi\omega)}{\mathcal{B}(X \to J/\psi\rho)} = 1.0 \pm 0.5, \qquad (46)$$

$$R_{\psi\gamma} = \frac{\mathcal{B}(X \to J/\psi\gamma)}{\mathcal{B}(X \to J/\psi\rho)} = 0.33 \pm 0.12, \qquad (47)$$

$$R_{\psi'\gamma} = \frac{\mathcal{B}(X \to \psi'\gamma)}{\mathcal{B}(X \to J/\psi\rho)} = 1.1 \pm 0.4. \qquad (48)$$

It is interesting that another narrow structure was found in the $D^0\bar{D}^0\pi^0$ [72] or the $D^0\bar{D}^{*0}$ [73] invariant mass spectrum near 3875 MeV, which is a little higher than that in (45), in the decays $B^{+/0} \to D^0\bar{D}^{*0}(\bar{D}^0\pi^0)K^{+/0}$. Recently, Belle [74] updated the measurement on $X(3875)$ and improved their fitting method and found

$$M(X(3875)) = 3872.6^{+0.5}_{-0.4} \pm 0.4 \text{ MeV}, \qquad (49)$$

which is consistent with that in (45). Provided that the two X-states are the same, one can extract the ratio

$$R_{DD^*} = \frac{\mathcal{B}(X \to D^0D^{*0})}{\mathcal{B}(X \to J/\psi\rho)} = 9 \pm 2 \qquad (50)$$

from the Belle data [74].

The $X(3872)$ is widely accepted as a molecule candidate of $D^0\bar{D}^{*0}$ in S-wave [75,76] since its mass is very close to the $D^0\bar{D}^{*0}$ threshold. This assignment can also give a natural explanation of the J^{PC} of $X(3872)$ and predict the ratio $R_{\psi\omega} \sim 1$ [76], which is in agreement with that in (46). However, as a loosely bound state of $D^0\bar{D}^{*0}$, it should be difficult to be produced in B-decays or in $p\text{-}p$ collision at the Tevatron. For example, a model calculation [77] shows that in the B^+ decay a molecule $X(3872)$ has a branching ratio $\mathcal{B}(B^+ \to XK^+) = (0.07 - 1.0) \times 10^{-4}$, whereas the experimental rate tends to exceed this upper limit. Furthermore, Belle Collaboration has observed $X(3872)$ in the neutral channel $B^0 \to X(J/\psi\pi^+\pi^-)K^0$ with 5.9σ significance and with a rate almost as large as that of the charged channel [69]

$$\frac{\mathcal{B}(B^0 \to X(3872)K^0)}{\mathcal{B}(B^+ \to X(3872)K^+)} = 0.82 \pm 0.22 \pm 0.05, \qquad (51)$$

which implies that $X(3872)$ is an isoscalar. The most serious problem of the molecular model, in our opinion, is that it is difficult for a loosely bound state to radiatively transit into exited charmonium, such as ψ', through quark annihilation or other mechanisms. Model calculations [76] predict the ratio

$$R_{\psi'\gamma/\psi\gamma} = \frac{\mathcal{B}(X \to \psi'\gamma)}{\mathcal{B}(X \to \psi\gamma)} \simeq 4 \times 10^{-3}, \qquad (52)$$

whereas the experimental value of this ratio [71] is

$$R^{\text{ex}}_{\psi'\gamma/\psi\gamma} = 3.4 \pm 1.4. \qquad (53)$$

Most of the above problems for the molecular model can be resolved if one can assign $X(3872)$ as a conventional charmonium. As a $J^{PC} = 1^{++}$ state, the only candidate is the χ'_{c1} whose mass is about 3.90 GeV in our model (see Table I). The 30 MeV difference between the predicted mass and the experimental one in (45) can be further reduced if the coupled channel effects are taken into account [25]. It is the S-wave coupling of χ'_{c1} to $D^0\bar{D}^{*0}$ that tends to lower the mass of χ'_{c1} towards the threshold of $D^0\bar{D}^{*0}$. This is related to the cusp effect at the $D^0\bar{D}^{*0}$ threshold [78].

