PHYSICAL REVIEW D **79,** 084033 (2009)

Boson stars with repulsive self-interactions

Pratik Agnihotri,[1,2] Jürgen Schaffner-Bielich,[3] and Igor N. Mishustin[4,5]
[1]*Indian Institute of Technology, Kanpur, India 208016*
[2]*Institut für Theoretische Physik, J. W. Goethe Universität, Max von Laue-Straße 1, D-60438 Frankfurt am Main, Germany*
[3]*Institut für Theoretische Physik, Ruprecht-Karls-Universität, Philosophenweg 16, D-69120 Heidelberg, Germany*
[4]*Frankfurt Institute for Advanced Studies, J. W. Goethe Universität, Ruth-Moufang-Straße 1, D-60438 Frankfurt am Main, Germany*
[5]*The Kurchatov Institute, Russian Research Center, 123182 Moscow, Russia*
(Received 16 December 2008; published 22 April 2009)

The properties of compact stars made of massive bosons with a repulsive self-interaction mediated by vector mesons are studied within the mean-field approximation and general relativity. We demonstrate that there exists a scaling property for the mass-radius curve for arbitrary boson masses and interaction strengths which results in a universal mass-radius relation. The radius remains nearly constant for a wide range of compact star masses. The maximum stable mass and radius of boson stars are determined by the interaction strength and scale with the Landau mass and radius. Both the maximum mass and the corresponding radius increase linearly with the interaction strength so that they can be radically different compared to the other families of boson stars where interactions are ignored.

DOI: 10.1103/PhysRevD.79.084033 PACS numbers: 05.30.Jp, 04.40.−b, 04.40.Dg, 95.35.+d

I. INTRODUCTION

White dwarfs, neutron, and quark stars, collectively dubbed compact stars, are the final result of stellar evolution. White dwarfs are stabilized by the Fermi degeneracy pressure. There exists an upper limit for the mass of a white dwarf, the Chandrasekhar mass, which is about 1.4 times the mass of the Sun [1]. Beyond this limit the white dwarf is unstable against gravitational collapse. Neutron stars are stable mainly due to the repulsive nature of the interactions between nucleons. Therefore, the precise value of the maximum mass for a neutron star is less certain, but is presumably close to the predictions based on the Landau consideration [2]. As shown in Ref. [3], Landau's argument can be extended to a general compact star made of fermions, a fermion star, with arbitrary fermion mass and interaction strength.

In the following we are studying compact stars made of bosons. Unlike fermion stars, boson stars have no observational evidence, yet. Besides this the existence of any stable scalar particle has never been experimentally verified. Wheeler [4] introduced a gravitational electromagnetic entity called a geon. The gravitational attraction of its own field energy confines the geon in a certain region. Later Kaup [5] solved the Klein-Gordon Einstein equations for scalar fields and found a new class of solutions for gravitating objects. These boson stars are stable with respect to spherically symmetric gravitational collapse. Ruffini and Bonazzola [6] demonstrated that boson stars describe a family of self-gravitating scalar field configurations within general relativity. In Ref. [7] Takasugi and Yoshimura calculated boson stars within an approach similar to the one conventionally adopted for neutron stars by solving the Tolman-Oppenheimer-Volkoff (TOV) equation [8–10] with a separate equation of state describing the properties of matter. Boson stars with self-interactions have been also considered, in particular, as candidates for dark matter [11]. For reviews on boson stars we refer to [12–14].

In the following, we consider boson stars as localized, gravitationally bound objects made of self-interacting bosons at zero temperature. The interactions between the bosons is described by vector meson exchange in the relativistic mean-field approximation. The resulting equation of state is used as input to solve the TOV equation for boson stars, similar to the approach of Ref. [7] but with an equation of state based on a field-theoretical approach. We demonstrate that there are scaling relations for the mass-radius curve. In particular, we show that the maximum mass is controlled by the interaction strength and the Landau mass, not by the boson mass. We compare our results to previous works and to the case of fermion stars with self-interactions.

