PHYSICAL REVIEW D **79,** 093006 (2009)

Texture specific mass matrices with Dirac neutrinos and their implications

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(Received 3 March 2009; published 14 May 2009)

Considering Dirac neutrinos and Fritzsch-like texture six zero and five zero mass matrices, detailed predictions for cases pertaining to normal/inverted hierarchy as well as degenerate scenario of neutrino masses have been carried out. All the cases considered here pertaining to inverted hierarchy and degenerate scenario of neutrino masses are ruled out by the existing data. For the normal hierarchy cases, the lower limit of m_{ν_1} and of s_{13} as well as the range of Dirac-like CP violating phase δ_l would have implications for the texture specific cases considered here.

DOI: 10.1103/PhysRevD.79.093006 PACS numbers: 14.60.Pq, 12.15.Ff

I. INTRODUCTION

In the last few years, apart from establishing the hypothesis of neutrino oscillations, impressive advances have been made in understanding the phenomenology of neutrino oscillations through solar neutrino experiments [1], atmospheric neutrino experiments [2], reactor based experiments [3], and accelerator based experiments [4]. Ever since the observation of neutrino oscillations, there has been an explosive amount of activity both at the theoretical as well as the experimental fronts in understanding the problem of neutrino masses and mixings. In the case of neutrinos, neither the mixing angles nor the neutrino masses show any hierarchy, this being in sharp contrast to the distinct hierarchy shown by quark masses and mixing angles. In fact, the two mixing angles governing solar and atmospheric neutrino oscillations look to be rather large, the third angle may be very small compared to these. Further, at present there is no consensus about neutrino masses which may show normal/inverted hierarchy or may even be degenerate. Furthermore, the situation becomes complicated when one realizes that neutrino masses are much smaller than the charged fermion masses as well, as it is yet not clear whether neutrinos are Dirac or Majorana particles.

In the absence of a convincing fermion flavor theory, several approaches have been considered [5] to understand the fermion mass generation problem, e.g., radiative mechanisms, texture zeros, flavor symmetries, seesaw mechanism, extra dimensions, etc. In this context, texture specific mass matrices have gotten a good deal of attention in the literature, in particular, Fritzsch-like texture specific mass matrices seem to be very helpful in understanding the pattern of quark mixings and CP violation [5,6]. Taking clues from the success of these texture specific mass matrices in the context of quarks, several attempts [7,8] have been made to consider similar lepton mass matrices for explaining the pattern of neutrino masses and mixings by using the seesaw mechanism [9] given by

$$M_\nu = -M_{\nu D}^T (M_R)^{-1} M_{\nu D}, \tag{1}$$

where $M_{\nu D}$ and M_R are, respectively, the Dirac neutrino mass matrix and the right-handed Majorana neutrino mass matrix. In order to analyze the implications of mass matrix M_ν, it is perhaps more desirable to impose texture structure on $M_{\nu D}$, however in some of the attempts [8] texture structure has been imposed on M_ν itself. It may be mentioned that although several analyses have been carried out by considering neutrinos to be Majorana particles, yet similar attempts have not been carried out for Dirac neutrinos which have not yet been ruled out by experiment [10]. In this context, several authors have examined the possibility of Dirac neutrinos having small masses [11] as well as their compatibility with the supersymmetric grand unified theories (GUTs) [12]. This, therefore, motivates one to consider texture specific mass matrices with Dirac neutrinos, which are compatible with GUTs [5,7], as an alternative to the Majorana picture.

The possibility of having zero textures in the mass matrices of Dirac neutrinos, with the charged leptons in the flavor basis, has also been considered [13]. In fact, in [13] a very interesting and intensive analysis has been carried out, wherein they find that for the general form of the Dirac neutrino mass matrix with texture five and four zeros, with the charged leptons being in the flavor basis, one can accommodate the current data including the possibility of one massless neutrino, however without incorporating CP violation. Taking clues from [13] and keeping in mind broad principle like quark-lepton symmetry [14], as well as in view of the fact that Fritzsch-like texture specific mass matrices provide a good deal of success in understanding the quark mixing phenomenon, we have considered similar texture specific mass matrices for the case of Dirac neutrinos also. In this context, Fritzsch-like texture six and five zero mass matrices provide the simplest possibility of texture specific mass matrices. It needs to be

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mentioned that the texture specific mass matrices considered in the present work are quite different as compared to those considered in [13], which is explained in the sequel.

In the present paper, for the case of Dirac neutrinos, we have investigated 15 distinct possibilities of texture six zero and five zero mass matrices for normal/inverted hierarchy as well as degenerate scenario of neutrino masses. The analysis has been carried out by imposing Fritzsch-like texture structure on Dirac neutrino mass matrices as well as on charged lepton mass matrices. For the sake of completion, we have also investigated the cases corresponding to charged leptons being in the flavor basis. Further, detailed dependence of mixing angles on the lightest neutrino mass as well as the parameter space available to the phases of mass matrices have also been investigated. Furthermore, several phenomenological quantities such as Jarlskog's rephasing invariant parameter in the leptonic sector J_l and the corresponding CP violating Dirac-like phase δ_l have also been calculated for different cases.

The detailed plan of the paper is as follows. In Sec. II, we detail the essentials of the formalism connecting the mass matrix to the neutrino mixing matrix. Inputs used in the present analysis have been given in Sec. III. For texture six zero as well as texture five zero mass matrices, Sec. IV discusses the calculations pertaining to inverted hierarchy and degenerate scenario of neutrino masses, whereas Sec. V details the analysis for normal hierarchy of neutrino masses. Finally, Sec. VI summarizes our conclusions.

