

Minimal simple de Sitter solutions

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We show that the minimal set of necessary ingredients to construct explicit, four-dimensional de Sitter solutions from IIA string theory at tree level are O_6 -planes, nonzero Romans mass parameter, form fluxes, and negative internal curvature. To illustrate our general results, we construct such minimal simple de Sitter solutions from an orientifold compactification of compact hyperbolic spaces. In this case there are only two moduli and we demonstrate that they are stabilized to a sufficiently weakly coupled and large volume regime. We also discuss generalizations of the scenario to more general metric flux constructions.

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I. INTRODUCTION

Among the most surprising and puzzling discoveries in modern physics is the apparent acceleration of our current universe [1,2]. These striking cosmological observations, together with the associated conceptual issues in quantum gravity, have fueled a decade of studies of de Sitter (dS) space from string theory. By now, different strategies in constructing metastable de Sitter vacua have been suggested in various string theory limits [3–12]. Such continuing efforts in scenario building also lend support to the picture of a string landscape [13–15] realizing, in an interesting microphysical way, Weinberg’s earlier insight on the cosmological constant problem [16].

In light of the observational evidence for an accelerating universe and the need for a concrete formulation of a dS/CFT (conformal field theory) correspondence [17], it is of pragmatic importance to construct *explicit* de Sitter solutions from string theory. Attempts to construct fully explicit examples are severely hampered by a myriad of moduli. For example, in many constructions such as [3], nonperturbative effects which are difficult to explicitly compute are often invoked to stabilize moduli. Although in other setups, purely “perturbative” ingredients, e.g., fluxes, are shown to be sufficient in stabilizing all geometric moduli [18], such ingredients only lead us to an anti-de Sitter (AdS) minimum. Additional supersymmetry breaking localized sources, such as anti-D-branes [3], KK5-branes, and/or NS5-branes [10,19], are then intro-

duced to uplift the vacuum energy.¹ Other than backreaction of these extended objects, the explicit supersymmetry breaking sources introduced by such uplifting branes make it hard to analyze and control the corrections to the moduli stabilizing potential. It thus remains a major challenge to construct fully explicit and controllable de Sitter vacua from string theory, especially ones which admit not only a 4D effective field theory description but can be analyzed at the level of 10D supergravity equations of motion.

Recently, an interesting scenario was suggested in [10] by cleverly combining the virtues of [4,6] and of [18]. The strategy of using manifolds with negative scalar curvature to generate a positive energy density explored in [4,6] was maintained, but with the internal space replaced by a simpler compactification (more precisely, a particular twisted torus being a product of two Nil 3-manifolds [10]), such that the machineries developed for toroidal orientifolds in [18] can be easily generalized and applied. It was further argued that with additional ingredients including KK5-branes and discrete Wilson lines which are supported on such internal spaces, a metastable de Sitter vacuum with small cosmological constant can be obtained. Furthermore, this construction has already motivated new inflationary scenarios which can give rise to detectable

¹One can consider, instead of branes, uplifting by 4D effective field theory ingredients such as a D-term [20]. However, such an uplifting D-term vanishes in the absence of an F-term [21] so one still needs additional ingredients for completeness of the model.

tensor modes [22]. In view of these potential applications, it is of interest to understand what really makes such constructions tick.

In this paper, we determine what is the minimal set of ingredients that is truly necessary for the construction of metastable de Sitter vacua. Upon closer investigation, we found simpler ways to construct such solutions without invoking the aforementioned localized uplifting sources and discrete Wilson lines. For simplicity, we use hyperbolic spaces as an example to illustrate that such *minimal simple de Sitter solutions* exist. Hyperbolic spaces admit no deformations other than an overall rescaling of their sizes, making it easier to search for vacua. In principle, one can extend our analysis to other negatively Ricci curved compactifications which admit deformations other than the omnipresent dilaton and volume moduli. In fact, such extension may be useful for constructing de Sitter vacua whose energy scale of supersymmetry breaking is parametrically below the compactification scale.² However, these additional moduli typically lead to new runaway directions unless the scalar potential has the right moduli dependence whose criteria we will briefly sketch below. We leave the search for such examples for future work.

Various no-go theorems exist for the construction of stable de Sitter vacua [19], based on the consideration of two-dimensional slices in the full moduli space parametrized by the volume and the dilaton moduli of the compactification.³ By revisiting the assumptions made in the arguments, we find that the minimal set of ingredients needed to add to the usual Ramond (RR) and Neveu-Schwarz (NSNS) fluxes and O6/D6 sources to construct metastable dS vacua in type IIA string theory are geometric fluxes and a nonvanishing Romans parameter.

The simplicity of our constructions also motivates us to go beyond the schematic mechanisms demonstrated in [10], and to carry out fully explicit computations for complete models. Our explicit systematic analysis also makes it clear what the minimal ingredients are that are needed to stabilize all moduli, and the role each ingredient (KK5-branes, discrete Wilson lines, etc.) plays in forbidding new runaway directions.

The minimalism of our de Sitter solutions has several advantages. First of all, the fact that only three-level ingredients are invoked makes it easier to calculate from first principle the moduli stabilizing potential, including numerical factors. As we will also demonstrate, the moduli are stabilized to a sufficiently large volume, weak coupling regime so it is self-consistent to ignore higher corrections. Secondly, the absence of uplifting branes frees us from the concern of their backreaction which is notoriously difficult to compute. The only backreaction in our model is coming

from the O6-plane. The backreaction of the fluxes is incorporated in the 4D effective theory in the reductions we consider here. Finally, the simplicity and explicitness of our solutions enable us to analyze the system directly from the perspective of the 10D equations of motion without the crutch of 4D effective field theories.

II. SCALAR POTENTIAL BY DIMENSIONAL REDUCTION

We will dimensionally reduce massive IIA supergravity, with the action in the string frame given by (in the conventions of [10])

$$S = \frac{1}{2\kappa_{10}^2} \int e^{-2\phi} \left(\star \mathcal{R} + 4 \star d\phi \wedge d\phi - \frac{1}{2} \star H_3 \wedge H_3 \right) - \star F_2 \wedge F_2 - \star F_4 \wedge F_4 - \star m^2 + \text{CS} + (\text{sources}), \quad (1)$$

where $2\kappa_{10}^2 = (2\pi)^7 (\alpha')^4$ and the field strengths are defined as

$$H_3 = dB_2, \quad (2)$$

$$F_2 = dC_1 + mB_2, \quad (3)$$

$$F_4 = dC_3 - C_1 \wedge H_3 - \frac{m}{2} B \wedge B, \quad (4)$$

and the Chern-Simons (CS) term reads

$$- dC_3 \wedge dC_3 \wedge B_2 + \frac{m}{3} B \wedge B \wedge B \wedge dC_3 - \frac{m^2}{20} B \wedge B \wedge B \wedge B \wedge B. \quad (5)$$

In this paper we are working in the supergravity (plus localized sources such as D-branes and O-planes) limit; for explicit solutions, we can and will explicitly check whether this assumption is valid, finding that we are indeed in the large volume and (marginally) small string coupling limit, justifying our usage of the tree-level IIA supergravity action and its dimensionally reduced effective potential (1). We will be calling this the tree-level limit, in contrast to other moduli stabilization techniques such as KKLT [3] which require explicit 4-dimensional nonperturbative effects.

