

Axial-vector coupling and chiral anomaly in the spectral quark model

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(Received 16 December 2008; revised manuscript received 24 March 2009; published 7 April 2009)

We studied the Adler-Bardeen-Bell-Jackiw anomaly in the context of a finite chiral quark model known as the spectral quark model. Within this model, we obtain the general nonlocal form of the axial vertex compatible with a nonvanishing axial coupling, in the chiral limit. The triangle anomaly is computed and we show that the obtained dependence of the axial vertex with the spectral mass is necessary to ensure both finiteness and the correct violation of the chiral Ward-Takahashi identity.

DOI: 10.1103/PhysRevD.79.085002

PACS numbers: 11.30.Rd, 11.40.Ha, 12.38.Lg

I. INTRODUCTION

The nonperturbative low-energy behavior of quantum chromodynamics (QCD) is well described by a series of effective models [1–3] in both zero and finite temperature and densities [4–22]. It is assumed that these models could result from the suppression of the high energy degrees of freedom of QCD (such as gluons), and a scale that defines the validity of the model has to be introduced. In some of these models, the quarks remain as the only degrees of freedom and, as these chiral quark models are usually nonrenormalizable, in contrast with QCD itself and with mesonic models, these scales (or cutoff) are kept finite throughout the calculations. As a consequence, a series of problems [23,24] emerge as a reflex of this process. Nevertheless, the light hadrons phenomenology is successfully described by almost all these models, that, together with the nonperturbative nature of QCD at this scale, justifies their employment.

Some of the results that are jeopardized by the introduction of a cutoff scale are the anomaly dependent results. In particular, the anomalous transition form factor $F_{\gamma^* \pi^0 \gamma}(Q^2)$, for the process $\gamma^* \rightarrow \pi^0 \gamma$, can only be correctly reproduced in the limit $Q^2 = 0$ when no regulator is introduced [23]. In contrast, a finite regularization is necessary to keep the models finite, and it is also necessary to reproduce the expected QCD factorization at large momenta, $Q^2 F_{\gamma^* \pi^0 \gamma}(Q^2) \rightarrow 2f_\pi$ [24], where f_π is the pion weak decay constant.

Perturbatively, the anomaly appears as an ambiguity, represented by surface terms, due to the infinities of the perturbative calculations. Regularization of the Feynman integrals fixes, *a priori*, the result of the computation of these surface terms, but the undetermined nature of the ambiguity appears as different results for different regularizations. If the indeterminacy of these terms is kept up to the end of the calculation, as occurs in some recent regularization schemes [25–27], it can be shown that the simultaneous transversality for massless fermions in the axial and vector channels is broken [28]. On one hand, it is possible to fix the ambiguity in order to ensure the conser-

vation of the electromagnetic Ward-Takahashi identity (violating the chiral Ward-Takahashi identity), as required in QCD in order to explain the anomalous decays. On the other hand, it is also possible to fix the ambiguity in order to satisfy the chiral Ward-Takahashi identity and to violate the electromagnetic one, as in t'Hoofts proton decay calculation [29].

This undetermined character of the anomaly [30] and the implications of the presence of ambiguous terms in Quantum Field Theory calculations are in the heart of some recent controversies in the study of Lorentz and CPT violations in QED [31–36]. The picture in chiral quark models is somewhat different. Their divergent nonrenormalizable character implies in a dependence with an specific regularization scheme, which, *a priori*, fixes the ambiguous integrals. So, one cannot expect, in these models, the undetermined character of the anomaly to manifest itself. The regularization schemes employed usually fix the vector gauge symmetry, and the transversality of the vector currents is guaranteed from the very beginning, reproducing for this reason the expected final result.

A recent chiral quark model, namely, the spectral quark model (SQM) [37–41], has some interesting features that can be explored to study the presence of the undetermined character of the anomaly, as well as to correct some of the fails on chiral quark models. For example, it was shown [37] that the SQM solves the conflict between the anomaly normalization condition, $F_{\gamma^* \pi^0 \gamma}(0) = \frac{1}{4\pi f_\pi}$, and the factorization at large momenta $Q^2 F_{\gamma^* \pi^0 \gamma}(Q^2) \rightarrow 2f_\pi$. The spectral quark model is based on the Lehmann representation for the quark propagator [42], and in the solutions for the chiral and electromagnetic Ward-Takahashi identities using the gauge technique [43,44], resulting in a finite quark model. Being finite, one can speculate (i) if the model correctly reproduces the anomalous results, (ii) if the freedom on the choice of the Ward-Takahashi identity to be violated by the anomaly is still present, as discussed above, and under which conditions one can obtain the expected violation of the chiral Ward-Takahashi identity and conservation of the electromagnetic ones in QCD.

Question (i) is already positively answered in Ref. [37]. An important ingredient to this answer is the spectral

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version of the Goldberger-Treiman relation [45], a fact that we will stress later. In this paper, we intend to explore the answers to question (ii). As we shall see, the specific form of the axial-vector coupling plays an important role in this issue.

