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Isolated horizons, *p*-form matter fields, topology, and the black-hole/string correspondence principle

Tomáš Liko*

Institute for Gravitation and the Cosmos, Pennsylvania State University, University Park, Pennsylvania 16802, USA

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We study the mechanics of D-dimensional isolated horizons (IHs) for Einstein gravity in the presence of arbitrary p-form matter fields. This generalizes the analysis of Copsey and Horowitz to nonstationary spacetimes and therefore the local first law in $D > 4$ dimensions to include nonmonopolar (dipole) charges. The only requirement for the local first law to hold is that the action has to be differentiable. The resulting conserved charges are all intrinsic to the horizon and are independent of the topology of the horizon cross sections. We explicitly calculate the local charges for five-dimensional black holes and black rings that are relevant within the context of superstring theory. We conclude with some comments on the black-hole/string correspondence principle and argue that IHs (or some other quasilocal variant) should play a fundamental role in superstring theory.

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I. INTRODUCTION

The advent of superstring theory revolutionized our view of the Universe, for example, with the requirement of extra spatial dimensions. The natural question that should be investigated is the following: *What properties of black holes in four dimensions carry over to higher-dimensional spacetimes?* More specifically, we should ask the following question: *What are the generic features of black holes in higher-dimensional spacetimes in general, and within the superstring theory context in particular?* An ideal method for investigating such questions is to employ a covariant phase space framework that includes all black-hole solutions to the equations of motion for a given action principle.

Such a framework does exist and is known as the isolated horizon (IH) framework [1]. The classical theory of IHs was motivated by earlier considerations of trapping horizons [2], but the framework is considerably different as covariant phase space methods [3–6] are employed in the case of IHs. All the quantities that appear in the first law of IH mechanics are defined intrinsically at the horizon. The concept of such a surface generalizes the notion of a Killing horizon to much more general, and therefore physical, spacetimes that may include external radiation fields that are dynamical. Examples of such systems in general relativity are given by the so-called Robinson-Trautman spacetimes [7,8].

The focus of this paper is to examine the consequences of the IH boundary conditions on the covariant phase space of solutions to the equations of motion in the presence of generic p-form matter fields and to determine the conserved charges from the symplectic structure. Among other results, we find that the natural conserved charge associated with the matter term for the electric dual of Einstein-Maxwell theory with dilaton that arises from the symplectic structure is the electric dipole charge, not the magnetic monopolar charge that one would expect. This work generalizes two sets of constructions: the first law of Copsey and Horowitz [9] is generalized to nonstationary spacetimes and the IH framework in $D > 4$ dimensions [10–12] is extended to include nonmonopolar charges.

We consider a D-dimensional manifold $\mathcal{M}$ bounded by two spacelike partial Cauchy surfaces, M_1 and M_2, which are asymptotically related by a time translation and extend from the internal boundary Δ [with $\Delta \cap M \cong \mathbb{S}^{D-2}$ for some compact $(D-2)$-space $\mathbb{S}^{D-2}$ with positive constant curvature] to the boundary at infinity τ_∞.

In the first-order formulation of general relativity, the action for the theory that we consider is given by

$$S = \frac{1}{2\kappa_D}\int_{\mathcal{M}} \Sigma_{IJ}\wedge\Omega^{IJ} + \mathcal{L}_{\mathrm{M}}[\Phi, \mathcal{F}; \mathcal{A}] - \frac{1}{2\kappa_D} \times \int_{\tau_\infty} \Sigma_{IJ}\wedge A^{IJ}. \tag{1}$$

Here, $\kappa_D = 8\pi G_D$ with G_D the D-dimensional gravitational constant. This action depends on the coframe e^I, the gravitational $SO(D-1, 1)$ connection $A^I{}_J$, the scalar field Φ and the generic p-form field $\mathcal{F} = d\mathcal{A}$ (with p an integer such that $2 \le p \le D-2$). The coframe determines the metric $g_{ab} = \eta_{IJ} e_a{}^I \otimes e_b{}^J$, $(D-2)$-form $\Sigma_{IJ} = [1/(D-2)!]\epsilon_{IJK_1\ldots K_{D-2}} e^{K_1}\wedge\cdots\wedge e^{K_{D-2}}$ and spacetime volume form $\boldsymbol{\epsilon} = e^0\wedge\cdots\wedge e^{D-1}$, where $\epsilon_{I_1\ldots I_D}$ is the totally antisymmetric Levi-Civita tensor. The connection determines the curvature 2-form

