PHYSICAL REVIEW D **79,** 083532 (2009)

Nonthermal dark matter from cosmic strings

Yanou Cui[1,2] and David E. Morrissey[1]

[1]*Michigan Center for Theoretical Physics (MCTP), Physics Department, University of Michigan, Ann Arbor, Michigan 48109, USA*
[2]*Kavli Institute for Theoretical Physics, University of California, Santa Barbara, California 93106-4030, USA*
(Received 19 August 2008; published 29 April 2009)

Cosmic strings can be created in the early universe during symmetry-breaking phase transitions, such as might arise if the gauge structure of the standard model is extended by additional $U(1)$ factors at high energies. Cosmic strings presented in the early universe form a network of long horizon-length segments, as well as a population of closed string loops. The closed loops are unstable against decay, and can be a source of nonthermal particle production. In this work we compute the density of weakly-interacting massive particle dark matter formed by the decay of gauge theory cosmic string loops derived from a network of long strings in the scaling regime or under the influence of frictional forces. We find that for symmetry-breaking scales larger than 10^{10} GeV, this mechanism has the potential to account for the observed relic density of dark matter. For symmetry-breaking scales lower than this, the density of dark matter created by loop decays from a scaling string network lies below the observed value. In particular, the cosmic strings originating from a $U(1)$ gauge symmetry broken near the electroweak scale, that could lead to a massive Z' gauge boson observable at the LHC, produces a negligibly small dark matter relic density by this mechanism.

DOI: 10.1103/PhysRevD.79.083532 PACS numbers: 98.80.Cq, 95.35.+d

I. INTRODUCTION

Cosmic strings are one-dimensional topological defects [1–3]. They can be formed during symmetry-breaking phase transitions in the early universe [4], such as might arise if the gauge symmetry group of the standard model (SM) is enlarged by additional $U(1)$ factors at high energies [5–7]. Such gauge factors appear naturally in many models of physics beyond the SM. A typical cosmic string network consists of horizon-length *long strings* as well as smaller closed *string loops*. The long strings carry a net conserved topological charge and are stable, while closed string loops do not have a net charge and can decay away. In the present work we study the production of dark matter by the decays of cosmic string loops [8,9].

The evolution of cosmic string networks in the early universe has been studied extensively. It is found that the long strings in the network stretch with the Hubble expansion, as well as form closed string loops by the process of *reconnection* when they intersect themselves or each other. After an initial transient, these two processes balance out such that the string network reaches a *scaling* regime, in which the total energy density of the network makes up a small, constant fraction of the dominant matter or radiation energy density of the universe [10–15]. This string energy fraction is largely independent of the initial conditions, and is about $G\mu$ relative to the critical density, where $G = 1/(8\pi M_{\rm Pl}^2)$ is Newton's constant and μ is the string tension. In terms of the scale of spontaneous symmetry breaking η from which the strings arise, the tension is on the order of

$$\mu \simeq \eta^2. \tag{1.1}$$

Cosmic string scaling can be modified if the strings have significant interactions with the thermal background that lead to an effective frictional force on the strings. The importance of friction to the evolution of a string network depends on the pattern of symmetry breaking as well as the temperature of the surrounding plasma.

A possible decay product of the string loops formed by a scaling cosmic string network is dark matter (DM) [16]. Indirect evidence for cold dark matter has been obtained from a number of sources. Together, they indicate a dark matter density of [17]

$$\Omega_{\rm DM}h^2 = 0.1143 \pm 0.0034. \tag{1.2}$$

This dark matter density can be explained by the existence of a new weakly-interacting stable particle with a mass on the order of the electroweak scale, a weakly-interacting massive particle (WIMP) [16]. Such particles are common in extensions of the SM that stabilize the electroweak scale against quantum corrections such as supersymmetry [18], universal extra dimensions [19], and little Higgs models with T-parity [20].

String loops can produce DM and other particles in a number of ways. For local cosmic strings, corresponding to the spontaneous breakdown of a gauge symmetry, the direct emission of particles by the strings is thought to be suppressed [21].[1] Instead, loops lose most of their energy by oscillating and emitting gravitational radiation [23]. However, when the loop radius shrinks down to the order of the string width, it will self-annihilate into its constituent fields [8]. The subsequent decays of these states can pro-

[1]However, see Refs. [14,22] for arguments to the contrary.

duce dark matter. An even larger source of string annihilation and particle production by closed loops is the formation of *cusps* [24]. These are points on a loop where the string segment folds back on itself and briefly approaches the speed of light [25,26]. In the vicinity of a cusp, a small portion of the string will self-annihilate creating particles [24,27]. In many simple solutions for the motion of a string loop, a cusp is generally found to occur about once per loop oscillation period [25,26,28–31].

The amount of dark matter created by a cosmic string network depends on the initial size of the string loops that are formed by the network. While the evolution of the long horizon-length strings is well understood, the details of loop formation are less clear. These details are closely related to the spectrum of small fluctuations on long strings. Significant advances have been made recently in this direction, both in numerical simulations [32–34], as well as in analytic models [35–40]. In the present work we will mostly adopt the results of Refs. [36–39] to characterize the initial loop size spectrum.

In the picture of loop formation (in the scaling regime) that emerges from Refs. [36–39], fluctuations on long strings are created near the horizon scale $d_H \sim t$, which is also the scale that characterizes the long string network in the scaling regime. After they are formed, these fluctuations grow less quickly than the horizon, and thereby shrink relative to the characteristic scale of the long string network. The fluctuation spectrum that emerges is a power law in the fluctuation size that increases going to smaller scales. This power law is eventually cut off well below the horizon by gravitational radiation damping, which erases very small fluctuations. The cutoff occurs when the fluctuation size falls below the gravitational radiation scale [36]

$$l_{\rm GW} = \Gamma(G\mu)^{1+2\chi}t, \tag{1.3}$$

where $\Gamma \simeq 50$ is a constant, t is the cosmic time, and $\chi = 0.10$ during radiation and 0.25 during the matter era. The small fluctuation spectrum on long strings is therefore peaked near $l_{\rm GW}$. This peak implies that $l_{\rm GW}$ sets the typical initial loop size ℓ_i [36–39],

$$\ell_i \simeq l_{\rm GW}. \tag{1.4}$$

Both the number and the energy density of the loops formed are dominated by loops of this initial size.[2] However, recent simulations (and the analytic model of Ref. [40]) also point toward a significant loop population near the horizon scale, with $\ell_i \simeq (0.1)t_i$ [32–34]. We will therefore consider larger loops as well in our analysis.

