Estimation of cosmological parameters from neutral hydrogen observations of the post-reionization epoch

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The emission from neutral hydrogen (HI) clouds in the post-reionization era ($z \leq 6$), too faint to be individually detected, is present as a diffuse background in all low frequency radio observations below 1420 MHz. The angular and frequency fluctuations of this radiation (~ 1 mK) are an important future probe of the large-scale structures in the Universe. We show that such observations are a very effective probe of the background cosmological model and the perturbed Universe. In our study we focus on the possibility of determining the redshift-space distortion parameter β, coordinate distance r_ν, and its derivative with redshift r'_ν. Using reasonable estimates for the observational uncertainties and configurations representative of the ongoing and upcoming radio interferometers, we predict parameter estimation at a precision comparable with supernova Ia observations and galaxy redshift surveys, across a wide range in redshift that is only partially accessed by other probes. Future HI observations of the post-reionization era present a new technique, complementing several existing ones, to probe the expansion history and to elucidate the nature of the dark energy.

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I. INTRODUCTION

Determining the expansion history of our Universe and parametrizing the constituents of the Universe at a high level of precision are currently some of the most important goals in cosmology. While high-redshift ($z \leq 2$) supernova Ia observations (e.g. [1,2]) and galaxy surveys ($z \leq 1$) (e.g. [3]) probe the local universe, and cosmic microwave background radiation observations (e.g. [4,5]) probe the recombination era ($z \sim 1000$), the expansion history is largely unconstrained across the vast intervening redshift range. Observations of redshifted 21 cm radiation from neutral hydrogen (HI) hold the potential of probing the Universe over a large redshift range ($20 \geq z \geq 0$): from the dark ages to the present epoch (e.g. [6,7]). Such observations can possibly be realized at several redshifts, using the currently functioning giant meterwave radio telescope (GMRT) [8]. Several new telescopes are currently being built with such observations in mind (e.g. murschison widefield array [9] and low frequency array [10]). Such observations will map out the large-scale HI distribution at high redshifts. It has recently been proposed [11,12] that baryon acoustic oscillations (BAO) in the redshifted 21 cm signal from the post-reionization era ($z \leq 6$) is a very sensitive probe of the dark energy. The BAO is a relatively small ($\sim 10\%$–15%) feature that sits on the HI large-scale structure (LSS) power spectrum. In this paper we investigate the possibility of probing the expansion history in the post-reionization era using the HI LSS power spectrum without reference to the BAO. Unless otherwise stated we use the parameters $(\Omega_{m0}, \Omega_{\Lambda 0}, \Omega_b h^2, h, n_s, \sigma_8) = (0.3, 0.7, 0.024, 0.7, 1.0, 1.0)$ referred to as the lambda cold dark matter (LCDM) model in our analysis.

At redshifts $z \leq 6$, the bulk of the neutral gas is in clouds that have HI column densities in excess of 2×10^{20} atoms/cm^2 [13–15]. These high column density clouds are observed as damped Lyman-α absorption lines seen in quasar spectra. These observations indicate that the ratio of the density $\rho_{\rm gas}(z)$ of neutral gas to the present critical density $\rho_{\rm crit}$, of the Universe has a nearly constant value $\rho_{\rm gas}(z)/\rho_{\rm crit} \sim 10^{-3}$, over a large redshift range $0 \leq z \leq 3.5$. This implies that the mean neutral fraction of the hydrogen gas is $\bar{x}_{\rm HI} = 50\Omega_{\rm gas}h^2(0.02/\Omega_b h^2) = 2.45 \times 10^{-2}$, which we adopt for the entire redshift range $z \leq 6$. We note that this assumption is likely to be invalid at high redshits $z \simeq 6$. Given the large uncertainty in $\Omega_{\rm HI}$ we adopt a constant value to make fiducial predictions. The redshifted 21 cm radiation from the HI in this redshift range will be seen in emission. The emission from individual clouds (< 10 μJy) is too weak to be detected with existing instruments unless the image is significantly magnified by gravitational lensing [16]. The collective emission from the undetected clouds appears as a very faint background in all radio observations at frequencies below 1420 MHz. The fluctuations in this background with angle and frequency is a direct probe of the HI distribution at the redshift z where the radiation originated. It is possible to probe the HI

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power spectrum at high redshifts by quantifying the fluctuations in this radiation [17,18].