II. SCALING RELATIONS FOR COMPACT STARS

We assume a spherically symmetric and static configuration where the energy-momentum tensor is that of a perfect fluid at rest. Then the star structure can be obtained by solving the TOV equations, which can be conveniently written as

$$\frac{dp}{dr} = -\frac{GM\rho}{r^2}\left(1 + \frac{p}{\rho}\right)\left(1 + \frac{4\pi r^3 p}{M}\right)\left(1 - \frac{2GM}{r}\right)^{-1} \quad (1)$$

with

$$\frac{dM}{dr} = 4\pi r^2 \rho. \quad (2)$$

Additionally, one needs an equation of state, $p(\rho)$, which describes the microscopic properties of the stellar matter. These coupled differential equations for the pressure $p(r)$

and the mass profile $M(r)$ are integrated from $r = 0$ with some central value p_0 to a point where the pressure vanishes $p(R) = 0$ which defines the radius R and the total mass $M(R)$ of the boson star.

The TOV equations have a similar scaling behavior as the one for Newtonian hydrostatic equilibrium (see e.g. [3]). Just the gravitational constant G and the boson mass m_b are used to rewrite the TOV equations together with the equation of state in dimensionless form. Solving the dimensionless TOV equation for a certain class of equations of state allows for deriving general solutions by just rescaling the results by appropriate dimensionful parameters.

The first relativistic correction factor in Eq. (1), i.e. $(1 + p/\rho)$, can be scaled by choosing $p' = p/\rho_o$ and $\rho' = \rho/\rho_o$. Here ρ_o is a common factor with dimension of mass to the fourth power. For the other two relativistic factors in Eq. (1) we introduce the dimensionless mass $M' = M/a$ and the dimensionless radius $r' = r/b$. For a dimensionless expression one has to set

$$\frac{b^3\rho_o}{a} = 1 \quad \text{and} \quad \frac{a}{M_P^2 b} = 1 \tag{3}$$

that leads to the following relations:

$$a = \frac{M_P^3}{\sqrt{\rho_o}} \quad \text{and} \quad b = \frac{M_P}{\sqrt{\rho_o}} \tag{4}$$

where $M_P = \sqrt{\hbar c/G}$ is the Planck mass. Note that the Newtonian terms do not give any additional constraint. Choosing $\rho_o = m_b^4$, the rescaling factors are $a = M_P^3/m_b^2$ and $b = M_P/m_b^2$, which coincide with the expressions of the maximum mass and the radius for compact stars introduced by Landau [2],

$$M_L = \frac{M_P^3}{m_b^2} \quad \text{and} \quad R_L = \frac{M_P}{m_b^2}. \tag{5}$$

We note in passing that for the Massachusetts Institute of Technology bag equation of state these scaling factors are $\rho_o = B$ so that the maximum mass and the corresponding radius scale as $B^{-1/2}$, a well-known result for quark stars [15].

III. MESON EXCHANGE MODEL FOR INTERACTING BOSONS

We describe the interactions between scalar bosons by the exchange of vector mesons. For a scalar field ϕ and a vector field V_μ the Lagrangian reads

$$\mathcal{L} = \mathcal{D}_\mu^* \phi^* \mathcal{D}^\mu \phi - m_b{}^2 \phi^* \phi - \tfrac{1}{4} V_{\mu\nu} V^{\mu\nu} + \tfrac{1}{2} m_v^2 V_\mu V^\mu \tag{6}$$

with $V_{\mu\nu} = \partial_\mu V_\nu - \partial_\nu V_\mu$. The boson field is coupled to the vector field by a minimal coupling scheme

$$\mathcal{D}_\mu = \partial_\mu + i g_{v\phi} V_\mu, \tag{7}$$

where $g_{v\phi}$ is the ϕ-V coupling strength. Note that the vector field has a quadratic coupling term to the scalar field in the Lagrangian which ensures that the vector field is coupled to a conserved current (see below). We treat the vector field as a classical field. In static bulk matter the spatial components of the vector field vanish and the equation of motion for the scalar field reads

$$[\mathcal{D}_\mu^* \mathcal{D}^\mu + m_b{}^2]\phi(x) = 0. \tag{8}$$

In the mean-field approximation, after expanding into plane waves, we obtain for the lowest energy mode $k = 0$:

$$\omega_\phi = m_b + g_{v\phi} V_0. \tag{9}$$

Note that the vector interaction between the scalar particles is repulsive which ensures the overall stability of self-interacting boson matter. The vector field is determined from the equation