[2]For smaller values of η and at very early times, the gravitational damping length $l_{\rm GW}$ can fall below the width of the string, $w \sim \eta^{-1}$. In this case, we expect that small fluctuations are cut off at $\ell \sim w > l_{\rm GW}$ by direct particle emission as suggested in Ref. [14,22].

The primary goal of this paper is to compute the dark matter density generated by a network of gauge theory cosmic strings in the scaling regime. We also calculate the dark matter generated when *frictional* interactions of the strings with the background plasma modify the evolution of the network. A necessary condition for the phenomenological viability of a cosmic string network is that the dark matter density it generates not exceed the observational bound. Our results therefore provide a constraint on theories of new physics that lead to the formation of cosmic strings, such as models containing new $U(1)$ gauge groups that arise frequently in superstring theory constructions and in models of grand unification [5–7]. Our results also motivate certain classes of gauge extensions of the standard model that are able to generate the observed dark matter relic density by cosmic string loop decay. While we concentrate on the dark matter produced by a network of cosmic strings, our methods can also be applied to computing the densities of other cosmologically interesting particles arising from cosmic string loop decays, such as moduli and gravitinos.

We find that for symmetry-breaking scales η larger than 10^{10} GeV, the decays of cosmic string loops derived from a network of long cosmic strings that are scaling or dominated by friction can potentially generate (more than) enough cold dark matter to account for the observed relic density. The precise density depends on the typical initial loop size and the effective branching fraction of the loop decays into cold dark matter, in addition to η. Note that $\eta = 10^{10}$ GeV corresponds to $G\mu \simeq 10^{-18}$, for which the standard cosmic string signatures such as gravitational radiation and gravitational lensing are expected to be unobservably weak [2,3]. When η is much smaller than 10^{10} GeV, including those scales for which the massive Z' gauge boson associated with the symmetry breaking could be visible at the LHC, the density of nonthermal dark matter produced by this mechanism is well below the observed value. Thus, such models are not constrained by dark matter overproduction from scaling string loops.

This paper is organized as follows. In Sec. II we derive a general formula for the production of dark matter by cosmic string loops. In Sec. III, we apply this formula to compute the amount of dark matter produced by string loops in the scaling regime. In Sec. IV we extend our results to compute the dark matter density from cosmic string loops when friction plays an important role in the evolution of the string network. Finally, Sec. V is reserved for our conclusions.

Before proceeding, let us point out that the production of dark matter by the decays of cosmic string loops was considered previously for *ordinary* Abelian Higgs model cosmic strings in Ref. [8], for cosmic strings derived from a supersymmetric flat direction in Ref. [41], and for cosmic strings associated with aspects of superstring theory in Refs. [9,42,43]. We concentrate on ordinary cosmic strings in the present work, and we update and extend the results of Ref. [8] in a number of ways. Most importantly, we focus on cusping as the primary source of particle production by cosmic strings. We also make use of the recent results on the size distribution of string loops when they are formed from Refs. [36–39], and we consider the evolution of cosmic string networks both with and without friction. When friction is relevant, we extend Ref. [8] by using the analytic model of Ref. [13] to describe the long string network.

II. A FORMULA FOR DARK MATTER FROM COSMIC STRINGS

Let us denote the initial *invariant* length of a string loop formed at cosmic time t_i by ℓ_i. The invariant length ℓ of a loop is defined in relation to its energy in the cosmological frame by

$$E_{\rm loop} = \mu\ell. \tag{2.1}$$

If the loop is boosted with speed v, the invariant length ℓ will exceed the proper length of the loop in its rest frame by a factor of $\gamma = 1/\sqrt{1-v^2}$.

The key quantity characterizing loop formation is

$$r(\ell_i, t_i)d\ell_i dt_i, \tag{2.2}$$

the number density of string loops formed in the time interval $(t_i, t_i + dt_i)$ with initial length in the range $(\ell_i, \ell_i + d\ell_i)$. The form of the function $r(\ell_i, t_i)$ is constrained by the evolution of the long string network. In particular, the total rate at which energy is transferred from the long string network to loops is typically a known quantity [13]. We can use this fact to impose the constraint

$$\frac{d\rho_{\rm loop}}{dt_i} = \int d\ell_i \mu \ell_i r(\ell_i, t_i), \tag{2.3}$$

where $d\rho_{\rm loop}/dt$ is the rate of energy transfer into loops from the long string network.

Consider the evolution of a loop of initial size ℓ_i formed at time t_i. The length of this loop is described by the function

$$\ell(t; \ell_i, t_i), \tag{2.4}$$

which is the solution to the equation

$$\mu\frac{d\ell}{dt} = -P_{\rm tot}, \quad \text{with} \quad \ell(t_i; \ell_i, t_i) = \ell_i, \tag{2.5}$$

where $P_{\rm tot}$ is the total rate of energy loss from the loop. We will assume that $P_{\rm tot}$ is positive so that ℓ is monotonically decreasing in time. Under this assumption, the loop will eventually decay away completely at the time $t_{\rm co}(\ell_i, t_i)$, defined implicitly by

$$0 = \ell(t_{\rm co}; \ell_i, t_i). \tag{2.6}$$

While the loop is decaying, it will emit a fraction of its energy in the form of (cold) dark matter. We will write this fraction as $P_{\rm DM} \leq P_{\rm tot}$.

To find the total rate of dark matter production by the collection of string loops, consider loops with initial size in the range $(\ell_i, \ell_i + d\ell_i)$ formed in the time interval $(t_i, t_i + dt_i)$. There are $r(\ell_i, t_i)d\ell_i dt_i$ such loops per unit volume initially. As time goes on, this collection of loops is diluted by the cosmic expansion by a factor of a^{-3}. This is the *only* modification of their density until they decay away completely at time $t_{\rm co}(\ell_i, t_i)$. At time $\tilde{t}$, this collection of loops will produce dark matter at the rate $P_{\rm DM}$. The dark matter will thermalize if it is produced before the freeze-out time $\tilde{t} < t_{\rm fo}$, and redshift as a^{-3} after this time. Thus, we have

$$\Delta\rho_{\rm DM} = \int d\ell_i \int_{t_\eta}^{t_0} dt_i \int_{t_{\rm fo}}^{\bar{t}_{\rm co}(\ell_i, t_i)} d\tilde{t} r(\ell_i, t_i) \times \left[\frac{a(t_i)}{a(\tilde{t})}\right]^3 P_{\rm DM}\left[\frac{a(\tilde{t})}{a(t_0)}\right]^3 \Theta(t_{\rm co} - t_{\rm fo}). \tag{2.7}$$