Nonlocal axial-vector vertex for constituent quarks allows the presence of an axial-vector coupling constant g_A , and leading order N_c effects can result in $g_A \neq 1$ [46], even in the large N_c limit. In contrast with the pseudoscalar pion quark coupling constant $g_{\pi qq}$, which vanishes in the chiral limit [45], one should not expect the axial coupling constant to vanish at this limit—model estimates to g_A lie in the range $0.4 < g_A < 0.9$ [47], a result that is compatible with the axial-vector coupling constant of the nucleon, $G_A/G_V = 1.2670 \pm 0.0035$, deduced from neutral beta decay measurements [48] and from the nonrelativistic relation $G_A = \frac{5}{3}g_A$. In the context of the SQM, a dependence of g_A with positive powers of the spectral mass ω , as occurs for the pion quark coupling constant via the Goldberger-Treiman relation, would be desirable, since positive momenta of the spectral distribution $\rho(\omega)$ guarantees the finiteness of the amplitudes. We will show that a spectral $g_A(\omega)$ with these characteristics is compatible with a nonvanishing axial coupling in the chiral limit.

In this paper, we: (i) discuss the role of the spectral version of the Goldberger-Treiman relation in the pseudoscalar two point function, showing its importance to the finiteness of this amplitude and to the obtaining of the Nambu-Goldstone mode; (ii) obtain a more general expression for the axial vector vertex in the context of the spectral quark model, which includes the possibility of a nonunitary axial coupling and (iii) by applying the gauge technique, obtain the probability amplitude for the axial-vector-vector process in the SQM, including nonunitary axial coupling, and show that a dependence on the spectral mass for the axial coupling can generate an ambiguity free result, preserving the vector Ward identity and violating the axial one, as expected for QCD.

This paper is organized as follows: in Sec. II we briefly review the spectral quark model and some of its consequences. In Sec. III, we analyze the role of the Goldberger and Treiman relation in the finiteness of the pseudoscalar two point function. In Sec. IV we present the construction of a general form of the axial vertex in the SQM compatible with a nonvanishing axial coupling in the chiral limit. In Sec. V, we compute the axial-vector-vector amplitude by employing the axial vertex obtained in Sec. IV. We discuss the violation of the chiral Ward-Takahashi identity, and the role of the axial vertex on this result. Finally, in Sec. VI we present the conclusions.

II. THE MODEL

The spectral quark model is based on the introduction of the generalized Lehmann representation for the quark propagator

$$S(p) = \int_C d\omega \frac{\rho(\omega)}{\not{p} - \omega}, \quad (1)$$

where ω is the spectral mass of the quark and $\rho(\omega)$ is the spectral distribution, that acts as a regulator. The ω integral is supposed to be evaluated in a suitable complex contour C (suppressed in our notation, from now on).

The spectral function $\rho(\omega)$ needs not to be completely determined, although it is possible to find an explicit form to it if the vector-meson dominance of the pion form factor is assumed [37,49]. For the reproduction of most of the light mesons phenomenology, it is sufficient to know some of the moments of the quark spectral function, determined via physical conditions such as normalization of the quark propagator, which implies in

$$\rho_0 = \int d\omega \rho(\omega) = 1, \quad (2)$$

and finiteness of hadronic observables, implying in

$$\rho_n = \int d\omega \rho(\omega) \omega^n = 0, \quad (3)$$

for $n = 1, 2, 3$, or 4. Physical observables are proportional to the negative moments

$$\rho_{-n} = \int d\omega \rho(\omega) \frac{1}{\omega^n} \quad (4)$$

or to the logarithmic moments

$$\rho'_n = \int d\omega \rho(\omega) \omega^n \log(\omega^2). \quad (5)$$

Finiteness is guaranteed by the vanishing of the positive moments, Eq. (3). As a consequence, negative and log moments can be fixed from the finite values of hadronic observables such as the quark condensate for one flavor

$$\langle \bar{q}q \rangle = -\frac{N_c}{4\pi^2} \rho'_3, \quad (6)$$

the vacuum energy density

$$B = -\frac{3N_c}{16\pi^2} \rho'_4, \quad (7)$$

and so on. The quark propagator (1) can be parametrized as

$$S(p) = Z(p) \frac{\not{p} + m(p)}{p^2 - m^2(p)}, \quad (8)$$

with the mass function given by

$$m(p) = \frac{\int d\omega \frac{\omega \rho(\omega)}{p^2 - \omega^2}}{\int d\omega \frac{\rho(\omega)}{p^2 - \omega^2}} \quad (9)$$

and the wave function renormalization by

$$Z(p) = (p^2 - m^2(p)) \int d\omega \frac{\rho(\omega)}{p^2 - \omega^2}. \quad (10)$$

In what follows, we will refer to the mass function at $p^2 = 0$ as $m = m(0)$. The results presented here will not depend explicitly on m , but it will be necessary in order to make contact with the standard representation of the partial conservation of the axial current (PCAC).

The detailed determination of the spectral function moments, as well as the development of the SQM to the low-energy hadron phenomenology is presented in [37], and is not the aim of the present contribution.