$$m_v^2 V_0 = 2 g_{v\phi}(\omega_\phi - g_{v\phi} V_0)\phi^*\phi = 2 g_{v\phi} m_b \phi^* \phi \tag{10}$$

where we have used the dispersion relation for the ϕ field equation (9). The conserved current for the scalar field can be obtained from the Lagrangian (8)

$$\begin{aligned} J_\mu &= i\Big(\phi^* \frac{\partial \mathcal{L}}{\partial^\mu \phi^*} - \frac{\partial \mathcal{L}}{\partial^\mu \phi}\phi\Big) \\ &= \phi^* i \partial_\mu \phi - (i\partial_\mu \phi^*)\phi + 2 g_{v\phi} V_\mu \phi^* \phi. \end{aligned} \tag{11}$$

The number density of bosons

$$n_b = J_0 = 2(\omega_\phi - g_{v\phi} V_0)\phi^*\phi = 2 m_b \phi^* \phi \tag{12}$$

is just the source term for the vector field. The total energy density of the boson matter can be determined from the energy-momentum tensor

$$\rho = 2 m_b^2 \phi^* \phi + \frac{1}{2} m_v^2 V_0^2 = m_b n_b + \frac{g_{v\phi}^2}{2 m_v^2} n_b^2 \tag{13}$$

where the equation of motion for the vector field has been used. The pressure is given just by the vector field contribution

$$p = \frac{1}{2} m_v^2 V_0^2 = \frac{g_{v\phi}^2}{2 m_v^2} n_b^2. \tag{14}$$

Note that these expressions are thermodynamically consistent as can be checked by using the thermodynamic relation

$$p = n_b^2 \frac{d(\rho/n_b)}{d n_b}. \tag{15}$$

The form of the interaction is actually similar to the one used for interacting fermions and the corresponding Fermi stars in [3]. For the rescaled TOV equations we introduce the dimensionless interaction parameter $y = m_b/m_I$, where $m_I = \sqrt{2} m_v / g_{v\phi}$, and set $\rho' = \rho/m_b^4$ and $p' = p/m_b^4$. The equation of state for interacting boson matter

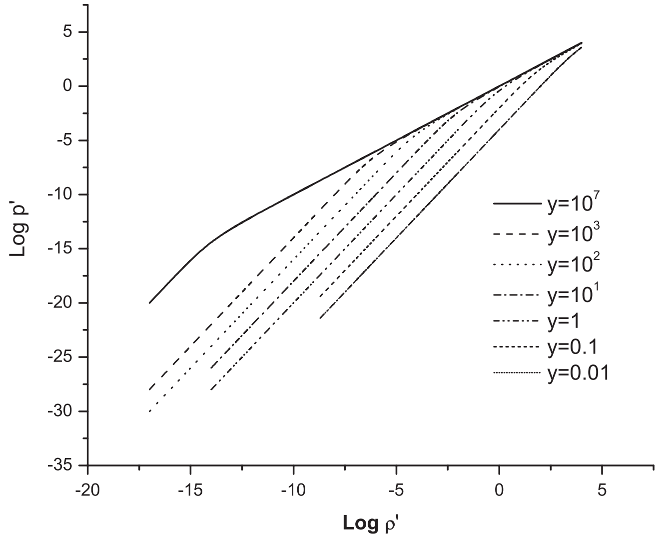

FIG. 1. Double logarithmic plot of the dimensionless pressure versus the dimensionless energy density for different interaction strengths y.