The integral over t_i runs between the string network formation time t_η and the present time $t_0 \simeq 4 \times 10^{41}\ \mathrm{GeV}^{-1}$. Typically, t_η is on the order of

$$t_\eta \simeq \frac{M_{\rm Pl}}{\eta^2}, \tag{2.8}$$

where η is the scale of spontaneous symmetry breaking. The integration over the decay time $\tilde{t}$ runs from the freeze-out time $t_{\rm fo}$ to either $t_{\rm co}$ or t_0, whichever is smaller. Thus, we have defined $\bar{t}_{\rm co}(\ell_i, t_i)$ by

$$\bar{t}_{\rm co}(\ell_i, t_i) = \begin{cases} t_{\rm co}(\ell_i, t_i) & : t_{\rm co} < t_0 \\ t_0 & : t_{\rm co} \geq t_0 \end{cases}. \tag{2.9}$$

Any restriction on the integration limits of ℓ_i is encoded in the support of the function $r(\ell_i, t_i)$.

It is often the case that $P_{\rm DM}$ and $P_{\rm tot}$ depend on $\tilde{t}$ only through $\tilde{\ell} = \ell(\tilde{t}; \ell_i, t_i)$. If so, it is convenient to change the integration variable from $\tilde{t}$ to $\tilde{\ell}$. The corresponding Jacobian is simply $(\partial\tilde{\ell}/\partial\tilde{t})^{-1} = -\mu/P_{\rm tot}$, where $P_{\rm tot}$ is the total power released by a loop of length $\tilde{\ell}$. It follows that

$$\Delta\rho_{\rm DM} = \int d\ell_i \int_{t_\eta}^{t_0} dt_i \int_{\ell_x(\ell_i, t_i)}^{\bar{\ell}_{\rm fo}(\ell_i, t_i)} d\tilde{\ell} r(\ell_i, t_i) \times \left[\frac{a(t_i)}{a(t_0)}\right]^3 \mu \frac{P_{\rm DM}}{P_{\rm tot}}. \tag{2.10}$$

The limits on the $\tilde{\ell}$ integration depend on the functions

$$\bar{\ell}_{\rm fo}(\ell_i, t_i) = \begin{cases} \ell(t_{\rm fo}; \ell_i, t_i) & : t_i < t_{\rm fo} \\ \ell_i & : t_i > t_{\rm fo} \end{cases}, \tag{2.11}$$

as well as

$$\ell_x(\ell_i, t_i) = \begin{cases} 0 & : t_{\rm co}(\ell_i, t_i) < t_0 \\ \ell(t_0; \ell_i, t_i) & : t_{\rm co}(\ell_i, t_i) \geq t_0 \end{cases}. \tag{2.12}$$

Equations (2.7) and (2.10) are our main results. We will apply them to two interesting special cases below.

III. DARK MATTER PRODUCTION FROM A SCALING NETWORK

As a first application of our main result, Eq. (2.7), we compute the dark matter density produced by a network of cosmic strings in the scaling regime due to the cusping of closed string loops. To simplify the analysis, we focus on monochromatic loop formation distributions with

$$\ell_i = \alpha t_i. \tag{3.1}$$

This relation implies $r(\ell_i, t_i) \propto \delta(\ell_i - \alpha t_i)$. More general distributions can be obtained by introducing a weight function and summing the final result over different values of α.

In the string scaling regime, the rate at which the long string network transfers energy into loops is [13]

$$\frac{d\rho_{\rm loop}}{dt_i} = \zeta \mu t_i^{-3}, \tag{3.2}$$

with $\zeta \simeq 10$ a constant characterizing the mean properties of the long string network. Imposing the constraint of Eq. (2.3) on the loop formation rate $r(\ell_i, t_i)$, we obtain

$$r(\ell_i, t_i) = \frac{\zeta}{\alpha} t_i^{-4} \delta(\ell_i - \alpha t_i). \tag{3.3}$$

All that remains to do is to specify the evolution of the loop length and the power emitted as dark matter, and apply Eq. (2.7).

Cosmic string loops lose energy to gravitational radiation as well as cusping, and shrink as a result. The rate of energy emission into gravity waves is [3]

$$P_{\rm grav} = \Gamma G \mu^2, \tag{3.4}$$

where μ is the string tension, $G = 1/(8\pi M_{\rm Pl}^2)$, and $\Gamma \simeq 50$ is a dimensionless constant [28].

Loops also lose energy when they form cusps, which are points on the loops that briefly fold back upon themselves and approach the speed of light. Near the apex of a cusp, a portion of the string overlaps itself and self-annihilates. Summing over many cusps, the net rate at which a loop loses energy to cusping is given by [27]

$$P_{\rm cusp} = \mu p_c \sqrt{\frac{w}{\ell}}, \tag{3.5}$$

where $w \sim \eta^{-1}$ is the string width, and p_c is the probability for a cusp to form per period of loop oscillation. Several studies suggest $p_c \simeq 1$ [26,28–31]. Comparing the cusp power of Eq. (3.5) to the gravitational wave power of Eq. (3.4), we see that cusping is the dominant energy-loss mechanism by loops when they are shorter than $\ell < \ell_=$ with

$$\ell_= = w\left(\frac{p_c}{\Gamma G \mu}\right)^2. \tag{3.6}$$

The particles produced by cusping can decay into dark matter. Provided $p_c \gtrsim \Gamma G \mu$, we expect cusping to be the dominant source of particle production and dark matter from cosmic strings.

