

Eötvös branes

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The high value of brane tension has a crucial role in recovering Einstein's general relativity at low energies. In the framework of a recently developed formalism with variable brane tension, one can pose the question of whether it was always that high. In analogy with fluid membranes, in this paper we allow for temperature-dependent brane tension, according to the corresponding law established by Eötvös. For cosmological branes this assumption leads to several immediate consequences: (a) The brane universe was created at a finite temperature T_c and scale factor a_{\min} . (b) Both the brane tension and the four-dimensional gravitational coupling "constant" increase with the scale factor from zero to asymptotic values. (c) The four-dimensional cosmological constant evolves with a , starting with a huge negative value, passing through zero, finally reaching a small positive value. Such a scale-factor-dependent cosmological constant is able to generate a surplus of attraction at small a (as dark matter does) and a late-time repulsion at large a (dark energy). In the particular toy model discussed here, the evolution of the brane tension is compensated by energy interchange between the brane and the fifth dimension, such that the continuity equation holds for the cosmological fluid. The resulting cosmology closely mimics the standard model at late times, a decelerated phase being followed by an accelerated expansion. The energy absorption of the brane drives the five-dimensional space-time towards maximal symmetry, becoming anti-de Sitter.

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I. INTRODUCTION

Physics aims for a unified description of nature, tracing back all physical laws to four fundamental interactions. While three of them are quantized, and to certain extent further unified, gravity is still best described classically. In contrast with the rest of the interactions about evolving fields on a flat background, gravity is perceived as the dynamics of geometry. String theory attempts to unify all interactions on different grounds, its basic objects being open or closed strings and higher-dimensional objects, called branes. The codimension one brane world theory (generalizing the early Randall-Sundrum model [1]) carries the original geometric spirit of general relativity, incorporating arbitrary curvature and matter (for a review see [2]). The extra dimension is both noncompact and curved (the remaining dimensions required by string and M-theory can still be thought of as compactified). Gravity acts in five dimensions according to Einstein's equation with a negative cosmological constant $\tilde{\kappa}^2 \tilde{\Lambda}$, while standard model fields are confined to the brane, a time-evolving three-dimensional spacelike hypersurface.

The projection of the five-dimensional (5D) Einstein equation onto the brane gives an effective Einstein equation, which in the most generic case reads [3]

$$G_{ab} = -\Lambda g_{ab} + \kappa^2 T_{ab} + \tilde{\kappa}^4 S_{ab} - \tilde{\mathcal{E}}_{ab} + \tilde{L}_{ab}^{TF} + \tilde{\mathcal{P}}_{ab}, \quad (1)$$

with T_{ab} , the brane energy-momentum tensor; S_{ab} , a quadratic expression in T_{ab} ; and $\tilde{\mathcal{E}}_{ab}$, the electric part of the 5D Weyl tensor with respect to the brane normal [4], averaged over the two sides of the brane. The source term \tilde{L}_{ab}^{TF} originates in the asymmetric embedding of the brane and $\tilde{\mathcal{P}}_{ab}$ is the pullback of generic nonstandard model fields in five dimensions [3]. The four-dimensional (4D) and gravitational coupling constants κ^2 and $\tilde{\kappa}^2$ are related as $6\kappa^2 = \tilde{\kappa}^4 \lambda$, with λ the brane tension. The 4D cosmological "constant" Λ , apart from contributions of the asymmetric embedding and nonstandard model 5D fields, is defined as $2\Lambda_0 = \kappa^2 \lambda + \tilde{\kappa}^2 \tilde{\Lambda}$.

Reference [3] gives the most generic form of the gravitational dynamics involving asymmetric embedding [5] and nonstatic 5D space-time due to radiation fields [6–15]. Besides cosmological applications in branes embedded into 5D black hole space-times [16,17] or into their horizon regions [18,19] other aspects of brane world models have been discussed, including black hole brane worlds [20–22], gravitational collapse on the brane [23–28], stellar models [29–31], galactic dynamics [32], the dynamics of clusters of galaxies [33], light deflection [34,35], and solar system tests [36].

