Tribimaximal mixing in a viable family symmetry unified model with an extended seesaw mechanism

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(Received 3 March 2009; published 8 May 2009)

We present a grand unified model based on $SO(10)$ with a $\Delta(27)$ family symmetry. Fermion masses and mixings are fitted and agree well with experimental values. An extended seesaw mechanism plays a key role in the generation of the leptonic mixing, which is approximately tribimaximal.

DOI: 10.1103/PhysRevD.79.093001 PACS numbers: 12.15.Ff, 11.30.Hv, 12.10.−g, 14.60.Pq

I. INTRODUCTION

The observed masses and mixings of the fermions and the existence of three families of fermions are left unexplained by the standard model (SM). This is just one of many theoretical motivations for going beyond the SM, either through grand unified theory (GUT) extensions or by adding a family symmetry (FS), usually using supersymmetry (SUSY) to keep the hierarchy problem under control. Adding a FS to justify the patterns of fermion parameters is motivated by the observation of leptonic mixing consistent with, and in fact well approximated by, tribimaximal (TBM) mixing [1–6].

There are many FS models in the literature that obtain mixing that is close to TBM for the leptons, but there are relatively few that simultaneously justify the large leptonic mixing and the strong hierarchy and small mixing of the quark sector. Ambitious FS models that tackle both the lepton and quark sectors usually do so using the Froggatt-Nielsen (FN) mechanism [7] in the context of a SUSY GUT symmetry commuting with the FS [8–15]. In particular, considering an underlying $SO(10) \times G_F$ structure is highly constraining as the left-handed (LH) and right-handed (RH) SM fermions must then transform the same under G_F, to be unified consistently into a single multiplet [the 16 of $SO(10)$].

Implementing the seesaw mechanism [16–21] in a nonminimal way [22–25] requires an enlarged field content. GUT FS models use multiple familon fields to break the FS, so requiring the neutrino sector to be minimal without considering the context is not readily justified—for example, the added freedom in the neutrino sector may enable a reduction in the number of familons needed. It is thus interesting to consider the possible benefits that can be derived from combining a FS with the extended seesaw. Recently, in [26], those two ingredients are used to provide an explicit realization of the screening mechanism [27].

The SUSY GUT model we present relies on the (extended) seesaw mechanism and on a discrete FS to obtain TBM mixing. Subtly, the nonminimal structure of the neutrino sector allows some freedom in choosing the field content of the model [e.g. our model does not include any 45 representations, which are ubiquitous in other $SO(10) \times G_F$ models [9,10,13,15]].

II. THE MODEL

The superfields and their representations under the symmetry content of the model are summarized in Table I. We start with an underlying $SU(3)_F$ FS as in practice the effects of considering its discrete subgroup $\Delta(27)$ are relevant for the vacuum expectation value (VEV) alignment only (see Sec. III A).

The $U(1)_F$ charge of ϕ_O is specified such that $\phi_{23}\phi_{23}$ has the same overall charge assignment as $\phi_O\phi_O\phi_0$.

The singlets s and $\bar{s}$ enlarge the neutrino sector leading to the extended seesaw. Ψ contains the SM fermions and the RH neutrinos. The η fields Ψ_η and $\bar{\Psi}_\eta$ serve as FN-like messenger fields and behave as a fourth heavy family that mixes with the third family of the SM fermions.

The Higgs sector breaks $SO(10)$ down to the SM gauge group. The VEV of a Higgs field is generically denoted as v_A^F, the label F denoting the field and the label A denoting the $SO(10)$ breaking direction with respect to $SU(5)$. For example, v_{75}^Σ. In this notation it is useful to keep in mind the $SU(5)$ representations within the $SO(10)$ representations that contain them [Σ is a 210 of $SO(10)$, containing a 75 with respect to $SU(5)$].

The familons are $SO(10)$ singlets and break the FS when they acquire a nonvanishing VEV, and are generically denoted as ϕ_A ($\langle\phi_A\rangle$ for the corresponding VEV). The label in ϕ_0 denotes the field is an $SU(3)_F$ singlet [note it is however charged under $U(1)_F$]. The labels $A = 3$, 23, and 123 serve to identify the direction of the respective VEVs (the numbers identify which entries do not vanish), and the label in ϕ_O denotes "orthogonal" (its VEV is orthogonal to both the 23 and 123 VEVs). Specifically, the VEV directions are given by

$$\langle\phi_3\rangle \propto (0, 0, 1), \qquad \langle\phi_{23}\rangle \propto (0, 1, -1),$$
$$\langle\phi_{123}\rangle \propto (1, 1, 1), \qquad \langle\phi_O\rangle \propto (2, -1, -1). \tag{1}$$

While the Lagrangian must be invariant under $\Delta(27)$, in practice the terms allowed by the discrete FS [and not by

TABLE I. Chiral superfields and their charges. The $U(1)_F$ charges of 10s of $SO(10)$ and of the $\bar{3}$s of $SU(3)_F$ must be unique (e.g. $q_{23} \neq q_{123}$).

Matter fields	Ψ	Ψ_η	$\bar{\Psi}_\eta$	s	$\bar{s}$
$SO(10)$	16	16	$\overline{16}$	1	1
$SU(3)_F$	3	1	1	3	$\bar{3}$
$U(1)_F$	1	q_η	$-q_\eta$	q_s	$-q_s$

Familons	ϕ_{23}	ϕ_{123}	ϕ_3	ϕ_O	ϕ_0
$SO(10)$	1	1	1	1	1
$SU(3)_F$	$\bar{3}$	$\bar{3}$	$\bar{3}$	$\bar{3}$	1
$U(1)_F$	q_{23}	q_{123}	$-3q_s - 2q_{123}$	$q_{23} - q_s - q_{123}$	$2q_s + 2q_{123}$

Higgs fields	φ	φ'	$\tilde{\varphi}$	ξ	ρ	Σ
$SO(10)$	10	10	10	$\overline{16}$	$\overline{126}$	210
$SU(3)_F$	1	1	1	1	1	1
$U(1)_F$	$-2q_\eta$	$3q_s + 2q_{123} - q_\eta$	$2q_s + 2q_{123} - 2q_{23}$	$-q_s - 2q_{123}$	$-2q_{23}$	$3q_s + 2q_{123} + q_\eta$