If loops lose energy to gravitational radiation and cusping, the loop length evolves according to

$$\mu \frac{d\ell}{dt} = -P_{\rm tot} = -\Gamma G \mu^2 - \mu p_c \sqrt{\frac{w}{\ell}}. \tag{3.7}$$

The solution of this equation subject to the initial condition $\ell(t_i) = \ell_i$ is given implicitly by

$$t - t_i = \frac{\ell_=}{\Gamma G \mu}\Bigg[\left(\frac{\ell_i - \ell}{\ell_=}\right) - 2\left(\sqrt{\frac{\ell_i}{\ell_=}} - \sqrt{\frac{\ell_i}{\ell_=}}\right) + 2\ln\left(\frac{1 + \sqrt{\ell_i/\ell_=}}{1 + \sqrt{\ell/\ell_=}}\right)\Bigg]. \tag{3.8}$$

Suppose that a fraction $\epsilon_{\rm cusp}$ of the energy emitted by a string loop cusp (eventually) takes the form of cold dark matter. It follows that

$$P_{\rm DM} = \epsilon_{\rm cusp} P_{\rm cusp}. \tag{3.9}$$

Putting this into Eq. (2.10) and making use of Eq. (3.3), we find the dark matter density due to cusping to be

$$\begin{aligned}\Delta\rho_{\rm DM} &= \int_0^\infty d\ell_i \int_{t_\eta}^{t_0} dt_i \int_{\ell_x(\ell_i,t_i)}^{\ell_{\rm fo}(\ell_i,t_i)} d\tilde{\ell}\, r(\ell_i, t_i) \times \left(\frac{a_i}{a_0}\right)^3 \mu \frac{\epsilon_{\rm cusp} P_{\rm cusp}(\tilde{\ell})}{P_{\rm tot}(\tilde{\ell})} \\ &= \int_{t_\eta}^{t_0} dt_i \frac{\zeta}{\alpha} t_i^{-4} \left(\frac{a_i}{a_0}\right)^3 2\epsilon_{\rm cusp}\mu \ell_= \Bigg[\sqrt{\frac{\ell}{\ell_=}} - \ln(1 + \sqrt{\ell/\ell_=})\Bigg]_{\ell_x(\alpha t_i,t_i)}^{\bar{\ell}_{fo}(\alpha t_i,t_i)} \Theta(\bar{\ell}_{\rm fo})\Theta(\ell_x).\end{aligned} \tag{3.10}$$

The Θ functions in this expression account for the fact that only those loops that decay after $t_{\rm fo}$ and before t_0 can contribute to the dark matter density. Numerically, we find that the integrand of Eq. (3.10) is a steeply falling function of t_i that is cut off at small t_i by the $\Theta(\bar{\ell}_{\rm fo})$ condition. It follows that the dominant contribution to the dark matter density comes from loops formed at the smallest possible value of t_i such that they decay shortly after $t_{\rm fo}$. In this sense, the dark matter density is created nearly instantaneously at $t_{\rm fo}$.

A. Dark matter in the scaling regime

To be concrete, we evaluate Eq. (3.10) assuming a weakly-interacting massive dark matter particle with a freeze-out time of $t_{\rm fo} = 2 \times 10^{16}\ {\rm GeV}^{-1}$. This corresponds approximately to a freeze-out temperature of

5 GeV, which is in the range expected for a stable, weakly-interacting particle with a mass on the order of 100 GeV. We take the universe to be radiation dominated prior to $t_{\rm fo}$, and we assume a standard cosmological evolution afterward. We also set the loop network parameter to $\zeta = 10$, the cusping probability to $p_c = 1$, and the branching fraction into dark matter equal to unity, $\epsilon_{\rm cusp} = 1$. In realistic models $\epsilon_{\rm cusp}$ can be much smaller than unity. Our results can therefore be interpreted as providing an upper bound on $\epsilon_{\rm cusp}$ within the underlying gauge theory model.

In Fig. 1 we show the dark matter relic density due to cusping as a function of the initial loop size parameter α, normalized to $\Gamma G\mu$, with the other parameter values as described above. We normalize α to $\Gamma G\mu$ because, in the absence of cusping, a loop is long-lived relative to the Hubble time at formation ($\sim t_i$) provided $\alpha/\Gamma G\mu > 1$, and short-lived otherwise. From this plot, we see that increasing the initial loop size α well above $\Gamma G\mu$ increases the final dark matter density. The reason for this is that the integration over t_i in Eq. (3.10) is dominated by the earliest times for which loops produced at t_i decay after $t_{\rm fo}$. This is due primarily to the rapid falloff of the scaling loop density with t_i, as can be seen in Eq. (3.3). Since larger loops are longer-lived, they can be formed much earlier, when the loop density is higher, and still decay after $t_{\rm fo}$ and contribute to the DM relic density. For $\eta \gtrsim 3 \times 10^{13}$ GeV we find that gravitational radiation dominates the evolution of the large loops generating the dark matter density, while for $\eta \lesssim 3 \times 10^{13}$ GeV cusping is always dominant.

As $\alpha/\Gamma G\mu$ decreases, the dark matter density curves flatten out for lower values of the vacuum expectation value (VEV) η. In these flat regions, cusping dominates the evolution of the loops contributing to the dark matter, and these loops are short-lived even though $\alpha/\Gamma G\mu > 1$. More generally, the evolution of loops relevant to dark matter is dominated by cusping whenever $\eta \lesssim 3 \times 10^{13}$ GeV. When the VEV is larger, $\eta \gtrsim 3 \times 10^{13}$ GeV, gravitational radiation can become more important than cusping. Loops then become short-lived only when $\alpha/\Gamma G\mu < 1$, and the resulting density of DM is smaller since most of the loop energy is emitted in the form of gravity waves. When α becomes very small, cusping again takes over and the curves flatten out once more. Note that this flattening also suggests that our results are applicable if particles are produced directly by long cosmic strings, as proposed in Refs. [14,22]

In Fig. 2 we show the dependence of the dark matter density from string cusping as a function of the symmetry-breaking VEV η. The other string model parameters are as described above. This plot shows a general increase in the dark matter density up to large values of η, and then a falloff. Short-lived loops, with $\alpha \leq \Gamma G\mu$, and smaller values of η decay mostly through cusping at time $t_{\rm fo}$. Using Eq. (2.10), it is possible to show that $\Omega_{\rm DM}$ increases as η^2 in this case. As η is increased further, gravitational radiation becomes more important than cusping, and loops lose most of their energy to gravity waves instead of dark matter. Thus, the dark matter density falls for these very large values of η, with $\Omega_{\rm DM} \propto (\alpha\eta)^{-1/2}$ in this regime. The precise value of η at which the crossover occurs depends on the value of α, with smaller initial loops being more prone to cusping. For very large initial loops, $\alpha = 0.1$, a similar crossover occurs close to $\eta \simeq 3 \times 10^{13}$ GeV. When η is smaller than this transition value, the evolution of loops relevant to DM is dominated by cusping. For η above this critical value, the evolution of these large loops is controlled by gravitational radiation and the fraction of DM generated through cusping is reduced.