The charmonium candidates of $X(3872)$ were suggested [79,80] soon after it was found. However, these suggestions were almost given up after the isospin-violating decay $X \to J/\psi\rho$ was confirmed. Because of the coupled channel effects, the χ'_{c1} will mix with nearby opened $D^0\bar{D}^{*0}$ component. Such a mixed charmonium model for $X(3872)$ was proposed in Refs. [81,82]. Differing from the molecular models, the $D^0\bar{D}^{*0}$ component mixed in the 1^{++} charmonium is just a hadronic description for effects of the vacuum polarization induced by the dynamical quark pair creation and annihilation. Thus, the mixed chamonium is as compact as the conventional charmonium. As a result, the production rates of $X(3872)$ should be large and equal in both the neutral and charged channels in B meson decays [81]. The production rate of $X(3872)$ in $p\text{-}p$ collisions at the Tevatron should also be large, comparable to that of $\chi_{c1}(1P)$ [but somewhat reduced due to a smaller $c\bar{c}$ norm in the mixed charmonium model of $X(3872)$].

Using the final state rescattering mechanism, one may explain the isospin-violating decay $X(3872) \to J/\psi\rho$ [83]. The isospin violation, which is implied by the ratio $R_{\psi\omega}$ in (46), is expected to be mainly due to the difference between the thresholds of $D^0\bar{D}^{*0}$ and D^+D^{*-}, and the larger phase space of $J/\psi\rho$ than that of $J/\psi\omega$ also favors the $J/\psi\rho$ decay [82,83]. In addition, the ratio R_{DD^*} in (50) may also be accounted for provided that the $X(3872)$ lies below or just a little amount above the $D^0\bar{D}^{*0}$ threshold [83].

The E1 transition rates of χ'_{c1} are sensitive to the relativistic corrections due to the node in the $2P$ wave function, especially for the one $\chi'_{c1} \to J/\psi\gamma$. In our model, after relativistic corrections are taken into account, the transition widths $\Gamma(\chi'_{c1} \to J/\psi(\psi')\gamma) = 45(60)$ KeV (see Table IV). The corresponding ratio $R_{\psi'\gamma/\psi\gamma} \simeq 1.33$, which is much larger than the one predicted by the molecular model in (52) and in rough agreement with the experimental value (53). Different treatments or different parameters in the relativistic corrections can result in very different estimations for the rate of $\chi'_{c1} \to J/\psi\gamma$ ($2P$-$1S$ transition), while the rate of $\chi'_{c1} \to \psi'\gamma$ ($2P$-$2S$ transition) can only be changed a little. For example, Ref. [79] gives

$\Gamma(\chi'_{c1} \rightarrow J/\psi(\psi')\gamma) = 11(64)$ KeV, and the corresponding ratio $R_{\psi'\gamma/\psi\gamma} \simeq 6$. Thus, in the mixed charmonium model for $X(3872)$, the expected range of the ratio may be

$$R_{\psi'\gamma/\psi\gamma} = 1.3 - 6.0. \tag{54}$$

If we use the calculated $\Gamma(\chi'_{c1} \rightarrow \psi'\gamma) = 60$ KeV as input for $X(3872) \rightarrow \psi'\gamma$, and use the experimental results (46)–(48) and (50), as well as the width of decay to light hadrons (assuming $\Gamma(\chi_{c1}(2P) \rightarrow$ light hadrons) $\approx \Gamma(\chi_{c1}(1P) \rightarrow$ light hadrons) ≈ 600 KeV), we will get the total width of $X(3872)$ to be about 1400 ± 300 KeV, which is compatible with the measurement [it can be further reduced when the $c\bar{c}$ norm in $X(3872)$ is smaller than 1].