A classical fluid membrane needs tension to exist. Similarly its higher-dimensional counterpart, the 3-brane, as it evolves, remains a hypersurface due to the brane tension. The strongest bound on the minimal value of λ was derived by combining the results of tabletop experiments on possible deviations from Newton's law, which probe gravity at submillimeter scales [37] with the known

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value of the 4D Planck constant. In the 2-brane model [38] this gives [11] $\lambda > 138.59 \text{ TeV}^4$. A much milder limit $\lambda \gtrsim 1 \text{ MeV}^4$ arises from the constraint that the dominance of S_{ab} ends before the big bang nucleosynthesis (BBN) [39]. From astrophysical considerations on brane neutron stars an intermediate value $\lambda > 5 \times 10^8 \text{ MeV}^4$ was derived [29]. (All these limiting values are for $c = 1 = \hbar$. In units $c = 1 = G$ the corresponding minimal values of the brane tension are $\lambda_{\text{tabletop}} = 4.2 \times 10^{-119} \text{ eV}^{-2}$, $\lambda_{\text{BBN}} = 3 \times 10^{-145} \text{ eV}^{-2}$, and $\lambda_{\text{astro}} = 1.5 \times 10^{-136} \text{ eV}^{-2}$, respectively [28].) For typical stellar densities the condition $\rho_{\text{star}}/\lambda \ll 1$ is obeyed with any of these bounds.

In a cosmological context the brane represents our observable Universe. Cosmic expansion is realized through the movement of the brane in the warped extra dimension. During cosmological evolution the temperature of the brane changes drastically. Cosmological branes cool down from a very hot early universe (whose thermal radiation is able to create a black hole in the fifth dimension [6]), to the present day's low temperature of the cosmic microwave background.

In Sec. II we explore the possibility of a variable brane tension, discussed in detail in Ref. [40], by introducing a toy model, in which the brane tension literally follows the temperature dependence of the fluid tensions established for membranes. Such a model becomes particularly simple when we assume the continuity equation. We discuss the emerging cosmological model in Sec. III. A numerical solution of this toy model, which is compatible with cosmological observations, is presented in Sec. IV. Finally, Sec. V contains the concluding remarks.

Throughout the paper we follow the notations of Ref. [40].

II. EÖTVÖS BRANES

How justified is it to assume a constant brane tension during cosmological evolution, which spans over such a wide range of temperatures? The tension of classical membranes depends on temperature, according to Eötvös' law [41]

$$\lambda_{\text{fluid}} = K(T_c - T), \quad (2)$$

K being a constant and T_c a critical temperature representing the highest temperature for which the membrane exists.

Motivated by this analogy, the covariant gravitational dynamics on the brane were analyzed in detail for variable tension brane worlds, with the brane asymmetrically embedded into the 5D space-time, both the latter and the brane containing arbitrary sources [40]. After establishing the covariant dynamics on the brane in a generic setup, a specialization to a cosmological situation was presented, considering a Friedmann brane asymmetrically embedded into a 5D space-time—containing radiation.

As a first attempt to discuss in more detail the consequences of a temperature-dependent brane tension, we adopt Eötvös' law

$$\lambda = \lambda_{\text{lt}} - \frac{6l}{\tilde{\kappa}^4 a}, \quad (3)$$

where we have employed the standard relation $T \propto a^{-1}$. We have also denoted $KT_c = \lambda_{\text{lt}}$ and written the constant in the second term in a suitable form, such that the 4D coupling constant takes the simple expression

$$\kappa^2 = \kappa_{\text{lt}}^2 - \frac{l}{a}, \quad (4)$$

with

$$\kappa_{\text{lt}}^2 = \frac{\tilde{\kappa}^4 \lambda_{\text{lt}}}{6}. \quad (5)$$

The subscript ‘‘lt’’ refers to late time, as the second terms of both λ and κ^2 go to zero with $a \rightarrow \infty$.