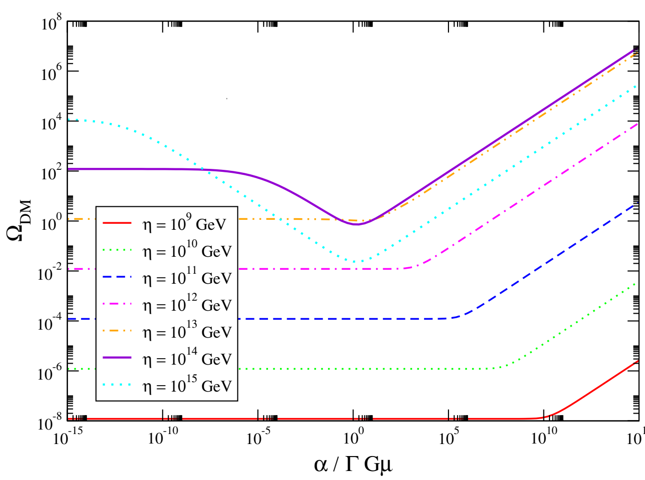

FIG. 1 (color online). Dark matter density due to loop cusping for $\epsilon_{\rm cusp} = 1$, $p_c = 1$, $\zeta = 10$, and $t_{\rm fo} = 2 \times 10^{16}$ GeV^{-1} as a function of the initial loop size parameter α. The various lines correspond to different values of the symmetry-breaking VEV η.

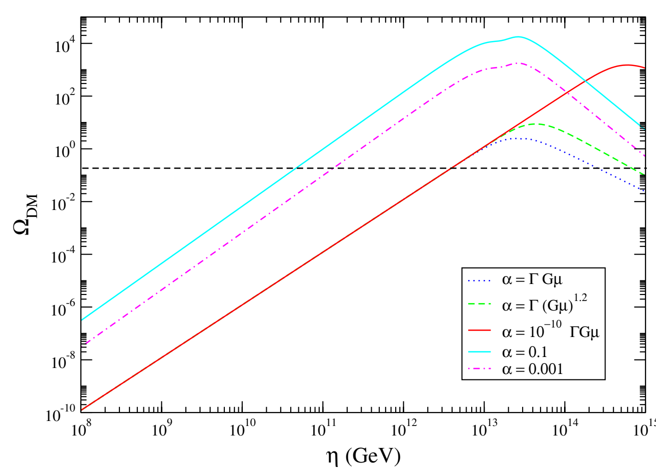

FIG. 2 (color online). Dark matter density due to loop cusping for $\epsilon_{\rm cusp} = 1$, $\zeta = 10$, $p_c = 1$, and $t_{\rm fo} = 2 \times 10^{16}$ GeV^{-1} as a function of the symmetry-breaking VEV η. The various curves correspond to different values of the initial loop size parameter α. The black dashed horizontal line denotes the observed dark matter relic density.

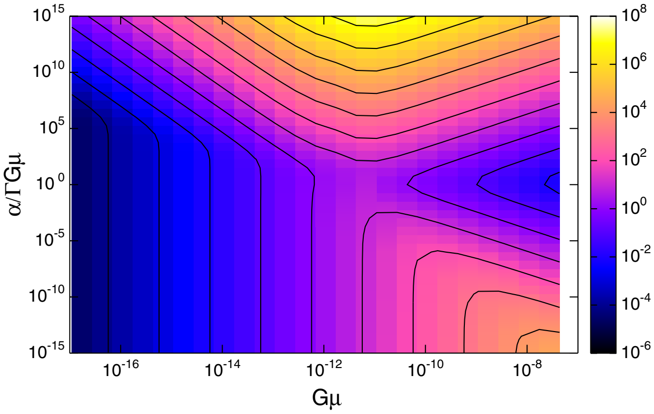

FIG. 3 (color online). Contours of the dark matter relic density $\Omega_{\rm DM}$ from cosmic string decays as a function of the normalized string tension $G\mu$ and the typical initial loop length $\alpha/\Gamma G\mu$. As above, we have taken $\epsilon_{\rm cusp} = 1$, $p_c = 1$, $\zeta = 10$, and $t_{\rm fo} = 2 \times 10^{16}\ {\rm GeV}^{-1}$.

To compare our results for dark matter to other cosmological constraints on cosmic strings, we show in Fig. 3 contours of the dark matter relic density from string loop decays as a function of the initial loop size $\alpha/\Gamma G\mu$ and the more commonly used string tension $G\mu \simeq \eta^2/M_{\rm Pl}^2$ variable.[3] Other string and network parameters are taken as above. This figure also summarizes the content of Figs. 1 and 2. In particular, we see the crossover from cusp domination to gravitational radiation domination of loop decays that occurs for large initial loop sizes when $\eta \simeq 3 \times 10^{13}$ GeV, corresponding to $G\mu \sim 10^{-11}$. For larger values of η, very small loops continue to decay primarily through cusping, but gravitational radiation dominates otherwise.

By way of comparison to other bounds on cosmic strings, cosmic microwave background data constrains the tension of cosmic strings to $G\mu \lesssim 2 \times 10^{-7}$, corresponding to $\eta \lesssim 5 \times 10^{15}$ GeV due to their effect on the temperature power spectrum [46–48] and B-mode polarization [49,50]. Proposed direct searches using pattern recognition methods on cosmic microwave background maps may further improve this bound by 2 orders of magnitude [51–53]. Cosmic strings are also constrained by searches for gravitational radiation [44]. For large loops, $\alpha \sim 0.1$, pulsar timing bounds limit $G\mu \lesssim 10^{-9}$ [45,54], although these bounds weaken considerably if only a small fraction of initial loops are large [37]. Our results for dark matter from cosmic strings shown in Fig. 3 are frequently more constraining than these other cosmological bounds when $\epsilon_{\rm cusp} = 1$.

B. Potential additional effects

In the discussion above we have computed the density of dark matter produced by decaying string loops derived from a scaling cosmic string network. Several additional effects could potentially modify the ultimate relic density beyond what we have included in our analysis above. These include deviations of the loop cusping probability away from unity, contributions from loops formed before the network has attained scaling, and the possibility of boosted loops and cusp decay products. We discuss here how these effects could modify our previous results.