$$\Omega^I{}_J = dA^I{}_J + A^I{}_K\wedge A^K{}_J = \tfrac{1}{2}R^I{}_{JKL}e^K\wedge e^L, \tag{2}$$

with $R^I{}_{JKL}$ as the Riemann tensor. In this paper, spacetime indices $a, b, \ldots \in \{0, \ldots, D-1\}$ are raised and lowered using the metric g_{ab} and internal indices $I, J, \ldots \in \{0, \ldots, D-1\}$ are raised and lowered using the

*liko@gravity.psu.edu

Minkowski metric $\eta_{IJ} = \mathrm{diag}(-1, \ldots, 1)$. The boundary term at the timelike cylinder τ_∞ at infinity is required in order that the action be differentiable. It is the natural boundary term associated with the first-order action principle. Important properties of this boundary term are discussed in [13–15].

II. HORIZON STRUCTURES AND DIFFERENTIABILITY OF THE ACTION

Let us remind the reader of the basic definition of a rotating weakly isolated horizon [11,16,17], with a suitable generalization of the boundary conditions tailored to include the presence of p-form matter fields.

Definition I. A rotating weakly isolated horizon (WIH) Δ is a null surface and has a degenerate metric q_{ab} with signature $0 + \ldots +$ (with $D-2$ nondegenerate spatial directions) along with an equivalence class of null normals $[\ell]$ (the equivalence relation being defined by $\ell' = z\ell$ for some constant z) and spacelike rotational vector fields ϕ^a_ι ($\iota \in \{1, \ldots, \lfloor (D-1)/2 \rfloor \| \lfloor\cdot\rfloor$ denotes "integer value of"$\}$) such that the following conditions hold: (1) the expansion $\theta_{(\ell)}$ of ℓ_a vanishes on Δ; (2) the field equations hold on Δ; (3) the stress-energy tensor is such that the vector $-T^a{}_b\ell^b$ is a future-directed and causal vector; (4) $\mathcal{L}_\ell \omega_a = 0$ and $\mathcal{L}_\ell \underleftarrow{\mathcal{A}} = 0$ for all $\ell \in [\ell]$ (see below); (5) ϕ^a_ι satisfy $\mathcal{L}_\phi q_{ab} = \mathcal{L}_\phi \ell_a = \mathcal{L}_\phi \omega_a = \mathcal{L}_\phi \underleftarrow{\mathcal{A}} = \mathcal{L}_\phi \underleftarrow{\mathcal{F}} = 0$.

The first three conditions determine the intrinsic geometry of Δ. Since ℓ is normal to Δ the associated null congruence is necessarily twist-free and geodesic. By condition (1) that congruence is nonexpanding. Then the Raychaudhuri equation implies that $T_{ab}\ell^a\ell^b = -\sigma_{ab}\sigma^{ab}$, with σ_{ab} the shear tensor, and applying the energy condition (3) we find that $\sigma_{ab} = 0$.

In addition, the vanishing of the expansion, twist and shear imply that [16]

$$\nabla_{\underleftarrow{a}} \ell_b \approx \omega_a \ell_b, \tag{3}$$

with "$\approx$" denoting equality restricted to Δ and the underarrow indicating pullback to Δ. Thus the 1-form ω is the natural connection induced on the horizon. The conditions also imply that

$$\ell \lrcorner \underleftarrow{\mathcal{F}} = 0. \tag{4}$$

This property will play an important role in the derivation of the first law with nonmonopolar charges for black rings. We emphasize that this condition is a consequence of the boundary conditions and not an assumption.

Condition (5) captures the notion of a WIH rotating with angular velocities Ω_ι whereby the rotational vector fields ϕ^a_ι are symmetries of Δ. For a multidimensional rotating WIH, a suitable evolution vector field on the covariant phase space is given by [11,17]

$$\xi^a = z\ell^a + \sum_{\iota=1}^{\lfloor (D-1)/2 \rfloor} \Omega_\iota \phi^a_\iota. \tag{5}$$

This vector field is spacelike in general and becomes null when all angular momenta are zero.

