Hawking radiation, covariant boundary conditions, and vacuum states

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The basic characteristics of the covariant chiral current $\langle J_\mu \rangle$ and the covariant chiral energy-momentum tensor $\langle T_{\mu\nu} \rangle$ are obtained from a chiral effective action. These results are used to justify the covariant boundary condition used in recent approaches of computing the Hawking flux from chiral gauge and gravitational anomalies. We also discuss a connection of our results with the conventional calculation of nonchiral currents and stress tensors in different (Unruh, Hartle-Hawking and Boulware) states.

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I. INTRODUCTION

The motivation of this paper is to provide a clear understanding of the covariant boundary condition used in the recent analysis [1–5] of deriving the Hawking flux using chiral gauge and gravitational anomalies. Besides this we also reveal certain new features in chiral currents and energy-momentum tensors which are useful in exhibiting their connection with the standard nonchiral expressions.

Long ago, Hawking [6] proposed an idea that black holes evaporate, due to quantum particle creation, and behave like thermal bodies with an appropriate temperature. This is essentially a consequence of quantization of matter in a background spacetime having an event horizon. There are several approaches to derive the Hawking effect [7–10]. Recently, Wilczek and collaborators [1,11] gave an interesting method to compute the Hawking fluxes using chiral gauge and gravitational (diffeomorphism) anomalies. It rests on the fact that the effective theory near the event horizon is a two-dimensional chiral theory which, therefore, has gauge and gravitational anomalies. This method is expected to hold in any dimensions. In this sense it is distinct from the trace anomaly method [10] which was formulated in two dimensions.[1] However, an unpleasant feature of [1,11] was that whereas the expressions for chiral anomalies were taken to be consistent, the boundary conditions required to fix the arbitrary constants were covariant. This was rectified by us [3] and a simplified derivation using only covariant forms was presented. It might be recalled that there are two types of chiral anomalies—covariant and consistent. Covariant anomalies transform covariantly under the gauge or general coordinate transformation but do not satisfy the Wess-Zumino consistency condition. Consistent anomalies, on the contrary, behave the other way. Covariant and consistent expressions are related by local counterterms [14–18].

In another new development also based on chiral gauge and gravitational anomalies, Hawking fluxes were obtained by us [4,5]. Contrary to the earlier approaches [1–3,11] a splitting of the space in different regions (near to and away from the horizon) using discontinuous (step) functions was avoided. This split, apart from requiring the necessity of both the normal and anomalous Ward identities, poses certain conceptual issues [19]. In [4,5] the only input was the structure of the covariant anomaly while retaining the original covariant boundary condition, i.e the vanishing of the covariant current/energy-momentum (EM) tensor at the event horizon.

It is thus clear that the covariant boundary condition plays an important role in the computation of Hawking fluxes. However, a precise understanding of this boundary condition is still missing. Here we give a detailed analysis for this particular choice of boundary condition. It turns out that, with this choice of covariant boundary condition, the components for covariant current/EM tensors (J^r, $T^r{}_t$) obtained from solving the anomaly equation match exactly with the expectation values of the current/EM tensors, obtained from the chiral effective action, taken by imposing the regularity condition on the outgoing modes at the future horizon. Furthermore, we discuss the connection of our results with those found by a standard use of boundary conditions on nonchiral (anomaly free) currents and EM tensors. Indeed we are able to show that our results are equivalent to the choice of the Unruh vacuum for a nonchiral theory. This choice, it may be recalled, is natural for discussing Hawking flux.

In Sec. II we provide a generalization of our recent approach [4,5] of computing fluxes. The covariant current/EM tensor following from a chiral effective action, suitably modified by a local counterterm, are obtained in Sec. III. The role of chirality in imposing constraints on the structure of the current/EM tensor is elucidated. The arbitrary coefficients in J_μ, $T_{\mu\nu}$ are fixed by imposing appropriate regularity conditions on the outgoing modes at the future event horizon (Sec. IV). Here we also discuss the relation of the results obtained for a chiral theory, subjected to the regularity conditions, with those found in a nonchiral theory in different vacua. Some examples are given in Sec. V. Our concluding remarks are contained in Sec. VI. Finally, there is an appendix discussing the connection

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[1]For a connection of the trace anomaly method with the diffeomorphism anomaly approach, see [12,13]

between the trace anomaly and gravitational anomaly for a $(1 + 1)$ dimensional chiral theory.

II. CHARGE AND ENERGY FLUX FROM COVARIANT ANOMALY

Consider a generic spherically symmetric black hole represented by the metric,

$$ds^2 = f(r)dt^2 - \frac{1}{h(r)}dr^2 - r^2(d\theta^2 + \sin^2\theta d\phi^2), \quad (1)$$

where $f(r)$ and $h(r)$ are the metric coefficients. The event horizon is defined by $f(r_h) = h(r_h) = 0$. Also, in the asymptotic limit the metric (1) become Minkowskian i.e $f(r \to \infty) = h(r \to \infty) = 1$ and $f''(r \to \infty) = f'''(r \to \infty) = h''(r \to \infty) = h'''(r \to \infty) = 0$. Now consider quantum fields (scalar or fermionic) propagating on this background. It was shown that [1,11], by using a dimensional reduction technique, the effective field theory near the event horizon becomes two dimensional with the metric given by the $r - t$ section of (1)