II. FORMULATION

The multifrequency angular power spectrum $C_\ell(\Delta\nu)$ [19] quantifies the statistics of the HI signal as a joint function of the angular multipole ℓ and the frequency separation $\Delta\nu$. We define the angular power spectrum $C_\ell = C_\ell(0)$ and the frequency decorrelation function

$$\kappa_\ell(\Delta\nu) = \frac{C_\ell(\Delta\nu)}{C_\ell(0)}, \quad (1)$$

to separately characterize the angular and the $\Delta\nu$ dependence, respectively. The latter quantifies whether the HI signal at two different frequencies ν and $\nu + \Delta\nu$ is correlated $\kappa_\ell(\Delta\nu) \sim 1$ or uncorrelated $\kappa_\ell(\Delta\nu) \sim 0$. The function $C_\ell(\Delta\nu)$ can be estimated directly from observations without reference to a cosmological model (e.g. [20]). However, it is necessary to assume a background cosmological model in order to interpret $C_\ell(\Delta\nu)$ in terms of the three-dimensional LSS HI power spectrum. On the large scales of interest here, it is reasonable to assume that HI traces the dark matter with a possible linear bias b, whereby the three-dimensional HI power spectrum is $b^2P(k)$, where $P(k)$ is the dark matter power spectrum at the redshift where HI signal originated. We have [19]

$$C_l(\Delta\nu) = \frac{\bar{T}^2}{\pi r_\nu^2}\int_0^\infty dk_\parallel \cos(k_\parallel r'_\nu \Delta\nu) P_{\rm HI}(\mathbf{k}), \quad (2)$$

where the three-dimensional wave vector $\mathbf{k}$ has been decomposed into components $k_\parallel$ and l/r_ν, along the line of sight and in the plane of the sky, respectively. The comoving distance r_ν is the distance at which the HI radiation originated. Note that $(1+z)^{-1}r_\nu = d_{\rm A}(z)$ is the angular diameter distance and $r'_\nu = dr_\nu/d\nu$. The temperature occurring in Eq. (2) is given by

$$\bar{T}(z) = 4.0\ {\rm mK}(1+z)^2\left(\frac{\Omega_b h^2}{0.02}\right)\left(\frac{0.7}{h}\right)\frac{H_0}{H(z)}, \quad (3)$$

and $P_{\rm HI}(\mathbf{k})$ is the three-dimensional power spectrum of the "21 cm radiation efficiency in redshift space," which in this situation is given by

$$P_{\rm HI}(\mathbf{k}) = \bar{x}_{\rm HI}^2 b^2 (1+\beta\mu^2)^2 P(k). \quad (4)$$

The term $(1+\beta\mu^2)^2$ arises due to HI peculiar velocities [17,21], which we assume to be determined by the dark matter. This is the familiar redshift-space distortion seen in galaxy redshift surveys, where $\mu = k_\parallel/k$ and $\beta = f(z)/b$ is the linear distortion parameter, which is the ratio of $f(z)$ that quantifies the growth rate of linear perturbations, and b the linear bias which we assume to be unity throughout. Note that we have assumed that the HI spin temperature is much larger than the temperature of the cosmic microwave background radiation, and the HI is seen in emission.