The dimensional reduction of the action (1) leads to several terms which contribute to the effective 4-dimensional scalar potential.

$$V = V_{\text{metric}} + V_3^{\text{NS}} + \sum_p V_p^{\text{RR}} + V_{\text{O6}} + V_{\text{D6}} + V_{\text{NS5}} + V_{\text{KK5}}, \quad (6)$$

where we schematically denoted the contributions coming from the metric flux, the B-field flux, the RR fluxes, space-filling O6-planes, D6-branes, KK5-, and NS5-branes. Before delving into the details of these contributions to

²For hyperbolic spaces which are rigid, supersymmetry is broken at the compactification scale.

³See also [23] for more advanced no-go theorems in the effective field theory approach.

the potential, let us consider some general properties of the potential (6) and its capacity for de Sitter vacua.

We are largely interested in two (real, dimensionless) moduli, the volume modulus ρ and the dilaton τ , defined by

$$\rho \equiv (\text{Vol})^{1/3}, \quad \tau \equiv e^{-\phi}(\text{Vol})^{1/2}. \quad (7)$$

Additional moduli exist, but depend on the specific model under consideration.

Recently, it was shown that a “no-go theorem” exists for inflation and de Sitter vacua in IIA string theory [19]. The no-go theorem proves that for a *vanilla* subset of the possible contributions to the scalar potential in (6) (namely, NSNS fluxes, RR fluxes, and O6/D6 sources) there is a bound on the derivatives of the potential,

$$-\rho \frac{\partial V}{\partial \rho} - 3\tau \frac{\partial V}{\partial \tau} = 9V + \sum_p p V_p \geq 9V. \quad (8)$$

For inflation, this leads to a bound on the slow roll parameter $\epsilon \geq \frac{27}{13}$ whenever $V > 0$. For vacua, we find from (8) that $V = -(\sum_p p V_p)/9$. As long as $V_p > 0$, we cannot obtain de Sitter vacua in these models.

A simple way to check for de Sitter vacua in IIA models by simple inspection of the scalar potential is to arrange the contributions to the potential in the following way:

$$V = a(\rho, M)\tau^{-2} - b(\rho, M)\tau^{-3} + c(\rho, M)\tau^{-4}. \quad (9)$$

The quantity $a(\rho, M)$ contains contributions from the curvature of the internal manifold, NS 3-form flux, NS 5-branes, and KK 5-branes,

$$a(\rho, M) = \frac{\tilde{C}_f(M)}{\rho} + \frac{\tilde{A}_{\text{KK5}}(M)}{\rho} + \frac{\tilde{A}_{\text{NS5}}(M)}{\rho^2} + \frac{\tilde{A}_{H3}(M)}{\rho^3}. \quad (10)$$

The quantity $b(\rho, M)$ contains contributions from O6-planes and D6-branes,

$$b(\rho, M) = +n_{\text{O6}}f(M) - n_{\text{D6}}g(M), \quad (11)$$

where f and g are functions of moduli different from ρ and τ . The quantity $c(\rho, M)$ contains contributions from the RR fluxes (and by extension, fractional Wilson lines),

$$c(\rho, M) = \rho^3 \tilde{m}^2 + \rho \tilde{A}_2(M) + \frac{\tilde{A}_4^{\text{elec}}(M)}{\rho} + \frac{\tilde{A}_6(M)}{\rho^3}. \quad (12)$$

As described in [10] and discussed in more detail in the Appendix, finding de Sitter vacua with small cosmological constant (cc) is as easy as using the a , b , c quantities to search for critical points of

$$\frac{4ac}{b^2} \approx 1. \quad (13)$$

In particular, writing

$$\frac{4ac}{b^2} = 1 + \delta(\rho, M) \quad (14)$$

and denoting minimized quantities with a subscript 0, for $\delta_0 \approx 0$ we have a vacuum solution with positive vacuum energy

$$V_{\min} \approx \left(\frac{b_0}{2c_0}\right)^4 c_0 \delta_0. \quad (15)$$

It is straightforward to show the no-go theorem for de Sitter vacua using this formalism. In particular, restricting only to NSNS and RR fluxes and O6/D6 sources we find that the critical quantity takes the form

$$\frac{4ac}{b^2} = (\text{const}) \sum_p \rho^{-p} \tilde{A}_p(M). \quad (16)$$

It is clear that the minimum of (16) in the ρ direction is a runaway, $\rho \rightarrow \infty$, with $4ac/b^2 \rightarrow 0$. Thus, de Sitter vacua cannot exist with these ingredients.

In order to evade the no-go theorem of [19] we need to introduce different energy sources with different functional dependence on the moduli (ρ , τ). Ideally, one would like to not include *all* possible additional sources—it would be helpful to know what is the minimal set of additional ingredients needed in order to allow for de Sitter vacua.

Let us allow the possibility of nonzero, negative curvature of the internal space in the scalar potential. Indeed, we find now that the no-go theorem of [19] does not apply. More precisely, the critical quantity (13) becomes

$$\frac{4ac}{b^2} = (\text{const}) \sum_p \tilde{A}_p [\tilde{C}_f \rho^{2-p} + \tilde{A}_{H3} \rho^{-p}]. \quad (17)$$

For simplicity, let us consider the coefficients \tilde{A}_p , \tilde{C}_f , and \tilde{A}_{H3} to be pure constants, independent of any other moduli. We now see that (17) no longer has a runaway potential for ρ as long as $\tilde{A}_p \neq 0$ for $p < 2$, so including negative internal curvature and nonzero $p < 2$ flux are the minimal additional ingredients needed for de Sitter vacua. In IIA the latter statement translates into a requirement that the IIA Romans mass parameter is nonzero (in IIB this requirement suggests that RR F_1 flux is a necessary ingredient), so we have a minimal set of requirements for de Sitter vacua in IIA: In order to build de Sitter vacua at tree level in IIA, in addition to the usual RR and NSNS fluxes and O6/D6 sources, one must minimally have negative curvature spaces and nonzero Romans parameter.