We do not fix the fields at the inner boundary Δ, so we need to determine explicitly the surface terms for which the action (1) will be differentiable. To this end, let $\Psi \in \{e, A, \Phi, \mathcal{F}\}$ denote the set of field variables. Then, taking the first variation of (1) gives

$$\delta S = \frac{1}{2\kappa_D} \int_{\mathcal{M}} E[\Psi]\delta\Psi - \frac{1}{2\kappa_D} \int_\Delta J[\Psi, \delta\Psi], \tag{6}$$

with $E[\Psi] = 0$ representing the equations of motion and $J[\Psi, \delta\Psi]$ representing a linear combination of gravitational and matter-field surface terms. In the present case, we have that

$$J[\Psi, \delta\Psi] = \Sigma_{IJ} \wedge \delta A^{IJ} + \mathbf{Y} \wedge \delta\mathcal{A}; \tag{7}$$

here we defined $\mathbf{Y} = \mathbb{D}\mathcal{L}_{\mathrm{M}}/\mathbb{D}\mathcal{F}$ as the functional derivative of the Lagrangian density $\mathcal{L}_{\mathrm{M}}$ with respect to $\mathcal{F}$.

It turns out that the pullback of J to Δ vanishes, and therefore the action (1) is indeed differentiable and the equations of motion $E[\Psi] = 0$ follow from the variational principle $\delta S = 0$. In particular, the pullback of the gravitational surface term is given by [12]

$$\Sigma \wedge \delta \underleftarrow{A} \approx \tilde{\boldsymbol{\epsilon}} \wedge \delta\omega, \tag{8}$$

with $\tilde{\boldsymbol{\epsilon}} = \vartheta^{(1)} \wedge \ldots \wedge \vartheta^{(D-2)}$ the area element of the cross section $\mathbb{S}^{D-2}$ of the horizon, and $\vartheta_{(i)}$ ($i \in \{2, \ldots, D-1\}$) are $D-2$ spacelike vectors adapted to $\mathbb{S}^{D-2}$ that satisfy the orthogonality condition $\vartheta_{(i)} \cdot \vartheta_{(j)} = \delta_{ij}$. The key property of Δ is that the variation of ℓ is proportional to ℓ itself. Then from the WIH condition (4) it follows that $\mathcal{L}_\ell \delta\omega = 0$. However, ω is held fixed on $M_{\{1,2\}}$ which means that $\delta\omega = 0$ on the initial and final cross-sections of Δ (i.e. on $M_1 \cap \Delta$ and on $M_2 \cap \Delta$), and because $\delta\omega$ is Lie dragged on Δ it follows that $\tilde{\boldsymbol{\epsilon}} \wedge \delta\omega \approx 0$. The same argument also holds for the matter-field part of the surface term: from condition (4) $\mathcal{L}_\ell \underleftarrow{\mathcal{A}} = 0$, and with $\delta\ell \propto \ell$ on Δ it follows that $\mathbf{Y} \wedge \delta \underleftarrow{\mathcal{A}} \approx 0$, whence

$$J[\Psi, \delta\Psi]|_\Delta \approx 0. \tag{9}$$

Therefore, in the presence of an internal null boundary Δ satisfying the conditions of Definition I, the action (1) is differentiable.

III. COVARIANT PHASE SPACE AND CONSERVED CHARGES

As in the previous papers on IHs (e.g. [11,12,16,17]), the first law follows directly from applying standard covariant phase space methods [3–6]. The symplectic current is obtained from antisymmetrizing the second variation of the surface term; integrating over the boundary $M_1 \cup M_2 \cup \Delta$ (because the asymptotic conditions ensure that the integral over τ_∞ vanishes) gives the symplectic structure $\mathbf{\Omega} \equiv \mathbf{\Omega}(\delta_1, \delta_2)$. The first law then follows directly from evaluating the symplectic structure at (δ, δ_ξ).