III. RESULTS AND CONCLUSIONS

The expected signal $C_l(\Delta\nu)$ from a few representative redshifts, calculated for the LCDM model, is plotted in Fig. 1, and in Fig. 2 we have plotted the frequency decorrelation function $\kappa_\ell(\Delta\nu)$ as a function of $\Delta\nu$, for a fixed redshift $z = 3.0$ and for $\ell = 100, 1000$, and $10\,000$. The HI signal ($\sqrt{l(l+1)C_l/2\pi}$) is smaller than $\sim$1 mK, and it decreases with increasing l. The shape or ℓ dependence is decided by the shape of $P(k)$ at all comoving wavenumbers $k \geq \ell/r_\nu$. The signal at two different frequencies ν and $\nu + \Delta\nu$ decorrelates rapidly with increasing $\Delta\nu$ and $\kappa_\ell(\Delta\nu) < 0.1$ at $\Delta\nu > 5$ MHz. The decorrelation occurs at a smaller $\Delta\nu$ for the larger multipoles (Fig. 2). Defining $\Delta\nu_{1/2}$ [19] such that $\kappa_\ell(\Delta\nu_{1/2}) = 1/2$, and $|\kappa_\ell(\Delta\nu)| \leq 1/2$ for $\Delta\nu > \Delta\nu_{1/2}$, we find that it is reasonably well approximated as $\Delta\nu_{1/2} \approx 1\ {\rm MHz}(\ell/100)^{-0.7}$ at the redshifts of interest. The value of $\kappa_\ell(\Delta\nu)$ falls rapidly for $\Delta\nu > \Delta\nu_{1/2}$, and we use $\Delta\nu_{1/2}$ to estimate the frequency separation beyond which the HI signal is uncorrelated. While the HI signal at a frequency separation $\Delta\nu >$ 5 MHz is definitely expected to be uncorrelated, the foregrounds are expected to be highly correlated even at frequency separations larger than this (e.g. [22]). This should in principle allow the HI signal to be separated from the foregrounds, which are a few orders of magnitude larger (e.g. [23,24]).

It is clear from Eq. (2) that $C_\ell(\Delta\nu)$ depends on the background cosmological model through the parameters (β, r_ν, r'_ν). Assuming that the dark matter power spectrum $P(k)$ is known *a priori*, observations of $C_\ell(\Delta\nu)$ can be used to determine the values of these three parameters. It is convenient to replace r'_ν with the dimensionless parameter

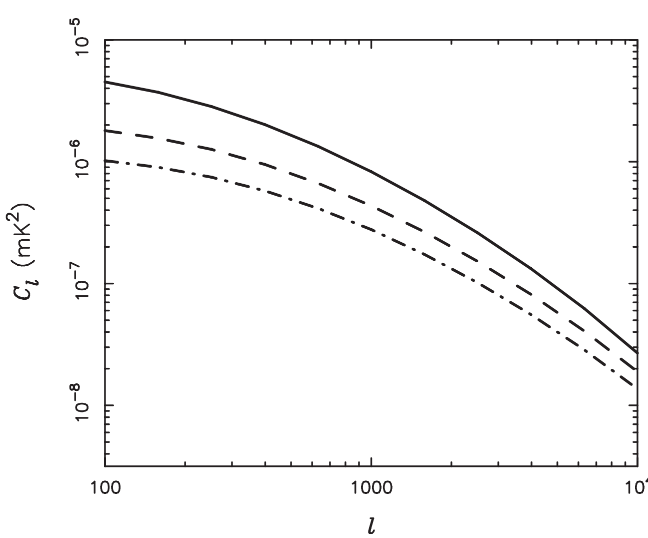

FIG. 1. Here we plot $C_l(0)$ at redshifts $z = \{1.5, 3.0, 4.5\}$. The signal decreases monotonically with increasing redshift, so the lowest plot is for the highest redshift. We assume the bias to be $b = 1$ throughout.

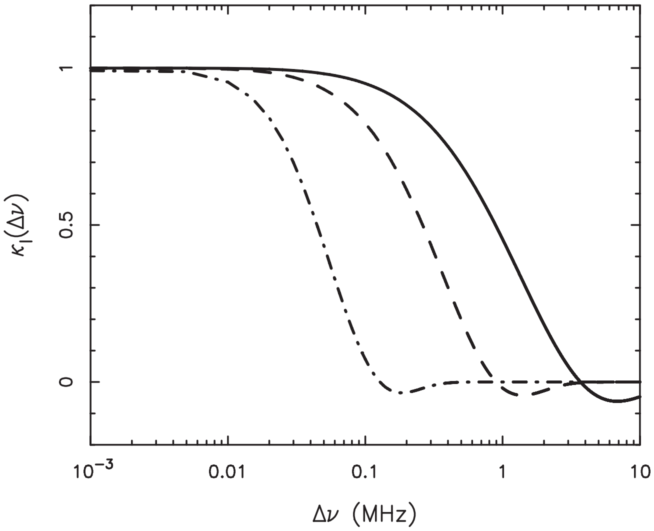

FIG. 2. Here we plot the frequency decorrelation function $\kappa_\ell(\Delta\nu)$ as a function of $\Delta\nu$, for a fixed redshift $z = 3.0$ and $\ell = \{100, 1000, 10000\}$. The signal declines more sharply for higher value of ℓ.