A simple intuitive way to understand this result is to investigate the behavior of the potential (9) as a function of τ , as shown in Fig. 1. Without the negative curvature, the potential has an AdS minimum. Adding in the negative curvature acts as an uplifting term (slightly shifting the minimum of the potential), lifting the AdS minimum to a dS one. Clearly, we see a limitation on the amount of negative curvature we can turn on in these models and still obtain stable de Sitter vacua—for too large of the curva-

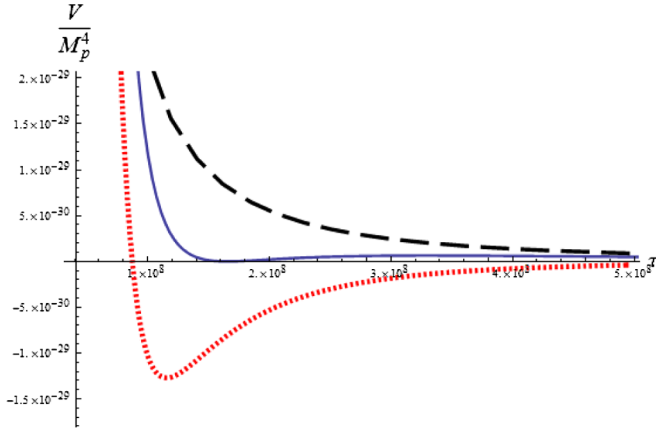


FIG. 1 (color online). Simple stable de Sitter solutions (solid line) can be constructed by uplifting a stable AdS minimum (dotted line) by negative curvature (dashed line).

ture, the de Sitter minimum of Fig. 1 disappears and the potential has a runaway to a Minkowski vacuum at $\tau \rightarrow \infty$. Notice that the argument given above did not require that the solution is in the large volume and weak coupling regime—that is an additional constraint that must be imposed upon candidate solutions and depends on the details of the construction.

Below we will discuss a simple model based on the compact 3-hyperboloid, which has constant negative curvature. Since the compact 3-hyperboloid is rigid, it only has an overall scale modulus ρ ; we will see that indeed stable dS vacua can be found for this model by minimizing the quantity (17).

III. DE SITTER VACUA FOR HYPERBOLIC SPACE

We will be considering massive IIA supergravity with sources. While it is not yet clear how massive IIA supergravity emerges from a perturbative string theory description (see [24] for some discussion), we will assume that such a supergravity limit exists. For some interesting recent work on a string/M theory description of the Romans parameter see [25].

We will start with the action (1) with the string frame metric

$$ds_{10}^2 = g_{\mu\nu}^{(s)} dx^\mu dx^\nu + g_{mn} dy^m dy^n \quad (18)$$

$$= g_{\mu\nu}^{(s)} dx^\mu dx^\nu + \alpha' \rho d\tilde{s}_6^2, \quad (19)$$

where we factored out the overall breathing mode ρ of the internal space,

$$\alpha' \rho \equiv \left[\frac{\int \sqrt{g_6}}{\int \sqrt{\tilde{g}_6}} \right]^{1/3} \quad (20)$$

and we take the internal space to be the product of two identical compact maximally symmetric 3-hyperboloids,

$$d\tilde{s}_6^2 = d\mathbb{H}_3^2(\Lambda) + d\mathbb{H}_3^2(\Lambda). \quad (21)$$

The 3-hyperboloid with curvature $\mathcal{R} = -\Lambda$ has the metric

$$d\mathbb{H}_3^2(\Lambda) = \frac{6}{\Lambda} (d\varphi^2 + \sinh^2(\varphi) d\Omega_2^2), \quad (22)$$

where φ runs over the real line. Algebraically, this space is the coset $\text{SO}(3,1)/\text{SO}(3)$. We will compactify the space by using discrete $\text{SO}(3,1)$ identifications. By rescaling ρ we can always take $\Lambda = 1$.⁴

The dimensionless volume of the internal space associated with $d\tilde{s}_6^2$ is a constant, determined by the details of the compactification of the space,

$$\tilde{V}_6 = \int_{(\mathbb{H}_3 \times \mathbb{H}_3)/\mathbb{Z}_2} \epsilon_3 \wedge \tilde{\epsilon}_3 = \frac{e^{2\alpha}}{2}, \quad (23)$$

where we defined the volume elements on the 3-hyperboloids as $\epsilon_3, \tilde{\epsilon}_3$, respectively, and we denoted the volume of the individual 3-hyperboloids as $e^\alpha \geq 1$. The number α is discrete and bounded from below. It is related to the specific discrete identifications one can make [26]. In the integration domain we inserted a \mathbb{Z}_2 from the O6-plane involution. Similarly, we will define the total physical volume of the internal space as

$$V_6 \equiv \int \sqrt{g_6}. \quad (24)$$

Clearly, from (20) the modulus ρ is related to these volumes as

$$\alpha' \rho = (V_6/\tilde{V}_6)^{1/3}. \quad (25)$$

Finally, we will define the dimensionless field τ as

$$\tau \equiv e^{-\phi} \rho^{3/2}, \quad (26)$$

where $e^{-\phi} = g_s^{-1}$ is the 10-dimensional string coupling which appears in (1).

The dimensional reduction of the 10-dimensional Ricci scalar with the ansatz (19) leads to the noncanonical form for the 4D Ricci scalar,

$$\int d^4x \sqrt{g_4^{(s)}} \left(\frac{\tau^2 \alpha'^3 \tilde{V}_6}{2\kappa_{10}^2} \mathcal{R}_4^{(s)} + \dots \right), \quad (27)$$

where \dots includes terms depending on the curvature of the internal space. In order to bring the 4-dimensional curvature term into canonical Einstein-Hilbert form $S_{\text{EH}} = \int \sqrt{|g|} 1/2 M_{\text{pl}}^2 \mathcal{R}_4$ we will make a conformal transformation $g_{\mu\nu}^{(s)} = (\tau/\tau_0)^{-2} g_{\mu\nu}^{(E)}$, where τ_0 is the stabilized value of τ in the vacuum. Altogether, this gives a 10D metric in a 4D Einstein frame and canonical 4D Planck mass as

$$ds_{10}^2 = (\tau/\tau_0)^{-2} g_{\mu\nu}^{(E)} dx^\mu dx^\nu + \rho \alpha' d\tilde{s}_6^2, \quad (28)$$

⁴The overall curvature of the internal space in units of α' equals $-2/\rho$.

$$M_p^2 = \frac{\tilde{V}_6 \alpha'^3 \tau_0^2}{\kappa_{10}^2} = \frac{V_{6,0}}{\kappa_{10}^2 g_{s,0}^2}, \quad (29)$$

with $V_{6,0}$ and $g_{s,0}$ are the stabilized values of the total internal volume and 10D string coupling, respectively.

We further consider an O-plane spacetime action R that mirrors one 3-hyperboloid to the other:

$$(z_1, z_2, z_3, \tilde{z}_1, \tilde{z}_2, \tilde{z}_3) \leftrightarrow (\tilde{z}_1, \tilde{z}_2, \tilde{z}_3, z_1, z_2, z_3). \quad (30)$$

This corresponds to an O6-plane that is 4D space filling and wraps the three-cycle in the internal manifold formed by the submanifold Σ_3^S invariant under the orientifold action:

$$\Sigma_3^S = (z_1, z_2, z_3, z_1, z_2, z_3). \quad (31)$$

A. Scalar potential

Dimensionally reducing (1) on the above space leads to a scalar potential for the moduli. Since the space is maximally symmetric there are no deformations of the space aside from the overall volume modulus ρ .⁵ Further, there are no nontrivial 2- or 4-cycles, so we cannot turn on F_2 or F_4 flux in the internal space, nor can we have flux moduli descending from the gauge potentials. Thus, we see that *there are only two moduli in this model, ρ and τ* . As we will see, it is due to the simplicity of this background that we will easily find stable de Sitter solutions.