$$ds^2 = f(r)dt^2 - \frac{1}{h(r)}dr^2. \quad (2)$$

Note that $\sqrt{-g} = \sqrt{-\det g_{\mu\nu}} = \sqrt{\frac{f}{h}} \neq 1$ (unless $f(r) = h(r)$). On this two dimensional background, the modes which are going in to the black hole (for example left moving modes) are lost and the effective theory become chiral. Two-dimensional chiral theory possesses gravitational anomaly and, if gauge fields are present, also gauge anomaly [14–18,20,21]. Hawking radiation, which is necessary to cancel these anomalies, were obtained by solving the anomalous Ward identity near the horizon and the usual (i.e anomaly free) conservation equations which are valid far away from the horizon [1,11]. This approach used consistent forms for gauge and gravitational anomaly. However, the boundary condition used to fix the arbitrary constants was covariant. As already stated, a reformulation of this approach using only covariant structures was given by us [3,4]. An efficient and economical way to obtain the Hawking flux was discussed in [5] where the computation involved only the expressions for anomalous covariant Ward identities and the covariant boundary conditions. The splitting of space into two regions [1–3,11] is avoided. Here we would first generalize this new approach for the generic black hole (2). This would also help in setting up the conventions and introduce certain equations that are essential for the subsequent analysis.

As already stated the effective theory near the event horizon is a two-dimensional chiral theory. The relevant contribution comes from the outgoing (right moving) modes only. For these modes the expression for covariant gauge anomaly is given by [17,18],

$$\nabla_\mu J^\mu = -\frac{e^2}{4\pi\sqrt{-g}}\epsilon^{\alpha\beta}F_{\alpha\beta} \quad (3)$$

$\epsilon^{01} = -\epsilon^{10} = 1$, $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$, and the gauge potential is defined as $A_t = -\frac{Q}{r}$. For a static background, the above equation becomes,

$$\partial_r(\sqrt{-g}J^r) = \frac{e^2}{2\pi}\partial_r A_t. \quad (4)$$

Solving this equation we get

$$\sqrt{-g}J^r = c_H + \frac{e^2}{2\pi}[A_t(r) - A_t(r_h)]. \quad (5)$$

Here c_H is an integration constant which can be fixed by imposing the covariant boundary condition i.e. covariant current (J^r) must vanish at the event horizon,

$$J^r(r = r_h) = 0. \quad (6)$$

Hence we get $c_H = 0$ and the expression for the current becomes,

$$J^r = \frac{e^2}{2\pi\sqrt{-g}}[A_t(r) - A_t(r_h)]. \quad (7)$$

Note that the Hawking flux is measured at infinity where there is no anomaly. This necessitated a split of space into two distinct regions—one near the horizon and one away from it—and the use of two Ward identities [1–3,11]. This is redundant if we observe that the anomaly (4) vanishes at the asymptotic infinity. Consequently, in this approach, the flux is directly obtained from the asymptotic infinity limit of (7):

$$\text{Charge flux} = J^r(r \to \infty) = -\frac{e^2 A_t(r_h)}{2\pi} = \frac{e^2 Q}{2\pi r_h}. \quad (8)$$

This reproduces the familiar expression for the charge flux [1,3–5]. Next, we consider the expression for the two dimensional covariant gravitational Ward identity [1–4],

$$\nabla_\mu T^{\mu\nu} = J_\mu F^{\mu\nu} + \frac{\epsilon^{\nu\mu}}{96\pi\sqrt{-g}}\nabla_\mu R, \quad (9)$$

where the first term is the classical contribution (Lorentz force) and the second is the covariant gravitational anomaly [20–22]. Here R is the Ricci scalar and for the metric (2) it is given by

$$R = \frac{f''h}{f} + \frac{f'h'}{2f} - \frac{f'^2h}{2f^2}. \quad (10)$$

By simplifying (9) we get, in the static background,

$$\partial_r(\sqrt{-g}T^r{}_t) = \partial_r N_t^r(r) - \frac{e^2 A_t(r_h)}{2\pi}\partial_r A_t(r) + \partial_r\left(\frac{e^2 A_t^2(r)}{4\pi}\right), \quad (11)$$

where

$$N_t^r = \frac{1}{96\pi}\left(hf'' + \frac{f'h'}{2} - \frac{f'^2 h}{f}\right). \quad (12)$$

The solution for (11) is given by

$$\sqrt{-g}T^r{}_t = b_H + [N_t^r(r) - N_t^r(r_h)] + \frac{e^2A_t^2(r_h)}{4\pi} - \frac{e^2}{2\pi}A_t(r_h)A_t(r) + \frac{e^2A_t^2(r)}{4\pi}. \quad (13)$$

Here b_H is an integration constant. Implementing the covariant boundary condition, namely, the vanishing of covariant EM tensor at the event horizon,

$$T^r{}_t(r = r_h) = 0 \quad (14)$$

yields $b_H = 0$. Hence (13) reads

$$\sqrt{-g}T^r{}_t(r) = [N_t^r(r) - N_t^r(r_h)] + \frac{e^2}{4\pi}[A_t(r) - A_t(r_h)]^2. \quad (15)$$

Since the covariant gravitational anomaly vanishes asymptotically, we can compute the energy flux as before by taking the asymptotic limit of (15)

$$\text{energy flux} = T^r{}_t(r \to \infty) = -N_t^r(r_h) + \frac{e^2A_t^2(r_h)}{4\pi} = \frac{1}{192\pi}f'(r_h)g'(r_h) + \frac{e^2Q^2}{4\pi r_h^2}. \quad (16)$$

This reproduces the expression for the Hawking flux found by using the anomaly cancelling approach of [1–3].