[25],

$$p(z) = \frac{d\ln[r_\nu(z)]}{d\ln(z)}. \tag{5}$$

Figure 3 shows the variation of the three parameters (β, r_ν, p) across the redshift range $z \leq 6$ for the LCDM model.

We separately consider parameter estimation using C_ℓ and $\kappa_\ell(\Delta\nu)$. The former does not depend on p. The amplitude $A = (\bar{T}\bar{x}_{\rm HI}b)^2/\pi r_\nu^2$ of C_ℓ is uncertain, and we consider the joint estimation of three parameters (A, β, r_ν)

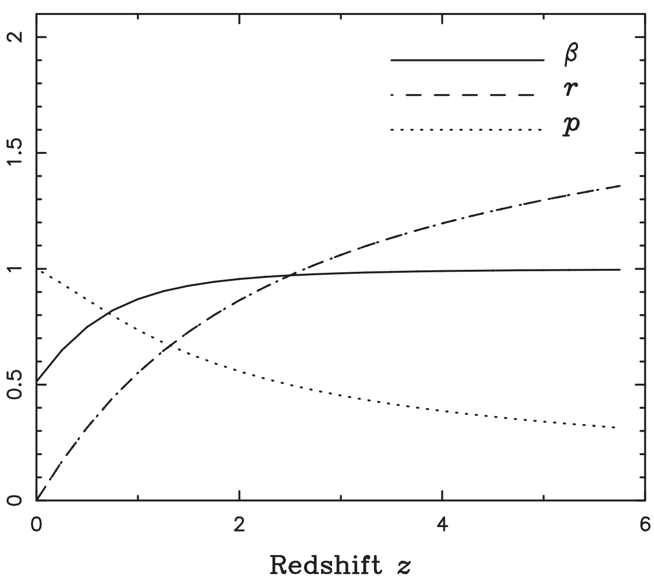

FIG. 3. Here we plot the parameters (β, r, p) as a function of redshift z for the concordance LCDM model. The parameter $r = r_\nu/(6000\ \mathrm{Mpc})$.

from observations of C_ℓ. The value of $\kappa_\ell(\Delta\nu)$ is insensitive to the amplitude A, leaving three parameters (β, r_ν, p) that can be jointly estimated from this. We use the Fisher matrix (e.g. [26]) to determine the accuracy at which these parameters can be estimated.

Parameter estimation depends on two distinct aspects of the observing instrument. The first is the ℓ range i.e. $\ell_{\rm min}$, $\ell_{\rm max}$, and the sampling interval $\Delta\ell$, which corresponds to the smallest ℓ spacing at which we have independent estimates of $C_\ell(\Delta\nu)$. This is determined by the instrument's field of view and is inversely related to it. The second is the observational uncertainty in $C_\ell(\Delta\nu)$. This is a sum, in quadrature, of the instrumental noise and the cosmic variance. The cosmic variance contribution $\delta C_\ell/C_\ell = \sqrt{2/((2\ell+1)\mathrm{f}\Delta\ell)}$ (f is the fraction of sky observed) is further reduced because the large frequency bandwidth $\Delta\nu_B$ provides several independent estimates of C_ℓ. We assume that δC_ℓ is reduced by a factor $\sqrt{\Delta\nu_B/\Delta\nu_{1/2}}$ because of this. The instrumental uncertainties were estimated using relations [20] between δC_ℓ and the noise in the individual visibilities measured in radio-interferometric observations. For this we assume that the baselines in the radio-interferometric array have a uniform coverage in the Fourier space.

We consider three different instrumental configurations for parameter estimation.