To proceed with the computation of N -point functions in the SQM, the vertex functions are defined as particular solutions of the relevant Ward-Takahashi identities for unamputated Green functions, obtained by applying the gauge technique [44]. This allows the obtaining of linear solutions, whereas the use of amputated Green functions would imply in the appearance of nonlinear solutions to the Ward-Takahashi identities. The vector Ward-Takahashi identity (VWI), for the vector-quark-quark vertex $\Lambda_V^{\mu a}$, reads

$$(p' - p)_\mu \Lambda_V^{\mu a}(p', p) = S(p') \frac{\lambda^a}{2} - \frac{\lambda^a}{2} S(p), \quad (11)$$

where $S(p)$ is given by Eq. (1) and λ_a are the Pauli matrices (we are assuming a $SU(2)$ flavor symmetry). A solution to Eq. (11), up to transverse pieces, is

$$\Lambda_V^{\mu a}(p', p) = \int d\omega \rho(\omega) \frac{i}{\not{p}' - \omega} \gamma^\mu \frac{\lambda^a}{2} \frac{i}{\not{p} - \omega}. \quad (12)$$

The axial vector Ward identity (AWI) reads:

$$(p' - p)_\mu \Lambda_5^{\mu a}(p', p) = S(p') \frac{\lambda^a}{2} \gamma_5 + \gamma_5 \frac{\lambda^a}{2} S(p), \quad (13)$$

and one possible solution to Eq. (13) for the axial vector to quarks coupling is

$$\Lambda_5^{\mu a}(p', p) = \int d\omega \rho(\omega) \frac{i}{\not{p}' - \omega} \left(\gamma^\mu - \frac{2\omega q^\mu}{q^2} \right) \times \frac{\gamma_5 \lambda^a}{2} \frac{i}{\not{p} - \omega}, \quad (14)$$

with $q = p' - p$. One can identify the pion pole in (14), the dominant term as $q^2 \rightarrow 0$. For massless quarks, it cannot be done with a non spectral propagator, since in this case we have $\rho(\omega) = \delta(\omega)$, and the consequent vanishing of the pole term.

III. THE PSEUDOSCALAR TWO POINT FUNCTION

The pion to quarks coupling can be obtained from the axial vector vertex near the pion pole by using

$$\Lambda_5^{\mu a}(p', p)|_{q \rightarrow 0} = -2f_\pi \frac{q^\mu}{q^2} \Lambda_\pi^a(p', p), \quad (15)$$

resulting in

$$\Lambda_\pi^a(p', p) = \int d\omega \rho(\omega) \frac{i}{\not{p}' - \omega} \gamma_5 \frac{\omega}{f_\pi} \frac{\lambda^a}{2} \frac{i}{\not{p} - \omega}. \quad (16)$$

An important consequence of the appearance of the pion pole in solution (14) is the obtaining of the spectral version of the Goldberger-Treiman relation $g_\pi(\omega) = \frac{\omega}{f_\pi}$. By closing the quark line in the pion vertex, Eq. (16), we can obtain the pseudoscalar two-point function

$$\Pi_{\text{PS}}(q) = \int d\omega \rho(\omega) \int_p \text{Tr} \left\{ \frac{i}{\not{p} + \not{q} - \omega} \gamma_5 \frac{\omega}{f_\pi} \frac{\lambda^a}{2} \times \frac{i}{\not{p} - \omega} \gamma_5 \right\}, \quad (17)$$

where \int_p stands for $\int \frac{d^4 p}{(2\pi)^4}$. Because of the spectral conditions, Eq. (3), the divergent terms arising from the computation of Eq. (17) vanishes, and the final result is finite. In the intermediary calculation, however, an auxiliary regularization scheme is necessary in order to compute the loop momentum integral before the evaluation of the integral over the spectral mass ω . The use of a gauge invariant regularization scheme would enforce the *a priori* preservation of the gauge Ward-Takahashi identities. It is interesting, however, to explore how the correct violation of the AWI in QCD can be obtained in a regularization independent way. We thus choose to work with the sharp-cutoff regularization scheme, a scheme that violates the VWI, as is widely known.

In fact, employing the covariant sharp cutoff regularization scheme to compute the p integral in (17), one obtains

$$\begin{aligned} \Pi_{\text{PS}}(q) = & \frac{-i}{4\pi^2} \int d\omega \rho(\omega) \frac{\omega}{f_\pi} \left\{ 2\Lambda^2 - 2\omega^2 \ln\left(\frac{\Lambda^2}{\omega^2}\right) \right. \\ & + q^2 \left(1 - \ln\left(\frac{\Lambda^2}{\omega^2}\right) \right) + 2q^2 \left(-1 \right. \\ & \left. \left. + \sqrt{1 - \frac{4\omega^2}{q^2}} \tanh^{-1}\left(\frac{1}{\sqrt{1 - \frac{4\omega^2}{q^2}}}\right) \right) \right\}. \quad (18) \end{aligned}$$

From the first term on the right hand side of Eq. (18) one can clearly see that the Goldberger-Treiman relation is important in order to make the pseudoscalar two-point function finite. If the coupling between the pion and the quarks was not dependent on ω , this term would be divergent in the limit $\Lambda \rightarrow \infty$.