II. CONSTRUCTION OF PMNS MATRIX FROM MASS MATRICES

To begin with, we present the modified Fritzsch-like matrices, e.g.,

$$M_l = \begin{pmatrix} 0 & A_l & 0 \\ A_l^* & D_l & B_l \\ 0 & B_l^* & C_l \end{pmatrix}, \qquad M_{\nu D} = \begin{pmatrix} 0 & A_\nu & 0 \\ A_\nu^* & D_\nu & B_\nu \\ 0 & B_\nu^* & C_\nu \end{pmatrix}, \tag{2}$$

M_l and $M_{\nu D}$ respectively corresponding to Dirac-like charged lepton and neutrino mass matrices. It may be noted that each of the above matrix is texture two zero type with $A_{l(\nu)} = |A_{l(\nu)}| e^{i\alpha_{l(\nu)}}$ and $B_{l(\nu)} = |B_{l(\nu)}| e^{i\beta_{l(\nu)}}$, in case these are symmetric then $A^*_{l(\nu)}$ and $B^*_{l(\nu)}$ should be replaced by $A_{l(\nu)}$ and $B_{l(\nu)}$, as well as $C_{l(\nu)}$ and $D_{l(\nu)}$ should, respectively, be defined as $C_{l(\nu)} = |C_{l(\nu)}| e^{i\gamma_{l(\nu)}}$ and $D_{l(\nu)} = |D_{l(\nu)}| e^{i\omega_{l(\nu)}}$.

The texture six zero matrices can be obtained from the above mentioned matrices by taking both D_l and D_ν to be zero, which reduces the matrices M_l and $M_{\nu D}$ each to texture three zero type. Texture five zero matrices can be obtained by taking either $D_l = 0$ and $D_\nu \neq 0$ or $D_\nu = 0$ and $D_l \neq 0$, thereby, giving rise to two possible cases of texture five zero matrices, referred to as texture five zero $D_l = 0$ case pertaining to M_l texture three zero type and $M_{\nu D}$ texture two zero type and texture five zero $D_\nu = 0$ case pertaining to M_l texture two zero type and $M_{\nu D}$ texture three zero type. It may be added that the above formulation of texture zeros of mass matrices is quite different as compared to that considered in [13], wherein the authors have examined the minimal allowed structure of the Dirac neutrino mass matrix, referred to as m_ν by them and $M_{\nu D}$ by us. They have carried out the analysis by considering charged lepton mass matrix to be diagonal and incorporating up to five zero entries in m_ν alone, unlike the way we have defined texture zero mass matrices above, essentially involving both M_l and $M_{\nu D}$.

To fix the notations and conventions as well as to facilitate the understanding of inverted hierarchy case and its relationship to the normal hierarchy case, we detail the formalism connecting the mass matrix to the neutrino mixing matrix. The mass matrices M_l and $M_{\nu D}$ given in Eq. (2), for Hermitian as well as symmetric case, can be exactly diagonalized. Details of the Hermitian case can be looked up in our earlier work [6], the symmetric case can similarly be worked out. To facilitate diagonalization, the mass matrix M_k, where $k = l, \nu D$, can be expressed as

$$M_k = Q_k M_k^r P_k \tag{3}$$

or

$$M_k^r = Q_k^\dagger M_k P_k^\dagger, \tag{4}$$

where M_k^r is a real symmetric matrix with real eigenvalues and Q_k and P_k are diagonal phase matrices. For the Hermitian case $Q_k = P_k^\dagger$, whereas for the symmetric case under certain conditions $Q_k = P_k$. In general, the real matrix M_k^r is diagonalized by the orthogonal transformation O_k, e.g.,

$$M_k^{\rm diag} = O_k^T M_k^r O_k, \tag{5}$$

which on using Eq. (4) can be rewritten as

$$M_k^{\rm diag} = O_k^T Q_k^\dagger M_k P_k^\dagger O_k. \tag{6}$$

To facilitate the construction of diagonalization transformations for different hierarchies, we introduce a diagonal phase matrix ξ_k defined as diag$(1, e^{i\pi}, 1)$ for the case of normal hierarchy and as diag$(1, e^{i\pi}, e^{i\pi})$ for the case of inverted hierarchy. Equation (6) can now be written as

$$\xi_k M_k^{\rm diag} = O_k^T Q_k^\dagger M_k P_k^\dagger O_k, \tag{7}$$

which can also be expressed as

$$M_k^{\rm diag} = \xi_k^\dagger O_k^T Q_k^\dagger M_k P_k^\dagger O_k. \tag{8}$$

Making use of the fact that $O_k^* = O_k$ it can be further expressed as

$$M_k^{\rm diag} = (Q_k O_k \xi_k)^\dagger M_k (P_k^\dagger O_k), \tag{9}$$

from which one gets

$$M_k = Q_k O_k \xi_k M_k^{\rm diag} O_k^T P_k. \tag{10}$$

The case of leptons is fairly straight forward, for the Dirac neutrinos the diagonalizing transformation is hierarchy specific. To clarify this point further, in analogy with Eq. (10), we can express $M_{\nu D}$ as

$$M_{\nu D} = Q_{\nu D} O_{\nu D} \xi_{\nu D} M_{\nu D}^{\rm diag} O_{\nu D}^T P_{\nu D}. \tag{11}$$

The lepton mixing matrix, obtained from the matrices used for diagonalizing the mass matrices M_l and $M_{\nu D}$, is expressed as

$$U = (Q_l O_l \xi_l)^\dagger (P_{\nu D} O_{\nu D} \xi_{\nu D}). \tag{12}$$

Eliminating the phase matrices ξ_l and $\xi_{\nu D}$ by redefinition of the charged lepton and Dirac neutrinos, the above equation becomes

$$U = O_l^\dagger Q_l P_{\nu D} O_{\nu D}, \tag{13}$$

where $Q_l P_{\nu D}$, without loss of generality, can be taken as $(e^{i\phi_1}, 1, e^{i\phi_2})$, ϕ_1 and ϕ_2 being related to the phases of mass matrices and can be treated as free parameters.