Such a brane with temperature-dependent tension cannot exist below the scale factor $a_{\text{min}} = l/\kappa_{\text{lt}}^2$: first, because the tension would become negative, leading to the destruction of the brane; and second, because the gravitational constant would also become negative below this limit, leading to antigravity on the brane. In terms of a_{min} the 4D coupling constant and the brane tension can be conveniently expressed as

$$\kappa^2 = \kappa_{\text{lt}}^2 \left(1 - \frac{a_{\text{min}}}{a}\right), \quad (6)$$

$$\lambda = \lambda_{\text{lt}} \left(1 - \frac{a_{\text{min}}}{a}\right). \quad (7)$$

Both increase from zero to their asymptotic late-time values (Fig. 1). We also note that

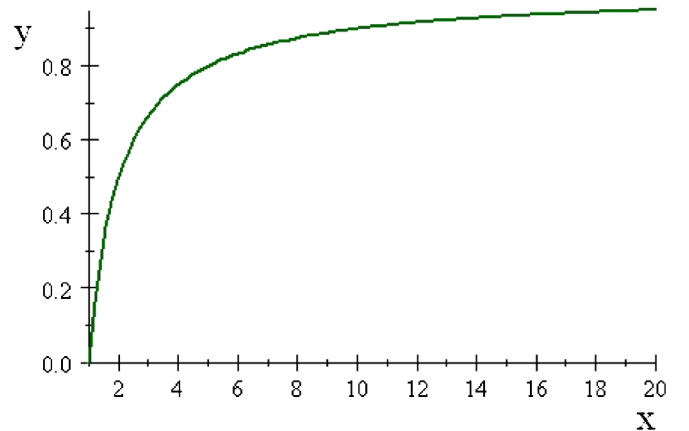


FIG. 1 (color online). The gravitational constant and brane tension normalized to their late-time values ($\kappa^2/\kappa_{\text{lt}}^2 = \lambda/\lambda_{\text{lt}}$) represented as functions of $x = a/a_{\text{min}}$. Both the gravitational constant and brane tension are zero at scale factor a_{min} and asymptotically increase to their late-time values as $a \rightarrow \infty$.

$$\tilde{\kappa}^4 = \frac{6\kappa^2}{\lambda} = \frac{6\kappa_{\text{lt}}^2}{\lambda_{\text{lt}}}. \quad (8)$$

As the brane tension increases with scale factor according to the Eötvös law, we may call this model an Eötvös brane world. The limits derived for the brane world tension from nucleosynthesis constraints in the case of an Eötvös brane refer to the value of the brane tension at the time of nucleosynthesis, and consequently, its present day value is higher than for constant tension branes.

The Λ_0 contribution to the 4D cosmological constant evolves according to

$$\Lambda_0 = \Lambda_{\text{lt}} - \kappa_{\text{lt}}^2 \lambda_{\text{lt}} \frac{a_{\text{min}}}{a} \left(1 - \frac{a_{\text{min}}}{2a}\right), \quad (9)$$

(Fig. 2) with its present day (late-time) value given by

$$2\Lambda_{\text{lt}} = \kappa_{\text{lt}}^2 \lambda_{\text{lt}} + \tilde{\kappa}^2 \tilde{\Lambda}. \quad (10)$$

As can be seen from Fig. 2, when the brane is formed at temperature T_c , the contribution Λ_0 to the cosmological constant is negative

$$\Lambda_c = \Lambda_{\text{lt}} - \frac{\kappa_{\text{lt}}^2 \lambda_{\text{lt}}}{2} = \frac{\tilde{\kappa}^2 \tilde{\Lambda}}{2} < 0. \quad (11)$$

Then, as the brane world universe cools down with increasing a , the factor in the second term of Eq. (9) obeys

$$\frac{d}{da} \left[\frac{a_{\text{min}}}{a} \left(1 - \frac{a_{\text{min}}}{2a}\right) \right] = -\frac{a_{\text{min}}}{a^2} \left(1 - \frac{a_{\text{min}}}{a}\right) < 0. \quad (12)$$

Therefore the second term of Eq. (9) is positive, resulting in an increasing Λ_0 throughout the cosmological evolution,