It is now clear that the covariant boundary conditions play a crucial role in the computation of Hawking fluxes using chiral gauge and gravitational anomalies, either in the approach based on the anomaly cancelling mechanism [1–3] or in the more direct approach [5] reviewed here. Therefore it is worthwhile to study it in some detail. We adopt the following strategy. The expressions for the expectation values of the covariant current and EM tensor will be deduced from the chiral effective action, suitably modified by a local counterterm. Local structures are obtained by introducing auxiliary variables whose solutions contain arbitrary constants. These constants are fixed by imposing regularity conditions on the outgoing modes at the future event horizon. The final results are found to match exactly with the corresponding expressions for the covariant current (7) and EM tensor (15), which were derived by using the covariant boundary conditions (6) and (14). Subsequently we show that our results are consistent with the imposition of the Unruh vacuum on usual (nonchiral) expressions.

III. COVARIANT CURRENT AND EM TENSOR FROM CHIRAL EFFECTIVE ACTION

The two-dimensional chiral effective action [4,23] is defined as,

$$\Gamma_{(H)} = -\frac{1}{3}z(\omega) + z(A), \quad (17)$$

where A_μ and ω_μ are the gauge field and the spin connection, respectively, and,

$$z(v) = \frac{1}{4\pi}\int d^2x d^2y \epsilon^{\mu\nu}\partial_\mu v_\nu(x)\Delta^{-1}(x,y)\partial_\rho[(\epsilon^{\rho\sigma} + \sqrt{-g}g^{\rho\sigma})v_\sigma(y)]. \quad (18)$$

Here Δ^{-1} is the inverse of d'Alembertian $\Delta = \nabla^\mu\nabla_\mu = \frac{1}{\sqrt{-g}}\partial_\mu(\sqrt{-g}g^{\mu\nu}\partial_\nu)$. From a variation of this effective action the energy-momentum tensor and the gauge current are computed. These are shown in the literature [14–18,20,21] as consistent forms. To get their covariant forms in which we are interested, however, appropriate local polynomials have to be added. This is possible since energy-momentum tensors and currents are only defined modulo local polynomials. We obtain,

$$\delta\Gamma_H = \int d^2x\sqrt{-g}\left(\frac{1}{2}\delta g_{\mu\nu}T^{\mu\nu} + \delta A_\mu J^\mu\right) + l, \quad (19)$$

where the local polynomial is given by [23],

$$l = \frac{1}{4\pi}\int d^2x \epsilon^{\mu\nu}\left(A_\mu\delta A_\nu - \frac{1}{3}w_\mu\delta w_\nu - \frac{1}{24}Re_\mu^a\delta e_\nu^a\right) \quad (20)$$

The covariant energy-momentum tensor $T^{\mu\nu}$ and the covariant gauge current J^μ are read off from the above relations as [4,23],

$$T^\mu{}_\nu = \frac{e^2}{4\pi}(D^\mu BD_\nu B) + \frac{1}{4\pi}\left(\frac{1}{48}D^\mu GD_\nu G - \frac{1}{24}D^\mu D_\nu G + \frac{1}{24}\delta_\nu^\mu R\right) \quad (21)$$

$$J^\mu = -\frac{e^2}{2\pi}D^\mu B. \quad (22)$$

Note the presence of the chiral covariant derivative D_μ expressed in terms of the usual covariant derivative ∇_μ,

$$D_\mu = \nabla_\mu - \bar{\epsilon}_{\mu\nu}\nabla^\nu = -\bar{\epsilon}_{\mu\nu}D^\nu, \quad (23)$$

where $\bar{\epsilon}_{\mu\nu} = \sqrt{-g}\epsilon_{\mu\nu}$ and $\bar{\epsilon}^{\mu\nu} = \frac{1}{\sqrt{-g}}\epsilon^{\mu\nu}$. The auxiliary fields B and G in (21) and (22) are defined as

$$B(x) = \int d^2y\sqrt{-g}\Delta^{-1}(x,y)\bar{\epsilon}^{\mu\nu}\partial_\mu A_\nu(y) \quad (24)$$

$$G(x) = \int d^2y\sqrt{-g}\Delta^{-1}(x,y)R(y) \tag{25}$$

so that they satisfy

$$\Delta B(x) = \bar{\epsilon}^{\mu\nu}\partial_\mu A_\nu(x) \tag{26}$$

$$\Delta G(x) = R(x), \tag{27}$$

where R is given by (10).