To understand the relationship between diagonalizing transformations for different hierarchies of neutrino masses as well as their relationship with the charged lepton case, we reproduce the general diagonalizing transformation O_k. The elements of O_k can figure with different phase possibilities, however these possibilities are related to each other through phase matrices Q_l and $P_{\nu D}$. For the present work, we have chosen the possibility,

$$O_k = \begin{pmatrix} O_k(11) & O_k(12) & O_k(13) \\ O_k(21) & -O_k(22) & O_k(23) \\ -O_k(31) & O_k(32) & O_k(33) \end{pmatrix}, \tag{14}$$

where

$$O_k(11) = \sqrt{\frac{m_2 m_3 (m_3 - m_2 - D_k)}{(m_1 - m_2 + m_3 - D_k)(m_3 - m_1)(m_1 + m_2)}}$$

$$O_k(12) = \sqrt{\frac{m_1 m_3 (m_1 + m_3 - D_k)}{(m_1 - m_2 + m_3 - D_k)(m_2 + m_3)(m_1 + m_2)}}$$

$$O_k(13) = \sqrt{\frac{m_1 m_2 (m_2 - m_1 + D_k)}{(m_1 - m_2 + m_3 - D_k)(m_2 + m_3)(m_3 - m_1)}}$$

$$O_k(21) = \sqrt{\frac{m_1 (m_3 - m_2 - D_k)}{(m_3 - m_1)(m_1 + m_2)}}$$

$$O_k(22) = \sqrt{\frac{m_2 (m_1 + m_3 - D_k)}{(m_2 + m_3)(m_1 + m_2)}}$$

$$O_k(23) = \sqrt{\frac{m_3 (m_2 - m_1 + D_k)}{(m_2 + m_3)(m_3 - m_1)}}$$

$$O_k(31) = \sqrt{\frac{m_1 (m_2 - m_1 + D_k)(m_1 + m_3 - D_k)}{(m_1 - m_2 + m_3 - D_k)(m_1 + m_2)(m_3 - m_1)}}$$

$$O_k(32) = \sqrt{\frac{m_2 (m_2 - m_1 + D_k)(m_3 - m_2 - D_k)}{(m_1 - m_2 + m_3 - D_k)(m_2 + m_3)(m_1 + m_2)}}$$

$$O_k(33) = \sqrt{\frac{m_3 (m_3 - m_2 - D_k)(m_1 + m_3 - D_k)}{(m_1 - m_2 + m_3 - D_k)(m_3 - m_1)(m_2 + m_3)}}, \tag{15}$$

m_1, $-m_2$, m_3 being the eigenvalues of M_k. In the case of charged leptons, because of the hierarchy $m_e \ll m_\mu \ll m_\tau$, the mass eigenstates can be approximated, respectively, to the flavor eigenstates as has been considered by several authors [15,16]. Using the approximation, $m_{l1} \simeq m_e$, $m_{l2} \simeq m_\mu$, and $m_{l3} \simeq m_\tau$, the first element of the matrix O_l can be obtained from the corresponding element of Eq. (15) by replacing m_1, $-m_2$, m_3 with m_e, $-m_\mu$, m_τ, e.g.,

$$O_l(11) = \sqrt{\frac{m_\mu m_\tau (m_\tau - m_\mu - D_l)}{(m_e - m_\mu + m_\tau - D_l)(m_\tau - m_e)(m_e + m_\mu)}}. \tag{16}$$

For normal hierarchy defined as $m_{\nu_1} < m_{\nu_2} \ll m_{\nu_3}$, as well as for the corresponding degenerate case given by $m_{\nu_1} \lesssim m_{\nu_2} \sim m_{\nu_3}$, Eq. (15) can also be used to obtain the first element of diagonalizing transformation for Dirac neutrinos. This element can be obtained from the corresponding element of Eq. (15) by replacing m_1, $-m_2$, m_3 with $m_{\nu 1}$, $-m_{\nu 2}$, $m_{\nu 3}$ and is given by

$$O_{\nu D}(11) = \sqrt{\frac{m_{\nu_2} m_{\nu 3} (m_{\nu 3} - m_{\nu 2} - D_\nu)}{(m_{\nu 1} - m_{\nu 2} + m_{\nu 3} - D_\nu)(m_{\nu 3} - m_{\nu 1})(m_{\nu 1} + m_{\nu 2})}}, \tag{17}$$

where m_{ν_1}, m_{ν_2} and m_{ν_3} are neutrino masses.

In the same manner, one can obtain the elements of diagonalizing transformation for the inverted hierarchy case defined as $m_{\nu_3} \ll m_{\nu_1} < m_{\nu_2}$ as well as for the corresponding degenerate case given by $m_{\nu_3} \sim m_{\nu_1} \lesssim m_{\nu_2}$. The corresponding first element, obtained by replacing m_1, $-m_2$, m_3 with $m_{\nu 1}$, $-m_{\nu 2}$, $-m_{\nu 3}$ in Eq. (15), is given by

$$O_{\nu D}(11) = \sqrt{\frac{m_{\nu_2} m_{\nu 3}(m_{\nu 3} + m_{\nu 2} + D_\nu)}{(-m_{\nu 1} + m_{\nu 2} + m_{\nu 3} + D_\nu)(m_{\nu 3} + m_{\nu 1})(m_{\nu 1} + m_{\nu 2})}}. \tag{18}$$

The other elements of diagonalizing transformations in the case of neutrinos as well as charged leptons can similarly be found. Detailed expressions of the Pontecorvo-Maki-Nakagawa-Sakata (PMNS) matrix elements [17] have been presented in the appendix.