(A) The currently functional GMRT has too few antennas for cosmological parameter estimation. We consider an enhanced GMRT-like instrument with a substantially larger number of antennas ($N = 120$), each identical to those of the existing GMRT. The antennas have a relatively small field of view ($\theta_{\rm FWHM} \sim 0.8°$ at 610 MHz) and the array has relatively large baselines spanning $\ell_{\rm min} = 500$ to $\ell_{\rm max} = 10\,000$ with $\Delta\ell = 100$.

(B) Many upcoming instruments like murschison wide-field array have a large number of small-sized antennas. The principle aim of this instrument is to map the epoch of reionization and therefore it does not operate in the range of frequencies of interest to us here. We consider here a similar wide-field instrument operating at higher frequencies for our study. The antennas are assumed to have a large field of view ($\theta_{\rm FWHM} \sim 5°$ at 610 MHz), and the array is expected to be quite compact spanning $\ell_{\rm min} = 100$ to $\ell_{\rm max} = 2000$ with $\Delta\ell = 20$, with the number of antennas $N = 500$.

(C) This is a future wide-field instrument with $N = 5000$ antennas.

For each of these configurations, we assume that only a single primary beam is observed at a given time. Note that future instruments could have the capability of simultaneously observing in several (N_p) primary beams. This would cause a $1/\sqrt{N_p}$ reduction of the error estimates presented here.

We present results for 2 yr of observation for A and B, and 1000 h for C. Throughout we assume frequency channels adjusted to $\Delta\nu_{1/2}$, a bandwidth $\Delta\nu_B = 32$ MHz, and that a single field is observed for the entire duration. For parameter estimation we use $\delta\kappa_\ell(\Delta\nu) = \sqrt{2}\delta C_\ell/C_\ell$.

We find that observations of C_ℓ impose very poor constraints on the parameters β and r_ν, and we do not show these here. The accuracy is considerable higher for $\kappa_\ell(\Delta\nu)$, which captures the three-dimensional clustering of the HI as compared to C_ℓ, which quantifies only the angular dependence. Figure 4 shows the predicted estimates for the parameters β, r_ν, and p at various redshifts. Further, we find that a compact, wide-field array (B, C) is considerably more sensitive to these parameter as compared to case A.

Considering the three parameters individually:

Redshift-space distortion parameter: β. This has traditionally been measured from galaxy redshift surveys [27–30], with uncertainties in the range $0.1 \leq \Delta\beta/\beta \leq 0.2$. These observations have, to date, been restricted to $z \leq 1$. Future galaxy surveys are expected to achieve higher redshifts and smaller uncertainties. Galaxy surveys have the drawback that at very high redshifts they probe only the most luminous objects, which are expected to be highly biased. HI observations do not have this limitation and could provide high precision ($\Delta\beta/\beta < 0.1$) estimates over a large redshift range.

Coordinate distance r_ν: The most direct measurement of the coordinate distance comes from supernova type Ia observations for $z \leq 2$. Current Sn Ia observations give $\Delta r_\nu/r_\nu \simeq 0.07$ [31] for a single supernova. The statistical error in the coordinate distance can be further reduced by observing a large number of supernovae in a small redshift bin; thus the fundamental limitation of this technique is due to unknown systematics in the supernovae themselves, since it is certainly possible that supernovae at high redshifts are different. Figure 4 shows that the HI method might have the potential to enable a precise measurement of the coordinate distance up to much larger redshifts. Furthermore, such a complimentary probe will also help in ascertaining systematics in the supernova probe.

Derivative of coordinate distance p: This quantifies the Alcock-Paczynski effect [32], which is well accepted as a means to study the expansion history at high z, though such observations have not been possible to date. Observations of redshifted 21 cm radiation hold the potential of measuring the Alcock-Paczynski effect [25,33,34]. The parameter p is not affected by the overall amplitude A and the bias b, and is a sensitive probe of the spatial curvature (Fig. 3). Our estimates indicate that it will be possible to measure p with an accuracy $\Delta p/p \sim 0.03$ over a large z range.