After applying the spectral conditions (3) on Eq. (18), we get

$$\begin{aligned} \Pi_{\text{PS}}(q) = & \frac{-i}{4\pi^2} \int d\omega \rho(\omega) \frac{\omega}{f_\pi} \left\{ (q^2 + 2\omega^2) \ln(\omega^2) \right. \\ & \left. + 2q^2 \sqrt{1 - \frac{4\omega^2}{q^2}} \tanh^{-1}\left(\frac{1}{\sqrt{1 - \frac{4\omega^2}{q^2}}}\right) \right\}. \quad (19) \end{aligned}$$

From now on, all momentum integrals will be computed employing the covariant sharp cutoff regularization

scheme. The use of this regularization to compute the pseudoscalar two point function introduces surface terms that could, in principle, break the Nambu-Goldstone mode [50]. It also generates dependence on the arbitrary choice of the momentum routing in the loop. In a renormalizable theory it will be no problem: regularization and symmetries fix these ambiguities [35,51,52]. This is not the case for purely fermionic chiral quark models. It is interesting, however, to observe how the spectral regularization corrects this fail: the surface terms that emerges from this computation are

$$\begin{aligned} 2q_\mu q_\nu \int d\omega \rho(\omega) \frac{\omega}{f_\pi} \int_p \left(\frac{g^{\mu\nu}}{(p^2 - \omega^2)^2} - 4 \frac{p^\mu p^\nu}{(p^2 - \omega^2)^3} \right) \\ = 2q_\mu q_\nu \int d\omega \rho(\omega) \frac{\omega}{f_\pi} g^{\mu\nu} \omega^2 \frac{i}{2(4\pi)^2 \omega^2} = 0 \end{aligned} \quad (20)$$

The details on the computation of surface terms will be presented on Sec. V, in the context of the chiral anomaly. As we can see, the spectral condition (3) guarantees that Eq. (20) vanishes, since it depends on $\rho_1 = \int d\omega \omega \rho(\omega) = 0$. The role played by the Goldberger-Treiman coupling ω/f_π becomes clear—if it was not present, then the pseudoscalar two point function, Eq. (19), would be divergent and dependent of the ambiguous result of the momentum integral in the left side of Eq. (20). This feature—the dependence of the coupling with the spectral mass *via* the Goldberger-Treiman relation—suggests that a similar dependence on the spectral mass for the axial-vector coupling could be important in the study of the chiral anomaly in the context of the spectral quark model.

IV. THE AXIAL-VECTOR COUPLING IN SQM

Equation (14) is one of the possible solutions to the AWI, Eq. (13). Its functional form suggests that, for this ansatz, the axial coupling g_A is unitary. As mentioned before, there are mechanisms that generate contributions to $1 - g_A$ of order N_c^0 , such as the $\pi - A_1$ mixing mechanism [46], present in chiral models such as the σ -model [1] and the NJL model [2]. A general form to the axial vertex that allows $g_A \neq 1$, including pseudoscalar and pseudo-vector pion couplings is [53]

$$\begin{aligned} \Lambda_A^{\mu a}(p', p) = \int d\omega \rho(\omega) \frac{i}{\not{p}' - \omega} (g_A(q^2, \omega) \gamma^\mu \\ + h_A(q^2, \omega) q^\mu + f_A(q^2, \omega) q^\mu \not{q}) \\ \times \gamma_5 \frac{\lambda^a}{2} \frac{i}{\not{p} - \omega}, \end{aligned} \quad (21)$$

where we introduced the spectral form factors $g_A(q^2, \omega)$, $h_A(q^2, \omega)$ and $f_A(q^2, \omega)$. One can recognize that $g_A(q^2, \omega) = 1$, $h_A(q^2, \omega) = -2\omega/q^2$ and $f_A(q^2, \omega) = 0$ in the ansatz (14). The evaluation of (13) with (21) gives

$$h_A(q^2, \omega) = -2\omega/q^2 \quad (22)$$

and

$$f_A(q^2, \omega) = \frac{1 - g_A(q^2, \omega)}{q^2}. \quad (23)$$

In what follows we will assume that the axial coupling depends on the spectral mass, but not on the exchanged momenta, i.e., $g_A(q^2, \omega) = g_A(\omega)$. One can recognize the poles associated to the Goldstone pion in the second and third terms on the right hand side of Eq. (21) whereas the first term is associated to the axial-vector coupling. Let us denote this term as $\Lambda_{g_A}^{\mu a}(p', p)$, with

$$\Lambda_{g_A}^{\mu a}(p', p) = \int d\omega \rho(\omega) g_A(\omega) \frac{i}{\not{p}' - \omega} \gamma^\mu \gamma^5 \frac{\tau_a}{2} \frac{i}{\not{p} - \omega} \quad (24)$$

For on-shell massless quarks, Dirac equation implies $\not{p}' = \not{p} = 0$. Thus, we get for the axial-vector coupling with on-shell quarks

$$\Lambda_{g_A}^{\mu a}(p', p)|_{\text{on shell}} = - \left\{ \int d\omega \rho(\omega) \frac{g_A(\omega)}{\omega^2} \right\} \gamma^\mu \gamma^5 \frac{\tau_a}{2} \quad (25)$$

Equation (25) gives us some insight about the functional form of $g_A(\omega)$: if it was proportional to any positive integer power of the spectral mass ω greater than 2 the axial-vector coupling for on-shell quarks would be zero. Yet, assuming $g_A(\omega) = \alpha \omega^n$ with $n \leq 2$ and α an arbitrary constant, we obtain a nonvanishing axial coupling in this limit. From our previous analysis of the pseudoscalar two-point function, a coupling constant proportional to an integer positive power of ω (i.e., $n = 1$ or $n = 2$) would be desirable in order to render finite some amplitudes, and also to avoid regularization ambiguities. In the next section we will see that this feature is also necessary in the computation of the chiral anomaly in SQM, in order to make it free from ambiguities.