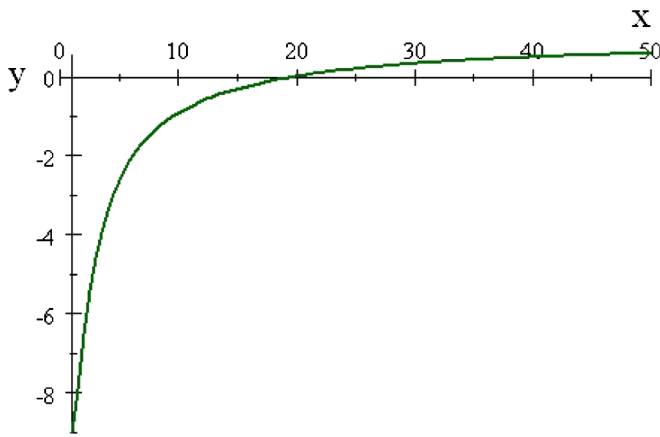


FIG. 2 (color online). The cosmological constant normalized to its late-time value ($\Lambda_0/\Lambda_{\text{lt}}$), represented as a function of $x = a/a_{\text{min}}$ for the parameter value $L = 1 + \tilde{\kappa}^2 \tilde{\Lambda}/\kappa_{\text{lt}}^2 \lambda_{\text{lt}} = 2\Lambda_{\text{lt}}/\kappa_{\text{lt}}^2 \lambda_{\text{lt}} = 0.1$ (The parameter L obeys the inequalities $0 < L < 1$ due to the negativity of $\tilde{\Lambda}$ and the positivity of Λ_{lt} .) The represented normalized cosmological constant starts at high negative values, then it becomes positive, increasing asymptotically to 1 as $x \rightarrow \infty$.

from $\Lambda_c < 0$ to a positive Λ_{lt} for $a \rightarrow \infty$, obeying $\Lambda_{\text{lt}} \ll -\Lambda_c$.

These features imply the following modifications on the physics of the early brane world universe, first discussed in Ref. [42]: (a) brane world effects for an Eötvös brane are more dominant than for a constant tension brane, due to the initial smallness of the brane tension (this also implies that the typical brane world source term ρ^2/λ , arising from S_{ab} , dominates for a longer time); (b) due to the initial smallness of κ^2 , gravity is initially quite weak; and (c) the huge negative value of the cosmological constant generates an apparent gravitational attraction.

In order to have a small Λ_{lt} , the values of λ_{lt} and $\tilde{\Lambda}$ have to be almost perfectly fine tuned. As the astrophysical lower limit refers to λ_{lt} , we can safely assume the usual high negative value for the 5D cosmological constant $\tilde{\kappa}^2 \tilde{\Lambda}$. Consequently, the initial 4D cosmological constant Λ_c and its late-time value Λ_{lt} obey $-\Lambda_c \gg \Lambda_{\text{lt}}$. Therefore the 4D cosmological constant at early times represents a huge contribution in the balance of sources.

III. COSMOLOGY

According to the Stefan-Boltzmann law the energy density of the cosmic microwave background (which defines the temperature T) is proportional to the fourth power of T , and further, according to the assumption $T \propto a^{-1}$, to a^{-4} . This is possible only if the continuity equation holds.

Because of the variable brane tension and possible existence of a nonstandard model energy-momentum tensor \tilde{T}_{cd} in the fifth dimension, the energy density of the cosmological fluid, however, obeys a more sophisticated balance equation [40]:

$$\dot{\rho} + 3\frac{\dot{a}}{a}(\rho + p) = -\dot{\lambda} + \Delta(u^c n^d \tilde{T}_{cd}). \quad (13)$$

Here u is the 4-velocity of the fluid flow lines, Δ denotes the difference taken on the right and left sides of the brane, while a dot represents the derivative with respect to cosmological time τ . Note that the normal vectors on the two sides of the brane are $n_R = n$ and $n_L = -n$; therefore the second term on the right-hand side of Eq. (13) can be nonvanishing even in the symmetric case. In order to have a continuity equation on the brane, the condition

$$\dot{\lambda} = \Delta(u^c n^d \tilde{T}_{cd}) \quad (14)$$

should hold. For an expanding (collapsing) universe $\dot{\lambda} = (d\lambda/da)\dot{a} > 0$ (< 0), therefore $\Delta(u^c n^d \tilde{T}_{cd}) > 0$ (< 0) as well, corresponding to a brane absorbing energy from (radiating energy into) the 5D space-time.