$SU(3)_F$] require distinct messengers, and are either absent or present only at higher order such that they are strongly suppressed [the $\Delta(27)$ invariants in the real potential are also very small, but as the only terms that distinguish the VEV directions they cannot be neglected in the alignment discussion]. The Lagrangian invariant under the symmetry content in Table I is given by

$$\begin{aligned}\mathcal{L}_Y = &\frac{1}{\Lambda^2}(\phi_{23}\Psi)\rho(\phi_{23}\Psi) + \frac{1}{\Lambda^2}(\phi_{123}\Psi)\xi(\phi_{123}s)\\ &+ \frac{1}{\Lambda^3}(\phi_O\Psi)\rho(\phi_O\Psi)\phi_0 + M_s(\bar{s}s) + \frac{1}{\Lambda}(\bar{s}\Psi)\xi\phi_0\\ &+ \frac{1}{\Lambda}(\phi_3\Psi)\Sigma\bar{\Psi}_\eta + M_\eta\bar{\Psi}_\eta\Psi_\eta + \frac{1}{\Lambda^2}(\phi_O\Psi)\tilde{\varphi}(\phi_O\Psi)\\ &+ \frac{1}{\Lambda^3}(\phi_3 s)(\phi_3 s)\phi_0^2 + \frac{1}{\Lambda}(\phi_3\Psi)\varphi'\Psi_\eta + \Psi_\eta\varphi\Psi_\eta.\end{aligned} \tag{2}$$

The $U(1)_F$ assignments ensure that any undesirable terms are absent or sufficiently suppressed. Parentheses denote the $SU(3)_F$ invariant contractions $(\phi_{23}\Psi) = \phi_{23}^i\Psi_i$, with other contractions not allowed or suppressed by the messenger content [e.g. $(3 \otimes 3) \otimes (\bar{3} \otimes \bar{3})$ contractions are absent]. The nonrenormalizable terms have associated the cutoff scale Λ, assumed to be $\Lambda \sim 10^{17}$ GeV $> M_{\rm GUT} \sim 2 \times 10^{16}$ GeV. The other mass scales are the η messengers mass and the singlet masses, $M_\eta \sim M_s \sim 10^{12}$–$10^{14}$ GeV.

A. Neutrino masses and the extended seesaw

To obtain viable leptonic mixing, we aim to generalize the method described in detail in [28] to extended seesaw mechanisms. Before proceeding with this generalization, we use component notation explicitly in order to illustrate how one can achieve TBM neutrino mixing through a type I seesaw. In a type I seesaw the effective neutrino mass matrix is given in component notation by

$$(m_\nu)^{ij} = -(m_D)^{il}(M_R^{-1})_{lk}(m_D^T)^{kj}. \tag{3}$$

m_D is the neutrino Dirac matrix and M_R is the heavy RH Majorana neutrino mass matrix.

In FS models the mass matrices are typically given by some combination of the familon VEVs. Specifically in the type of model considered here the Dirac mass can be written as

$$(m_D)^{il} = \langle\phi_A^i\rangle^T\langle\phi_C^l\rangle. \tag{4}$$

We have omitted any proportionality constants that have no family index structure. The components of m_D are clearly given by the familon VEV family structure. Inserting m_D^{il} into Eq. (3) we have

$$(m_\nu)^{ij} = -\langle\phi_A^i\rangle^T(\langle\phi_C^l\rangle(M_R^{-1})_{lk}\langle\phi_C^k\rangle^T)\langle\phi_A^j\rangle. \tag{5}$$

Note that the quantity $a = \langle\phi_C^l\rangle(M_R^{-1})_{lk}\langle\phi_C^k\rangle^T$ is just a constant with no index structure, and therefore

$$(m^\nu)^{ij} = -a\langle\phi_A^i\rangle^T\langle\phi_A^j\rangle. \tag{6}$$

Unless $a = 0, \langle\phi_A\rangle$ is an eigenstate of m^ν and the details of $\langle\phi_C\rangle$ and M_R only serve to determine the corresponding eigenvalue.

Generalizing, with

$$(m_D)^{il} = a'\langle\phi_A^i\rangle^T\langle\phi_C^l\rangle + b'\langle\phi_B^i\rangle^T\langle\phi_D^l\rangle, \tag{7}$$

we have the corresponding effective neutrino matrix:

$$(m_\nu)^{ij} = -[a\langle\phi_A^i\rangle^T\langle\phi_A^j\rangle + b\langle\phi_B^i\rangle^T\langle\phi_B^j\rangle + c\langle\phi_A^i\rangle^T\langle\phi_B^j\rangle + d\langle\phi_B^i\rangle^T\langle\phi_A^j\rangle], \tag{8}$$

where a, b, c, and d are constants involving the products of the respective VEVs $\langle\phi_C\rangle$ and $\langle\phi_D\rangle$ with M_R^{-1}. From Eq. (8) we conclude that as long as $\langle\phi_A\rangle$ and $\langle\phi_B\rangle$ are orthogonal, they are both eigenstates of m_ν provided that $c = d = 0$. The natural expectation in GUT FS models is that M_R is structured similarly to m_D (in terms of being analogously formed by familon VEVs), and then one can identify which combinations of familons in M_R lead to $c = d = 0$.

After establishing how the method works for a type I seesaw, it is straightforward to apply it to the extended seesaw: if the extended seesaw gives as a result Eq. (8) with generalized a, b and $c = d = 0$ numbers we can still easily identify the eigenvectors. The difference is that instead of the type I relation [e.g. $a = \langle\phi_C^l\rangle(M_R^{-1})_{lk}\langle\phi_C^k\rangle^T$] these numbers will be in general more complicated products of the respective intervening familon VEVs and the relevant neutrino matrices of the extended seesaw. Although the following details may be somewhat complicated due to the intricacies of the GUT and of the extended seesaw, the basic idea is rather simple—we want to obtain TBM mixing in the neutrino sector directly from two orthogonal VEVs $\langle\phi_{123}\rangle$ and $\langle\phi_{23}\rangle$. In the particular realization we consider, the trimaximal eigenstate is obtained through a linear seesaw that starts from the term $(\phi_{123}\Psi)(\phi_{123}s)$ (this eigenstate has to arise through the extended seesaw as the starting term involves the singlet s). In contrast, the bimaximal eigenstate is obtained through both type I and type II seesaws resulting from $(\phi_{23}\Psi)(\phi_{23}\Psi)$. In this particular realization we also produce the orthogonal eigenstate explicitly from $(\phi_O\Psi)(\phi_O\Psi)$ (similarly to the bimaximal state).