As a simple consistency check the covariant Ward identities (3) and (9) are obtained from (21) and (22). For example, using (22), (24), and (26), we find,

$$\nabla_\mu J^\mu = -\frac{e^2}{2\pi}\Delta B = -\frac{e^2}{2\pi}\bar{\epsilon}^{\mu\nu}\partial_\mu A_\nu = \frac{-e^2}{4\pi\sqrt{-g}}\epsilon^{\mu\nu}F_{\mu\nu} \tag{28}$$

reproducing (3). Note also the existence of the covariant trace anomaly[2] following from (21),

$$T^\mu{}_\mu = \frac{R}{48\pi}. \tag{29}$$

The chiral nature of the current (22) and the stress tensor (21) are revealed by the following conditions,

$$J_\mu = -\bar{\epsilon}_{\mu\nu}J^\nu \tag{30}$$

$$T_{\mu\nu} = -\frac{1}{2}(\bar{\epsilon}_{\mu\rho}T^\rho{}_\nu + \bar{\epsilon}_{\nu\rho}T^\rho{}_\mu) + \frac{g_{\mu\nu}}{2}T^\alpha{}_\alpha, \tag{31}$$

which are a consequence of the presence of the chiral derivative (23). This may be compared with the definitions of J_μ and $T_{\mu\nu}$, obtained from a Polyakov type action valid for a vector theory, which do not satisfy the chiral properties (30) and (31). These properties constrain the structure of J_μ, $T_{\mu\nu}$.

After solving (26) and (27) we get,

$$B(x) = B_o(r) - at + b; \qquad \partial_r B_o = \frac{A_t(r) + c}{\sqrt{fh}} \tag{32}$$

and

$$G = G_o(r) - 4pt + q; \qquad \partial_r G_o = -\frac{1}{\sqrt{fh}}\left(\frac{f'}{\sqrt{-g}} + z\right), \tag{33}$$

where a, b, c, p, q, and z are constants. Now, by substituting (32) in (22) we obtain,

$$J^r(r) = \frac{e^2}{2\pi\sqrt{-g}}[A_t(r) + c + a] \tag{34}$$

$$J^t(r) = \frac{e^2}{2\pi f}[A_t(r) + c + a] = \frac{\sqrt{-g}}{f}J^r. \tag{35}$$

Observe that there is only one independent component of J_μ which is a consequence of (30). Likewise, by using (32) and (33) in (21) we find

$$T^r{}_t = \frac{e^2}{4\pi\sqrt{-g}}\bar{A}_t^2(r) + \frac{1}{12\pi\sqrt{-g}}\bar{P}^2(r) + \frac{1}{24\pi\sqrt{-g}}\left[\frac{f'}{\sqrt{-g}}\bar{P}(r) + \bar{Q}(r)\right] \tag{36}$$

$$T^r{}_r = \frac{R}{96\pi} - \frac{\sqrt{-g}}{f}T^r{}_t \tag{37}$$

$$T^t{}_t = -T^r{}_r + \frac{R}{48\pi} \tag{38}$$

with $\bar{A}_t(r)$, $\bar{P}(r)$ and $\bar{Q}(r)$ defined as

$$\bar{A}_t(r) = A_t(r) + c + a \tag{39}$$

$$\bar{P}(r) = p - \frac{1}{4}\left(\frac{f'}{\sqrt{-g}} + z\right) \tag{40}$$

$$\bar{Q}(r) = \frac{1}{4}hf'' - \frac{f'}{8}\left(\frac{hf'}{f} - h'\right). \tag{41}$$

Relation (38) is a consequence of the trace anomaly (29) while (37) follows from the chirality criterion (31). The $r - t$ component of the EM tensor (36) calculated above is same as the one given in [24].

To further illuminate the chiral nature, we transform the various components of current/EM tensor to null coordinates given by

$$v = t + r*; \qquad u = t - r* \tag{42}$$

$$\frac{dr}{dr*} = \sqrt{fh}. \tag{43}$$

The metric (2) in these coordinates looks like

$$ds^2 = \frac{f(r)}{2}(dudv + dvdu). \tag{44}$$

Finally, the expressions for the current and EM tensors in these coordinates are given by,

$$J_u = \frac{1}{2}[J_t - \sqrt{fh}J_r] = \frac{e^2}{2\pi}\bar{A}_t(r) \tag{45}$$

$$J_v = \frac{1}{2}[J_t + \sqrt{fh}J_r] = 0 \tag{46}$$

and

[2]Observe that the chiral theory has both a diffeomorphism anomaly (9) and a trace anomaly (29). This is distinct from the vector case where there is only a trace anomaly $T^\mu{}_\mu = \frac{R}{24\pi}$. No diffeomorphism anomaly exists. See the appendix for more details.

$$T_{uu} = \frac{1}{4}[fT^t{}_t - fT^r{}_r + 2\sqrt{-g}T^r{}_t] = \frac{e^2}{4\pi}\bar{A}_t^2(r) + \frac{1}{12\pi}\bar{P}^2(r) + \frac{1}{24\pi}\left[\frac{f'}{\sqrt{-g}}\bar{P}(r) + \bar{Q}(r)\right] \tag{47}$$

$$T_{uv} = \frac{f}{4}[T^t{}_t + T^r{}_r] = \frac{1}{192\pi}fR \tag{48}$$

$$T_{vv} = \frac{1}{4}[fT^t{}_t - fT^r{}_r - 2\sqrt{-g}T^r{}_t] = 0, \tag{49}$$

where extensive use has been made of (34) to (38). We now observe that, due to the chiral property, the J_v and T_{vv} components vanish everywhere. These correspond to the ingoing modes and are compatible with the fact, stated earlier, that the near horizon theory is a two dimensional chiral theory where the ingoing modes are lost. Also, the structure of T_{uv} is fixed by the trace anomaly. Only the J_u and T_{uu} components involve the undetermined constants. These will now be determined by considering various vacuum states.