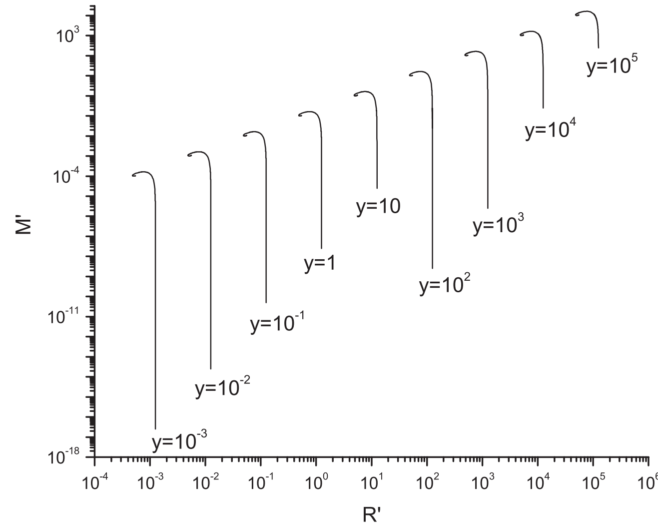

FIG. 2. The dimensionless mass M' is plotted versus the dimensionless radius R' for different interaction strengths y ranging from 10^{-3} to 10^5. Note that each mass-radius curve terminates in a spiral at the left end, which is not visible in the double logarithmic plot, but can be seen in the linear plot of Fig. 4.

can be summarized to be of the simple form

$$p' = y^2 n_b'^2 \quad \text{and} \quad \rho' = n_b' + y^2 n_b'^2 \tag{16}$$

with the dimensionless number density $n_b' = n_b/m_b^3$. It is possible to represent the equation of state in a polytropic form $p = \rho^\gamma$ for certain limits. For low densities, one approaches $p \propto \rho^2$, a polytrope with $\gamma = 2$. For high densities, one has an equation of state of the form $p = \rho$ with $\gamma = 1$, which is the stiffest possible equation of state first discussed by Zel'dovich [16]. The switch between those two limiting cases is controlled by the interaction strength y. The larger y, the lower is the energy density to approach the causal limit $p = \rho$.

Figure 1 depicts the dimensionless pressure versus the dimensionless energy density. The values of the interaction strength y are chosen between 10^{-2} to 10^7. There are two different slopes for small and large values of ρ' corresponding to the above mentioned limits. The point where the slope changes shifts to lower densities with increasing interaction strength y, but for low densities the slope is $\gamma = 2$. At high ρ' all curves merge to the limiting curve with a slope of $\gamma = 1$.

IV. SCALING RELATION FOR BOSON STARS WITH SELF-INTERACTION

We use the dimensionless equation of state to solve the dimensionless TOV equations. The equation of state depends only on the interaction strength y. Figure 2 shows the double logarithmic plot of the dimensionless mass M' versus the dimensionless radius R' for different interaction strengths y ranging from 10^{-3} to 10^5. One observes that each mass-radius curve contains a constant radius part over a wide range of masses. Also, the curves are very similar and seem to be just shifted to larger masses and radii with increasing interaction strength.

This interesting behavior can be explained by considering the equation of state described by a polytrope $p \sim \rho^\gamma$ with $\gamma = 2$. In general, the solution to the Lane-Emden equation, see e.g. [17], results in a mass-radius relation of the form $M' \propto \rho_c^{(3\gamma-4)/2}$ and $R' \propto \rho_c^{(\gamma-2)/2}$, where ρ_c is the central energy density. Hence, at low densities and large radius $R \gg 2GM$, where effects from general relativity can be ignored, the mass of the star increases linearly with ρ_c while the radius R' remains constant for $\gamma = 2$ that explains the peculiar form of the mass-radius curves.

There exists another interesting feature of the mass-radius curves which reflects the scaling properties of the equation of state and the TOV equations. To illustrate this we plot in Fig. 3 the dimensionless maximum mass $M'_{\max}$ as a function of the interaction strength y. It is interesting to see that the maximum mass $M'_{\max}$ scales linearly with the interaction strength y.

Therefore, one can conclude that by proper rescaling all mass-radius curves can be reduced to one universal mass-radius curve. Indeed, dividing the dimensionless mass M' and the corresponding radius R' by the interaction strength y results in an unique mass-radius relation as depicted in Fig. 4. This graph looks rather similar to the mass-radius curve of a strongly interacting fermion star [3]. There the maximum mass is constant for weak interactions ($y \ll 1$) and increases linearly in y for strong interactions ($y \gg 1$). Note that the part of the curve to the left of the maximum mass represents unstable configurations; only the star configurations at the maximum and to the right of it can exist.