In order to consider in detail how the seesaw proceeds, we write the full neutrino mass matrix M_ν. We do not need to consider Ψ_η mixing in the neutrino sector (the η mixing is considered in detail in Sec. II B) due to the VEVs of the Higgs sector—particularly, $\langle\Sigma\rangle$ develops only along the 75 of $SU(5)$. We start in the $(\nu, \nu^c, s, \bar{s})$ basis (each of these fields is a triplet under the FS). It is convenient to write the 12×12 M_ν as a 4×4 block matrix (each block is 3×3):

$$M_\nu = \begin{pmatrix} m_{LL} & . & . & . \\ m_{RL} & m_{RR} & . & . \\ m_{SL} & m_{SR} & m_{SS} & . \\ m_{\bar{S}L} & m_{\bar{S}R} & m_{\bar{S}S} & 0 \end{pmatrix}, \tag{9}$$

with

$$\begin{aligned} m_{LL} &= v_{15}^\rho(\langle\phi_{23}\rangle^T\langle\phi_{23}\rangle/\Lambda^2 + \langle\phi_O\rangle^T\langle\phi_O\rangle\langle\phi_0\rangle/\Lambda^3), \\ m_{RL} &= v_5^\rho(\langle\phi_{23}\rangle^T\langle\phi_{23}\rangle/\Lambda^2 + \langle\phi_O\rangle^T\langle\phi_O\rangle\langle\phi_0\rangle/\Lambda^3), \\ m_{RR} &= v_1^\rho(\langle\phi_{23}\rangle^T\langle\phi_{23}\rangle/\Lambda^2 + \langle\phi_O\rangle^T\langle\phi_O\rangle\langle\phi_0\rangle/\Lambda^3), \\ m_{SL} &= v_5^\xi\langle\phi_{123}\rangle^T\langle\phi_{123}\rangle/\Lambda^2, \\ m_{SR} &= v_1^\xi\langle\phi_{123}\rangle^T\langle\phi_{123}\rangle/\Lambda^2, \\ m_{SS} &= \langle\phi_0\rangle^2\langle\phi_3\rangle^T\langle\phi_3\rangle/\Lambda^3, \\ m_{\bar{S}L} &= v_5^\xi I\langle\phi_0\rangle/\Lambda, \\ m_{\bar{S}R} &= v_1^\xi I\langle\phi_0\rangle/\Lambda, \\ m_{\bar{S}S} &= M_s I. \end{aligned} \tag{10}$$

I is the 3×3 identity matrix. Note that $m_{SS} \propto \mathrm{Diag}(0, 0, 1)$. M_ν is symmetric and we use dots in redundant blocks. The Higgs VEVs follow the notation discussed in the introduction, with the subscript labels corresponding to the VEV that projects the appropriate components of the matter fields in each block [e.g. v_1^ρ and v_1^ξ appear in the RH neutrino blocks, correspond to $SU(5)$ singlets, and project the RH neutrino component of Ψ, ν^c].

We assume that $m_{\bar{S}S} > m_{RR} > m_{SS}, m_{SR}, m_{\bar{S}R}$. We first consider the 9×9 sub-block that leaves out the first three (ν) rows and columns and go into the basis in which $\bar{s}$ and s form a Dirac spinor. Continuing to use the 3×3 blocks defined above

$$\begin{pmatrix} m_{RR} & . & . \\ m_{SR} & m_{SS} & . \\ m_{\bar{S}R} & m_{\bar{S}S} & 0 \end{pmatrix} \tag{11}$$

becomes approximately

$$\begin{pmatrix} M_{RR} & . & . \\ 0 & M_s I + m_{SS}/2 & . \\ 0 & m_{SS}/2 & -M_s I + m_{SS}/2 \end{pmatrix}, \tag{12}$$

with

$$M_{RR} = m_{RR} + \tilde{M},$$
$$\tilde{M} = \frac{2}{M_s} m_{SR}^T m_{\bar{S}R} + \frac{2}{M_s^2}[m_{\bar{S}R}^T m_{SS} m_{\bar{S}R}].$$

Note that the combination inside square brackets, while seemingly complicated, is simply proportional to m_{SS}.

Consistently reintroducing the ν part of M_ν, we have in the new basis

$$M_\nu \simeq \begin{pmatrix} m_{LL} & . & . & . \\ m_{RL} & M_{RR} & . & . \\ (m_{SL} + m_{\bar{S}L})/\sqrt{2} & 0 & M_s I + m_{SS}/2 & . \\ (m_{SL} - m_{\bar{S}L})/\sqrt{2} & 0 & m_{SS}/2 & -M_s I + m_{SS}/2 \end{pmatrix}, \tag{14}$$

from where we can read off the 3×3 light Majorana neutrino mass matrix structure m_ν:

$$m_\nu = m_{LL} + m_{RL}^T M_{RR}^{-1} m_{RL} + \frac{2}{M_s} m_{SL}^T m_{\bar{S}L} + \frac{2}{M_s^2}[m_{\bar{S}L}^T m_{SS} m_{\bar{S}L}]. \quad (15)$$

The third term is the linear seesaw contribution arising by the extra singlets and produces the candidate trimaximal eigenstate rather trivially, as its structure is

$$\langle\phi_{123}\rangle^T\langle\phi_{123}\rangle, \quad (16)$$

as the other matrices involved in the term are proportional to I.

The first term in Eq. (15) is the type II seesaw contribution, of the form

$$a''\langle\phi_{23}\rangle^T\langle\phi_{23}\rangle + b''\langle\phi_O\rangle^T\langle\phi_O\rangle. \quad (17)$$

The Dirac mass matrix that enters in the second term of Eq. (15) presents the general structure given in Eq. (6) since from Eq. (10) we have [similarly to Eq. (7)]

$$m_{RL} = a'\langle\phi_{23}\rangle^T\langle\phi_{23}\rangle + b'\langle\phi_O\rangle^T\langle\phi_O\rangle. \quad (18)$$

The orthogonality between $\langle\phi_{23}\rangle$ and $\langle\phi_O\rangle$ ensures that the coefficients equivalent to the c and d of Eq. (8) vanish and therefore the contribution to m_ν arising by the second term presents the same structure as m_{LL}:

$$a\langle\phi_{23}\rangle^T\langle\phi_{23}\rangle + b\langle\phi_O\rangle^T\langle\phi_O\rangle. \quad (19)$$

The resulting effect is that we obtain a candidate bimaximal eigenstate, and also explicitly a candidate third eigenstate in TBM mixing (orthogonal to both the trimaximal and bimaximal eigenstates).