V. THE CHIRAL ANOMALY

The mechanism of the anomalous symmetry breakdown was codiscovered by Bell and Jackiw [54] and Adler [55]. This violation is related to a probability amplitude that cannot satisfy, simultaneously, the gauge and chiral symmetries. Nevertheless, which symmetry is violated is a model dependent result: in the anomalous pion decay, the VWI are to be conserved and the AWI is violated, whereas in the t'Hooft calculation of the proton decay [29], the situation is opposite—the global gauge symmetry is violated and the chiral symmetry is preserved.

The probability amplitude for the axial-vector-vector process is represented by the triangle diagram, depicted on Fig. 1. In order to compute it in the SQM, one needs to find a spectral representation to the vertex function with one axial and one vector current, $\Lambda_{AV}^{\mu a; \nu b}$, that fulfills the $SU(2)$ vector and axial vector Ward identities.

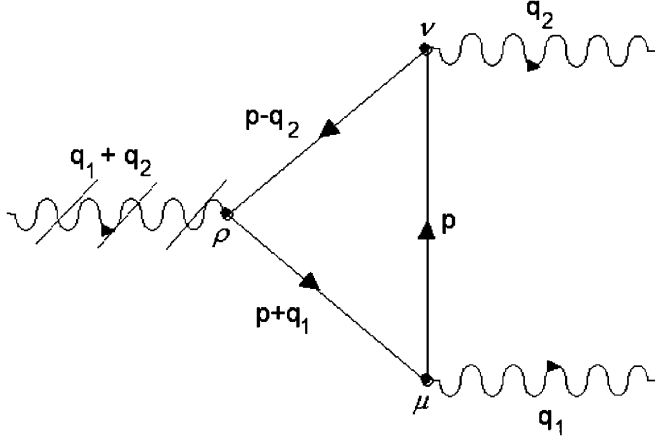


FIG. 1. The axial-vector-vector 3-point function. Wavy lines correspond to vector currents and wavy slashed line corresponds to the axial vector current.

For nonlocal axial vertex in the chiral limit, the axial Ward identity reads:

$$\Lambda_{AV}^{\mu a; vb}(p', q', p, q) = \int d\omega \rho(\omega) \left\{ \frac{i}{\not{p}' - \omega} \gamma^\nu \frac{\tau^b}{2} \frac{i}{\not{p} + \not{q} - \omega} \left(g_A \gamma^\mu - 2\omega \frac{q^\mu}{q^2} + (1 - g_A) \frac{q^\mu \not{q}}{q^2} \right) \gamma^5 \frac{\tau_a}{2} \frac{i}{\not{p} - \omega} \right. \\ \left. + \frac{i}{\not{p}' - \omega} \left(g_A \gamma^\mu - 2\omega \frac{q^\mu}{q^2} + (1 - g_A) \frac{q^\mu \not{q}}{q^2} \right) \gamma^5 \frac{\tau_a}{2} \frac{i}{\not{p}' - \not{q} - \omega} \gamma^\nu \frac{\tau^b}{2} \frac{i}{\not{p} - \omega} \right\}, \quad (28)$$

with $p' + q' = p + q$. In fact, replacing Eq. (28) in Eq. (26) one obtains:

$$-iq_\mu \Lambda_{AV}^{\mu a; vb}(p', q', p, q) = i\epsilon_{bac} \Lambda_A^{\nu c}(p', p)|_{g_A=1} + \Lambda_V^{vb}(p', p + q) \gamma^5 \frac{\tau_a}{2} + \gamma^5 \frac{\tau_a}{2} \Lambda_V^{vb}(p' - q, p) \\ + i2\epsilon_{bac} \frac{q^\nu}{q^2} \int d\omega \rho(\omega) \omega \frac{i}{\not{p}' - \omega} \gamma^5 \frac{\tau^c}{2} \frac{i}{\not{p} - \omega} \quad (29)$$