In the present case the closed and conserved symplectic structure is given by

$$\mathbf{\Omega}(\delta_1, \delta_2) = \frac{1}{2\kappa_D}\int_M [\delta_{[1}\Sigma_{IJ} \wedge \delta_{2]}A^{IJ} - \delta_{[1}\mathbf{Y} \wedge \delta_{2]}\mathcal{A}] + \frac{1}{\kappa_D}\oint_{\mathbb{S}^{D-2}}[\delta_{[1}\tilde{\boldsymbol{\epsilon}} \wedge \delta_{2]}\psi + \delta_{[1}\mathbf{Y} \wedge \delta_{2]}\boldsymbol{\chi}]. \quad (10)$$

Here we defined the potential ψ for the surface gravity $\kappa_{(\ell)}$ and analogous $(p-2)$-form $\boldsymbol{\chi}$ for the $(p-2)$-form $\mathbf{\Phi}_{(\ell)}$ such that

$$\mathcal{L}_\ell \psi \approx \ell \lrcorner \omega = \kappa_{(\ell)} \quad \text{and} \quad \mathcal{L}_\ell \boldsymbol{\chi} \approx \ell \lrcorner \mathcal{A} = -\mathbf{\Phi}_{(\ell)}. \quad (11)$$

We find that evaluating the horizon integral at (δ, δ_ξ) is given by

$$\mathbf{\Omega}|_\Delta = \frac{1}{\kappa_D}\oint_{\mathbb{S}^{D-2}}\kappa_{(z\ell)}\delta\tilde{\boldsymbol{\epsilon}} + \frac{1}{\kappa_D}\oint_{\mathbb{S}^{D-2}}\mathbf{\Phi}_{(z\ell)} \wedge \delta\mathbf{Y} + \sum_{\iota=1}^{[(D-1)/2]}\frac{\Omega_\iota}{\kappa_D}\delta\oint_{\mathbb{S}^{D-2}}[(\phi_\iota \lrcorner \omega)\tilde{\boldsymbol{\epsilon}} + (\phi_\iota \lrcorner \mathcal{A}) \wedge \mathbf{Y}], \quad (12)$$

where we used $\kappa_{(z\ell)} = \mathcal{L}_{z\ell}\psi = z\ell \lrcorner \omega$ and $\mathbf{\Phi}_{(z\ell)} = \mathcal{L}_{z\ell}\boldsymbol{\chi} = z\ell \lrcorner \mathcal{A}$. These quantities are constant for any given horizon, but in general vary across the phase space from one point to another. This implies that (12) is in general *not* a total variation. However, if $\kappa_{(z\ell)}$, $\mathbf{\Phi}_{(z\ell)}$ and Ω_ι can be expressed as functions of the entropy $\mathcal{S}$, charge $\mathcal{Q}$, and angular momenta $\mathcal{J}_\iota$ defined by

$$\mathcal{S} = \frac{1}{4G_D}\oint_{\mathbb{S}^{D-2}}\tilde{\boldsymbol{\epsilon}}, \quad (13)$$

$$\mathcal{Q} = \frac{1}{8\pi G_D}\oint_{\mathbb{S}^{D-2}}\mathbf{Y}, \quad (14)$$

$$\mathcal{J}_\iota = \frac{1}{8\pi G_D}\oint_{\mathbb{S}^{D-2}}[(\phi_\iota \lrcorner \omega)\tilde{\boldsymbol{\epsilon}} + (\phi_\iota \lrcorner \mathcal{A}) \wedge \mathbf{Y}], \quad (15)$$

and satisfy the integrability conditions

$$\frac{\partial \kappa}{\partial \mathcal{J}} = \frac{\partial \Omega}{\partial \mathcal{S}}, \qquad \frac{\partial \kappa}{\partial \mathcal{Q}} = \frac{\partial \mathbf{\Phi}}{\partial \mathcal{S}}, \qquad \frac{\partial \Omega}{\partial \mathcal{Q}} = \frac{\partial \mathbf{\Phi}}{\partial \mathcal{J}}, \quad (16)$$

then there exists a function $\mathcal{E}$ such that [11,17]

$$\mathbf{\Omega}|_\Delta(\delta, \delta_\xi) = \delta\mathcal{E}. \quad (17)$$

In this case (12) becomes

$$\delta\mathcal{E} = \frac{\kappa_{(z\ell)}}{2\pi}\delta\mathcal{S} + \frac{1}{\kappa_D}\oint_{\mathbb{S}^{D-2}}\mathbf{\Phi}_{(z\ell)} \wedge \delta\mathbf{Y} + \sum_{\iota=1}^{[(D-1)/2]}\Omega_\iota\delta\mathcal{J}_\iota, \quad (18)$$

which is the first law (for a quasistatic process). Therefore, rotating WIHs in D-dimensional asymptotically flat spacetimes with generic p-form matter fields satisfy the first law.