The last term in Eq. (15) is proportional to Diag(0, 0, 1), incompatible with TBM mixing. Fortunately it is rather suppressed through what might be thought of as a generalization of sequential dominance [28–32]—the resulting magnitude is approximately $10^{-2}\sqrt{\Delta m^2_{\rm sol}}$, and therefore we can neglect it to good approximation. The ϕ_O candidate orthogonal eigenstate is also suppressed due to $\langle\phi_0\rangle$. The ϕ_{123} and ϕ_{23} states are naturally heavier in this scheme so a normal hierarchy is predicted for the effective neutrinos.

To summarize, we concluded that the effective neutrino mass matrix is given by

$$m_\nu \simeq \alpha\langle\phi_{23}\rangle^T\langle\phi_{23}\rangle + \beta\langle\phi_{123}\rangle^T\langle\phi_{123}\rangle + \gamma\langle\phi_O\rangle^T\langle\phi_O\rangle = \begin{pmatrix} \beta+4\gamma & \beta-2\gamma & \beta-2\gamma \\ \beta-2\gamma & \alpha+\beta+\gamma & -\alpha+\beta+\gamma \\ \beta-2\gamma & -\alpha+\beta+\gamma & \alpha+\beta+\gamma \end{pmatrix}, \quad (20)$$

where for clarity we absorbed the magnitude of the VEVs such that $\langle\phi_{23}\rangle$, $\langle\phi_{123}\rangle$, and $\langle\phi_O\rangle$ have integer entries (appropriately defining α, β, and γ). This form satisfies $(m_\nu)_{11} = (m_\nu)_{22} + (m_\nu)_{23} - (m_\nu)_{13}$ and it is diagonalized by TBM mixing, becoming $m_\nu^{\rm diag} = {\rm Diag}(6\gamma, 3\beta, 2\alpha)$ with eigenvalues $6\gamma \ll 3\beta < 2\alpha$.

In order to fit the neutrino mass splitting data we need $v_1^\xi \sim 10^{10}$–10^{12} GeV, $v_1^\rho \sim 10^{12}$–10^{14} GeV and $M_s \sim 10^{14}$ GeV, for familon VEVs satisfying $\langle\phi_0\rangle/\Lambda \sim \lambda^3$ and $\langle\phi_{23}^{2,3}\rangle/\Lambda \sim \lambda$, $\langle\phi_3^3\rangle/\Lambda \sim \sqrt{\lambda}$, where λ is the Cabibbo angle. As we shall see in the next sections these values for the familon VEVs are fixed once we take into account the charged fermion spectrum. The magnitude of $\langle\phi_{123}^i\rangle$ is not tightly constrained by phenomenology as its VEV is associated always with v_5^ξ. For alignment purposes we take $\langle\phi_{123}^i\rangle$ to be large compared to the other familon VEVs (see Sec. III A).

B. Charged fermion masses

We will now describe how the charged fermion mass hierarchies are obtained and how the quark mixing is generated. The SM fermions belong to the 16s of $SO(10)$. We denote the 10 of $SU(5)$ inside Ψ and $\bar{\Psi}_\eta$, as f_i and $\bar{f}_\eta$, respectively (this notation separates e.g. the lepton doublets L_i). When ϕ_3 and Σ develop their VEVs ($\langle\Sigma\rangle = v_{75}^\Sigma$), the term $(\phi_3\Psi)\Sigma\bar{\Psi}_\eta$ in the Lagrangian [Eq. (2)] becomes

$$\bar{f}_\eta(\alpha_f v_3 v_{75}^\Sigma f_3 + M_\eta f_\eta), \quad (21)$$

defining $v_3 = \langle\phi_3^3\rangle/\Lambda$ (the nonzero VEV of the $i=3$ component of ϕ_3^i). α_f is a Clebsch-Gordan factor and we assume $v_{75}^\Sigma, \langle\phi_3^3\rangle \sim 10^{16}$ GeV $\gg M_\eta$. Because of Σ, the mixing with the η field involves only Q_i, u_i^c, and e_i^c not involving d_i^c, and also not involving L_i which keeps the η mixing from strongly affecting the leptonic mixing angles. We define the heavy and light combinations

$$f_h = s_f f_\eta + c_f f_3, \qquad f_{l_3} = -s_f f_3 + c_f f_\eta, \quad (22)$$

with

$$s_f = \frac{M_\eta}{\sqrt{\alpha_f^2 v_3^2 (v_{75}^\Sigma)^2 + M_\eta^2}}, \qquad c_f = \frac{\alpha_f v_3 v_{75}^\Sigma}{\sqrt{\alpha_f^2 v_3^2 (v_{75}^\Sigma)^2 + M_\eta^2}}. \quad (23)$$

We mentioned already that some sectors have no η mixing, with $c_L = c_{d^c} = 0$. Furthermore the mixing only involves f_3 so $f_{l_{1,2}} \equiv f_{1,2}$.

With the η mixing establishing the light states, we considering for now just the terms $\Psi_\eta\varphi\Psi_\eta + \frac{1}{\Lambda}\times(\phi_3\Psi)\varphi'\Psi_\eta$ in the Lagrangian [Eq. (2)]. The desired vacuum configuration for the Higgs multiplets φ and φ' is $\langle\varphi\rangle = v_{5,\bar{5}}^\varphi, \langle\varphi'\rangle = v_{\bar{5}}^{\varphi'}$ (i.e. φ' has no 5 VEV, just $\bar{5}$). Going to the basis defined by Eq. (22) it is easy to see that the term $\Psi_\eta\varphi\Psi_\eta$ gives rise only to the following light-state mass term:

$$c_Q c_{u^c} v_5^\varphi u_{l_3} u_{l_3}^c, \quad (24)$$

that we identify with the top quark. The Clebsch-Gordan α_f are such that $c_Q = -c_{u^c}$ [33]. On the other hand, the term $\frac{1}{\Lambda}(\phi_3\Psi)\varphi'\Psi_\eta$ gives rise to two light-state mass terms:

$$c_Q v_3 v_{\bar{5}}^{\varphi'} d_{l_3} d_{l_3}^c + c_{e^c} v_3 v_{\bar{5}}^{\varphi'} e_{l_3} e_{l_3}^c, \tag{25}$$

which we identify as the bottom and the tau, respectively. Bottom and tau unification and the hierarchy between m_t and m_b is realized with

$$c_Q \simeq -c_{e^c} \simeq 1, \qquad v_3 v_{\bar{5}}^{\varphi'}/v_5^{\varphi} \simeq 1/10, \tag{26}$$

which requires that

$$M_\eta \ll v_3 v_{75}^{\Sigma}, \qquad s_Q = s_{u^c} \simeq 3\sqrt{2}\frac{M_\eta}{v_3 v_{75}^{\Sigma}}, \qquad s_{e^c} \simeq \sqrt{2}\frac{M_\eta}{v_3 v_{75}^{\Sigma}}, \tag{27}$$

having made explicit the Clebsch-Gordan coefficients α_f. It is convenient to define $r \equiv \frac{M_\eta}{v_3 v_{75}^{\Sigma}}$. Note that a term of the kind $(\phi_A\Psi)\varphi'\Psi_\eta$ would not change the top quark mass term even if $\langle\varphi'\rangle$ had a 5 VEV—the contribution would vanish as it is proportional to $(c_Q + c_{u^c}) = 0$.