The nature of $X(3872)$ can also be uncovered by the pole structure of the scattering amplitude involving the resonance near the $D^0 \bar{D}^{*0}$ threshold. This study is also needed to explain the different peak locations of $X(3872)$ in the $J/\psi\pi^+\pi^-$ and $D^0\bar{D}^0\pi^0/D^0\bar{D}^{*0}$ modes. Three groups [84–86] have devoted themselves to this study and the conclusions are quite different. One group [84] concludes that the $X(3872)$ tends to be a virtual state of $D^0\bar{D}^{*0}$, while another group's fit [85] favors the loosely bound state explanation. Most recently, with the Belle's new data [69,74], authors of Ref. [86] gave a more systematic study on this topic and found that there may need to be two near-threshold poles to account for the data, one from the $D^0\bar{D}^{*0}$ component and the other from the charmonium state χ'_{c1}.

To sum up for the discussion in this subsection, we find that the $\chi_{c1}(2P)$-dominated charmonium interpretation for the $X(3872)$ may account for (i) the E1 transition rates to J/ψ and $\psi(2S)$ and their ratio (53); (ii) the large production rates in B decays and equal rates for B^+ and B^0; (iii) the large production rate in $p - p$ collisions at the Tevatron; (iv) the isospin-violating decay to $J/\psi\rho$. Moreover, in the screened potential model the mass of $\chi_{c1}(2P)$ is predicted to take a lower value than the quenched potential model. However, though the mass of $\chi_{c1}(2P)$ can be lowered by coupling to $D^0\bar{D}^{*0}$, one cannot provide a quantitative explanation for the extreme closeness of $X(3872)$ to the $D^0\bar{D}^{*0}$ threshold (say, within 0.5 MeV), which is the most favorable motivation for the molecule interpretation.

E. $X(3940)$, $X(4160)$

The $X(3940)$ was found by the Belle Collaboration [6] in the recoiling spectrum of J/ψ in the e^+e^- annihilation process $e^+e^- \rightarrow J/\psi +$ anything and $e^+e^- \rightarrow J/\psi + D\bar{D}^*$. The latter was studied further with higher statistics by Belle [87]. The mass and width of $X(3940)$ are determined to be

$$M(X(3940)) = 3942^{+7}_{-6} \pm 6 \text{ MeV}, \tag{55}$$

$$\Gamma(X(3940)) = 37^{+26}_{-18} \pm 8 \text{ MeV}. \tag{56}$$

Meanwhile, they also found the $X(4160)$ in the $D^*\bar{D}^*$ mode in the process $e^+e^- \rightarrow J/\psi + D^*\bar{D}^*$ with a significance of 5.1σ. The mass and width of the $X(4160)$ are given by

$$M(X(4160)) = 4156^{+25}_{-20} \pm 15 \text{ MeV}, \tag{57}$$

$$\Gamma(X(4160)) = 139^{+111}_{-61} \pm 21 \text{ MeV}. \tag{58}$$

Besides, there is a structure around 3880 MeV in the $D\bar{D}$ spectrum in $e^+e^- \rightarrow J/\psi + D\bar{D}$. However, it is too wide to present a resonance shape sufficiently.

Both of the two X-states have large production rates in these processes [87]. This fact implies that the charge parities should be even since the charge odd state associated with J/ψ needs to be produced via two-photon fragmentation, which is expected to be highly suppressed [43]. On the other hand, the only known charmonium states that are produced in this way are η_c, η'_c, and χ_{c0} [88], and this double charmonium production phenomenon can be explained in the framework of nonrelativistic QCD [89]. The production rates of $X(3940)$ and $X(4160)$ [87] are both as large as those of η_c, η'_c, and χ_{c0}. This suggests that the two X-states could be either pseudoscalar like η_c or scalar like χ_{c0} (see Ref. [43] for more detailed discussions).