IV. VACUUM STATES

In a generic spacetime three different quantum states (vacua) [25] are defined by appropriately choosing 'in' and 'out' modes. This general picture is modified when dealing with a chiral theory since, as shown before, the 'in' modes always vanish. Consequently this leads to a simplification and conditions are imposed only on the 'out' modes. Moreover, these conditions have to be imposed on the horizon since the chiral theory is valid only there. The natural condition, leading to the occurrence of Hawking flux, is that a freely falling observer must see a finite amount of flux at the horizon. This implies that the current (EM tensor) in Kruskal coordinates must be regular at the future horizon. Effectively, this is the same condition on the 'out' modes in either the Unruh vacuum [26] or the Hartle-Hawking vacuum [27]. As far as our analysis is concerned this is sufficient to completely determine the form of J_μ or $T_{\mu\nu}$. We show that their structures are identical to those obtained in the previous section using the covariant boundary condition.

A more direct comparison with the conventional results obtained from Unruh or Hartle-Hawking states is possible. In that case one has to consider the nonchiral expressions [2,28] containing both 'in' and 'out' modes. We show that, at asymptotic infinity where the flux is measured, our expressions agree with that calculated from Unruh vacuum only. We discuss this in some detail.

A. Regularity conditions, Unruh and Hartle-Hawking vacua

In Kruskal coordinates U the current takes the form $J_U = -\frac{J_u}{\kappa U}$, where κ is the surface gravity. Since J_U is required to be finite at the future horizon where $U \to \sqrt{r - r_h}(r \to r_h)$, J_u must vanish at $r \to r_h$. Hence from (39) and (45) we have,

$$c + a = -A_t(r_h). \tag{50}$$

Similarly, imposing the condition that $T_{UU} = (\frac{1}{\kappa U})^2 T_{uu}$ must be finite at future horizon leads to $T_{uu}(r \to r_h) = 0$. This yields, from (39)–(41) and (47),

$$p = \frac{1}{4}(z \pm \sqrt{f'(r_h)h'(r_h)}). \tag{51}$$

Using (50) and (51) in Eqs. (34)–(38) we obtain the final expressions,

$$J^r(r) = \frac{e^2}{2\pi\sqrt{-g}}[A_t(r) - A_t(r_h)] \tag{52}$$

$$J^t(r) = \frac{\sqrt{-g}}{f}J^r(r) \tag{53}$$

for the current and the EM tensor,

$$\sqrt{-g}T^r{}_t = \frac{e^2}{4\pi}[A_t(r) - A_t(r_h)]^2 + [N_t^r(r) - N_t^r(r_h)] \tag{54}$$

while, $T^r{}_r$ and $T^t{}_t$ follow from (37) and (38) and N_t^r is given by (12).

The expressions for J^r (52) and $T^r{}_t$ (54) agree with the corresponding ones given in (7) and (15). This shows that the structures for the universal components J^r, $T^r{}_t$ obtained by solving the anomalous Ward identities (3) and (9) subjected to the covariant boundary conditions (6) and (14) exactly coincide with the results computed by demanding regularity at the future event horizon.

It is possible to compare our findings with conventional (nonchiral) computations where the Hawking flux is obtained in the Unruh vacuum. We begin by considering the conservation equations for a nonchiral theory that is valid away from the horizon. Such equations were earlier used in [1–3,11]. Conservation of the gauge current yields,[3]

$$\nabla_\mu\tilde{J}^\mu = \frac{1}{\sqrt{-g}}\partial_\mu(\sqrt{-g}\tilde{J}^\mu) = 0 \tag{55}$$

which, in a static background, leads to,

[3]We use a tilde (J^μ) to distinguish nonchiral expressions from chiral ones.

$$\tilde{J}^r = \frac{C_1}{\sqrt{-g}} \qquad (56)$$

where C_1 is some constant.

As is well know there is no regularization that simultaneously preserves the vector as well as axial vector gauge invariance. Indeed a vector gauge invariant regularization resulting in (55) yields the following axial anomaly,

$$\nabla_\mu \tilde{J}^{5\mu} = \frac{e^2}{2\pi\sqrt{-g}}\epsilon^{\mu\nu}F_{\mu\nu}; \qquad \tilde{J}^{5\mu} = \frac{1}{\sqrt{-g}}\epsilon^{\mu\nu}\tilde{J}_\nu. \qquad (57)$$

The solution of this Ward identity is given by,

$$\tilde{J}^t = -\frac{1}{f}\left[C_2 - \frac{e^2}{\pi}A_t(r)\right] \qquad (58)$$

where C_2 is another constant.