The remaining Yukawa terms contained in Eq. (2),

$$\frac{1}{\Lambda^2}(\Psi\phi_{23})\rho(\Psi\phi_{23}) + \frac{1}{\Lambda^2}(\Psi\phi_O)\tilde{\varphi}(\Psi\phi_O) + \frac{1}{\Lambda^3}(\Psi\phi_O)\rho(\Psi\phi_O)\phi_0, \tag{28}$$

contribute mass terms to the lighter generations. Both terms with ϕ_O are similar in structure and can be considered together [x_f in the following matrix—the distinct Higgs leads to family-specific factors that are different for each family as we see in Eq. (31)]. With the familon vacuum configuration $\frac{\langle\phi_{23}\rangle}{\Lambda} \equiv \alpha(0, 1, -1)$, $\frac{\langle\phi_O\rangle}{\Lambda} \equiv \epsilon(2, -1, -1)$ and $\frac{\langle\phi_0\rangle}{\Lambda} \equiv v_0$, the Dirac mass matrices present the general form

$$M_{LR}^f = \begin{pmatrix} 4x_f & -2x_f & -2s_{f^c}x_f \\ -2x_f & y_f + x_f & s_{f^c}(-y_f + x_f) \\ -2s_F x_f & s_F(-y_f + x_f) & s_F s_{f^c}(y_f + x_f) + z_f \end{pmatrix}. \tag{29}$$

x_f encodes the ϕ_O contributions, y_f the ϕ_{23} contributions, and z_f the leading order contribution to the third generation that was already discussed in detail. The desired Higgs VEV configuration is $v_5^{\tilde{\varphi}}$, $v_{\bar{5}}^{\tilde{\varphi}}$ for $\tilde{\varphi}$, while ρ develops $v_{\overline{45}}^{\rho}$, v_5^{ρ} in addition to the singlet (v_1^{ρ} as previously seen in Sec. II A). More specifically, for each charged fermion family we have

$$M_{LR}^u = \begin{pmatrix} 4\epsilon_u^2 v_5^{\tilde{\varphi}} & -2\epsilon_u^2 v_5^{\tilde{\varphi}} & -6\sqrt{2}r\epsilon_u^2 v_5^{\tilde{\varphi}} \\ -2\epsilon_u^2 v_5^{\tilde{\varphi}} & \alpha^2 v_5^{\rho} + \epsilon_u^2 v_5^{\tilde{\varphi}} & 3\sqrt{2}r(-\alpha^2 v_5^{\rho} + \epsilon_u^2 v_5^{\tilde{\varphi}}) \\ -6\sqrt{2}r\epsilon_u^2 v_5^{\tilde{\varphi}} & 3\sqrt{2}r(-\alpha^2 v_5^{\rho} + \epsilon_u^2 v_5^{\tilde{\varphi}}) & 18r^2(\alpha^2 v_5^{\rho} + \epsilon^2 v_5^{\tilde{\varphi}}) + v_5^{\varphi} \end{pmatrix},$$

$$M_{LR}^d = \begin{pmatrix} 4\epsilon_d^2 v_{\bar{5}}^{\tilde{\varphi}} & -2\epsilon_d^2 v_{\bar{5}}^{\tilde{\varphi}} & -2\epsilon_d^2 v_{\bar{5}}^{\tilde{\varphi}} \\ -2\epsilon_d^2 v_{\bar{5}}^{\tilde{\varphi}} & \alpha^2 v_{\overline{45}}^{\rho} + \epsilon_d^2 v_{\bar{5}}^{\tilde{\varphi}} & (-\alpha^2 v_{\overline{45}}^{\rho} + \epsilon_d^2 v_{\bar{5}}^{\tilde{\varphi}}) \\ -6\sqrt{2}r\epsilon_d^2 v_{\bar{5}}^{\tilde{\varphi}} & 3\sqrt{2}r(-\alpha^2 v_{\overline{45}}^{\rho} + \epsilon_d^2 v_{\bar{5}}^{\tilde{\varphi}}) & 3\sqrt{2}r(\alpha^2 v_{\overline{45}}^{\rho} + \epsilon_d^2 v_{\bar{5}}^{\tilde{\varphi}}) + v_{\bar{5}}^{\varphi'} \end{pmatrix}, \tag{30}$$

$$M_{LR}^l = \begin{pmatrix} 4\epsilon_l^2 v_{\bar{5}}^{\tilde{\varphi}} & -2\epsilon_l^2 v_{\bar{5}}^{\tilde{\varphi}} & -2\sqrt{2}r\epsilon_l^2 v_{\bar{5}}^{\tilde{\varphi}} \\ -2\epsilon_l^2 v_{\bar{5}}^{\tilde{\varphi}} & -3\alpha^2 v_{\overline{45}}^{\rho} + \epsilon_l^2 v_{\bar{5}}^{\tilde{\varphi}} & \sqrt{2}r(-3\alpha^2 v_{\overline{45}}^{\rho} + \epsilon_l^2 v_{\bar{5}}^{\tilde{\varphi}}) \\ -2\epsilon_l^2 v_{\bar{5}}^{\tilde{\varphi}} & (-3\alpha^2 v_{\overline{45}}^{\rho} + \epsilon_l^2 v_{\bar{5}}^{\tilde{\varphi}}) & \sqrt{2}r(3\alpha^2 v_{\overline{45}}^{\rho} + \epsilon_l^2 v_{\bar{5}}^{\tilde{\varphi}}) + v_{\bar{5}}^{\varphi'} \end{pmatrix},$$

noting that $r \equiv \frac{M_\eta}{v_3 v_{75}^{\Sigma}}$ appears along with the appropriate Clebsch-Gordan factors through s_{f^c} and s_F [Eq. (27)], due to η mixing. We have absorbed the complex Yukawa parameters in the Higgs scalar VEVs. We have also defined the family-specific

$$\epsilon_u^2 = \epsilon^2(1 + v_0 v_5^{\rho}/v_5^{\tilde{\varphi}}), \qquad \epsilon_d^2 \epsilon^2(1 + v_0 v_{\overline{45}}^{\rho}/v_{\bar{5}}^{\tilde{\varphi}}), \qquad \epsilon_l^2 = \epsilon^2(1 - 3v_0 v_{\overline{45}}^{\rho}/v_{\bar{5}}^{\tilde{\varphi}}), \tag{31}$$

to condense the two distinct (but similar) ϕ_O contributions encoded in x_f.