In our conventions the scalar potential V is defined as the object in the 4D action that appears as follows:

$$S = \int dx_4 \sqrt{g_4} \left(\frac{M_p^2}{2} R_4 - \frac{M_p^2}{2} G_{ij} \partial \phi^i \partial \phi^j - V(\phi) \right), \quad (32)$$

where we took the scalars to be dimensionless.

The potential energy contribution from the curvature is obtained by simply reducing the internal curvature part of the 10D Ricci scalar, and going to the 4D Einstein frame,

$$V_{\text{CURV}} = \frac{M_p^2 \tau_0^2}{\alpha'} \tau^{-2} \rho^{-1}. \quad (33)$$

In massive IIA, the 0-form flux contributes a potential energy,

$$V_{F_0} = \frac{M_p^2 \tau_0^2}{\alpha'} \tau^{-4} \rho^3 \frac{f_0^2}{16\pi^2}, \quad (34)$$

where we made use of the quantization of the 0-form flux

$$m_0 = \frac{f_0}{2\sqrt{2}\pi\sqrt{\alpha'}}, \quad f_0 \in \mathbb{Z}. \quad (35)$$

We will also include RR 6-form flux,

⁵The scalars that deform the shape are very massive and naturally stabilized at the value for which the hyperboloid possesses its maximum symmetry.

$$F_6 = 2k_6 \epsilon_3 \wedge \tilde{\epsilon}_3. \quad (36)$$

Using the quantization rule

$$\int_{(\mathbb{H} \times \mathbb{H})/\mathbb{Z}_2} F_6 = \frac{1}{\sqrt{2}} (2\pi\sqrt{\alpha'})^5 f_6, \quad f_6 \in \mathbb{Z}, \quad (37)$$

we find that

$$k_6 = \frac{(2\pi\sqrt{\alpha'})^5}{\sqrt{2}e^{2\alpha}} f_6 \quad (38)$$

which contributes a potential energy

$$V_{F_6} = \frac{M_p^2 \tau_0^2}{\alpha'} \tau^{-4} \rho^{-3} \frac{(2\pi)^{10} f_6^2}{e^{4\alpha}}. \quad (39)$$

Because of the orientifold projection, NSNS 3-form flux must thread a 3-cycle which is odd under the \mathbb{Z}_2 involution. We write

$$H_3 = p \epsilon_3^A \quad (40)$$

where ϵ_3^A is the corresponding form of the antisymmetric 3-cycle $\epsilon_3^A = \sqrt{2}(\epsilon_3 - \tilde{\epsilon}_3)$. One can show that the volume of this cycle is $\text{Vol}(\Sigma_3^A) = \frac{1}{2}\sqrt{8}e^\alpha \alpha'^{3/2}$. The field strength is related to the flux quantum number by

$$p = \frac{8\pi^2}{\sqrt{8}e^\alpha \sqrt{\alpha'}} h, \quad h \in \mathbb{Z}. \quad (41)$$

From this we find the contribution to the energy to be

$$V_{H_3} = \frac{M_p^2 \tau_0^2}{\alpha'} \frac{8\pi^4 h^2}{e^{2\alpha}} \tau^{-2} \rho^{-3} h^2. \quad (42)$$

To understand the effect of the O6-plane on the energy we recall that the Bogomol'nyi-Prasad-Sommerfield O6 source term in the IIA action is (string frame) [18]

$$2(2\pi)^{-6} l_s^{-7} \int_{\text{O6}} e^{-\Phi} \sqrt{|g|} - 2\sqrt{2}(2\pi)^{-6} l_s^{-7} \int_{\text{O6}} C_7. \quad (43)$$

The O-plane contributes to the potential energy via the first term in Eq. (43). We find

$$V_{\text{O6}} = -\frac{M_p^2 \tau_0^2}{\alpha'} e^{-\alpha} 4\sqrt{8}\pi\tau^{-3}, \quad (44)$$

where we made use of the fact that the \mathbb{Z}_2 -symmetric 3-cycle Σ_3^S wrapped by the O6-plane has the volume, $\text{Vol}(\Sigma_3^S) = \sqrt{8}e^\alpha l_s^3$.

The O6-plane also introduces a charge for C_7 through the second term in Eq. (43). This affects the Bianchi identity for F_2

$$dF_2 = m_0 H_3 + 2\pi\sqrt{2}l_s \Sigma_3^A, \quad (45)$$

$$dF_4 = -F_2 \wedge H_3, \quad (46)$$

where $\Sigma_3^A \sim \epsilon^A$ is the antisymmetric 3-form “orthogonal” to the cycle Σ_3^S that is wrapped by the O6-plane ($\Sigma_3^A = \star_6 \Sigma_3^S$). We also presented the Bianchi identity for F_4 . The

associated “tadpole relations” are found by integrating over a cycle and using Gauss’s law. This gives

$$\int_{\Sigma_3^i} m_0 H_3 = -2\pi\sqrt{2}l_s \int_{\Sigma_3^i} \Sigma_3^A, \quad (47)$$

$$\int_{\Sigma_5} F_2 \wedge H_3 = 0. \quad (48)$$

The tadpole relation for H_3 flux (47) becomes

$$f_0 h = 2. \quad (49)$$

The other tadpole relations are satisfied trivially.

The potential energy from the NSNS flux, in terms of the Romans flux parameter f_0 , is

$$V_{H_3} = \frac{M_p^2 \tau_0^2}{2} \frac{32\pi^4}{e^{2\alpha} f_0^2} \tau^{-2} \rho^{-3}, \quad (50)$$

where now f_0 can only be 1 or 2 in order to satisfy the tadpole relation (49).