In the null coordinates introduced in (42), (43), (45), and (46) the various components are defined as,

$$\tilde{J}_u = \frac{1}{2}\left[C_1 - C_2 + \frac{e^2}{\pi}A_t(r)\right], \qquad (59)$$

$$\tilde{J}_v = -\frac{1}{2}\left[C_1 + C_2 - \frac{e^2}{\pi}A_t(r)\right]. \qquad (60)$$

The constants C_1, C_2 are now determined by using appropriate boundary conditions corresponding to first, the Unruh state, and then, the Hartle-Hawking state. For the Unruh state $\tilde{J}_u(r \to r_h) = 0$ and $\tilde{J}_v(r \to \infty) = 0$ yield,

$$C_1 = -C_2 = -\frac{e^2}{2\pi}A_t(r_h), \qquad (61)$$

so that, reverting back to (r, t) coordinates, we obtain,

$$\tilde{J}^r = -\frac{e^2}{2\pi\sqrt{-g}}A_t(r_h), \qquad (62)$$

$$\tilde{J}^t = \frac{e^2}{\pi f}\left[A_t(r) - \frac{1}{2}A_t(r_h)\right]. \qquad (63)$$

The Hawking charge flux, identified with $\tilde{J}^r(r \to \infty)$, reproduces the desired result (8). Expectably, (62) and (63) differ from our relations (52) and (53) which are valid only near the horizon. However, at asymptotic infinity where the Hawking flux is measured, both expressions match, i.e.

$$\tilde{J}^r(r \to \infty) = J^r(r \to \infty), \qquad (64)$$

$$\tilde{J}^t(r \to \infty) = J^t(r \to \infty). \qquad (65)$$

All the above considerations follow identically for the stress tensor. Now the relevant conservation law is $\nabla_\mu \tilde{T}^{\mu\nu} = \tilde{J}_\mu F^{\mu\nu}$ and the trace anomaly is $T^\mu{}_\mu = \frac{R}{24\pi}$ (see also footnote 2) which have to be used instead of (55) and (57). Once again $\tilde{T}^\mu{}_\nu$ will not agree with our $T^\mu{}_\nu$ (54). However, at asymptotic infinity, all components agree:

$$\tilde{T}^\mu{}_\nu(r \to \infty) = T^\mu{}_\nu(r \to \infty), \qquad (66)$$

leading to the identification of the Hawking flux with $\tilde{T}^r{}_t(r \to \infty)$.

The equivalences (64)–(66) reveal the internal consistency of our approach. They are based on two issues. First, in the asymptotic limit the chiral anomalies (3) and (9) vanish and, secondly, the boundary conditions (6) and (14) get identified with the Unruh state that is appropriate for discussing Hawking effect. It is important to note that, asymptotically, all the components, and not just the universal component that yields the flux, agree.

In the Hartle-Hawking state, the conditions $\tilde{J}_u(r \to r_h) = 0$ and $\tilde{J}_v(r \to r_h) = 0$ yield,

$$C_1 = 0; \qquad C_2 = \frac{e^2}{\pi}A_t(r_h) \qquad (67)$$

so that,

$$\tilde{J}^r(r) = 0, \qquad (68)$$

$$\tilde{J}^t(r) = \frac{e^2}{\pi f}(A_t(r) - A_t(r_h)), \qquad (69)$$

Expectably, there is no Hawking (charge) flux now. The above expressions, even at asymptotic infinity, do not agree with our expressions (52) and (53).

B. Boulware vacuum

Apart from the Unruh and Hartle-Hawking vacua there is another vacuum named after Boulware [29] which closely resembles the Minkowski vacuum asymptotically. In this vacuum, there is no radiation in the asymptotic future. In other words this implies J^r and $T^r{}_t$ given in (34) and (36) must vanish at $r \to \infty$ limit. Therefore, for the Boulware vacuum, we get

$$c + a = 0 \qquad (70)$$

$$p = \frac{1}{4}z. \qquad (71)$$

By substituting (70) in (34) and (35) we have

$$J^r(r) = \frac{e^2}{2\pi\sqrt{-g}}A_t(r) \qquad (72)$$

$$J^t(r) = \frac{e^2}{2\pi f}A_t(r). \qquad (73)$$

Similarly, by substituting (70) and (71) in Eqs. (36)–(38), we get

$$T^r{}_t = \frac{e^2A_t^2(r)}{4\pi\sqrt{-g}} + \frac{1}{\sqrt{-g}}N_t^r(r) \qquad (74)$$

$$T^r{}_r = \frac{-e^2A_t^2(r)}{4\pi f} - \frac{1}{f}N_t^r(r) + \frac{R}{96\pi} \quad (75)$$

$$T^t{}_t = \frac{e^2A_t^2(r)}{4\pi f} + \frac{1}{f}N_t^r(r) + \frac{R}{96\pi}. \quad (76)$$

Observe that there is no radiation in the asymptotic region in the Boulware vacuum. Also, the trace anomaly (29) is reproduced since this is independent of the choice of quantum state. Further, we note that, in the Kruskal coordinates, J_U and T_{UU} components of current and EM tensors diverge at the horizon. This can be seen by substituting Eqs. (72) and (73) in (45). Then the expression for J_u in Boulware vacuum becomes,

$$J_u = \frac{e^2}{2\pi}A_t(r) \quad (77)$$

while, by putting (74)–(76) in (47), we obtain, for T_{uu}

$$T_{uu} = \frac{e^2A_t^2(r)}{4\pi} + N_t^r(r). \quad (78)$$

Note that in the limit $(r \to r_h)$ J_u and T_{uu} do not vanish. Hence, in the Kruskal coordinates, the current and EM tensor diverge. This is expected since the Boulware vacuum is not regular near the horizon.