The three charged fermion mass matrices of Eq. (30) are diagonalized by

$$U_L^{f\dagger} M_{LR}^f U_R^f = \mathrm{Diag}(m_1^f, m_2^f, m_3^f), \tag{32}$$

where $U_L^{u,d}$ give us the Cabibbo-Kobayashi-Maskawa (CKM) mixing matrix in the quark sector defined as $V_{\mathrm{CKM}} = U_L^{u\dagger} U_L^d$ while U_L^l produces corrections to TBM mixing in the lepton sector. In order to recover the typical FN textures for the charged fermion we need $\epsilon v^{\tilde{\varphi}}_{5,\bar{5}} < \alpha v^{\rho}_{5,\bar{45}} < v^{\varphi}_5, v^{\varphi'}_{\bar{5}}$, so we can use $\det(M^f_{LR} M^{f\dagger}_{LR}) = (m^f_1 m^f_2 m^f_3)^2$ to get approximated expressions for the charged fermion masses:

$$\begin{aligned}(m_u, m_c, m_t) &\simeq (4\epsilon_u^2 v^{\tilde{\varphi}}_5, \alpha^2 v^{\rho}_5, v^{\varphi}_5),\\ (m_d, m_s, m_b) &\simeq (4\epsilon_d^2 v^{\tilde{\varphi}}_{\bar{5}}, \alpha^2 v^{\rho}_{\bar{45}}, v^{\varphi'}_{\bar{5}}),\\ (m_e, m_\mu, m_\tau) &\simeq (4\epsilon_l^2 v^{\tilde{\varphi}}_{\bar{5}}, 3\alpha^2 v^{\rho}_{\bar{45}}, v^{\varphi'}_{\bar{5}}).\end{aligned} \tag{33}$$

Equation (33) correctly gives bottom-tau unification and $m_\mu \simeq 3m_s$. Moreover in order to have $m_\mu/m_\tau \sim \lambda^2$ we need $\alpha \sim \lambda$ for $v^{\rho}_{\bar{45}} \sim v^{\varphi'}_{\bar{5}}$, which in turn requires $v^{\rho}_5 \sim \lambda^2 v^{\varphi}_5$ to recover $m_c/m_t \sim \lambda^4$.

The LH mixing matrices are approximately given by

$$\begin{aligned}U_L^{u\dagger} &\simeq \begin{pmatrix} 1 & \mathcal{O}(\epsilon_u^2 v^{\tilde{\varphi}}_5/(\alpha^2 v^{\rho}_5)) & \mathcal{O}(10\epsilon_u^2 r v^{\tilde{\varphi}}_5/v^{\varphi}_5) \\ -\mathcal{O}(\epsilon_u^2 v^{\tilde{\varphi}}_5/(\alpha^2 v^{\rho}_5)) & 1 & \mathcal{O}(\alpha^2 v^{\rho}_5/v^{\varphi}_5) \\ -\mathcal{O}(10\epsilon_u^2 r v^{\tilde{\varphi}}_5/v^{\varphi}_5) & -\mathcal{O}(\alpha^2 v^{\rho}_5/v^{\varphi}_5) & 1 \end{pmatrix},\\ U_L^{d\dagger} &\simeq \begin{pmatrix} 1 & \mathcal{O}(10\epsilon_d^4 v^{\tilde{\varphi}}_{\bar{5}}/(\alpha^4 v^{\rho}_{\bar{45}})) & \mathcal{O}(\epsilon_d^2 v^{\tilde{\varphi}}_{\bar{5}}/v^{\varphi}_{\bar{5}}) \\ -\mathcal{O}(10\epsilon_d^4 v^{\tilde{\varphi}}_{\bar{5}}/(\alpha^4 v^{\rho}_{\bar{45}})) & 1 & \mathcal{O}(\alpha^2 v^{\rho}_{\bar{45}}/v^{\varphi'}_{\bar{5}}) \\ -\mathcal{O}(\epsilon_d^2 v^{\tilde{\varphi}}_{\bar{5}}/v^{\varphi}_{\bar{5}}) & -\mathcal{O}(\alpha^2 v^{\rho}_{\bar{45}}/v^{\varphi'}_{\bar{5}}) & 1 \end{pmatrix},\\ U_L^{l\dagger} &\simeq \begin{pmatrix} 1 & \mathcal{O}(\epsilon_l^2 v^{\tilde{\varphi}}_{\bar{5}}/(3\alpha^2 v^{\rho}_{\bar{45}})) & \mathcal{O}(10 r\epsilon_l^2 v^{\tilde{\varphi}}_{\bar{5}}/v^{\varphi'}_{\bar{5}}) \\ -\mathcal{O}(\epsilon_l^2 v^{\tilde{\varphi}}_{\bar{5}}/(3\alpha^2 v^{\rho}_{\bar{45}})) & 1 & \mathcal{O}(3\alpha^2 v^{\rho}_{\bar{45}}/v^{\varphi'}_{\bar{5}}) \\ -\mathcal{O}(10 r\epsilon_l^2 v^{\tilde{\varphi}}_{\bar{5}}/v^{\varphi}_{\bar{5}}) & -\mathcal{O}(3\alpha^2 v^{\rho}_{\bar{45}}/v^{\varphi'}_{\bar{5}}) & 1 \end{pmatrix}.\end{aligned} \tag{34}$$

Note that U_L^d and U_L^l have rather different 12 entries—the orthogonality between $\langle\phi_{23}\rangle$ and $\langle\phi_O\rangle$ cancels the contribution proportional to $\alpha^2\epsilon^2$ in the entry 12 of $M^d_{LR}M^{d\dagger}_{LR}$, but the η mixing of the third family of the RH leptons enables a $\alpha^2\epsilon^2$ contribution in the 12 of $M^l_{LR}M^{l\dagger}_{LR}$.