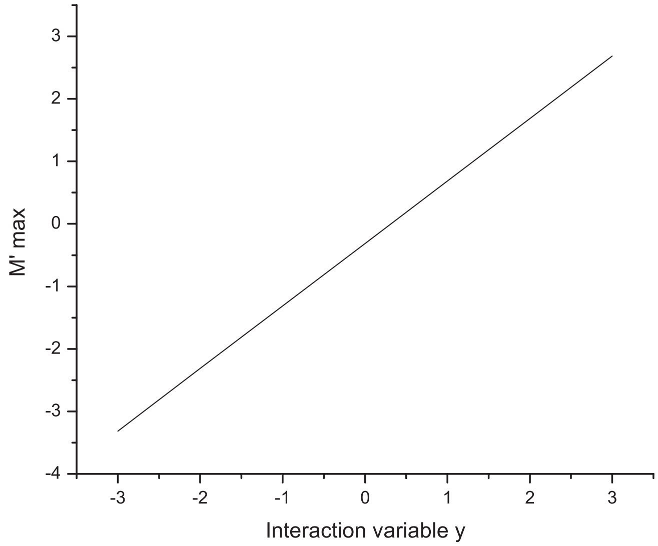

FIG. 3. The dimensionless maximum mass $M'_{\max}$ is plotted as a function of interaction strength y.

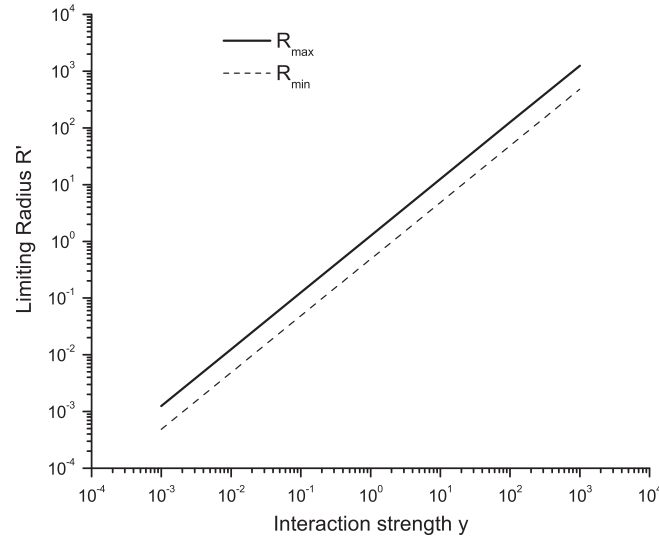

FIG. 5. Plot of the two limiting radii $R'_{\max}$ and $R'_{\min}$ for boson stars as a function of interaction strength y.

The existence of a maximum mass is determined by the change of the equation of state from a polytrope with $\gamma = 2$ to one with $\gamma = 1$. The latter value is lower than the critical value $\gamma_c = 4/3$ for stable compact stars (effects of general relativity will even increase this value slightly).

Figure 5 shows the two limiting radii for interacting boson stars as a function of the interaction strength y. Here, $R'_{\max}$ and $R'_{\min}$ denote the maximum and minimum radius for boson stars, respectively. $R'_{\min}$ stands for radius corresponding to the maximum mass configuration, while $R'_{\max}$ is the radius of stars with masses much smaller than the maximum mass. Both the maximum and the minimum radius vary linearly with the interaction strength y and the difference between the two radii is rather small, by a factor of about 0.61 independent of the interaction strength.

Figure 6 shows the normalized density profile of $\rho'(r)/\rho'(0)$ over the normalized radius r'/R' for different interaction strengths calculated for the maximum mass configuration. Again, there appears a universal curve independent of the interaction strength and the mass of the boson. The rate of the decrease of the density with the radius is then the same for all interaction strengths y. The density profile shows a small plateau in the core region up to a radius of about $r \sim 0.1R$ followed by a nearly linear decrease up to the surface of the boson star.