B. Searching for de Sitter vacua

Collecting terms as in (9), and factoring out the overall factor of $M_p^2 \tau_0^2 / \alpha'$, we have

$$\begin{aligned} \frac{\alpha'}{M_p^2 \tau_0^2} a(\rho) &= \frac{1}{\rho} + \frac{32\pi^4}{e^{2\alpha} f_0^2} \rho^{-3}, \\ \frac{\alpha'}{M_p^2 \tau_0^2} b(\rho) &= e^{-\alpha} 4\sqrt{8}\pi, \\ \frac{\alpha'}{M_p^2 \tau_0^2} c(\rho) &= \frac{f_0^2}{16\pi^2} \rho^3 + \frac{(2\pi)^{10} f_6^2}{e^{4\alpha}} \rho^{-3}. \end{aligned} \quad (51)$$

The scalar potential is thus explicitly calculable in terms of the microphysical parameters. As discussed earlier, to find de Sitter vacua we need only to find minima of the a , b , c quantity near unity,

$$\left. \frac{4ac}{b^2} \right|_{\min} \approx 1 + \delta \quad (52)$$

for $\delta \ll 1$. Using properties of these vacua, we have that

$$\tau = \frac{b}{2a} + \mathcal{O}(\delta), \quad (53)$$

from which we find that the stabilized value of the string coupling is related to the overall volume,

$$g_s = \frac{e^\alpha}{4\sqrt{2}\pi} \sqrt{\rho_0} + \frac{4\sqrt{2}\pi^3}{e^\alpha f_0^2} \frac{1}{(\sqrt{\rho_0})^3}. \quad (54)$$

There is not much freedom in tuning various quantities and, as a consequence, there is generically a tradeoff between having g_s small and having a separation of scales between the Kaluza-Klein (KK) masses and the moduli masses. We shall therefore examine a few solutions and focus on (i) the value of the string coupling, which deter-

mines whether string loop corrections can be consistently ignored, (ii) the value of the internal volume which determines whether α' corrections can be consistently ignored, and (iii) the masses of the moduli and the KK particles.

Before we proceed, let us elaborate further on point (iii). In performing our dimensional reduction to four dimensions, we truncated the KK tower of states from the internal space, keeping only the zero modes (which we have been calling “moduli”). This procedure is consistent if there exists a hierarchy of mass scales between the KK modes and the zero modes. If no hierarchy exists then the KK modes can contribute to the dynamics of the low energy theory and the simple dimensional reduction to the zero modes does not give a complete 4D effective theory.⁶ In our specific case, we expect setting the KK modes to zero is a *consistent truncation* in the sense of supergravity reduction. This is similar to the Freund-Rubin vacua. The vacuum solution exists from a 10D point of view but the KK modes are as important for the 4D physics as the moduli. In other words, we expect the 10D equations of motion are solved even though there is no clear separation of scales. We expect to return to a more complete analysis of the 10D equations of motion in future work.

Let us now look for an explicit solution. This can easily be done numerically for various choices of the parameters; one set of parameters which leads to a de Sitter vacua with small vacuum energy:

$$\begin{aligned} f_0 = 2, \quad \frac{4ac}{b^2} &\approx 1.03, \quad \frac{V_{\text{dS}}}{M_p^4} \approx 7.9 \times 10^{-5}, \\ f_6 = 8, \quad \rho_{\text{dS}} &\approx 90.614, \quad \tau_{\text{dS}} \approx 1.47 \times 10^3, \\ \alpha &\approx 0. \end{aligned}$$

The potential for these flux choices is shown in Fig. 2, which clearly illustrates a metastable de Sitter vacuum in the τ direction as discussed earlier. In units of α' , the Planck mass is thus,

$$M_p^2 \approx \frac{5.59}{\alpha'}. \quad (55)$$

Thus, we see that a main limitation of using a rigid compact 3-hyperboloid is that there are no parameterically small or large numbers with which to simultaneously make the volume large and the string coupling small. Instead, one must rely on the precise numerical factors in order to satisfy the consistency constraints. For the parameters given above, we find

⁶For example, in IIB flux compactifications on Calabi-Yau spaces the backreaction of the fluxes generates warping factors in the 10-dimensional metric, and in regions of strong warping there is generically no separation of scales between the moduli and the KK modes so one cannot consistently truncate the effective theory to the zero modes. Further, the warping ends up modifying the dimensional reduction procedure and affects the low energy effective theory and cannot be ignored [27–29].

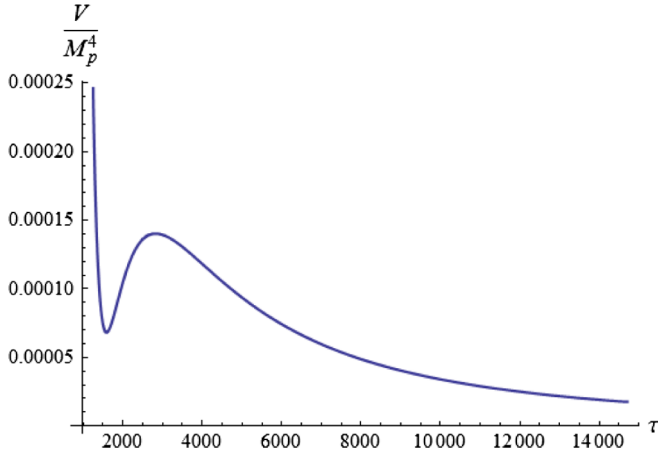


FIG. 2 (color online). Stable de Sitter vacua with small cosmological constant can be obtained with minimal tree-level ingredients when compactified on spaces of negative curvature.

$$V_6 = \frac{1}{2}e^{2\alpha}\rho_{\text{ds}}^3 \approx 3.72 \times 10^5, \quad g_s \approx 0.56, \quad (56)$$

so we find that our solution is marginally within the large volume and weak coupling regime. The actual string loop expansion parameter for type IIA orientifold in the present T-dual frame is g_s times a numerical factor which can be deduced (by T duality) from that of type I string theory discussed in [30,31]. Given the discrepancy in the precise numerical factor in the literature, we take the more conservative estimate⁷ about the region of validity of perturbation theory in [31] which suggests that the loop expansion parameter λ is $\lambda \leq \frac{g_s}{2\sqrt{2}}$. Thus, our solution is in the perturbative regime.

In order to compute the moduli masses, we first need to identify the appropriate canonically normalized scalar fields. It is straightforward to see that the fields ρ , τ have kinetic terms [19],

$$S_{\text{kinetic}} = \int \sqrt{g_4^{(E)}} \left[M_p^2 \frac{(\partial\tau)^2}{\tau^2} + \frac{3M_p^2}{4} \frac{(\partial\rho)^2}{\rho^2} \right] \quad (57)$$

so that the canonically normalized scalar fields are

$$\hat{\tau} = \sqrt{2}M_p \ln\tau, \quad (58)$$

$$\hat{\rho} = \sqrt{\frac{3}{2}}M_p \ln\rho. \quad (59)$$

The square root of eigenvalues of the mass matrix computed as $M_{ij} = \frac{1}{2}\partial_i\partial_j V$, for $i, j = \{\hat{\tau}, \hat{\rho}\}$, are, in units of $\alpha' = 1$,

$$\approx \{0.22, 0.076\}. \quad (60)$$

⁷We thank A. Hebecker and M. Trappetti for correspondence on this point.

The KK masses can be estimated in two ways: first, one can compute the longest length L around the 3-hyperboloid [26], and identify the KK mass as (again in units of α')

$$m_{\text{KK}} \sim \frac{c_n}{\rho_{\text{ds}}^{1/2} L} \sim 0.17, \quad (61)$$

where we found $L \sim 0.61$ in string units and c_n is some $\mathcal{O}(1)$ number. Alternatively, one can estimate the KK mass scale from the overall volume,

$$m_{\text{KK}} \sim \frac{a_n}{V_6^{1/6}} \sim \frac{a_n}{(\alpha'\rho_{\text{ds}})^{1/2} \tilde{V}_6^{1/6}} \sim 0.11. \quad (62)$$

Clearly the two estimates are not too far off.