III. INPUTS USED IN THE PRESENT ANALYSIS

Before going into the details of the analysis, we would like to mention some of the essentials pertaining to various inputs. Adopting the three neutrino framework, several authors [18–20] have presented updated information regarding the neutrino mass and mixing parameters obtained by carrying out detailed global analyses. The latest situation regarding masses and mixing angles at 3σ C.L. is summarized as follows [20],

$$\Delta m_{12}^2 = (7.14\text{–}8.19) \times 10^{-5}\ \text{eV}^2, \quad \Delta m_{23}^2 = (2.06\text{–}2.81) \times 10^{-3}\ \text{eV}^2, \tag{19}$$

$$\sin^2\theta_{12} = 0.263\text{–}0.375, \quad \sin^2\theta_{23} = 0.331\text{–}0.644, \quad \sin^2\theta_{13} \leq 0.046. \tag{20}$$

The above data reveals that at present not much is known about the hierarchy of neutrino masses as well as about their absolute values.

The masses and mixing angles, used in the analysis, have been constrained by the data given in Eqs. (19) and (20). For the purpose of calculations, we have taken the lightest neutrino mass, the phases ϕ_1, ϕ_2 and $D_{l,\nu}$ as free parameters, the other two masses are constrained by $\Delta m_{12}^2 = m_{\nu_2}^2 - m_{\nu_1}^2$ and $\Delta m_{23}^2 = m_{\nu_3}^2 - m_{\nu_2}^2$ in the normal hierarchy case and by $\Delta m_{23}^2 = m_{\nu_2}^2 - m_{\nu_3}^2$ in the inverted hierarchy case. It may be noted that lightest neutrino mass corresponds to m_{ν_1} for the normal hierarchy case and to m_{ν_3} for the inverted hierarchy case. In the case of normal hierarchy, the explored range for m_{ν_1} is taken to be 0.0001 eV–1.0 eV, which is essentially governed by the mixing angle s_{12}, related to the ratio $\frac{m_{\nu_1}}{m_{\nu_2}}$. For the inverted hierarchy case also we have taken the same range for m_{ν_3} as our conclusions remain unaffected even if the range is extended further. In the absence of any constraint on the phases, ϕ_1 and ϕ_2 have been given full variation from 0 to 2π. Although $D_{l,\nu}$ are free parameters, however, they have been constrained such that diagonalizing transformations, O_l and O_ν, always remain real, implying $D_l < m_{l_3} - m_{l_2}$ whereas $D_\nu < m_{\nu_3} - m_{\nu_2}$ for normal hierarchy and $D_\nu < m_{\nu_1} - m_{\nu_3}$ for inverted hierarchy.

We have carried out detailed calculations pertaining to texture six zero as well as two possible cases of texture five zero lepton mass matrices, e.g., $D_l = 0$ case and $D_\nu = 0$ case. Corresponding to each of these cases, we have considered three possibilities of neutrino masses having normal/inverted hierarchy or being degenerate. In addition to these nine possibilities, we have also considered those cases when the charged leptons are in the flavor basis. These possibilities sum up to 18, however, the texture five zero $D_\nu = 0$ case with charged leptons in the flavor basis reduces to the similar texture six zero case, hence the 18 possibilities reduce to 15 distinct cases.

IV. INVERTED HIERARCHY AND DEGENERATE SCENARIO OF NEUTRINO MASSES

A. Texture six zero mass matrices

To begin with, we first consider the cases pertaining to inverted hierarchy of neutrino masses as well as when neutrino masses are degenerate. Interestingly, we find that all the cases pertaining to inverted hierarchy and degenerate scenarios of neutrino masses seem to be ruled out. This can be concluded using the plots of the variation of the mixing angles with the lightest neutrino mass. For the texture six zero case, in Fig. 1, by giving full variations to other parameters, we have plotted the mixing angles against the lightest neutrino mass. The dotted lines and the dot-dashed lines depict the limits obtained assuming normal and inverted hierarchy, respectively, the solid horizontal lines show the 3σ limits of the plotted mixing angle as given in Eq. (20). A look at Fig. 1(a) shows that for $m_{\nu_1} \sim$ 0.0001 eV there is a slight overlap of the inverted hierarchy region with the experimental limits of the angle s_{12}. Similarly, Fig. 1(b) shows that again for $m_{\nu_1} \sim 0.1$–1 eV there is an overlap of the inverted hierarchy region with the experimental limits of the angle s_{13}. However, it is easily evident from Fig. 1(c) that inverted hierarchy is ruled out at 3σ C.L. by the experimental limits on the mixing angle s_{23}. It may be emphasized that in Figs. 1(a) and 1(b) a slight overlap of the inverted hierarchy region with the experimental limits of the two angles does not affect our conclusions regarding inverted hierarchy of neutrino masses