In general, we expect the loop cusping probability p_c to be on the order of unity.[4] However, it is also interesting to look at smaller values of p_c, such as might arise on large string loops with many kinks [30,31]. Figure 4 shows the effect of reducing p_c for small ($\alpha = \Gamma G\mu$) and large ($\alpha = 0.1$) initial loop sizes, and a range of values of the symmetry-breaking VEV η. All other parameters are as in the previous plots. For $\alpha = \Gamma G\mu$ the loops are necessarily short-lived. In this case, cusping is the dominant energy-loss mechanism at time $t_{\rm fo}$ for $\eta \lesssim 10^{12}$ GeV, and remains so even for lower values of p_c. Thus, reducing p_c does not alter the resulting dark matter density. When η is larger, gravitational radiation dominates the energy loss at time $t_{\rm fo}$, and reducing p_c therefore decreases the fraction of loop energy released as dark matter. With $\alpha = 0.1$, string loops can be long-lived. For smaller values of $\eta \lesssim 10^{12}$ GeV, lowering p_c can enhance the dark matter density. This occurs because cusping is the dominant energy-loss mechanism around time $t_{\rm fo}$ for these loops. Reducing p_c thus further increases the lifetime of these long-lived loops which, as discussed above, enhances the final dark matter density. For larger values of η, gravitational radiation dominates at time $t_{\rm fo}$, so that reducing p_c only decreases the fraction of energy released as dark matter without significantly altering the loop lifetime.

The dark matter density computed above includes only the contributions to the relic density from loops in the scaling regime. Loops created in the string-forming phase transition at time t_η and while the string network was evolving towards scaling will give an additional contribution to the dark matter density if they decay after $t_{\rm fo}$. The approach of a string network to scaling depends on the details of the phase transition, and a study of this process is beyond the scope of the present work. However, we can derive the condition for loops formed in the phase transition to decay before $t_{\rm fo}$. We obtain

$$\eta \gtrsim p_c^{-2/5}\left(\frac{\alpha}{0.1}\right)^{3/5}(7 \times 10^3\ {\rm GeV}). \qquad (3.11)$$

If this bound is satisfied and scaling is attained quickly

[3]The quantity $\alpha/\Gamma G\mu$ coincides with the variable $\epsilon = \alpha/G\mu$ used to characterize the initial size of cosmic string loops in Refs. [44]. See, for example, Fig. (2) of Ref. [45] for a comparison with gravitational wave bounds.

[4]The backreaction from string annihilation at a cusp can suppress its reoccurrence, but this need not prevent the formation of new cusps [27].

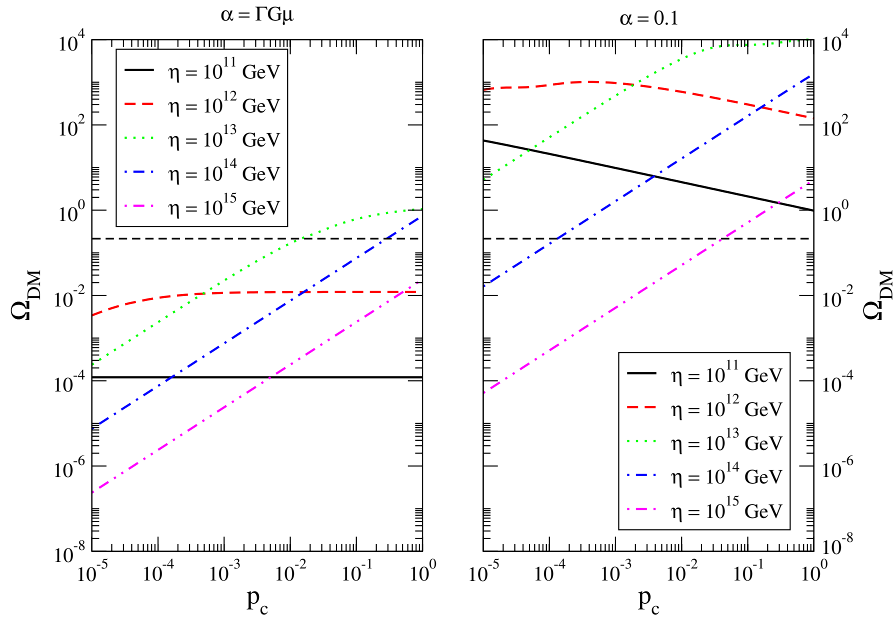

FIG. 4 (color online). Dark matter density due to loop cusping for $\epsilon_{\mathrm{cusp}} = 1$, $\zeta = 10$, and $t_{\mathrm{fo}} = 2 \times 10^{16}\ \mathrm{GeV}^{-1}$ as a function of the cusp formation probability p_c for several values of the symmetry-breaking scale η. The panel on the left corresponds to small initial loops with $\alpha = \Gamma G\mu$, while the panel on the right corresponds to large initial loops with $\alpha = 0.1$. The black dashed horizontal line indicates the observed dark matter relic density.

after the network is formed at t_η, loops formed in the scaling regime will give the dominant contribution to the dark matter density. This bound also indicates that our results are applicable to the phenomenologically interesting case of a $U(1)$ gauge symmetry broken only slightly above the electroweak scale provided $p_c \sim 1$ and $\alpha \ll 0.1$.

In our calculation of the dark matter relic density above, we implicitly treated the loops formed by a scaling string network as being nonrelativistic. However, the analysis of Refs. [36–39] indicate that small loops ($\alpha < \Gamma G\mu$) formed in the radiation era are boosted by an amount

$$\gamma \simeq \alpha^{-0.1}, \tag{3.12}$$

where $\gamma = 1/\sqrt{1 - v^2}$ is the boost factor relative to the cosmological background. This boost does not alter the cusping power, although the expression for this power in Eq. (3.5) picks up a factor of $\gamma^{1/2}$ (for short-lived loops). Since this boost factor is strongest for small loops that are short-lived relative to the Hubble time at formation, its most important effect is that the loop decay products can be boosted as well.

The loop decay products from a string cusp were treated as being nonrelativistic in the analysis above. Beyond the boost induced by the parent loops being relativistic, the decay products from a loop cusp can be very highly boosted due to the dynamics of the cusp itself, even if the loop is initially at rest. The size of this boost γ was estimated in Ref. [27] to be

$$\gamma = \sqrt{\frac{\ell}{w}}. \tag{3.13}$$

For the loops relevant to dark matter, this boost factor is generally much larger than the boost factor from loop formation, and we will therefore concentrate on it alone. On account of this boost, the cusp decay products including the dark matter might not thermalize immediately upon production. If not, they may thermalize at a later time, or even remain as hot dark matter.[5]

To estimate the effect of this possibly very large boost, we adopt a simple picture of the cusp decay products as consisting of a *bunch* of light states (including WIMP dark matter) moving together with a boost γ relative to the thermal background. In the rest frame of the bunch we assume very little relative motion, and that a fraction ϵ_1 of the total bunch energy consists of dark matter particles of mass $m \simeq 100$ GeV. The dark matter in the bunch will thermalize when $\sigma n_{\mathrm{tot}} \gtrsim H$, where $n_{\mathrm{tot}} \sim T^3$ is the total plasma number density of light states and σ is the cross section for the dark matter to scatter off the thermal background. To estimate the thermalization time, we take this cross section to be

$$\sigma = \frac{1}{8\pi}\frac{1}{s} = \frac{1}{8\pi}\frac{1}{\gamma^2 m^2}. \tag{3.14}$$

With this cross section, the dark matter decay products from a cusping loop with $\alpha = \Gamma G\mu$ or smaller formed at time t_{fo} will reach kinetic equilibrium nearly instantane-

[5]Let us also point out that our previous results can also be applied directly to the case of long strings fragmenting directly into particles, as suggested in Refs. [14,22]. In this situation, the estimated boost factors given above are likely not applicable.

ously after the cusp occurs when the symmetry-breaking scale lies below $\eta \lesssim 10^{12}$ GeV. In this case, our previous estimates for the dark matter density remain valid up to a reduction in the effective value of $\epsilon_{\rm cusp}$.