Equation (29) displays the expected partial conservation of the axial current. This can be verified by evaluating the last term in Eq. (29) with $\not{p}' = \not{p}$, resulting in

$$\left\{ 2i\epsilon_{bac} \frac{q^\nu}{q^2} \int d\omega \rho(\omega) \omega \frac{i}{\not{p}' - \omega} \gamma^5 \frac{\tau^c}{2} \frac{i}{\not{p} - \omega} \right\}_{\not{p}' = \not{p}} \\ = \left\{ 2i\epsilon_{bac} \frac{q^\nu}{q^2} \int d\omega \rho(\omega) \omega \right. \\ \left. \times \frac{\not{p}' = \not{p} - \omega^2 - \omega(\not{p}' - \not{p})}{(p'^2 - \omega^2)(p^2 - \omega^2)} \gamma^5 \frac{\tau^c}{2} \right\}_{\not{p}' = \not{p}} \\ = 2i\epsilon_{bac} \frac{q^\nu}{q^2} \gamma^5 \frac{\tau^c}{2} \int d\omega \rho(\omega) \frac{\omega}{(p^2 - \omega^2)} \\ = 2i\epsilon_{bac} m(p) \frac{q^\nu}{q^2} \gamma^5 \frac{\tau^c}{2} \int d\omega \rho(\omega) \frac{1}{(p^2 - \omega^2)} \quad (30)$$

where we have made use of Eq. (9). So, the last term of Eq. (29) shows the violation of the axial current conservation arising from the fermionic mass term, as usual. When chiral symmetry is restored, Eq. (30) will vanish, and the axial Ward identity, Eq. (26), will be fulfilled by the solution (28).

$$-iq_\mu \Lambda_{AV}^{\mu a; vb}(p', q', p, q) = i\epsilon_{bac} \Lambda_A^{\nu c}(p', p)|_{g_A=1} \\ + \Lambda_V^{vb}(p', p + q) \gamma^5 \frac{\tau_a}{2} \\ + \gamma^5 \frac{\tau_a}{2} \Lambda_V^{vb}(p' - q, p), \quad (26)$$

The first term on the right hand side of Eq. (26) is the axial vertex computed with $g_A = 1$. This term express the fact that the contraction of the axial vertex with the exchanged momentum does not depends on g_A , as can be easily checked by evaluating $q_\mu \Lambda_A^{\mu a}$ from Eq. (21) with the use of Eqs. (22) and (23).

The vector Ward identity reads:

$$-iq'_\nu \Lambda_{AV}^{\mu a; vb}(p', q', p, q) = i\epsilon_{bac} \Lambda_A^{\mu c}(p', p) \\ - \Lambda_A^{\mu b}(p + q, p) \frac{\tau_a}{2} - \frac{\tau_a}{2} \Lambda_A^{\mu b}(p', p' - q). \quad (27)$$

A solution that fulfills Eq. (26) and (27) in the chiral limit is given by:

In the SQM the probability amplitude for the axial-vector-vector process is obtained by closing the quark line in the unamputated two currents (axial and vector) vertex, Eq. (28). With the appropriated insertion of the charge matrices, and the momenta labels chosen as in Fig. 1, this probability amplitude is given by:

$$T_{\rho\mu\nu}^{AVV}(q_1, q_2) = - \int d\omega \rho(\omega) \int_p \text{Tr} [N_C i \tau^3 (g_A(\omega) \gamma_\rho \\ - \frac{2\omega}{q^2} q_\rho + \frac{1 - g_A(\omega)}{q^2} q_\rho \not{q}) \gamma^5 \frac{i}{\not{p} + \not{q}_1 - \omega} \\ \times i \hat{Q} \gamma_\mu \frac{i}{\not{p} - \omega} i \hat{Q} \gamma_\nu \frac{i}{\not{p} - \not{q}_2 - \omega}] \\ + \text{crossed terms}, \quad (31)$$

where \hat{Q} is the quark charge matrix, $q = q_1 + q_2$ and, for a $SU(2)$ model, τ^3 is the Pauli matrix σ_z . In order to compare with the PCAC relation, let us rewrite the term proportional to $\frac{q}{q^2} q_\rho$ as

$$T_{\mu\nu} = \frac{1}{m} \int d\omega \rho(\omega) \omega \int_p \text{Tr} \left\{ \frac{\gamma_5(\not{p} + \not{q}_1 + \omega)}{((p+q_1)^2 - \omega^2)} \right. \\ \left. \times \frac{\gamma_\mu(\not{p} + \omega) \gamma_\nu(\not{p} - \not{q}_2 + \omega)}{(p^2 - \omega^2)((p-q_2)^2 - \omega^2)} \right\} + \text{crossed terms} \quad (32)$$

where we have introduced the mass function at zero external momentum m in order to associate $T_{\mu\nu}$ with the standard representation of the neutral pion to two photons decay amplitude. Equation (31), of course, does not depend on m , and can be rewritten as

$$T_{\rho\mu\nu}^{AVV}(q_1, q_2) = \lambda \left(\hat{T}_{\rho\mu\nu} + \frac{q_\rho}{q^2} 2mT_{\mu\nu} + \frac{q_\rho}{q^2} T_{\mu\nu}^* \right), \quad (33)$$

with

$$\hat{T}_{\rho\mu\nu} = \int d\omega \rho(\omega) g_A(\omega) \int_p \text{Tr} \left\{ \frac{\gamma_\rho \gamma_5(\not{p} + \not{q}_1 + \omega)}{((p+q_1)^2 - \omega^2)} \right. \\ \left. \times \frac{\gamma_\mu(\not{p} + \omega) \gamma_\nu(\not{p} - \not{q}_2 + \omega)}{(p^2 - \omega^2)((p-q_2)^2 - \omega^2)} \right\} + \text{crossed terms} \quad (34)$$

and

$$T_{\mu\nu}^* = \int d\omega \rho(\omega) (1 - g_A(\omega)) \int_p \text{Tr} \left\{ \frac{\not{q}_1 \gamma_5(\not{p} + \not{q}_1 + \omega)}{((p+q_1)^2 - \omega^2)} \right. \\ \left. \times \frac{\gamma_\mu(\not{p} + \omega) \gamma_\nu(\not{p} - \not{q}_2 + \omega)}{(p^2 - \omega^2)((p-q_2)^2 - \omega^2)} \right\} + \text{crossed terms}. \quad (35)$$