Any 5D radiation field (in the geometrical optics approximation) has nonvanishing projection $u^c n^d \tilde{T}_{cd}$ [3], such that

$$\Delta(u^c n^d \tilde{T}_{cd}) = \frac{3}{\tilde{\kappa}^2 a^3} \sum_{I=L,R} \epsilon_I (-1)^{n_I} \beta_I \dot{v}_I^2 \quad (15)$$

at the location of the brane $r = a$. Here the function

$$\beta(v) = \epsilon \frac{dm}{dv} \quad (16)$$

is a measure of the (linear) energy density of radiation, v is a null coordinate, and $m(v)$ the mass function of the 5D space-time. The 5D space-time is Vaidya–anti-de Sitter (VAdS5), with line element

$$d\bar{s}^2 = -f(v, r)dv^2 + 2\epsilon dvdr + r^2[d\chi^2 + \chi^2(d\theta^2 + \sin^2\theta d\phi^2)], \quad (17)$$

where for a spatially flat brane

$$f(v, r) = -\frac{2m(v)}{r^2} - \frac{\tilde{\kappa}^2 \tilde{\Lambda}}{6} r^2. \quad (18)$$

The radiation is ingoing (towards $r = 0$) for $\epsilon = 1$ and outgoing for $\epsilon = -1$, while η takes the value 1 if the region contains $r = 0$, and 0 otherwise. (The null coordinate v is outgoing for $\epsilon = 1$ and ingoing for $\epsilon = -1$.) Therefore, for an energy-absorbing brane the following combinations are allowed: $(\eta = 1, \epsilon = -1)$ and $(\eta = 0, \epsilon = 1)$, thus $(-1)^\eta \epsilon = 1$, while for a radiating brane either $(\eta = 1, \epsilon = 1)$ or $(\eta = 0, \epsilon = -1)$ hold, thus $(-1)^\eta \epsilon = -1$. The derivatives \dot{v} and \dot{r} are related as [40]

$$f\dot{v} = \epsilon\dot{r} + (-1)^{\eta+1}(\dot{r}^2 + f)^{1/2}, \quad (19)$$

the sign of \dot{v} being given by $(-1)^{\eta+1}$. From Eqs. (7) and (14) and the relation between β and m , by defining $(dm/dv)\dot{v} = \dot{m}$ (which allows for the introduction of $m(\tau)$ in the equation), finally inserting the expression (19) evaluated on the brane in place of the remaining factor \dot{v} , we find

$$2a\dot{a} = \frac{\tilde{\kappa}^2}{\kappa_{\text{lt}}^2 a_{\text{min}}} \sum_{I=L,R} \dot{m}_I \frac{\epsilon_I (-1)^{\eta_I} \dot{a} - (\dot{a}^2 + f_I)^{1/2}}{f_I}. \quad (20)$$

We chose the simplest case of a symmetrically embedded brane. By employing the identity

$$\epsilon(-1)^\eta \dot{a} - (\dot{a}^2 + f)^{1/2} = -f[\epsilon(-1)^\eta \dot{a} + (\dot{a}^2 + f)^{1/2}]^{-1}, \quad (21)$$

Eq. (20) can be rewritten as

$$\frac{\dot{m}}{a^2} = -\left(\frac{\kappa_{\text{lt}}^2 \lambda_{\text{lt}}}{6}\right)^{1/2} \frac{a_{\text{min}}}{a} \dot{a} [\epsilon(-1)^\eta \dot{a} + (\dot{a}^2 + f)^{1/2}]. \quad (22)$$

It is remarkable that the above equation depends only on the combined sign $\epsilon(-1)^\eta$. For a brane in the $f > 0$ region Eq. (22) implies $\dot{m} \dot{a} < 0$. Indeed, the brane should absorb (emit) radiation during expansion (contraction), and consequently the mass of the bounded 5D region decreases (increases). The positivity of the radiation energy density $0 < \beta(v) = \epsilon dm/dv = \epsilon \dot{m} \dot{v}^{-1}$ implies $\epsilon(-1)^{\eta+1} \text{sgn}(\dot{m}) > 0$, confirming $\epsilon(-1)^\eta = 1$ during expansion and $\epsilon(-1)^\eta = -1$ during contraction.