If the cusp decay products do not thermalize instantaneously, they may still thermalize at a later time t_T as their momentum redshifts. To compute the final density of cold dark matter in this case, it is only necessary to add a factor of $a(\tilde{t})/a(t_T)$ to the integrand of Eq. (2.7) and modify the limits of integration such that $t_T > t_{\rm fo}$ rather than $\tilde{t} > t_{\rm fo}$. Doing so, we find that the resulting dark matter density coincides with what we found previously (without the boost) up to an overall factor of order unity even when instantaneous thermalization does not occur. The additional redshift $a(\tilde{t})/a(t_T)$ is cancelled nearly exactly by the delay in thermalization, which allows loops contributing to the dark matter density to be formed at earlier times when the rate of loop production was larger. Similar to before, we also find that the dominant contribution to the cold dark matter density comes from loop decay products that thermalize shortly after $t_{\rm fo}$. Thus, even when the cusp decay products do not thermalize immediately, our previous estimates still provide a strong upper bound on the cosmic string-induced dark matter density.

The prospect of highly boosted decay products also raises several new issues. Since a large fraction of the energy of the cusp typically goes into the kinetic energy of the decay products, the fraction of this energy that ends up as *cold* dark matter can be suppressed. If kinetic thermalization proceeds exclusively through elastic collisions, we would have $\epsilon_{\rm cusp} = \epsilon_1/\gamma(t_T)$, where $\gamma(t_T)$ is the boost factor at the time of thermalization. The actual value of $\epsilon_{\rm cusp}$ can be larger than this if inelastic collisions during thermalization generate additional dark matter particles as secondaries. In general we expect a significant suppression of $\epsilon_{\rm cusp}$ relative to the nonboosted case.

Highly boosted loop decay products can also generate relics that have not thermalized by the present time (or the time of matter-radiation equality). In general, the resulting densities of hot dark matter are on the order of $G\mu$ and are safely small for lower values of the symmetry-breaking scale. However, a reliable precise estimate is complicated by the fact that for extremely large boosts, the cusp decay products will naively have super-Planckian momenta. A full analysis of this issue is beyond the scope of the present work.

In summary, our results indicate that the density of WIMP cold dark matter generated by string loops from cusping with $p_c = 1$ in the scaling regime is safely below the observed value for $\eta \lesssim 10^{10}$ GeV ($G\mu \simeq 10^{-18}$). For values of the symmetry-breaking scale larger than this, the string-induced dark matter density depends on the initial loop size. Loops of initial size $\alpha = \Gamma(G\mu)^{1.2}$, motivated by the results of Refs. [36–39], can create as much or more than the observed dark matter density for $\eta \gtrsim 5 \times 10^{12}$ GeV ($G\mu \simeq 10^{-13}$) if the branching fraction $\epsilon_{\rm cusp}$ is on the order of unity. Larger loops, with $\alpha \sim 0.1$, can lead to a much greater dark matter density. If such large loops are typical, the branching fraction into dark matter $\epsilon_{\rm cusp}$ must be significantly less than unity if the cosmic strings are to avoid overproducing dark matter. This is natural for large loops, whose cusps generate very highly boosted decay products, leading to much of the cusp energy being lost to kinetic energy that is transferred to the thermal bath rather than cold dark matter. Let us also emphasize that for models with a new $U(1)$ gauge symmetry broken only slightly above the electroweak scale, the cold dark matter produced by the corresponding cosmic string network in the scaling regime is negligibly small for both large and small initial loop sizes.

IV. DARK MATTER PRODUCTION WITH FRICTION

The evolution of a cosmic string network can be modified if the strings have significant interactions with the thermal background. As we will discuss below, the relevance of these interactions to the string network depends on the details of the symmetry breaking from which the cosmic strings arise. Such interactions, when present, tend to slow the motion of the strings by creating an effective frictional force on them [13,55,56]. This in turn changes the density and rate of growth of the network. The frictional forces on strings decrease as the universe cools, and eventually become unimportant relevant to the Hubble damping from the expansion of spacetime. Frictional effects also change the way cosmic string loops form and decay, and can enhance the total rate of loop formation. In this section we apply the result of Eq. (2.7) to compute the density of dark matter created by a cosmic string network evolving under the influence of frictional forces.

For local (gauge) cosmic strings, the dominant interaction between the strings and the thermal background comes from Aharonov-Bohm scattering [57]. This results from the phase change experienced by the charged particle as it is transported around the string. The effective frictional force induced by this scattering can be characterized by a *friction length* ℓ_f. Friction becomes unimportant when $\ell_f(t)$ grows larger than the Hubble length. The friction length due to Aharonov-Bohm scattering is given by [13,55]

$$\ell_f = \frac{\mu}{\beta T^3}, \tag{4.1}$$

where the dimensionless quantity β is [55]

$$\beta = \frac{2\zeta(3)}{\pi^2} \sum_a b_a \sin^2(\pi\nu_a), \tag{4.2}$$

with the sum running over relativistic degrees of freedom, and $b_a = 1(3/4)$ for bosons (fermions). The value of ν_a is related to the charge Q_a of the light particle species a. If the underlying $U(1)$ gauge symmetry (subgroup) is broken by the condensation of a field with charge Q, it is equal to Q_a/Q. Therefore we expect β to be of order unity when some of the ν_a are noninteger, and zero otherwise. When Aharonov-Bohm scattering vanishes, *Everett scattering* of charged particles off the strings will be the dominant source of frictional interactions [58]. The corresponding friction length is similar to that for (nontrivial) Aharonov-Bohm scattering but is enhanced by a factor of $\ln^2(T/\eta)$. Frictional interactions will be completely irrelevant when all the light states in the theory are uncharged under the broken gauge group. In the present section, we will assume that Aharonov-Bohm scattering is the dominant source of frictional interactions with $\beta = 1$.