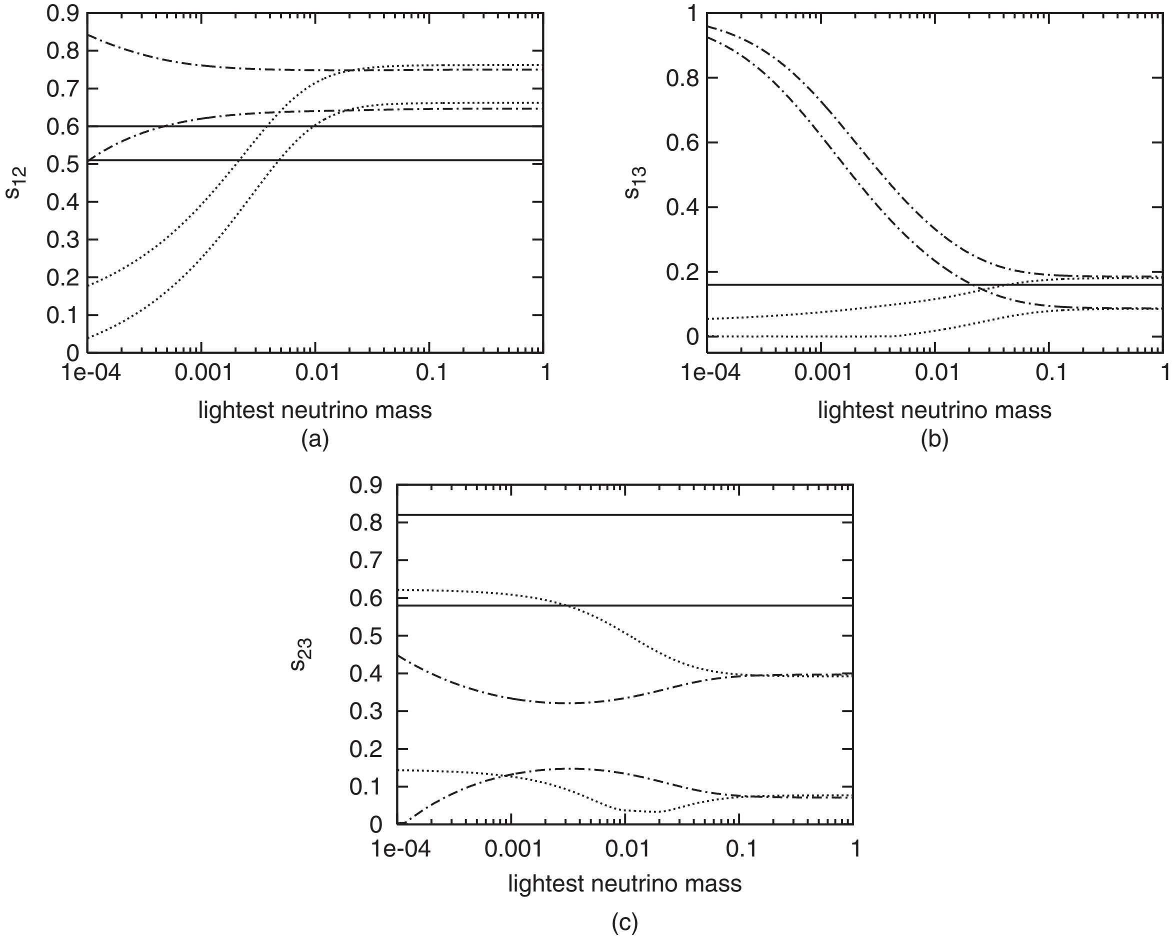

FIG. 1. Plots showing variation of mixing angles s_{12}, s_{13}, and s_{23} with lightest neutrino mass for texture six zero case of Dirac neutrinos. The dotted lines and the dot-dashed lines depict the limits obtained assuming normal and inverted hierarchy, respectively, the solid horizontal lines show the 3σ limits of s_{23} given in Eq. (20).

since to rule it out it is sufficient to do so from any one of the graphs.

One can easily check that degenerate scenarios characterized by either $m_{\nu_1} \lesssim m_{\nu_2} \sim m_{\nu_3} \sim 0.1$ eV or $m_{\nu_3} \sim m_{\nu_1} \lesssim m_{\nu_2} \sim 0.1$ eV are clearly ruled out from Figs. 1(a) and 1(c). This can be understood by noting that around 0.1 eV, the limits obtained assuming normal and inverted hierarchies have no overlap with the experimental limits of angles s_{12} and s_{23}.

B. Texture five zero mass matrices

Coming to the texture five zero cases, we first discuss the case when $D_l = 0$ and $D_\nu \neq 0$. In Fig. 2 we have plotted the mixing angles against the lightest neutrino mass for both normal and inverted hierarchy for a particular value of $D_\nu = \sqrt{m_{\nu_3}}$. A look at Figs. 2(a) and 2(c) reveals that the region pertaining to inverted hierarchy, depicted by dot-dashed lines, shows an overlap with the experimental limits on s_{12} and s_{23}, respectively. The graph of s_{13} versus the lightest neutrino mass, shown in Fig. 2(b) immediately rules out inverted hierarchy by experimental limits on angle s_{13}.

In Fig. 3 we have plotted allowed parameter space for the three mixing angles in the D_ν-lightest neutrino mass plane, for texture five zero $D_l = 0$ case. This allows us to extend our results to other acceptable values of D_ν and study their implications. Figure 3 reveals that the allowed parameter spaces of the three mixing angles show an overlap only when $D_\nu \sim 0$, which leads to the present texture six zero case, wherein degenerate scenario has already been ruled out. Therefore, again one can easily conclude that inverted hierarchy as well as degenerate scenarios are ruled out for texture five zero $D_l = 0$ case, not only for $D_\nu = \sqrt{m_{\nu_3}}$ but also for its other allowed values.

For the texture five zero $D_\nu = 0$ and $D_l \neq 0$ case, the plots of mixing angles against the lightest neutrino mass are shown in Fig. 4. Interestingly, these graphs are very similar to Fig. 1 pertaining to the texture six zero case of Dirac neutrinos. Therefore, arguments similar to the ones