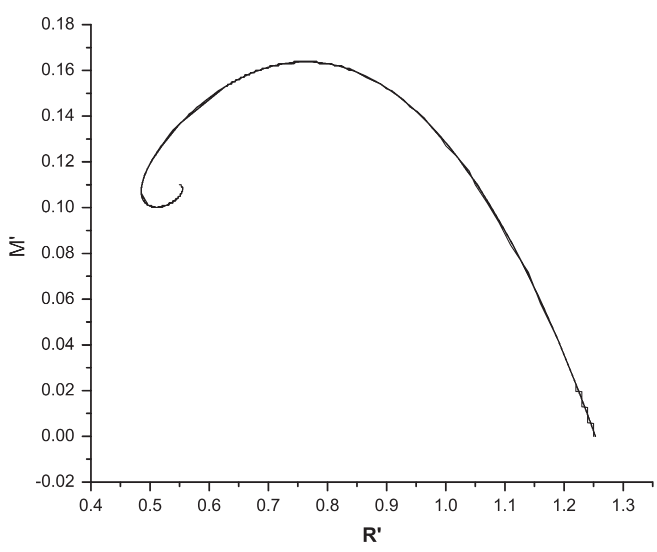

FIG. 4. The dimensionless mass M' is plotted versus the dimensionless radius R' dividing both by the interaction strength y. The result is a universal mass-radius relation.

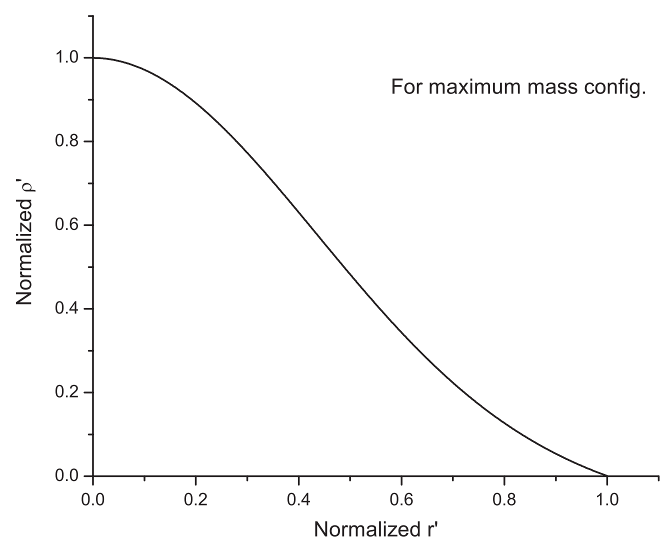

FIG. 6. The variation of the normalized density $\rho'(r)/\rho'(0)$ with the normalized radius r'/R' for different interaction strengths showing that there is only one universal curve.

The scaling behavior observed for interacting boson stars follows in a straightforward way from our discussion above on the scaling features of the TOV equations. The maximum mass configuration is determined by the equation of state at high central densities, where $p = \rho$. The pressure and the energy density, p and ρ, depend then on the interaction strength as y^2. Rescaling both, the energy density and the pressure, by the factor m_b^4/y^2 gives the modified Landau mass and Landau radius

$$M_L^b = y \cdot \frac{M_P^3}{m_b^2} \quad \text{and} \quad R_L^b = y \cdot \frac{M_P}{m_b^2} \tag{17}$$

for compact stars with interacting bosons. The maximum mass and the corresponding radius have to increase linearly with the interaction strength y. This is in complete agreement with the results of Ref. [11] where the self-interaction term of the form $\lambda\phi^4$ was used to describe the interactions between bosons. Also for this type of interaction, the maximum mass and the corresponding radius were found to scale with the interaction strength and the Landau mass as $M_{\rm max} \propto \lambda^{1/2} M_L$. The relation for the maximum mass is compatible with our findings by realizing that the dimensionless coupling constant λ can be associated with our interaction strength y^2. In addition to the case of scalar self-interaction, we find that for vector interactions the scaling property is even more general as the whole mass-radius curve can be described by a universal curve when using the modified Landau mass and radius. We note that compact stars made of fermions with vector interactions [3] reveal the same scaling feature of the mass-radius curve for large interaction strengths.