We also have

$$\lambda = N_c \text{Tr} \{ \tau^3 \hat{Q} \hat{Q} \} = 1. \quad (36)$$

It is interesting to note that, after taking the Dirac traces, Eq. (32) is logarithmically divergent, and thus does not present surface terms in its computation. Equation (35) is quadratically divergent, and in principle should present surface terms. However, in the sharp cutoff regularization scheme these surface terms result zero. After a little algebra, we obtain

$$T_{\mu\nu}^* = \frac{1}{2\pi^2} \epsilon_{\rho\mu\nu\eta} q_1^\rho q_2^\eta \int d\omega \rho(\omega) (1 - g_A(\omega)), \quad (37)$$

and

$$2mT_{\mu\nu} = \frac{2}{4\pi^2} \epsilon_{\mu\nu\rho\eta} q_1^\eta q_2^\rho \int d\omega \rho(\omega). \quad (38)$$

Equation (34), however, presents non-null surface terms in its calculation. These surface terms come from the difference of logarithmically divergent integrals, the same integrals in p appearing in Eq. (20),

$$\int_p \left(\frac{g^{\mu\nu}}{(p^2 - \omega^2)^2} - 4 \frac{p^\mu p^\nu}{(p^2 - \omega^2)^3} \right). \quad (39)$$

As already discussed, this term corresponds to a regularization ambiguity, since it can result zero in some regularizations (e.g. gauge invariant Pauli-Villars) or finite in other ones ($= \frac{i}{2(4\pi^2)}$ in the sharp cutoff regularization [56]). So, introducing two Feynman parameters on Eq. (34) we obtain

$$\hat{T}_{\rho\mu\nu} = \int d\omega \rho(\omega) g_A(\omega) \int_0^1 dy \int_0^{1-y} dx \int_p \\ \times \frac{\text{Tr} \{ \gamma_\rho \gamma_5(\not{p} + \not{q}_1 + \omega) \gamma_\mu(\not{p} + \omega) \gamma_\nu(\not{p} - \not{q}_2 + \omega) \}}{((p+q_1x - q_2y)^2 - M^2(\omega, x, y))^3}, \quad (40)$$

with $M^2(\omega, x, y) = q_1^2 x(x-1) + q_2^2 y(y-1) - 2q_1 q_2 xy + \omega^2$ (the sharp cutoff regularization scheme is, as before, implicitly assumed). In several regularization schemes, as in the sharp-cutoff, we are not allowed to shift the p variable in Eq. (40), unless we introduce the corresponding surface term. Following the procedure employed in Ref. [25], we obtain

$$\hat{T}_{\rho\mu\nu} = S_{\rho\mu\nu} + \int d\omega \rho(\omega) g_A(\omega) \int_0^1 dy \int_0^{1-y} dx \int_p \\ \times \frac{1}{(p^2 - M^2(\omega, x, y))^3} \text{Tr} \{ \gamma_\rho \gamma_5(\not{p} + (1-x)\not{q}_1 \\ + y\not{q}_2 + \omega) \gamma_\mu(\not{p} - x\not{q}_1 + y\not{q}_2 + \omega) \gamma_\nu(\not{p} - x\not{q}_1 \\ - (1-y)\not{q}_2 + \omega) \}, \quad (41)$$

where $S_{\rho\mu\nu}$ is the surface term given by

$$S_{\rho\mu\nu} = \int d\omega \rho(\omega) g_A(\omega) \int_0^1 dy \int_0^{1-y} dx \int_p (q_1^\sigma x - q_2^\sigma y) \\ \times \frac{\partial}{\partial p^\sigma} \left[\frac{\text{Tr} \{ \gamma_\rho \gamma_5 \not{p} \gamma_\mu \not{p} \gamma_\nu \not{p} \}}{(p^2 - M^2(\omega, x, y))^3} \right] \quad (42)$$