The first law (18) holds for any rotating WIH in the presence of p-form matter fields, regardless of the topology of the horizon cross section. For WIHs in asymptotically flat spacetimes, there is a very strong constraint on the possible topologies. As was shown in [12], the integral of the scalar curvature of the horizon cross section is strictly positive. This implies that in four dimensions (together with the Gauss-Bonnet theorem) $\mathbb{S}^2 \cong S^2$ and that in five dimensions $\mathbb{S}^3$ can only be a finite connected sum of the three-sphere S^3 or of the ring $S^1 \times S^2$. These results on topology are in agreement with the recent extension of the Hawking topology theorem to higher dimensions [18–22].

In addition, we note that the first law (18) is the *equilibrium version* of the first law of black-hole mechanics. That is, (18) relates the infinitesimal changes of the conserved charges of two nearby WIHs within the covariant phase space of solutions. However, as was discussed in [23], a local first law such as (18) also has a natural interpretation as the *physical process* version of black-hole mechanics [24–26], whereby the infinitesimal changes of the conserved charges of a single black hole are related when a small mass is dropped into the horizon and the black hole is allowed to settle into a new equilibrium state.

An extension of the current framework to asymptotically anti-de Sitter (ADS) spacetimes, along the lines of [11], is straightforward. In the presence of a negative cosmological constant $\Lambda = -(D-1)(D-2)/(2L^2)$ the covariant phase space of WIHs is modified to include a set of conserved charges at the boundary at infinity $\mathcal{S}$ (with $\mathcal{S} \cap M \cong \mathbb{C}^{D-2}$ for some compact $(D-2)$-space $\mathbb{C}^{D-2}$) [11]. These are the Ashtekar-Magnon-Das (AMD) charges [27,28]

$$\mathcal{Q}_\xi^{(I)} = \frac{L}{8\pi G_D}\oint_{\mathbb{C}^{D-2}}\tilde{E}_{ab}k^a\tilde{u}^b\tilde{\boldsymbol{\epsilon}}, \quad (19)$$

with k^a a Killing vector field that generates a symmetry (i.e. time translation etc.), $\tilde{u}^a$ the unit timelike normal to $\mathbb{C}^{D-2}$, $\tilde{\boldsymbol{\epsilon}}$ the area form on $\mathbb{C}^{D-2}$ and $\tilde{E}_{ab}$ the leading-order electric part of the Weyl tensor $\tilde{C}_{abcd}$. Explicitly we have that

$$\tilde{E}_{ab} = \frac{1}{D-3}\Omega^{3-D}\tilde{C}_{abcd}\tilde{n}^c\tilde{n}^d, \quad (20)$$

where $\tilde{n}^a = \tilde{\nabla}^a\Omega$ and Ω is a function on the conformally completed manifold $\widehat{\mathcal{M}} \cong \mathcal{M} \cup \mathcal{S}$ that defines the unphysical metric $\tilde{g}_{ab}$ on $\mathcal{M}$ in terms of the physical spacetime metric g_{ab} via $\tilde{g}_{ab} = \Omega^2 g_{ab}$. As was shown in Appendix B of [29], inclusion of antisymmetric tensor fields in the action does not contribute anything to the charges at $\mathcal{I}$ because the fields fall off too quickly. In particular this implies that in the presence of generic p-form fields the charges at infinity are the AMD charges. It is important to keep in mind that the charges at $\mathcal{I}$ are the charges of the spacetime and are independent of the local charges at Δ.

IV. EXAMPLE THEORIES

The preceding analysis was rather abstract and technical. In this section we will apply the framework to two effective theories that arise within superstring theory. This will serve to illustrate the generality of the IH framework and will lead to some interesting surprises.