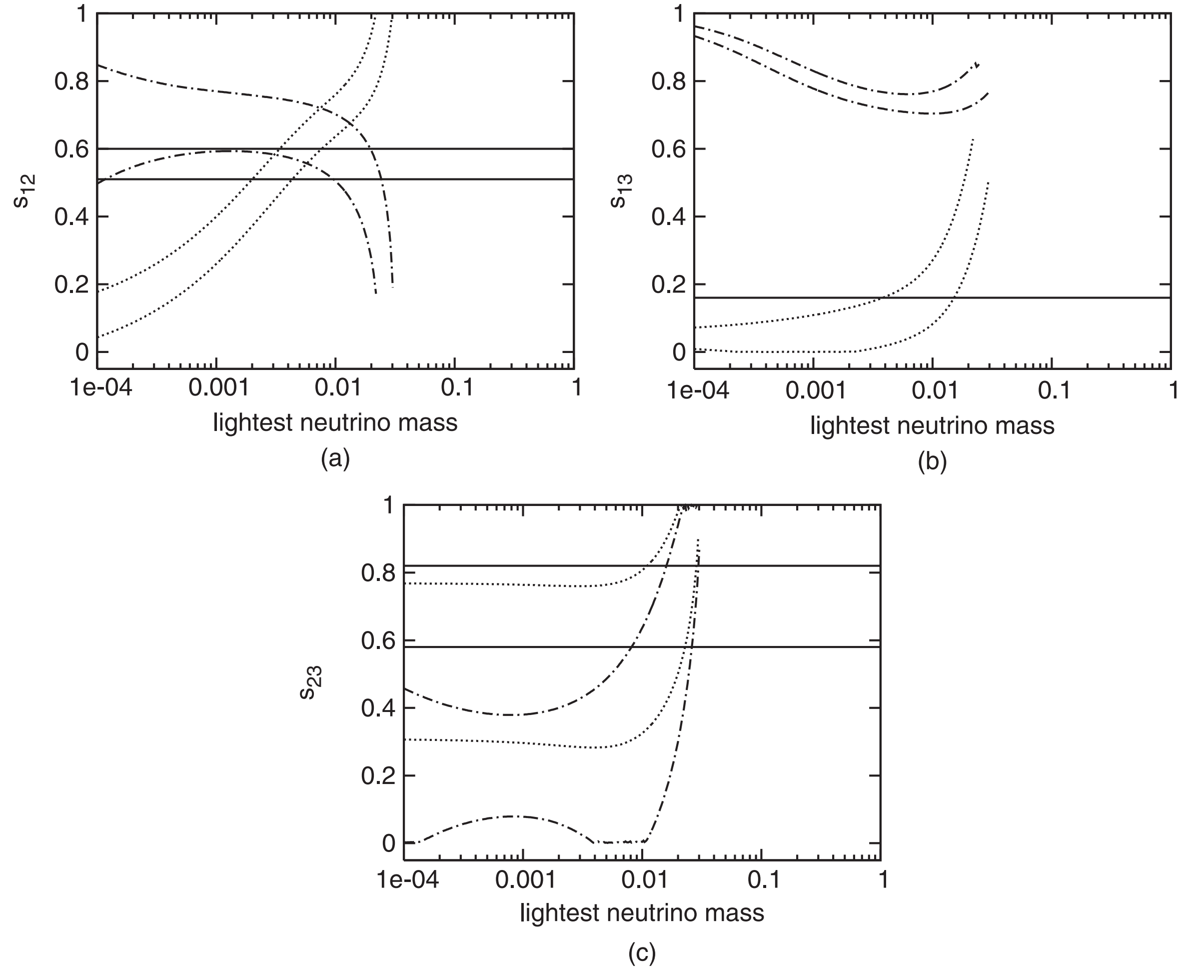

FIG. 2. Plots showing variation of mixing angles s_{12}, s_{13}, and s_{23} with lightest neutrino mass for texture five zero $D_l = 0$ case of Dirac neutrinos, with a value $D_\nu = \sqrt{m_{\nu_3}}$. The representations of the curves remain the same as in Fig. 1.

for the texture six zero case lead us to conclude that both inverted hierarchy as well as degenerate scenarios of neutrino masses are ruled out for this case as well. It may be mentioned that similarities observed in the mixing angles variation with the lightest neutrino mass for the texture five zero $D_\nu = 0$ and the texture six zero case can be understood by noting that a very strong hierarchy in the case of charged leptons reduces the texture five zero $D_\nu = 0$ case essentially to the texture six zero case only.

In case charged lepton mass matrices are considered to be in the flavor basis, one can easily find, using the above methodology, that all the cases pertaining to inverted hierarchy and the degenerate scenario of neutrino masses are again ruled out.

V. NORMAL HIERARCHY OF NEUTRINO MASSES

A. Texture six zero mass matrices

After considering the implications of the texture six zero as well as the two cases of texture six zero mass matrices on inverted hierarchy of neutrino masses as well as neu-

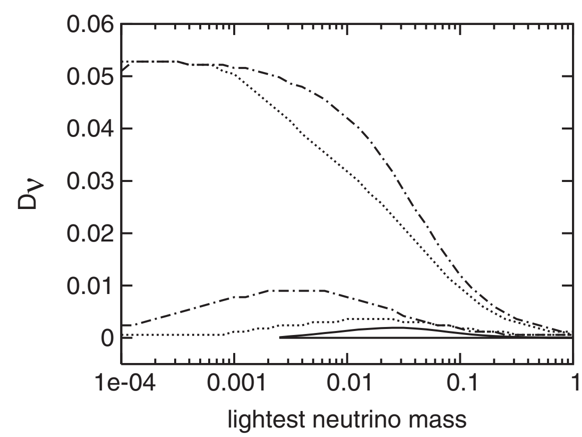

FIG. 3. Plots showing allowed parameter space for the three mixing angles in the D_ν-lightest neutrino mass plane, for texture five zero $D_l = 0$ case of Dirac neutrinos for the inverted hierarchy, with D_ν being varied from 0 to a value such that $D_\nu < \sqrt{m_{\nu_1}} - \sqrt{m_{\nu_3}}$. Dotted lines depict allowed parameter space for s_{12}, dot-dashed lines depict allowed parameter space for s_{23} and solid lines depict allowed parameter space for s_{13}.