With $v^{\rho}_{\bar{45}} \sim v^{\varphi'}_{\bar{5}}$ (previously chosen when fitting m_μ/m_τ) we automatically get from M^d_{LR} the correct magnitude for θ^d_{23} in Eq. (34):

$$\theta^d_{23} \sim \theta^{\rm CKM}_{23} \sim \lambda^2. \tag{35}$$

In order to fit the light family masses and the Cabibbo angle it is necessary that the latter arises from U^d_L. Remembering that $v_0 = \langle\phi_0\rangle/\Lambda \sim \lambda^3$ from the neutrino sector, we need $v^{\tilde{\varphi}}_5 \sim \lambda^2 v^{\rho}_5$, $v^{\tilde{\varphi}}_{\bar{5}} \sim (\lambda^3 - \lambda^2) v^{\rho}_{\bar{45}}$ and $\epsilon_u \sim \epsilon_d \sim \epsilon_l \sim \lambda$. Moreover, since $r \ll 1$, even $\theta^{\rm CKM}_{13}$ arises mainly by U^d_L. By substituting the values indicated into the expressions for $U_L^{u,d,l}$ given in Eq. (34) we get

$$\begin{aligned}&\theta^{\rm CKM}_{12} \sim \lambda, \qquad \theta^{\rm CKM}_{13} \sim \mathcal{O}(\lambda^5 - \lambda^4), \qquad \theta^l_{12} \sim \mathcal{O}(\lambda^2/3),\\ &\theta^l_{23} \sim \mathcal{O}(\lambda^2), \qquad \theta^l_{13} < \mathcal{O}(\lambda^5),\\ &\frac{m_u}{m_t} \sim \mathcal{O}(\lambda^6), \qquad \frac{m_d}{m_b} \sim \mathcal{O}(\lambda^4),\end{aligned} \tag{36}$$

where $\theta^l_{12,23,13}$ are the deviations of $U_L^{l\dagger}$ from the identity. From Eq. (36) we can estimate the amount of shifting of the lepton mixing from exact TBM mixing. At order $\mathcal{O}(\lambda^2)$ we get

$$U_{\rm lep} = \begin{pmatrix} \sqrt{\frac{2}{3}} - \frac{\lambda^2}{3\sqrt{6}} & \frac{1}{\sqrt{3}} + \frac{\lambda^2}{3\sqrt{3}} & -\frac{\lambda^2}{3\sqrt{2}} \\ -\frac{1}{\sqrt{6}} - (\sqrt{\frac{2}{27}} + \sqrt{\frac{3}{2}})\lambda^2 & \frac{1}{\sqrt{3}} - \frac{8\sqrt{3}}{9}\lambda^2 & -\frac{1}{\sqrt{2}} + \frac{3\sqrt{2}}{2}\lambda^2 \\ -\frac{1}{\sqrt{6}} + \sqrt{\frac{3}{2}}\lambda^2 & \frac{1}{\sqrt{3}} - \sqrt{3}\lambda^2 & \frac{1}{\sqrt{2}} + \frac{3\sqrt{2}}{2}\lambda^2 \end{pmatrix}, \tag{37}$$

that gives

$$\sin\theta^2_{12} = \frac{1}{3} + \frac{2}{9}\lambda^2 + \mathcal{O}(\lambda^4), \qquad \sin\theta^2_{23} = \frac{1}{2} - 3\lambda^2 + \mathcal{O}(\lambda^4), \qquad \sin\theta^2_{13} = \mathcal{O}(\lambda^4). \tag{38}$$

The comparison between the analytical expressions we get with the neutrino fit data [34] shows that we are inside the $2-\sigma$ range for all three angles.

Finally, the degeneracy between the down quark and the electron mass is solved by having $\epsilon^l \neq \epsilon^d$ and therefore m_e can be correctly fitted.

III. VACUUM ALIGNMENT

In the previous sections we assumed that $SO(10)$ is broken directly to $SU(3) \times SU(2) \times U(1)$ through the VEV of the 75 of $SU(5)$ contained in Σ and the VEVs of the SM singlets contained in ξ and ρ. In addition we assumed that $\langle\Sigma\rangle \gg \langle\xi\rangle, \langle\rho\rangle$ to recover the correct neutrino mass matrix and the absolute neutrino mass scale. The construction of the superpotential that reaches the correct breaking pattern goes beyond the purpose of this work. However it can be obtained using established strategies already applied in the literature [35–40]. Since we break $SO(10)$ directly to the SM the GUT scale of the model coincides with the $SU(5)$ one, that is, $M_{\rm GUT} \sim 2 \times 10^{16}$ GeV. However the presence of the 210 with respect to the minimal $SO(10)$ GUT model [38] and the requirement of preserving the gauge coupling pertubativity forces the model cutoff scale Λ to be approximately 10^{17}, a few orders below the usual one. In principle familons and Higgs scalars could have different cutoff scales, Λ_G and Λ_F, respectively, but we assume that they coincide ($\Lambda_G = \Lambda_F = \Lambda$).

Familon alignment

The pattern of VEVs displayed in Eq. (1) plays a crucial role in our model, and we now discuss how to obtain it. A relatively simple way to obtain the desired relies on the use of a discrete non-Abelian subgroup of $SU(3)_F$ [alternatively, in $SU(3)_F$ it is possible to obtain the pattern by adding several alignment fields, as in [9]]. The alignment mechanism we use is based on Δ (27), belonging to the $\Delta(3n^2)$ family of groups [41], and the method proposed here is rather similar to the one originally presented in [10]. Higher-order invariant terms can arise in the scalar potential through SUSY breaking soft terms and break the degeneracy of VEVs that would exist in the continuous group—these invariants are allowed by the discrete FS [but not by $SU(3)_F$]. These terms are very small but are the only terms that distinguish VEV directions and so must be considered in the alignment discussion. On the other hand, the Yukawa superpotential is approximately invariant under $SU(3)_F$: the higher-order terms can be neglected compared to the terms allowed by the continuous FS, such that the Lagrangian is given by Eq. (2) to good approximation.