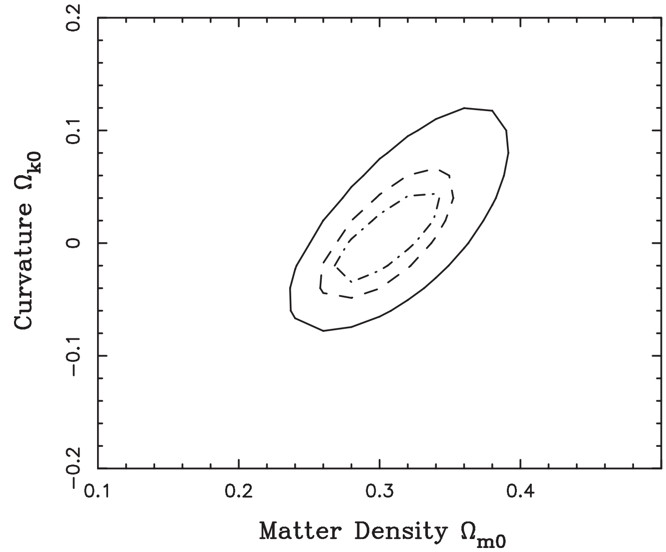

FIG. 5. Expected one-sigma confidence regions for the parameters Ω_{m0} and Ω_{k0}, based on estimated errors for observations of p, corresponding to Fig. 4, at $z = 1$ and $z = 3$.

The parameters (β, r_ν, p) chosen for our analysis occur naturally when we interpret $C_\ell(\Delta\nu)$ in terms of the three-dimensional dark matter power spectrum $P(k)$. Further, these parameters are very general in that they do not refer to any specific model for either the dark energy or the dark matter, and are valid even in models with alternate theories of gravity (e.g. [35,36]). In fact, observations of these three parameters at different redshifts can in principle be used to distinguish between these possibilities.

For the purpose of this paper, we illustrate the cosmological parameter estimation by considering the simplest LCDM model, with two unknown parameters Ω_{m0} and Ω_{k0}, and $\Omega_{\Lambda 0} = 1 - \Omega_{m0} - \Omega_{k0}$. In Fig. 5 we plot the 1-σ confidence interval for the estimation of Ω_{m0} and Ω_{k0},

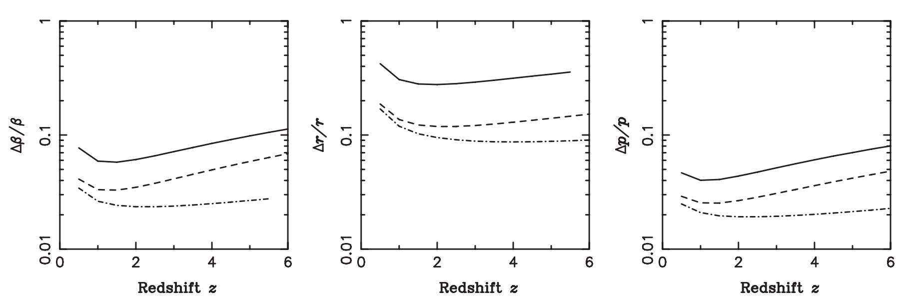

FIG. 4. Expected one-sigma fractional errors for parameter estimation at different redshifts for the LCDM model. The curves in each panel correspond, from top to bottom, to the cases A, B, and C, respectively.

using two measurements of p alone, i.e. only one of the three parameters measured at two different redshifts $z = 1$ and $z = 3$. Note that p is insensitive to H_0 and hence it is not considered as an additional parameter here. It is possible to combine measurements of all three parameters i.e. β, r, and p to improve the constraints on cosmological parameters. We shall undertake this and also a detailed analysis for quantifying the precision that can be achieved by combining different data sets (cosmic microwave background radiation, galaxy surveys) for a more complicated dark energy model in a future work.

In conclusion, HI observations of the post-reionization era can, in principle, determine the expansion history at a high level of precision and thereby constrain cosmological models. Neither the wide-field instrument with number of antennas $N = 500$ or any conceivable up-gradation of the existing GMRT will be in a position to carry out such observations, the observation time needed being too large. We find that an enhanced version of the wide-field instrument with 5000 antenna elements would be in a position to meaningfully constrain cosmological models. By combining different probes, we expect to achieve an unprecedented precision in the determination of cosmological parameters. This will be a step towards pinning down the precise nature of dark energy in the universe.

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