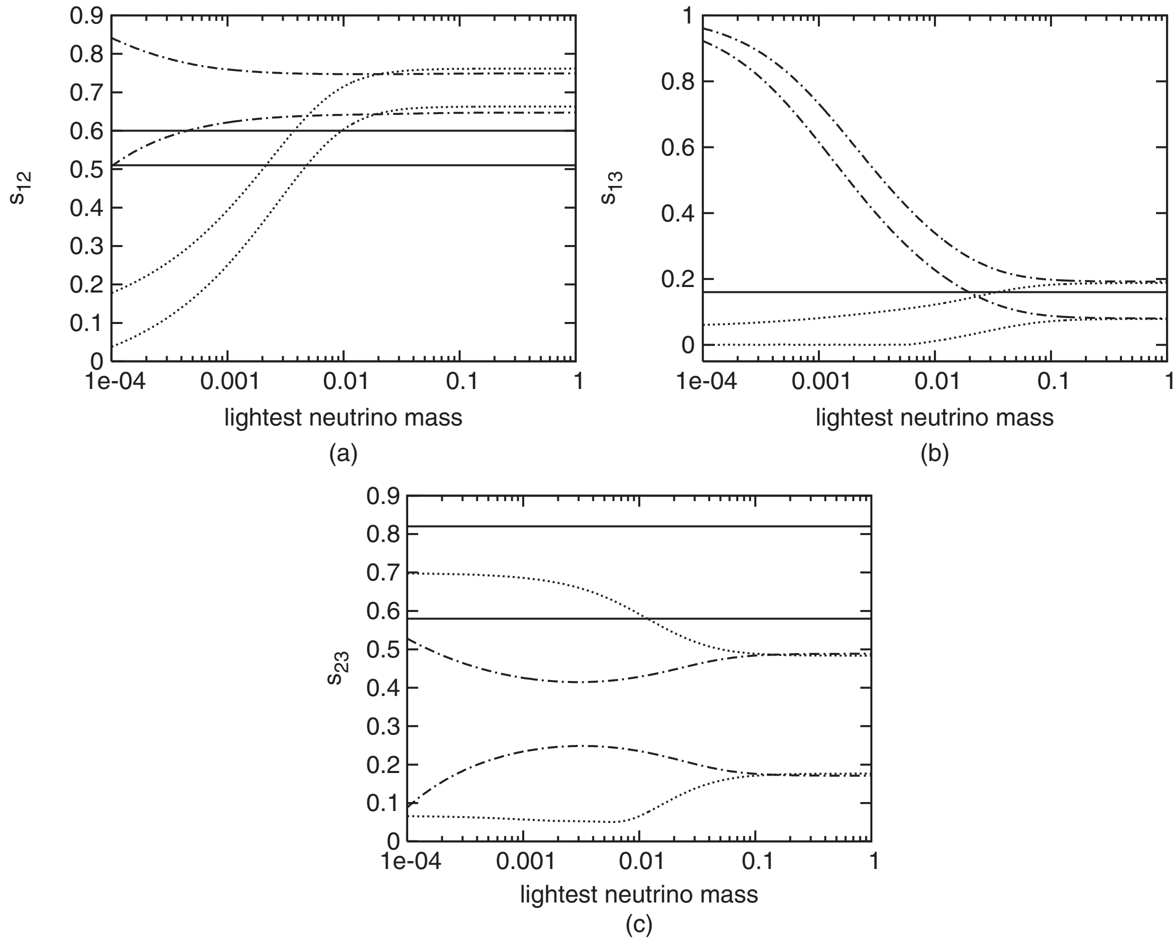

FIG. 4. Plots showing variation of mixing angles s_{12}, s_{13}, and s_{23} with lightest neutrino mass for texture six zero $D_\nu = 0$ case of Dirac neutrinos. The representations of the curves remain the same as in Fig. 1.

trino masses being degenerate, we come to case of normal hierarchy of neutrino masses. In Table I we have presented the viable ranges of neutrino masses, mixing angle s_{13}, Jarlskog's rephasing invariant parameter in the leptonic sector J_l and the Dirac-like CP violating phase in the leptonic sector δ_l. In the texture six zero case, the possibility of charged leptons being in the flavor basis is completely ruled out, therefore in the table we have presented the results corresponding to the case when M_l is considered texture specific. A general look at the table reveals several interesting points. In particular, for the texture six zero matrices, the viable ranges of masses m_{ν_1}, m_{ν_2}, and m_{ν_3} for Dirac neutrinos are quite narrow. Also, one can easily see that the range of the angle s_{13} is again quite narrow, particularly its upper limit being quite small. Therefore, a measurement of s_{13} would have direct implications for this

TABLE I. Calculated ranges for neutrino mass and mixing parameters obtained by varying ϕ_1 and ϕ_2 from 0 to 2π for the normal hierarchy cases of Dirac neutrinos. Inputs have been defined in the text. All masses are in eV.

	Six zero	Five zero $D_l = 0$ (M_l Three zero, $M_{\nu D}$ Two zero)	Five zero $D_\nu = 0$ (M_l two zero, $M_{\nu D}$ three zero)
m_{ν_1}	$\sim$0.0025	0.00032–0.0063	0.0025–0.0079
m_{ν_2}	0.0093–0.0096	0.0086–0.0112	0.0089–0.0122
m_{ν_3}	$\sim$0.0423	0.0421–0.0550	0.0422–0.0552
s_{13}	0.007–0.026	0.005–0.160	0.0001–0.135
J_l	$\sim$0–0.005	$\sim$0–0.027	$\sim$0–0.028
δ_l	3.6°– 69.2°	0°–80.2°	0°–90.0°

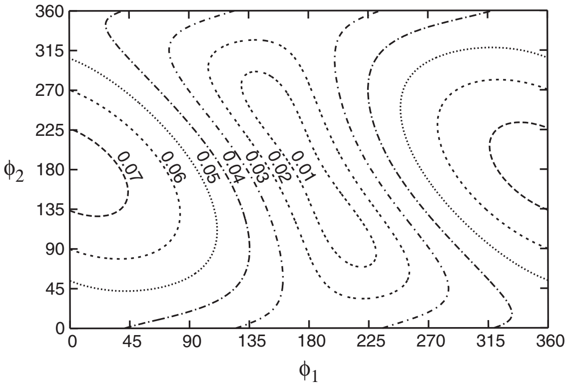

FIG. 5. The contours of s_{13} in $\phi_1 - \phi_2$ plane for texture six zero matrices for the normal hierarchy case of Dirac neutrinos.

case. Also, the range of Jarlskog's rephasing invariant parameter J_l is quite narrow for this case in comparison with its expectation from the mixing matrix [21]. For this case of texture six zero matrices, we have also examined the implications of the mixing angle s_{13} on the phases ϕ_1 and ϕ_2. In this context, in Fig. 5 we have plotted the contours for s_{13} in $\phi_1 - \phi_2$ plane. These contours indicate that the mixing angle s_{13} constrains both the phases ϕ_1 and ϕ_2. For example, if the lower limit of s_{13} around 0.07, then ϕ_1 lies in either the I or the IV quadrant and ϕ_2 lies between 135°–225°.