Let us consider first Einstein-Maxwell theory in five dimensions with electromagnetic Chern-Simons term. The action for this theory in five dimensions is given by

$$S = \frac{1}{2\kappa_5}\int_{\mathcal{M}} \Sigma_{IJ}\wedge\Omega^{IJ} - \frac{1}{4}\boldsymbol{F}\wedge\star\boldsymbol{F} - \frac{2}{3\sqrt{3}}\boldsymbol{A}\wedge\boldsymbol{F}\wedge\boldsymbol{F} - \frac{1}{2\kappa_5}\int_{\tau_\infty}\Sigma_{IJ}\wedge A^{IJ}. \tag{21}$$

Here, $\boldsymbol{F} = d\boldsymbol{A}$ is the field strength of the connection 1-form $\boldsymbol{A}$ and "$\star$" denotes the Hodge dual. The last term is a Chern-Simons (CS) term for the electromagnetic field. For this theory we take $\mathcal{F} = \boldsymbol{F}$ and $\mathcal{A} = \boldsymbol{A}$. Then $\boldsymbol{\Phi}_{(z\ell)} = -z\ell \lrcorner \boldsymbol{A} = \Phi_{(z\ell)}$ is just a scalar potential and $\boldsymbol{\Upsilon} = \star\boldsymbol{F} + [4/(3\sqrt{3})]\boldsymbol{A}\wedge\boldsymbol{A}$. The first law then takes the form

$$\delta\mathcal{E} = \frac{\kappa_{(z\ell)}}{2\pi}\delta\mathcal{S} + \Phi_{(z\ell)}\delta\mathcal{Q} + \sum_{\iota=1}^{\lfloor(D-1)/2\rfloor}\Omega_\iota\delta\mathcal{J}_\iota, \tag{22}$$

with electric charge $\mathcal{Q}$ given by

$$\mathcal{Q} = \frac{1}{8\pi G_5}\oint_{\mathbb{S}^3}\star\boldsymbol{F} + \frac{4}{3\sqrt{3}}\boldsymbol{A}\wedge\boldsymbol{F}. \tag{23}$$

This is the natural conserved charge for both $\mathbb{S}^3 \cong S^3$ and $\mathbb{S}^3 \cong S^1\times S^2$ topologies; it is a monopolar electric charge.

Let us now consider the electric dual of Einstein-Maxwell theory with dilaton in five dimensions. The action for this theory is given by

$$S = \frac{1}{2\kappa_5}\int_{\mathcal{M}}\Sigma_{IJ}\wedge\Omega^{IJ} - \frac{1}{12}e^{-\alpha\varphi}\boldsymbol{H}\wedge\star\boldsymbol{H} - \frac{1}{2}d\varphi\wedge\star d\varphi - \frac{1}{2\kappa_5}\int_{\tau_\infty}\Sigma_{IJ}\wedge A^{IJ}. \tag{24}$$

Here, φ is the dilaton field with coupling α, and $\boldsymbol{H} = d\boldsymbol{B}$ is the field strength of the 2-form $\boldsymbol{B}$. Because $\boldsymbol{H}$ is a 3-form, one expects to *define* a magnetic monopolar charge associated with black holes within this theory. However, this is not the case for IHs. As we will now show, the IH boundary conditions will give a dipolar electric charge that is conserved. For this theory we take $\mathcal{F} = \boldsymbol{H}$ and $\mathcal{A} = \boldsymbol{B}$. Then $\boldsymbol{\Phi}_{(z\ell)} = -z\ell\lrcorner\boldsymbol{B}$ is a 1-form potential, and $\boldsymbol{\Upsilon} = e^{-\alpha\varphi}\star\boldsymbol{H}$. The first law then takes the same form as (22), but with a charge $\mathcal{Q}$ that is radically different from the electric charge (23). Here we have

$$\oint_{\mathbb{S}^3}\boldsymbol{\Phi}_{(z\ell)}\wedge\delta\boldsymbol{\Upsilon} = \oint_{\mathbb{S}^3}(z\ell\lrcorner\boldsymbol{B})\wedge\delta(e^{-\alpha\varphi}\star\boldsymbol{H}). \tag{25}$$

The key observation is that $\boldsymbol{\Phi}_{(z\ell)}$ is a closed 1-form *at the horizon*. This follows from the Cartan identity $d(z\ell\lrcorner\underleftarrow{\boldsymbol{B}}) = \mathcal{L}_{z\ell}\underleftarrow{\boldsymbol{B}} - z\ell\lrcorner\underleftarrow{\boldsymbol{H}}$; pulling this identity back to the horizon gives

$$d(z\ell\lrcorner\underleftarrow{\boldsymbol{B}}) = \mathcal{L}_{z\ell}\underleftarrow{\boldsymbol{B}} - z\ell\lrcorner\underleftarrow{\boldsymbol{H}}. \tag{26}$$