Comparing these KK masses to the moduli masses, we see that there is no separation of scales between the KK masses and the moduli masses, as is expected since our model does not have many tunable parameters with which to create a separation of scales. This feature is an artifact of the simple example we have chosen for illustration, as the rigidity of the hyperbolic spaces also implies that there are fewer adjustable parameters (like fluxes over a variety of cycles) to separate these scales. It would be interesting to construct such examples from compactifications of other negatively Ricci curved spaces.

IV. DE SITTER VACUA FOR TWISTED 3-TORI

In the previous section we examined a very simple background which illustrates the minimal ingredients needed to construct stable, three-level de Sitter solutions as seen in (17). A key aspect of this construction is that the coefficients in (17) were constants, independent of the moduli. Unfortunately, models where these coefficients are moduli independent are not generic and may be marginally within the large volume, weak coupling regime at best.

We can extend our analysis to include a simple set of models in which the curvature of the internal space comes from a metric twist; some of these geometries can be viewed as T dual to spaces with NSNS flux. In fact, for metric twists which are a product of two twisted 3-tori $G_3 \times G_3$, all 3-dimensional Lie algebras were classified by Bianchi so it is possible to exhaust all possibilities (including the Nil manifold considered in [10]).

Let us now briefly consider the classification of twisted tori of the form $G_3 \times G_3$.

Given a parametrization of a Lie group G we can define the Maurer-Cartan forms via

$$g^{-1}dg = \eta^a T_a, \quad (63)$$

where the T_a are the generators of the Lie algebra \mathfrak{G} associated to the Lie group G . Clearly $d(g^{-1}dg) = -g^{-1}dg \wedge g^{-1}dg$ and hence we can read off the Maurer-Cartan equations

$$d\eta^a = -f_{bc}^a \eta^b \wedge \eta^c, \quad (64)$$

where f_{bc}^a are the structure constants of \mathfrak{G} . The metric on the Lie group is then defined via

$$ds^2 = M_{ab} \eta^a \otimes \eta^b, \quad (65)$$

where M is any symmetric nonsingular matrix. Since the η^a are left invariant (under $g \rightarrow \Omega g$) this metric has a left acting isometry group G_L . If M coincides with the Cartan-Killing metric we also have a right acting isometry such that in total we have $G_L \times G_R$.

For a clear discussion on the classification of 3-dimensional Lie algebras and the applications thereof in dimensional reduction of supergravity theories we refer the reader to [32]. The three-dimensional Lie algebras can be divided into two classes: class A and class B according to the following property of the Lie algebra:

$$\text{class A: } f_{nm}^n = 0, \quad \text{class B: } f_{nm}^n \neq 0. \quad (66)$$

It can be shown that reduction of the action on class B group manifolds is inconsistent. Instead one has to reduce the equations of motion and the result is that one obtains unusual theories with the property that they do not allow a Lagrangian description, there are only equations of motion [32].

The class A Lie algebras are taken from [32] and presented in Table I. The Q 's denote the metric flux through the following relation:

$$f_{bc}^a = \epsilon_{bcd} Q^{ad}, \quad Q = \begin{pmatrix} q_1 & & \\ & q_2 & \\ & & q_3 \end{pmatrix}. \quad (67)$$

It is straightforward to perform the dimensional reduction and search for stable de Sitter vacua for each of these twisted 3-tori. Recall that in order to uplift to de Sitter solutions the contribution to the potential energy from the curvature of the internal manifold must be positive, which implies that the stabilized value of the curvature of the manifold must be negative. Starting from the bottom of the list in Table I, the $\text{SO}(2,1)$, $\text{SO}(3)$, and $\text{ISO}(2)$ groups do not allow stable de Sitter vacua because the curvature for these spaces is not negative definite, so the stabilized curvature cannot uplift the AdS vacuum.

The only possible candidate backgrounds, then, are Heis_3 , studied in [10], and $\text{ISO}(1,1)$. We have studied

TABLE I. The different class A 3D Lie algebras. The Q_i represent the metric flux and are all positive numbers.

Bianchi type	Algebra	(q_1, q_2, q_3)
I	$U(1)^3$	$(0,0,0)$
II	Heis_3	$(0,0,Q_1)$
VI_0	$\text{ISO}(1,1)$	$(0, -Q_1, Q_2)$
VII_0	$\text{ISO}(2)$	$(0, Q_1, Q_2)$
VIII	$\text{SO}(2,1)$	$(Q_1, -Q_2, Q_3)$
IX	$\text{SO}(3)$	(Q_1, Q_2, Q_3)

both of these examples in detail, and have found that neither can support a stable de Sitter solution for the minimal ingredients given in (17). The reason is that the coefficients in (17) are now moduli dependent, so the stabilization of ρ depends on the stabilization of all of the other moduli as well. In particular, it is straightforward to show that for the minimal ingredients above, there always exists a runaway direction.⁸

As a simple illustration, let us suppose that there is one additional modulus ϕ , and that the orientifold and H_3 flux contributions are independent of this modulus (this will be the case for the twisted 3-tori in which the orientifold maps the 3-manifolds to each other); note also that the coefficient of the RR 0-form flux \tilde{A}_0 is also moduli independent, since it does not involve integrating over any cycles. In general, then, the coefficients have moduli dependence which we will parametrize as

$$\tilde{C}_f \sim \phi^n, \quad (68)$$

$$\tilde{A}_2 \sim \phi^{m_i}, \quad (69)$$

$$\tilde{A}_4 \sim \phi^{2\ell_i}, \quad (70)$$

where we will assume $n > 0$ without loss of generality, and we have allowed for the possibility of multiple different contributions of ϕ to the RR forms, parametrized by different powers m_i, ℓ_i . The quantity $4ac/b^2$ becomes

$$\begin{aligned} \frac{4ac}{b^2} \sim & \text{const} + a_1 \phi^n \rho^2 + a_2 \phi^{n+m_i} \\ & + \frac{(a_3 \phi^{n+2\ell_i} + a_4 \phi^{m_i})}{\rho^2} + \mathcal{O}(\rho^{-4}, \rho^{-6}) \end{aligned} \quad (71)$$

$$\sim a_1 \tilde{\phi}^n + \frac{a_2 \tilde{\phi}^{n+m_i} + a_4 \tilde{\phi}^{m_i}}{\rho^{2(1+m_i/n)}} + a_3 \frac{\tilde{\phi}^{n+2\ell_i}}{\rho^{4(1+\ell_i/n)}} \quad (72)$$

where in the third line we made the rescaling $\phi = \tilde{\phi}/\rho^{2/n}$. We ignored the constant and $\mathcal{O}(\rho^{-4}, \rho^{-6})$ contributions coming from the 6-form RR flux because these tend to destabilize $\rho \rightarrow \infty$, and so they will not be helpful in our stability analysis. That such a rescaling is possible is due to the extra moduli dependence in \tilde{C}_f ; this field redefinition can remove the manifestly positive power of ρ coming from the product of the RR 0-form and the geometric flux in (71). If it can be shown that stable de Sitter vacua do not exist for the field redefined quantity (72) then vacua will not exist for the original function (71) either.