B. Texture five zero mass matrices

Coming to the texture five zero mass matrices, we would like to mention that out of the two possible cases, for the texture five zero $D_\nu = 0$ case, again the possibility of M_l being in the flavor basis does not yield any results. However, for the $D_l = 0$ case, both the possibilities of M_l having Fritzsch-like structure as well as M_l being in the flavor basis yield viable ranges for the various phenomenological quantities. In Table I, we have presented the results corresponding to M_l having Fritzsch-like structure. A comparison of the texture five zero $D_l = 0$ case with the above mentioned texture six zero matrices reveals several interesting points. From the table one finds that going from texture six zero to texture five zero $D_l = 0$ case, the viable range of m_{ν_1} gets much broader, in the texture six zero case it being almost a unique value. Similarly, from almost a unique value of m_{ν_3} for the texture six zero matrices, now one gets a range of m_{ν_3}. Also, it may be seen that the upper limit of s_{13} is pushed considerably higher which can be understood by noting that s_{13} is quite sensitive to variations in D_ν. Similarly, as expected the ranges of J_l and δ_l become broader as compared to the texture six zero case. The Pontecorvo-Maki-Nakagawa-Sakata (PMNS) mixing matrix [17] obtained for the texture five zero $D_l = 0$ case is given by

$$U = \begin{pmatrix} 0.7897 - 0.8600 & 0.5036 - 0.5998 & 0.0054 - 0.1600 \\ 0.1838 - 0.4748 & 0.4859 - 0.7438 & 0.5726 - 0.8194 \\ 0.3107 - 0.5633 & 0.3974 - 0.6890 & 0.5650 - 0.8142 \end{pmatrix}. \quad (21)$$

For the case of M_l being in the flavor basis, the range of masses so obtained are $m_{\nu_1} = 0.0020$–0.0040, $m_{\nu_2} = 0.0088$–0.0100, and $m_{\nu_3} = 0.0422$–0.0548. The range of the mixing angle s_{13} is 0.0892–0.1594, indicating that the lower limit of s_{13} is considerably high which implies that refinements in the measurement of this angle would have consequences for this case of texture six zero mass matrices for Dirac neutrinos.

Considering the texture five zero $D_\nu = 0$ case the possibility of M_l having Fritzsch-like structure reveals several interesting facts, as can be seen from Table I. A comparison with the earlier cases shows both the lower and upper limits of m_{ν_1} have higher values. Interestingly, for this case the lower limit of s_{13} becomes almost 0, implying that measurement of this angle can lead to interesting implications for the texture specific mass matrices considered here. Also, it may be noted that out of all the three cases, this case has the widest range of the Dirac-like CP violating phase δ_l. The PMNS matrix corresponding to this case is quite similar to the one presented in Eq. (21), except for somewhat wider ranges of the elements U_{e3}, $U_{\mu 2}$, $U_{\tau 1}$, and $U_{\tau 2}$. This can be understood by noting that D_l can take much wider variation compared to D_ν.

It may be added that in case one carries out an exercise regarding the variation of phases ϕ_1 and ϕ_2 with respect to the mixing angle s_{13}, we find results similar to the ones obtained for the texture six zero case, hence we have not presented the same.

VI. SUMMARY AND CONCLUSIONS

To summarize, for Dirac neutrinos, using Fritzsch-like texture six zero and five zero mass matrices, detailed predictions for 15 distinct possible cases pertaining to normal/inverted hierarchy as well as degenerate scenario of neutrino masses have been carried out. Interestingly, all the presently considered cases pertaining to inverted hierarchy and the degenerate scenario seem to be ruled out.

In the normal hierarchy cases, when the charged lepton mass matrix M_l is assumed to be in flavor basis, the texture six zero and the texture five zero $D_\nu = 0$ case are again ruled out. For the viable texture six zero and five zero cases, we find that the ranges of the neutrino masses m_{ν_1}, m_{ν_2}, m_{ν_3} as well as of the mixing angle s_{13} would have implications for the texture specific cases considered here. Interestingly, the lower limit of s_{13} for the texture five zero

$D_\nu = 0$ case shows an appreciable difference as compared to the lower limits of s_{13} for the texture six zero and texture five zero $D_l = 0$ cases. Similarly, the Dirac-like CP violating phase δ_l shows interesting behavior, e.g., as compared to the texture six zero case, the texture five zero cases allow comparatively a larger range of δ_l, being widest in the texture five zero $D_\nu = 0$ case. The restricted range of δ_l, in spite of full variation to phases ϕ_1 and ϕ_2, seems to be due to texture structure, hence, any information about δ_l would have important implications.

ACKNOWLEDGMENTS

The authors would like to thank S. D. Sharma and Sanjeev Kumar for useful discussions. G. A. and M. G. would like to thank DST, Government of India and DAE, BRNS for financial support, respectively. G. A. would also like to thank the Chairman, Department of Physics for providing facilities to work in the department. M. R. and R. V. would like to thank the Director, UIET and the Director, RIEIT,, respectively, for providing facilities to work.

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