Then from Condition (4) of Definition I and Eq. (4) it immediately follows that the right hand side is zero. Because $d(z\ell\lrcorner\underleftarrow{\boldsymbol{B}}) \approx 0$ we conclude that at the horizon $z\ell\lrcorner\boldsymbol{B}$ is a closed 1-form and must therefore be the sum of an exact 1-form df and harmonic 1-form dh. That is,

$$z\ell\lrcorner\boldsymbol{B} \approx df + c\,dh, \tag{27}$$

with c a constant. The only nonzero contribution to the charge then comes from integrating h over S^1, otherwise the charge is zero [9] (see also [30,31]). Taking 2π to be the affine length of S^1, we conclude that

$$\oint_{S^1\times S^2} c\,dh\wedge\delta(e^{-\alpha\varphi}\star\boldsymbol{H}) = 2\pi c\,\delta\oint_{S^2}e^{-\alpha\varphi}\star\boldsymbol{H}, \tag{28}$$

whence the charge

$$\mathcal{Q} = \frac{1}{8\pi G_5}\oint_{S^2}e^{-\alpha\varphi}\star\boldsymbol{H}. \tag{29}$$

This is the natural conserved charge for the $\mathbb{S}^3 \cong S^1\times S^2$ topology. By contrast to the previous charge (23), however, (29) is a *dipole* electric charge.

The first law (18) that we obtained for IHs is in agreement with that which was found for stationary spacetimes [9]. However, we note that in the latter approach there also appeared a dipole charge in the first law for Einstein-Maxwell theory with electromagnetic Chern-Simons term. This charge does not appear in (22) which means that the dipole charge, although possible to define, is not a *conserved* charge for IHs. This is in agreement with what is known about the black ring solutions of Elvang *et al* [32–34].

The dipole charge (29) that we obtained for the electric dual of Einstein-Maxwell theory with dilaton is in agreement with that obtained for stationary spacetimes [9] for the dipole ring solution [35]. There the dipole charge is interpreted as an electric Kalb-Ramond charge localized on a fundamental string that winds around a contractible circle [36]. However, the other conserved charges are still measured at infinity. By contrast, here we have found a first law whereby *all* conserved charges, including the dipole charge, are localized at the source. This may have important consequences for the black-hole/string correspondence principle [37,38].

V. ISOLATED HORIZONS AND THE CORRESPONDENCE PRINCIPLE

The black-hole/string correspondence principle asserts that there is a smooth transition from a black hole to a string in the limit when the string coupling is decreased [37]. Let us briefly discuss two subtleties which suggest that IHs (or their nonequilibrium generalizations such as dynamical horizons [39,40]) should be the most appropriate framework for studying black-hole physics in superstring theory.

For the correspondence principle to work, the entropies of the black hole and string are required to be equal for a particular value of the string coupling constant, which ultimately means that the conserved charges of the two states must overlap [38]. However, the conserved charges of the black hole (other than the dipole charge) are typically measured at infinity (e.g. for Killing horizons), while the conserved charges of the string are localized on the string state; to define the conserved charges of the string *no reference needs to be made to infinity at all*. The conserved charges of the black hole should therefore *not* be defined at infinity!

In addition, specification of the conserved charges of the black hole requires an *a priori* knowledge of the internal topology, e.g. typically some density is integrated over a $(D-2)$-dimensional surface with some topology such as S^2 in four dimensions and S^3 or $S^1 \times S^2$ in five dimensions. At the transition point when the conserved charges are equal, however, the topology of the black hole is not really important because the spacetime loses its metric interpretation. Therefore, a framework for black holes should be employed that does not in any way rely on the internal topology of the horizon cross sections.

As we have shown in this paper, the IH framework together with covariant phase space methods can be used to derive a first law whereby all quantities are defined at the horizon. In order for this derivation to work we only require that the action be differentiable. The conserved charges of an IH in a specific theory naturally arise after the corresponding matter Lagrangian density is specified. Two important properties of IHs are that the conserved charges are intrinsic to the horizon and that there is no need to specify the topology of the horizon cross sections at any time. IHs should therefore be the norm rather than the exception in superstring theory.

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