The observation that the dominant decay mode of $X(3940)$ being $D\bar{D}^*$ and the lack of evidence for the $D\bar{D}$ decay mode [6,87] indicates that it cannot be a scalar but can be a good candidate for the $\eta_c(3S)$. The main problem is the low mass of $X(3940)$ as the $\eta_c(3S)$. Although lower than that in the quenched potential model [5] by 50 MeV or more, the mass of $\eta_c(3S)$ in our screened potential model (see Table I) is still larger than the observed mass (55) by about 50 MeV. Moreover, the mass splitting between $X(3940)$ and $\psi(4040)$ is larger than that between η'_c and ψ', which looks quite unnatural. But it may be due to the coupled-channel effects [7], which will further lower the $\eta_c(3S)$ mass hopefully.

The dominant decay mode of $X(4160)$ is $D^*\bar{D}^*$ [87], and the other modes, such as $D\bar{D}$ and $D\bar{D}^*$, were not seen. Thus the charmonium candidates can be $\eta_c(4S)$ or $\chi_{c0}(3P)$, whose masses are 4250 MeV and 4131 MeV in our model prediction, respectively. Evidently, the mass of $X(4160)$ in (57) favors the $\chi_{c0}(3P)$ candidate. The $\chi_{c0}(3P)$ cannot decay into $D\bar{D}^*$, and the decay $\chi_{c0}(3P) \rightarrow D\bar{D}$ is expected to be strongly suppressed by the form factor and the effects induced by the nodes of the $3P$ wave function, just like the case of suppressed $\psi(4040) \rightarrow D\bar{D}$ decay [5]. The main problem of this assignment may be why the $\chi_{c0}(2P)$ state is not found in the similar process. One possible account [43] is that the broad peak around 3880 MeV in the $D\bar{D}$ spectrum [87] mentioned above could be the missing $\chi_{c0}(2P)$ state, since its mass in our model is about 3842 MeV and just lies within the bump (note, however, that this bump might not be a resonance [90]). In addition, the measurements on angular distributions can be used to test the two

possible assignments, $\eta_c(4S)$ and $\chi_{c0}(3P)$, for the $X(4160)$ [43].

V. SUMMARY AND CONCLUSIONS

In this paper, we try to incorporate the color-screening (string breaking) effect due to light quark pair creation into the heavy quark-antiquark long-range confinement potential, and investigate the effects of screened potential on the spectrum of the charmonium, especially the higher charmonium. We calculate the masses, electromagnetic decays, and E1 transitions of charmonium states in the nonrelativistic screened potential model, and propose possible assignments for the newly discovered charmonium or charmoniumlike states, i.e., the so-called ''X, Y, Z'' mesons. We find that compared with the unscreened potential model, the masses predicted in the screened potential model are considerably lower for higher charmonium states. For example, the predicted $\chi_{c2}(2P)$ mass well agrees with that of the $Z(3930)$, and the mass of $\psi(5S)$ rather than $\psi(4S)$ is compatible with that of $\psi(4415)$. As a result of the compressed mass spectrum in our model, most of the X, Y, Z states might be accommodated in the conventional higher charmonia. In particular, the discovered four Y states in the ISR process, i.e., $Y(4008)$, $Y(4260)$, $Y(4320/4360)$, $Y(4660)$ may be assigned as the $\psi(3S)$, $\psi(4S)$, $\psi(3D)$, $\psi(6S)$ states, respectively. The $X(3940)$ and $X(4160)$ found in the double charmonium production in e^+e^- annihilation may be assigned as the $\eta_c(3S)$ and $\chi_{c0}(3P)$ states, respectively. Based on the calculation of E1 transition widths for $\chi_{c1}(2P) \rightarrow \gamma J/\psi$ and $\chi_{c1}(2P) \rightarrow \gamma \psi(2S)$ and other results, we argue that the $X(3872)$ may be a $\chi_{c1}(2P)$ dominated charmonium state with some admixture of the $D^0 \bar{D}^{*0}$ component. The problems encountered in these assignments and comparisons with other interpretations for these X, Y, Z mesons are discussed in detail. We emphasize that more theoretical and experimental investigations are urgently needed to clarify these assignments and other interesting interpretations. In particular, we hope experiments at BESIII and SuperBelle in the future will be crucially useful in searching for new hadrons including charmoniumlike states and testing the theoretical interpretations.

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