V. EXAMPLES

We discuss two explicit examples where the Hawking flux and the complete expressions for the covariant current/EM tensor are provided.

A. Reissner-Nordstrom black hole

For this black hole, the metric in the $r - t$ sector is given by

$$ds^2 = f(r)dt^2 - \frac{1}{f(r)}dr^2 \quad (79)$$

with

$$f(r) = 1 - \frac{2M}{r} + \frac{Q^2}{r^2} = \frac{(r - r_+)(r - r_-)}{r^2}, \quad (80)$$

where $r_\pm = M \pm \sqrt{M^2 - Q^2}$ are the outer and inner horizons. The gauge potential is given by $A_t = -\frac{Q}{r}$. Note that in this case $\sqrt{-g} = 1$. We can easily write the expressions for various components of current and EM tensor for Unruh, Hartle-Hawking, and Boulware vacua. As already discussed the results for Unruh and Hartle-Hawking vacua are identical. In this case we have from (52) and (53)

$$J^r(r) = \frac{e^2}{2\pi}[A_t(r) - A_t(r_+)] \quad (81)$$

$$J^t(r) = \frac{e^2r^2}{2\pi(r - r_+)(r - r_-)}[A_t(r) - A_t(r_+)]. \quad (82)$$

The charge flux, obtained from the asymptotic limit of (81) is,

$$J^r(r \to \infty) = -\frac{e^2}{2\pi}A_t(r_+) = \frac{e^2Q}{2\pi r_+} \quad (83)$$

reproducing the known result [1,3].

Similarly, the $r - t$ component of the covariant EM tensor from (54) is given by,

$$T^r{}_t = \frac{e^2}{4\pi}[A_t(r) - A_t(r_+)]^2 + \frac{1}{192\pi}[2f(r)f''(r) - f'^2(r) + f'^2(r_+)], \quad (84)$$

while, as before, the other components follow from (37) and (38). As usual, the energy flux obtained from the asymptotic limit of (84) yields,

$$T^r{}_t(r \to \infty) = \frac{e^2Q^2}{4\pi r_+^2} + \frac{1}{192\pi}\Big[\frac{2}{M + \sqrt{M^2 - Q^2}} \times (M^2 + M\sqrt{M^2 - Q^2} - Q^2)\Big]^2. \quad (85)$$

This reproduces the usual expression of energy flux coming from the Reissner-Nordstrom black hole [1,3].

For the Boulware vacuum, the expressions for current/EM tensors (72)–(76) are given by,

$$J^r = \frac{e^2}{2\pi}A_t(r) \quad (86)$$

$$J^t = \frac{e^2r^2}{2\pi(r - r_+)(r - r_-)}A_t(r) \quad (87)$$

$$T^r{}_t = \frac{e^2}{4\pi}A_t^2(r) + \frac{1}{192\pi}[2ff'' - f'^2(r)] \quad (88)$$

$$T^r{}_r = -\frac{r^2}{(r - r_+)(r - r_-)} \times \Big[\frac{e^2}{4\pi}A_t^2(r) + \frac{1}{192\pi}[2ff'' - f'^2(r)]\Big] + \frac{f''}{96\pi} \quad (89)$$

$$T^t{}_t = -T^r{}_r + \frac{f''}{48\pi} \quad (90)$$

As we can observe, by taking the asymptotic limit of (86) and (88), there are no Hawking fluxes.

B. Garfinkle-Horowitz- Strominger (GHS) black hole

The GHS black hole is a member of a family of solutions to low energy string theory [30,31]. The metric in the $r - t$ sector of this black hole is given by [32,33]

$$ds^2 = f(r)dt^2 - \frac{1}{h(r)}dr^2 \quad (91)$$

where

$$f(r) = \left(1 - \frac{2Me^{\phi_o}}{r}\right)\left(1 - \frac{Q^2 e^{3\pi_o}}{Mr}\right)^{-1} \tag{92}$$

$$h(r) = \left(1 - \frac{2Me^{\phi_o}}{r}\right)\left(1 - \frac{Q^2 e^{3\pi_o}}{Mr}\right), \tag{93}$$

with ϕ_o being the asymptotic constant value of the dilaton field. We consider the case when $Q^2 < 2e^{-2\phi_o M^2}$ for which the above metric describes a black hole with an event horizon [30,32,33],

$$r_h = 2Me^{\phi_o}. \tag{94}$$

Note that in this limit there is only one event horizon (94) and the gauge fields will not play any role in the subsequent analysis. In other words we have only gravitational anomaly in the theory. Also, this is an example with distinct $f(r)$, $h(r)$ so that $\sqrt{-g} \neq 1$. For this black hole we can write the complete expressions for current/EM tensors for the Unruh (Hartle-Hawking) (54) by substituting the values for $f(r)$ (92) and $h(r)$ (94). For the sake of simplicity, here we just give the asymptotic expression for T^r_t

$$T^r{}_t(r \to \infty) = \frac{1}{192\pi} f'(r_h)h'(r_h) = \frac{1}{768M^2 e^{2\phi_o}}, \tag{95}$$

which gives the usual expression for energy flux from GHS black hole [32,33].

For the Boulware vacuum, substituting (92) and (93) in (74) we note that there is no Hawking flux in the asymptotic region, as expected.