Of course, as usual, $T_{\rho\mu\nu}^{AVV}$ is finite, so all integrals can be computed and the connection limit $\Lambda \rightarrow \infty$ can be taken. After doing that, we have

$$\hat{T}_{\rho\mu\nu} = \frac{1}{4\pi^2} \left\{ \int d\omega \frac{g_A(\omega) \rho(\omega)}{6\omega^2} q_1^\eta q_2^\beta \{ \epsilon_{\rho\mu\beta\eta} (q_{2\nu} + 2q_{1\nu}) \right. \\ \left. + \epsilon_{\rho\beta\nu\eta} (2q_{2\mu} + q_{1\mu}) \} + \frac{2}{3} \epsilon_{\rho\mu\nu\eta} \int d\omega \rho(\omega) \right. \\ \left. \times g_A(\omega) (q_2^\eta - q_1^\eta) \right\} + S_{\rho\mu\nu} \quad (43)$$

with

$$S_{\rho\mu\nu} = \frac{1}{12\pi^2} \epsilon_{\rho\mu\nu\eta} \int d\omega \rho(\omega) g_A(\omega) (q_2^\eta - q_1^\eta) \quad (44)$$

From Eqs. (33), (37), and (43), the evaluation of the Ward-Takahashi identities in momentum space results in

$$q_1^\mu T_{\rho\mu\nu}^{AVV} = \frac{1}{4\pi^2} \frac{2}{3} q_1^\mu q_2^\eta \epsilon_{\mu\nu\rho\eta} \int d\omega \rho(\omega) g_A(\omega) \\ + q_1^\mu S_{\rho\mu\nu} \quad (45)$$

for the VWI, with a similar expression for $q_2^\nu T_{\rho\mu\nu}^{AVV}$, and

$$(q_1^\rho + q_2^\rho)T_{\rho\mu\nu}^{AVV} = \frac{2}{4\pi^2} q_1^\eta q_2^\rho \epsilon_{\mu\nu\rho\eta} \frac{1}{3} \int d\omega \rho(\omega) g_A(\omega) + (q_1^\rho + q_2^\rho)S_{\rho\mu\nu} + 2mT_{\mu\nu} - \frac{2}{4\pi^2} q_1^\eta q_2^\rho \epsilon_{\mu\nu\rho\eta}, \quad (46)$$

for the AWI.

Before performing the spectral mass integrals, the result is potentially ambiguous due the presence of the surface term $S_{\rho\mu\nu}$ in Eqs. (43), (45), and (46). For a constant (ω independent) g_A , the surface term, Eq. (44), depends on the choice of the intermediary regularization employed in the evaluation of Eq. (31), and it could generate different results in the Ward identity to be violated by the anomaly, as it is well known.

Nevertheless, from Eq. (44) we can see that the computation of the axial-vector-vector amplitude in the SQM with the axial vertex given by Eq. (21) and with $g_A = \alpha\omega$ (or $g_A = \alpha\omega^2$) is free from surface terms—the spectral condition, Eq. (3), ensures their vanishing, as well as the vanishing of the other terms proportional to $g_A(\omega)$ in Eqs. (45) and (46). In this case, the VWI is always preserved (i.e., is preserved in an ambiguity free way) and the AWI is violated. Hence, we clearly see that the choice of the specific dependence of the axial coupling with the spectral mass can result in an ambiguity free result, with the conservation of the VWI, and the violation of the chiral one, as expected for QCD. In this case, the vanishing of the spectral axial coupling in the zero spectral mass limit does not imply in the vanishing of the axial coupling itself. However, it is also possible to obtain the violation of the VWI, with conservation of the axial current, when the spectral axial coupling depends on the spectral mass with a power lower than 1. In this case, the presence of ambiguous terms implies in the freedom on choosing which Ward identity is to be violated.

VI. CONCLUSIONS

We have investigated the chiral anomaly in the context of the spectral quark model. We have proposed a generalized form of the four points vertex function with one axial

and one vector current which includes the possibility of a nonunitary, spectral mass dependent, axial coupling. This vertex function displays the expected partial conservation of the axial current, with the chiral Ward identity being violated by a term proportional to the mass function which vanishes at the chiral limit. The triangle anomaly was computed, taking into account the surface term that appears in its calculation, and we have shown that a dependence of the axial coupling on integer positive powers of the spectral mass is necessary in order to render the triangle amplitude free from ambiguities. In this case, the vector Ward identity is preserved, with the chiral Ward identity being violated as expected for QCD. We also remarked that the dependence of the pion to quarks couplings on the spectral mass, via the Goldberger-Treiman relation for the pseudoscalar coupling, and via the ansatz employed here for the axial coupling, are essential to the obtaining of a result free from divergences and regularization ambiguities.

In summary, we have shown that, in the context of the spectral quark model, the chiral anomaly computation can be carried out reproducing the expected result without any ambiguity introduced by regularizations schemes, if the axial vertex is treated as a nonlocal spectral mass dependent vertex. This shows to be compatible with a nonunitary axial-vector coupling, not vanishing at the chiral limit. Our result suggests that the spectral quark model provides an useful mechanism to justify why in QCD the anomaly violates the chiral symmetry, instead of violating gauge symmetry, as in the case of the non conservation of the baryonic number. However, we also discussed that the more general form for the axial vertex leaves room to the violation of the vector Ward Identity, complying the axial one. It could be interesting to analyze these features of the spectral quark model in the QCD chiral phase transition [57,58] or in the construction of spectral approaches to some recent applications of the chiral anomaly [59–62].

ACKNOWLEDGMENTS

This research was supported by CAPES-Brazil. The authors would like to thank H. Caldas for carefully reading the manuscript.

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