The Friedmann equation [3],

$$\frac{\dot{a}^2}{a^2} = \frac{\Lambda_0}{3} + \frac{\kappa^2 \rho}{3} \left(1 + \frac{\rho}{2\lambda}\right) + \frac{2m}{a^4}, \quad (23)$$

is not affected by the assumption of a variable brane tension [40], and can be used to eliminate \dot{a} from the right-hand side of Eq. (22). By inserting the a -dependent expressions of λ , κ^2 , and Λ_0 , the Friedmann equation becomes

$$\frac{\dot{a}^2}{a^2} = \frac{\Lambda_{\text{lt}}}{3} + \frac{\kappa_{\text{lt}}^2 \rho}{3} \left(1 + \frac{\rho}{2\lambda_{\text{lt}}}\right) + \frac{2m}{a^4} - \frac{\kappa_{\text{lt}}^2 \lambda_{\text{lt}}}{3} \frac{a_{\text{min}}}{a} \left(1 + \frac{\rho}{\lambda_{\text{lt}}} - \frac{a_{\text{min}}}{2a}\right). \quad (24)$$

The last term represents first and second order corrections in a_{min}/a to the constant tension brane world Friedmann equation. We have checked that the Raychaudhuri equation and twice-contracted Bianchi identity are consequences of Eqs. (13) and (24).

IV. NUMERICAL SOLUTION

The continuity equation gives $\rho = \rho_c (a_{\text{min}}/a)^n$ with $n = 3$ for matter and $n = 4$ for radiation. Here ρ_c is the density at the creation of the brane. Then, by denoting $\mathcal{T}^2 = 6/\kappa_{\text{lt}}^2 \lambda_{\text{lt}}$, we introduce the following dimensionless variables:

$$L = \frac{\Lambda_{\text{lt}} \mathcal{T}^2}{3}, \quad R = \frac{\rho_c}{\lambda_{\text{lt}}}, \quad (25)$$

$$x = \frac{a}{a_{\text{min}}}, \quad y = \frac{m \mathcal{T}^2}{a_{\text{min}}^4}, \quad t = \frac{\tau}{\mathcal{T}}.$$

The evolution [given by the system of Eqs. (22) and (24)] of the dimensionless variables x and y , in terms of the dimensionless time parameter t [the derivative with respect to t being denoted by a prime, and employing Eqs. (10) and (18) in the process], becomes

$$x'^2 = 1 - 2x + Lx^2 + \frac{R}{x^{n-2}} \left(2 - \frac{2}{x} + \frac{R}{x^n}\right) + \frac{2y}{x^2}, \quad (26)$$

$$\frac{y'}{x} = \epsilon(-1)^{\eta+1} x'^2 - x' \left[x'^2 + (1-L)x^2 - \frac{2y}{x^2} \right]^{1/2}. \quad (27)$$

The variable x increases from 1 and its present day value is $x_0 = z_{\text{max}} + 1 \gg z_{\text{BBN}} \approx 4.26 \times 10^9$ (where z_{max} corresponds to a_{min}). The parameters of the model obey

$$0 < L \ll 1 \quad (28)$$

(from the positivity and smallness of Λ_{lt} , compared to any of the $\kappa_{\text{lt}}^2 \lambda_{\text{lt}}$, $-\tilde{\kappa}^2 \tilde{\Lambda}$). From the dominance at present day of the ρ term over the correction terms containing a_{min}/a and over the $\rho^2/2\lambda_{\text{lt}}$ term of the Friedmann equation we obtain