The maximum mass of boson stars as obtained from numerical calculations is

$$M_{\rm max} \approx 0.164 y \cdot \frac{M_P^3}{m_b^2} \tag{18}$$

and the two limiting radii of boson stars are given by the expressions

$$R_{\rm max} \approx 1.252 y \cdot \frac{M_P}{m_b^2} \quad \text{and} \quad R_{\rm min} \approx 0.763 y \cdot \frac{M_P}{m_b^2}. \tag{19}$$

The above relations can be used to calculate the maximum mass and the maximum and minimum radii of boson stars for arbitrary interaction strength y and boson masses m_b. The values for the Landau mass and radius are $M_L = 1.632 M_\odot$ and $R_L = 2.410$ km, respectively, for a boson mass of $m_b = 1$ GeV. One recovers the same scaling relations as for the noninteracting case, see e.g. Ref. [7], by setting the interaction scale equal to the Planck mass, $m_I = M_P$,

$$M_{\rm max} \propto \frac{M_P^2}{m_b} \quad \text{and} \quad R_{\rm min} \propto \frac{1}{m_b}, \tag{20}$$

which are orders of magnitude smaller than for the case of realistic interactions. We note that our numerical prefactors are different from the ones of Takasugi and Yoshimura [7] while the scaling with the boson mass is the same. These authors adopt a different equation of state, where the pressure has the form as for degenerate configurations, e.g. in the low-density limit they recover that $p \propto \rho^{5/3}$. In our case the pressure is determined by interactions only and is proportional to the density squared.

Table I gives the maximum mass and the corresponding radius for four different cases of the interaction parameter. By setting $m_I = M_P$ or $y = m_b/M_P$ one recovers the case for ordinary boson stars with free bosons (see above) which are just gravitationally bound. The case $y = 1$ gives the Landau mass and radius of compact stars, which is nearly the same for boson stars and fermion stars when including interactions. Finally, we consider the case of interactions mediated by the weak interaction scale of about $m_I = 100$ GeV and the QCD scale of about $m_I = 100$ MeV. For the boson masses we choose the range from the electroweak scale to a typical mass of axions, $\sim 10^{-5}$ eV. We want to emphasize the following features of these calculations. For the free case $m_I = M_P$ one only reaches astrophysically interesting scales for boson masses of less

TABLE I. Order of magnitude scales of the maximum mass and the characteristic radius of compact stars made of different boson masses and interaction strengths. The first set corresponds to the free case by setting $y = m_b/M_P$ (the boson stars are just bound by gravity), and the second gives the Landau mass and radius by setting $y = 1$, which holds for boson and fermion stars. The third and last set lists the values for interaction mass scales of the standard model weak and strong interactions, i.e. of 100 GeV and 100 MeV, respectively.

Boson mass	100 GeV	1 GeV	1 MeV	1 keV	1 eV	10^{-5} eV	Interaction
$M_{\rm max}(M_\odot)$	10^{-22}	10^{-20}	10^{-17}	10^{-14}	10^{-11}	10^{-6}	$m_I = M_P$
R (km)	10^{-21}	10^{-19}	10^{-16}	10^{-13}	10^{-10}	10^{-5}	(Free case)
$M_{\rm max}(M_\odot)$	10^{-5}	0.1	10^{5}	10^{11}	10^{17}	10^{27}	$y = 1$
R (km)	10^{-4}	1	10^{6}	10^{12}	10^{18}	10^{28}	(Landau limit)
$M_{\rm max}(M_\odot)$	10^{-5}	10^{-3}	1	10^{3}	10^{6}	10^{11}	$m_I = 100$ GeV
R (km)	10^{-4}	10^{-2}	10	10^{4}	10^{7}	10^{12}	(Weak scale)
$M_{\rm max}(M_\odot)$	10^{-2}	1	10^{3}	10^{6}	10^{9}	10^{14}	$m_I = 100$ MeV
R (km)	0.1	10	10^{4}	10^{7}	10^{10}	10^{15}	(QCD scale)

than 10^{-5} eV. Maximum masses close to the ones for neutron stars, $M \sim 1M_\odot$, can be reached by boson stars for boson masses of around 1 GeV (Landau case), 1 MeV for bosons with weak interactions, and 1 GeV for strong interactions. The mass range of observed supermassive black holes, $M = 10^6$ to $10^9 M_\odot$, is found for boson masses between 1 keV and 1 MeV for the Landau case, 1 eV and below for weak interactions, and 1 eV to 1 keV for strong interactions. It is observed that the inclusion of interactions results in a wide range of possible masses and radii for boson stars, covering scales as small as a fraction of a solar mass and below a kilometer to scales of supermassive black holes. It is interesting to note that a boson with a mass of 100 GeV and with QCD-type interaction strengths gives star configurations with masses and radii as a neutron star. Surprisingly, for a boson star made of axions ($m_b \sim 10^{-5}$ eV) and self-interactions on the scale of 10^{12} GeV, one obtains a mass of about $30M_\odot$ with a radius of 200 km, i.e. the mass of compact objects found in binary systems which are attributed to light black holes. These values are orders of magnitude different compared to the case of boson stars with noninteracting axions, see also Ref. [7] and Table I.