As discussed in Sec. II, some of the familons acquire VEVs with larger magnitudes (namely, ϕ_{123}, but also ϕ_3). The leading D terms for these familons leads to a potential

$$V(\phi_A) = \alpha_A m^2 \sum_i |\phi_A^i|^2 + \beta_A m^2 \left|\sum_i |\phi_A^i|^2\right|^2 + \gamma_A m^2 \sum_i |\phi_A^i|^4, \tag{39}$$

where m is the gravitino mass. These are soft terms that arise only if SUSY is broken (which is why m^2 appears on every term). The coefficient α_A is radiatively driven negative near the scale Λ, triggering a VEV for ϕ_A. The second term is generated at one-loop order if the superpotential contains a term of the form $Y\Xi\sum_i \phi_A^i \chi_i$, where Y is a FS singlet, with χ_i (charged under the FS) and Ξ being massive chiral superfields (that go in the loop). The two first terms in Eq. (39) are invariant under the continuous group $SU(3)_F$ and, with α_A negative, generate $\langle\phi_A\rangle$ with a constant nonzero magnitude x of the order of Λ. The third term breaks $SU(3)_F$ but is consistent with $\Delta(27)$. It will be generated if the underlying theory contains a superpotential term of the form $Z\sum_i \phi_A^i \varphi^i \varphi^i$, where Z is a singlet of $\Delta(27)$ and φ^i is a massive chiral superfield (that goes in the loop) with the appropriate FS assignments. The resulting third term of Eq. (39) splits the vacuum degeneracy. The minimum for γ_A positive has $|\langle\phi_A^i\rangle| = x(1,1,1)/\sqrt{3}$ while for γ negative $|\langle\phi_A^i\rangle| = x(0,0,1)$ (the nonzero entry defines the preferred direction). The phases are unspecified as these terms do not establish any preferred phase.

This provides a mechanism to generate the vacuum alignment of ϕ_3 and ϕ_{123} as each will have a potential of the form in Eq. (39), provided they acquire large VEVs. The structure of Eq. (1) results if γ_3 is positive and γ_{123} is negative (and by definition $\langle\phi_3\rangle$ lies in the third direction). In order for the correct alignment to be reached, the terms featuring just the respective familon need to dominate over similar quartic terms mixing separate familons which may be present (e.g. $\phi_{123}^i \phi_{3_i}^\dagger \phi_3^j \phi_{123_j}^\dagger$ or $\phi_{23}^i \phi_{3_i}^\dagger \phi_3^j \phi_{23_j}^\dagger$). For this reason the magnitudes of $\langle\phi_{123}\rangle$ and $\langle\phi_3\rangle$ are required by naturalness to be somewhat larger than $\langle\phi_{23}\rangle$ and $\langle\phi_O\rangle$, which arise at a scale slightly smaller than Λ.

For ϕ_{23} to receive the correct alignment, we need to introduce an additional familon ϕ_1 which receives a large VEV of order Λ (just like ϕ_{123} and ϕ_3), with positive γ_1 and taking a direction which we define to be the first—$|\langle\phi_1^i\rangle| = x(1,0,0)$ (to justify why $\langle\phi_1\rangle$ and $\langle\phi_3\rangle$ have distinct directions, the mixed quartic terms involving ϕ_3 and ϕ_1 must favor their VEVs to be orthogonal by having a positive coefficient). The terms responsible for aligning ϕ_{23} in the desired direction arise just like the β quartics of Eq. (39), but are naturally dominant over the unmixed β_{23} term: the dominant terms must be $\beta_1' m^2 \phi_1^i \phi_{23_i}^\dagger \phi_{23}^j \phi_{1_j}^\dagger$ and $\beta_{123}' m^2 \phi_{123}^i \phi_{23_i}^\dagger \phi_{23}^j \phi_{123_j}^\dagger$. A positive β_1' term favors $\langle\phi_{23}^1\rangle = 0$. A positive β_{123}' term leads to the orthogonality of VEVs of ϕ_{123} and $\phi_{23}^\dagger$.

We introduce also an alignment field X. Because of the symmetry content, the only superpotential term directly relevant to our alignment purposes is $X\sum_i \phi_{23}^i \phi_{23}^i \phi_{23}^i$ [X has $U(1)_F$ of $-3q_{23}$]. This invariant is allowed by $\Delta(27)$ and the corresponding F term produces the vacuum condition $\sum_i (\phi_{23}^i)^3 = 0$. This condition is only satisfied for specific relative phases of the $\langle\phi_{23}\rangle$ components—the cube of the entries must close a triangle in the complex plane. With the soft terms favoring a nonvanishing VEV with

$\langle\phi_{23}^1\rangle = 0$, it is in fact a degenerate triangle and we conclude that one of the discrete sets of possible solutions is $\langle\phi_{23}^2\rangle = -\langle\phi_{23}^3\rangle$. In this case the correct orthogonality condition is obtained from β'_{123}, fixing the relative phases, with only the global phases remaining unknown.

Finally ϕ_O is aligned correctly if the dominant terms governing its alignment are $\beta''_1 m^2 \phi_1^i \phi_{O_i}^\dagger \phi_O^j \phi_{1_j}^\dagger$ with negative β''_1, and $\beta''_{123} m^2 \phi_{123}^i \phi_{O_i}^\dagger \phi_O^j \phi_{123_j}^\dagger$ with positive β''_{123}.

IV. CONCLUSION

In this paper we studied some of the possibilities provided by considering an extended seesaw scenario in a family symmetry grand unified model, presenting a specific case with phenomenologically viable fermion masses and mixings.

Neutrino mixing is tribimaximal from the combination of a specific realization of the extended seesaw mechanism with a specific vacuum alignment configuration (directly related to the structure of the discrete non-Abelian family symmetry used). The charged lepton mixing angles are small and produce slight deviations from tribimaximal mixing.

The charged fermion mass terms produce a structure that can fit the mass hierarchies and the CKM mixing angles (consistently with preserving near tribimaximal leptonic mixing, as described above).

The model is fairly complicated, with a large field content. However it demonstrates the potential benefits of considering extended seesaw realizations in this class of unified models with a family symmetry. In the model presented, the Higgs content is relatively less constrained: the phenomenologically required separation of the neutrino sector from the charged fermions—despite their unification in the same multiplet—is relatively easy to achieve by using a slightly enlarged matter content.

ACKNOWLEDGMENTS

The work of F. B. has been partially supported by MEC-Valencia MEC Grant No. FPA2008-00319/FPA, by European Commission Contracts No. MRTN-CT-2004-503369 and No. ILIAS/N6 RII3-CT-2004-506222 and by the foundation for Fundamental Research of Matter (FOM) and the National Organization for Scientific Research (NWO). The work of I. d. M. V. was supported by FCT under Grant No. SFRH/BPD/35919/2007. The work of I. d. M. V. was partially supported by FCT through the projects POCI/81919/2007, CERN/FP/83503/2008 and CFTP-FCT UNIT 777 which are partially funded through POCTI (FEDER) and by the Marie Curie RTN MRTN-CT-2006-035505. I. d. M. V. would like to thank IFIC-UV (Valencia) for their warm hospitality and support during a short stay when this project was initiated.

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