$$x_0^2 \ll R \ll x_0^3. \quad (29)$$

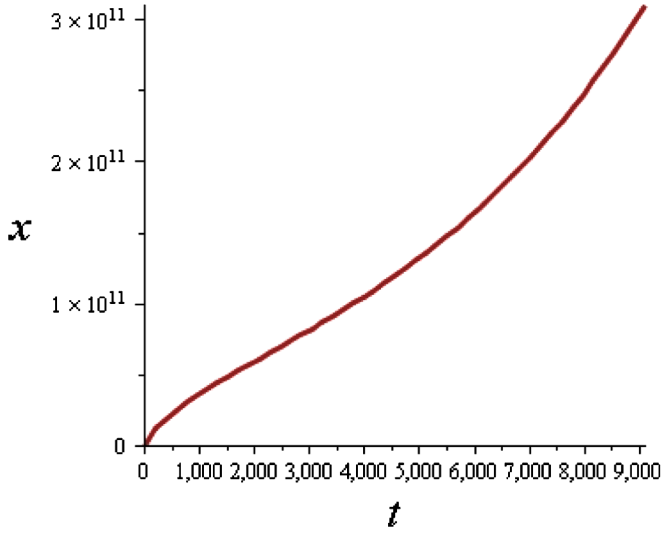


FIG. 3 (color online). The evolution of the scale factor for a matter dominated universe. The initial decelerated expansion is followed by an ever-accelerating phase. (Plot for $R = 10^{25}$, $y_c = 10^9 R$, and $L = 4R/x_0^3$, with the inflection point at about $x_0 \approx 10^{11}$.)

Since today there is approximately twice as much dark energy (represented by Λ_{H}) as matter,

$$L \approx 4R/x_0^3. \quad (30)$$

The present day contribution of the mass term to the Hubble expansion being also small [14], the condition

$$y_0 \ll Rx_0 \quad (31)$$

holds.

Numerical integration in this range of parameters gives an expanding universe, with an initial decelerated phase followed by an accelerated expansion (Fig. 3). Because of the energy absorption of the brane, the mass of the VAdS5 region decreases (Fig. 4). For the chosen parameters at approximately 3 times the time when the dominance of Λ_{H} over matter begins, the mass $m(\tau)$ will reach zero. With no mass left, the VAdS5 regions reduce to patches of 5D anti-de Sitter (AdS5) space-time and the expansion on the brane continues in a de Sitter phase.

V. CONCLUDING REMARKS

The variation of the brane tension introduces an additional degree of freedom in brane world models. The particular model discussed here, assuming the Eötvös law for the temperature dependence of the brane tension, balanced by the energy interchange between the brane and VAdS5 (such that the continuity equation holds), resulted

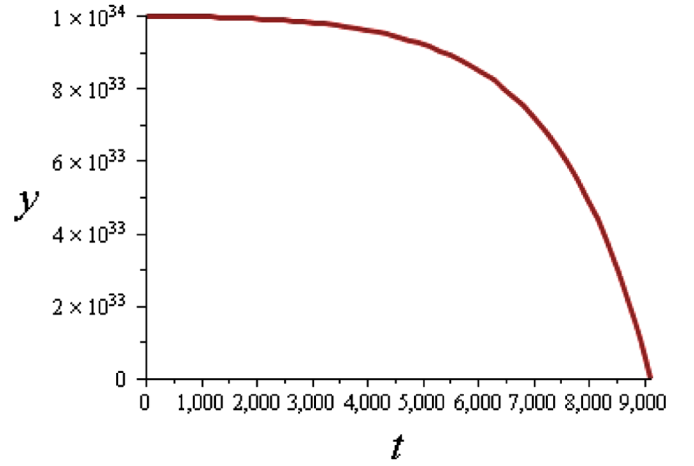


FIG. 4 (color online). The mass function of the 5D space-time decreases until reaching maximal symmetry (AdS5). Parameters as for Fig. 3.

in a monotonic increase with scale factor of the brane tension, gravitational coupling constant, and 4D cosmological constant. In the early universe both the brane tension and the 4D gravitational coupling constant are small, enhancing the dominance of brane world effects. The temperature-dependent 4D cosmological constant, being negative for small values of the scale factor, contributes to mutual attraction, while positive for large a , generates dark-energy-type repulsion.

We established the range of the model parameters allowed by the confrontation with observations, given by Eqs. (28)–(31). A particular configuration obeying these conditions, with $R = 10^{25}$, $x_0 = 10^{11}$, $y_0 = 10^{34}$ was represented in Figs. 3 and 4. For the allowed range the evolution of the fundamental constants basically occur in the very early universe preceding BBN, after which they asymptote to constant values. Still, fed by absorbed energy from the VAdS5 regions, they slightly evolve. This process eventually will consume $m(\tau)$, leaving maximally symmetric AdS5 space-time patches on the two sides of the brane, which further expands in a de Sitter phase.

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