A positive power of ρ can be regenerated in the other RR flux terms by the field redefinition if their moduli dependence is just right. In particular, note that if $n + m_i \geq 0$, $n + \ell_i \geq 0$ for all m_i, ℓ_i , then after the field redefinition we

⁸We would especially like to thank Xi Dong for pointing this out to us.

only have negative powers of ρ , and we have a runaway $\rho \rightarrow \infty$ in trying to minimize (72). Indeed, it is clear that in order for a positive power of ρ to be regenerated in the expression (72) we need at least one of the moduli dependences of the RR fluxes to satisfy $n + m_j < 0$ for some m_j (alternatively, the same condition applies to the ℓ_j).

For moduli coming from the metric of twisted 3-tori $M \sim \phi$, we generically expect contributions of the order

$$\begin{aligned}\tilde{C}_f &\sim M^2 \sim \phi^2 \Rightarrow n = 2, \\ \tilde{A}_2 &\sim M^{-2} \sim \phi^{-2} \Rightarrow m_i = -2, \\ \tilde{A}_4 &\sim M^{-4} \sim \phi^{-4} \Rightarrow \ell_i = -2.\end{aligned}$$

These scalings are the dominant scalings—for example, we generally have $m_i, \ell_i \geq -2$, but we will not have stronger dependence on ϕ than that listed above (e.g., we will not have A_2, A_4 scale with higher negative powers of ϕ than that listed above). Also, note that these are the same scalings that are expected from fractional Wilson lines as well, as they contribute to F_2, F_4 field strengths in similar ways as normal fluxes. Thus, we find that quite generally we expect $n + m_i, n + \ell_i \geq 0$ for twisted 3-tori, which suggests that with just metric flux, O6-planes, RR, and NSNS flux we do not have the sufficient ingredients to find stable de Sitter vacua for twisted 3-tori. Notice that while these ingredients were sufficient to find stable de Sitter vacua for the case when there were no additional moduli, when additional moduli are present we find that there generically exists a runaway direction $\rho \rightarrow \infty$.

As a more explicit check, let us consider the twisted 3-torus with ISO(1,1) metric twist. There are two metric moduli, L_1, L_2 . It turns out that the modulus L_2 only appears in the geometric flux; after minimizing with respect to L_2 , we find that the remaining moduli dependence on $\phi \equiv 1/L_1$ is

$$\begin{aligned}\tilde{C}_f &\sim 1/L_1^2 \sim \phi^2 \Rightarrow n = 2, \\ \tilde{A}_2 &\sim 1/L_1^4 + L_1^2 \sim \phi^4 + \phi^{-2} \Rightarrow m_1 = 4, \quad m_2 = -2, \\ \tilde{A}_4 &\sim L_1^4 + L_1^{-2} \sim \phi^2 + \phi^{-4} \Rightarrow \ell_1 = -2, \quad \ell_2 = 1.\end{aligned}$$

Thus we see that indeed $n + m_i \geq 0, n + \ell_i \geq 0$ for all m_i, ℓ_i in this example, so there does not exist a stable de Sitter solution for finite ρ : There always exists a field redefinition, discussed above, which removes the positive power of ρ such that it is manifest that no stable de Sitter vacua exist.

From the argument above it seems difficult to obtain the desired power of m_i, ℓ_i in the fluxes in order to have a stable de Sitter minimum. Instead, one can modify the moduli dependence in the “uplifting” energy by finding sources which have a different n dependence on ϕ . In particular, let us take a KK5 brane, which has the same ρ dependence as the geometric flux. The KK5 brane is wrapped on a 2-cycle in the internal space; thus it will pick up moduli dependence of the form,

$$\tilde{C}_{\text{KK5}} \sim \int_{\Sigma_{2,\text{KK5}}} \sqrt{g_{\text{ind}}(\Sigma_{2,\text{KK5}})} \sim M \sim \phi \Rightarrow n' = 1. \quad (73)$$

Combining the KK5-brane into the constructions above with geometric, NSNS, and RR fluxes, we see that we can now easily satisfy the requirement $n' + m_i < 0, n' + \ell_i < 0$ for the generic form of m_i, ℓ_i that we expect. Thus, negative curvature (e.g., geometric flux), NSNS and RR fluxes (including 0-form), O-planes, and KK5-branes appear to be necessary ingredients in order to find stable tree-level de Sitter vacua in IIA for twisted 3-tori. Unfortunately, it is not clear if constructions with KK5-branes in massive IIA are under complete control, particularly in twisted tori backgrounds, since backreaction can be severe.

V. DISCUSSION

It is a difficult problem to construct reliable stable de Sitter vacua in string theory. We have argued that the minimal ingredients needed to get de Sitter vacua in type IIA string theory are nonzero Romans parameter, RR, NSNS fluxes, and negative internal curvature. For the explicit example studied here in which the internal space is a pair of compact, maximally symmetric 3-hyperboloids, we have shown that one need only choose flux quanta appropriately to find de Sitter vacua with small cosmological constant. Because these solutions have very few tunable parameters, however, we find that our solution is marginally within the weak coupling regime, with $g_s \approx 0.5$.⁹

This simple model has just two moduli which are clearly stabilized in a dS minimum. Furthermore we found that there is no separation of scales in this simple model: the lightest KK modes are of the same order of mass as the moduli. This is a possible drawback if this solution should be considered as a semirealistic vacuum. But from the point of view of the dS/CFT correspondence [17] a separation of scales is not something that is required, in the same way that the $\text{AdS}_5 \times S^5$ solution has light KK modes.

We have also discussed the generalization of this simple model to more general metric fluxes, by considering all twisted, orientable, 6-tori of the form $G_3 \times G_3$, where G_3 represents the covering space. There are 5 such families of twisted tori (not including the normal torus): Heis₃, ISO(1,1), ISO(2), SO(2,1), and SO(3). As discussed in Sec. IV, only those twisted tori with negative definite curvature provide the necessary uplifting energy to create de Sitter solutions, immediately excluding ISO(2), SO(2,1), and SO(3). Constructions based on the Heis₃ background (also sometimes called the Nil 3-manifold) and ISO(1,1) background appear to require additional ingredients such as KK5 branes in order to stabilize all of the

⁹However, as pointed out earlier, we have ignored subtleties with the definition of perturbative string theory with nonzero Romans mass.

metric moduli, as in [10]. We showed why this is true by considering a simple field redefinition which leads to a runaway minimum for ρ unless KK5-branes are included.