V. SUMMARY

We have constructed an equation of state for a system of massive bosons interacting by the exchange of vector mesons. By solving the TOV equations for such boson stars, we have demonstrated that there exists a universal mass-radius curve independent of the boson mass and the interaction strength. The maximum mass and the corresponding radius of boson stars are scaled with the Landau mass and Landau radius times the interaction strength. For masses much smaller than the maximum mass, the radius stays constant and is only slightly larger than the one for the maximum mass configuration.

The maximum mass and the corresponding radius can be computed with the simple formulas $M_{\rm max} = 0.164y \cdot M_P^3/m_b^2$ and $R_{\rm min} = 0.763y \cdot M_P/m_b^2$ for any given boson mass m_b and interaction strength y. The possible masses and radii for boson stars can therefore cover a wide range and can be similar to the ones found for astrophysical compact objects, be it neutron stars or black hole candidates. For example, for a boson with QCD-type interaction strength and a boson mass of 100 GeV the maximum mass is $M_{\rm max} \sim 0.3M_\odot$ with a radius of about 2 km. A boson with a typical axionlike mass of 10^{-5} eV and an interaction scale of 10^{12} GeV will give a maximum mass of the boson star of $30M_\odot$ with a radius of 200 km. The compactness of boson stars for the maximum mass configuration is about $R/(2GM) \approx 2.3$ which is close to the value found for fermion stars $R/(2GM) \approx 2.4$ in Ref. [3]. It is interesting that these values are below the radius of the innermost stable circular orbit of nonrotating black holes $R/(2GM) = 3$.

Finally, we mention that the full problem addressed here involves solving the Einstein equations with the coupled system of Klein-Gordon and Proca equations which we leave to address as an interesting extension for future work.

ACKNOWLEDGMENTS

This work was supported by GSI Darmstadt and by the German Research Foundation (DFG) within the framework of the excellence initiative through the Heidelberg Graduate School of Fundamental Physics. I. M. acknowledges support from the DFG Grant No. 436 RUS 113/957/0-1 and the Russian Grant No. NS-3004.2008.2.

[1] S. Chandrasekhar, Astrophys. J. **74**, 81 (1931).
[2] L. Landau, Phys. Z. Sowjetunion **1**, 285 (1932).
[3] G. Narain, J. Schaffner-Bielich, and I. N. Mishustin, Phys. Rev. D **74**, 063003 (2006).
[4] J. A. Wheeler, Phys. Rev. **97**, 511 (1955).
[5] D. J. Kaup, Phys. Rev. **172**, 1331 (1968).
[6] R. Ruffini and S. Bonazzola, Phys. Rev. **187**, 1767 (1969).
[7] E. Takasugi and M. Yoshimura, Z. Phys. C **26**, 241 (1984).
[8] R. C. Tolman, *Relativity, Thermodynamics and Cosmology* (Oxford University Press, Oxford, 1934).
[9] R. C. Tolman, Phys. Rev. **55**, 364 (1939).
[10] J. R. Oppenheimer and G. M. Volkoff, Phys. Rev. **55**, 374 (1939).
[11] M. Colpi, S. L. Shapiro, and I. Wasserman, Phys. Rev. Lett. **57**, 2485 (1986).
[12] T. D. Lee and Y. Pang, Phys. Rep. **221**, 251 (1992).
[13] P. Jetzer, Phys. Rep. **220**, 163 (1992).
[14] F. E. Schunck and E. W. Mielke, Classical Quantum Gravity **20**, R301 (2003).
[15] E. Witten, Phys. Rev. D **30**, 272 (1984).
[16] Y. B. Zel'dovich, Zh. Eksp. Teor. Fiz. **41**, 1609 (1961).
[17] S. Weinberg, *Gravitation and Cosmology: Principles and Applications of the General Theory of Relativity* (John Wiley and Sons, New York, 1972).