VI. CONCLUSIONS

We have discussed in some details our method, briefly introduced in [5], of computing the Hawking flux using covariant gauge and gravitational anomalies. Contrary to earlier approaches, a split of space into distinct regions (near to and away from horizon) using step functions was avoided. This method is different from the anomaly cancelling mechanism of [1–3,11] although it uses identical (covariant) boundary conditions. It reinforces the crucial role of these boundary conditions, the study of which has been the principal objective of this paper.

In order to get a clean understanding of these boundary conditions we first computed the explicit structures of the covariant current $\langle J_\mu \rangle$ and the covariant energy-momentum tensor $\langle T_{\mu\nu} \rangle$ from the chiral (anomalous) effective action, appropriately modified by adding a local counterterm [4,23]. The chiral nature of these structures became more transparent by passing to the null coordinates. In these coordinates the contribution from the ingoing (left moving) modes was manifestly seen to vanish. The outgoing (right moving) modes involved arbitrary parameters which were fixed by imposing regularity conditions at the future horizon. No condition on the ingoing (left moving) modes was required as these were absent as a result of chirality. These findings by themselves are new. They are also different from the corresponding expressions for $\langle J_\mu \rangle$, $\langle T_{\mu\nu} \rangle$, obtained from the standard nonanomalous (Polyakov) action, satisfying $\nabla_\mu J^\mu = 0$, $\nabla_\mu T^{\mu\nu} = J_\mu F^{\mu\nu}$ and $T^\mu{}_\mu = \frac{R}{24\pi}$, implying the absence of any gauge or gravitational (diffeomorphism) anomaly. Only the trace anomaly is present. Details of the latter computation may be found in [2,28].

We have then established a direct connection of these results with the choice of the covariant boundary condition used in determining the Hawking flux from chiral gauge and gravitational anomalies [1–5]. The relevant universal component (J^r or $T^r{}_t$) obtained by solving the anomaly equation subject to the covariant boundary condition (6) and (14) agrees exactly with the result derived from imposing regularity condition on the outgoing modes at the future horizon: namely, a free falling observer sees a finite amount of flux at outer horizon indicating the possibility of Hawking radiation. Our findings, therefore, provide a clear justification of the covariant boundary condition.

Finally, we put our computations in a proper perspective by comparing our findings with the standard implementation of the various vacua states on nonchiral expressions. Specifically, we show that our results are compatible with the choice of Unruh vacuum for a nonchiral theory which eventually yields the Hawking flux.

APPENDIX

Unlike the case of vector theory, where the diffeomorphism invariance is kept intact in spite of the presence of trace anomaly, the chiral theory has both a diffeomorphism anomaly (gravitational anomaly) and a trace anomaly. In 1 + 1 dimensions it is possible to obtain a relation between the coefficients of the diffeomorphism anomaly and the trace anomaly by exploiting the chirality criterion.

To see this let us write the general structure of the covariant Ward identity,

$$\nabla_\mu T^\mu{}_\nu = J_\mu F^\mu{}_\nu + N_a \bar{\epsilon}_{\nu\mu} \nabla^\mu R, \tag{A1}$$

where N_a is an undetermined normalization. The functional form of the anomaly follows on grounds of dimensionality, covariance and parity. Likewise, the structure of the covariant trace anomaly is written as,

$$T^\mu{}_\mu = N_t R \tag{A2}$$

with N_t being the normalization. In the null coordinates (42) and (43) for $\nu = v$, the left hand side of (A1) becomes

$$\begin{aligned} \nabla_\mu T^\mu{}_v &= \nabla_u T^u{}_v + \nabla_v T^v{}_v = \nabla_u (g^{uv} T_{vv}) + \nabla_v (g^{uv} T_{uv}) \\ &= \nabla_v (g^{uv} T_{uv}), \end{aligned} \tag{A3}$$

where we have used the fact that for a chiral theory $T_{vv} = 0$ (see Eq. (49)). Also, in null coordinates, we have,

$$T_{uv} = \frac{1}{2}(g_{uv} T^v{}_v + g_{uv} T^u{}_u) = \frac{g_{uv}}{2} T^\mu{}_\mu = \frac{f}{4} T^\mu{}_\mu. \tag{A4}$$

By using (A2)–(A4) we obtain,

$$\nabla_\mu T^\mu{}_v = \frac{N_t}{2}\nabla_v R. \tag{A5}$$

where we used $g^{uv} = \frac{2}{f}$ (44).

The right-hand side of (A1) for $\nu = v$, with the use of the chirality constraint $J_v = 0$ (46), yields

$$J_\mu F^\mu{}_v + N_a \bar{\epsilon}_{v\mu}\nabla^\mu R = N_a \nabla_v R. \tag{A6}$$

Hence, by equating (A5) and (A6) we find a relationship between N_a and N_t

$$N_a = \frac{N_t}{2} \tag{A7}$$

which is compatible with (9) and (29) with $N_a = \frac{N_t}{2} = \frac{1}{96\pi}$. It is clear that chirality enforces both the conformal and diffeomorphism anomalies. The trivial (anomaly free) case $N_a = N_t = 0$ is ruled out because, using general arguments based on the unidirectional property of chirality, it is possible to prove the existence of the diffeomorphism anomaly in 1 + 1 dimensions [22].

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