While we have shown that minimal stable de Sitter solutions do not exist for manifolds which are a product of twisted 3-tori in IIA, our analysis does not rule out the possibility that the minimal set of ingredients can lead to stable de Sitter solutions for 6-manifolds with negative curvature which cannot be decomposed as the product of 3-tori. The benefit of simple dS solutions from twisted tori is that supersymmetry is not broken at the KK scale such that the dS solution is a spontaneous broken state in an $\mathcal{N} = 4$ gauged supergravity coupled to six vector multiplets [33–36]. The latter theories are rigid in the sense that the number of vector multiplets and the gauge group almost fully determine all interactions. The only freedom resides in the values of the gauge coupling constants and so called “de Roo–Wagemans angles” [aka $SU(1, 1)$ angles] [37]. Especially those angles play a central role in providing de Sitter vacua in extended supergravity [38–41]. This construction could offer a string theory embedding of the de Roo–Wagemans angles which were introduced in supergravity but their relation with string theory is so far only established in the examples of [42] (and effectively obtained in an $\mathcal{N} = 1$ context). Finally having the gauged supergravity description at hand allows an easier derivation of the explicit mass matrix, such that we can work out whether the minimal simple de Sitter vacua are stable with respect to all fluctuations. Embedding our constructions into 4D gauged supergravity also enables us to compute the gravitino mass, and check whether the bound in [43] applies.

It would also be interesting to study whether these simple de Sitter vacua exist in type IIB backgrounds as well. A geometric background can be looked at for using a similar derivation as in Sec. II to derive the necessary ingredients to obtain de Sitter vacua. From this it becomes apparent that the minimal ingredients are NSNS and geometric flux, Op-plane sources and F_1 flux (not necessarily all together). We thus expect minimal simple de Sitter solutions to exist in IIB as well, although additional ingredients may be needed in models with more than one moduli, as seen here. Nevertheless, this may give some insight into stabilization mechanisms for Kähler moduli in type IIB which do not rely on nonperturbative effects, as well as identify possible uplifting sources of energy.

Furthermore, it could be fruitful to use minimal de Sitter solutions as starting points to construct large-field inflation models [22,44] and particle physics constructions [45,46] explored in similar backgrounds. As the low energy chiral particle physics spectrum depends only on the topological data of cycles on which the D-branes are wrapped, the machineries developed for intersecting D-brane models [47] can be readily adopted to simple extensions of toroidal backgrounds. When embedded into a single framework,

these investigations may thus allow us to study the interplay between cosmology and particle physics.

Finally, the simplicity of our background solution $dS_4 \times \mathbb{H}_3 \times \mathbb{H}_3$ is of the same simplicity as the Freund-Rubin vacua $AdS_n \times S^m$ which allow explicit tests of the AdS/CFT conjecture. The background presented here is therefore a good starting point for testing a hypothetical dS/CFT correspondence.

We hope to return to all these exciting directions in the future.

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APPENDIX: a, b, c DE SITTER VACUA

In this Appendix we present a simple derivation, originally discussed in [10], that searching for de Sitter critical points with small vacuum energy of the potential

$$V = a(\phi^i)g^2 - b(\phi^i)g^3 + c(\phi^i)g^4 \quad (A1)$$

with respect to all of the moduli (g, ϕ^i) corresponds to finding critical points of the quantity $4ac/b^2 \approx 1$ as a function of the other moduli ϕ^i . Note that in this notation $g = 1/\tau$ corresponds to the potential used previously in (9).

Minimizing the potential (A1) in the g, ϕ^i directions leads to the equations,

$$(\partial_{\phi^i} a) - (\partial_{\phi^i} b)g_0 + (\partial_{\phi^i} c)g_0^2 = 0, \quad (A2)$$

$$\left(g^2 - \frac{3b}{4c}g + \frac{a}{2c} \right) \Big|_{g_0, \phi_0^i} = 0, \quad (A3)$$

where (g_0, ϕ_0^i) are the putative stabilized values of the moduli, and we will denote a_0, b_0, c_0 as the corresponding stabilized values of the functions appearing in the potential (A1). The expression (A3) can be solved for g_0 in terms of the stabilized values of a, b , and c

$$g_0 = \frac{3b_0}{8c_0} \pm \frac{b_0}{2} \sqrt{\frac{9}{16c_0^2} - \frac{1}{2c_0^2} \left(\frac{4a_0c_0}{b_0^2} \right)}. \quad (\text{A4})$$

When $\frac{4a_0c_0}{b_0^2} \simeq 1 + \delta$, with $\delta \ll 1$, we have two possible solutions

$$g_0 \simeq \frac{b_0}{2c_0} \left(1 + \frac{1}{8} \delta \right), \quad (\text{A5})$$

$$g_0 \simeq \frac{b_0}{4c_0} \left(1 - \frac{1}{4} \delta \right). \quad (\text{A6})$$

The solution $g_0 \simeq b_0/(2c_0)$ corresponds to a local minimum in the g direction with positive vacuum energy proportional to δ (and is thus the de Sitter vacua we are interested in)

$$V_{\min}|_{g_0=(b_0/2c_0)(1+1/8\delta), \phi_0^i} = g_0 c_0 \delta \quad (\text{A7})$$

so we will focus on this solution henceforth.

Now, searching for critical points of $4ac/b^2$ with respect to the moduli ϕ^i , we obtain (after some basic manipulation)

$$\partial_{\phi_i} a - \left(\frac{2a}{b} \right) \partial_{\phi_i} b + \frac{a}{c} \partial_{\phi_i} c = 0. \quad (\text{A8})$$

The approach of finding critical points of $4ac/b^2 \approx 1$ is equivalent to finding local de Sitter minima of the entire potential (A1) if solving the expression (A8) is identical to solving the expression (A2). It is clear that this is true only if

$$g_0 = \frac{2a_0}{b_0}, \quad g_0^2 = \frac{a_0}{c_0}. \quad (\text{A9})$$

But using $4a_0c_0/b_0^2 = 1 + \delta$, it is easy to see in fact that

$$\frac{2a_0}{b_0} = \frac{b_0}{2c_0} (1 + \delta), \quad \frac{a_0}{c_0} = \left(\frac{b_0}{2c_0} \right)^2 (1 + \delta) \quad (\text{A10})$$

which both imply the solution (A5), $g_0 = b_0/(2c_0) + \mathcal{O}(\delta)$. Thus, the critical point equations (A2) and (A8) are in fact identical when $4ac/b^2 \approx 1 + \delta$ when $\delta \ll 1$, so when searching for de Sitter vacua with small vacuum energy it is sufficient to search for critical points of $4ac/b^2 \approx 1$. The advantage of this approach is that in many cases (as discussed above) it is clear simply by inspection when $4ac/b^2$ cannot be minimized at all for finite values of the moduli; thus, these cases can be immediately ruled out as candidate de Sitter vacua without needing to solve the entire system of Eqs. (A2) and (A3) for every specific example.

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