Frictional effects on the long string network decouple when the friction length grows larger than the Hubble length. This occurs at the time t_* defined by the relation

$$H(t_*) = 1/\ell_f(t_*). \tag{4.3}$$

Friction is only relevant for the long string network when $t < t_*$. For $\beta = \mathcal{O}(1)$ and radiation domination, t_* has the parametric size[6]

$$t_* \simeq \frac{M_{\rm Pl}^3}{\eta^4}, \tag{4.4}$$

where η denotes the symmetry-breaking VEV. This is parametrically larger than the typical (radiation-era) formation time $t_\eta \sim M_{\rm Pl}/\eta^2$.

The evolution of a cosmic string network in the presence of friction was studied in Ref. [13]. In their analytic model, the long string network is characterized by an effective correlation length L and a mean velocity v. The energy density of the network is given in terms of these variables as

$$\rho_\infty = \frac{\mu}{L^2}. \tag{4.5}$$

If the initial string density is larger than the scaling density, as would be expected if the symmetry-breaking phase transition is second-order or weakly first-order, the string network evolves very quickly to the *Kibble regime* [2,13]. In this regime, with the universe assumed to be radiation dominated, the string network variables L and v have the parametric dependences [13]

$$L(t) \simeq \left(\frac{t}{t_*}\right)^{1/4} t, \tag{4.6}$$

$$v(t) \simeq \left(\frac{t}{t_*}\right)^{1/4}. \tag{4.7}$$

[6]Because of the many uncertainties involved in the description of cosmic strings within the friction-dominated regime, we only list and use here the leading parametric dependences of the string network parameters.

The Kibble regime only lasts while $t < t_*$. From Eqs. (4.1) and (4.4) we see that the friction length grows as

$$\ell_f \simeq \left(\frac{t}{t_*}\right)^{1/2} t. \tag{4.8}$$

As t approaches t_*, the friction length catches up to the long string length L as well as the horizon, and the string network transitions into the usual scaling regime with $L \propto t$ and $v \sim 1$.

During the friction-dominated Kibble regime, the rate at which energy is transferred from the long string network to loops is on the order of

$$\frac{d\rho_{\rm loop}}{dt_i} \simeq \mu\left(\frac{t_*}{t_i}\right)^{1/2} t_i^{-3}, \qquad t_i < t_*. \tag{4.9}$$

This is parametrically larger than during the scaling regime, as can be seen by comparing Eq. (4.9) with Eq. (3.2). Once a loop is formed, its subsequent evolution in the Kibble regime is also considerably different than during scaling. In particular, the typical initial loop size as well as the subsequent loop evolution are both strongly modified. Both of these effects can modify the resulting dark matter density.

A. Loop production and evolution with friction

The evolution of linear perturbations on long strings and closed string loops in the presence of friction was studied in Refs. [55,56]. In Ref. [56] it was found that linear fluctuations on a long string of wavelength larger than ℓ_f are overdamped and stretched. For wavelengths much smaller than ℓ_f, the damping time is on the order of ℓ_f, which is much longer than the typical period of oscillation but much shorter than the Hubble time. Hence these small fluctuations oscillate and lose energy to friction very quickly relative to the Hubble time. Given these results and the picture of loop formation of Ref. [39], we expect that the typical initial loop size during friction is on the order of ℓ_f.[7] Fluctuations smaller than ℓ_f are damped out quickly, while those larger than ℓ_f grow more slowly than the long string correlation length L, and therefore shrink relative to L. Thus, we expect that fluctuations build up near ℓ_f, which in turn sets the typical size of a loop when it is formed.

To model the evolution of string loops during friction, we will take the results of Ref. [56] for the evolution of a circular loop to be representative of the evolution of general loops. (Indeed, friction tends to make the loops more circular.) Ref. [56] finds that loops smaller than ℓ_f oscillate essentially freely and lose their energy to the thermal background according to

[7]We emphasize however that the picture of loop formation obtained in Ref. [39] was developed under the assumption of long string scaling, and did not consider the effects of friction.

$$\mu \frac{d\ell}{dt}\bigg|_{\text{friction}} \simeq -\mu \frac{\ell}{\ell_f}. \qquad (4.10)$$

It follows that such loops lose energy over the time scale ℓ_f, much less than the Hubble time in the friction regime. We do not expect these interactions between the strings and the thermal background to be a significant source of dark matter.

The motion of loops larger than ℓ_f is overdamped. They evolve according to

$$R\dot{R} \simeq -\frac{\ell_f}{2a}, \qquad (4.11)$$

where R is the comoving coordinate radius of the loop (and aR corresponds to the physical loop radius). The solution of Eq. (4.11) implies that loops of initial size smaller than the long string scale L shrink down to size ℓ_f in less than about a Hubble time. Once they do, the overdamping approximation of Eq. (4.11) breaks down, and the loops begin to oscillate and decay away.

In summary, loops are formed in the friction regime with a typical initial length close to ℓ_f. The loop formation rate per unit volume per unit length is

$$r(\ell_i, t_i) \simeq \left(\frac{t_*}{t_i}\right) t_i^{-4} \delta(\ell_i - \ell_f(t_i)), \qquad t_i < t_*. \qquad (4.12)$$

Any loop formed with initial length larger than ℓ_f will shrink down to length ℓ_f in less than about a Hubble time. Loops smaller than ℓ_f execute underdamped oscillations, transferring their energy to the thermal background and decaying away over the time scale ℓ_f. The loop length ℓ in this regime evolves according to

$$\frac{d\ell}{dt} = -\frac{\ell}{\ell_f} - \Gamma G\mu - p_c\sqrt{\frac{w}{\ell}}. \qquad (4.13)$$

Here, the first term comes from friction, the second from gravitational radiation, and the third from cusping.

Before going on to compute the dark matter density generated by the decaying loops, let us make note of the fact that at the end of the friction-dominated Kibble regime, the long string network is smoothed out nearly all the way up to the Hubble scale. In the loop formation picture of Ref. [36–39], small fluctuations on long strings giving rise to loops originating from fluctuations of Hubble size that have slowly shrunk. Thus it will take some time for a small scale structure to build up on the long strings, and the typical initial loop size will be initially larger than the gravitational damping length l_{GW}. We follow Ref. [14] and model this transitional period by writing the initial typical loop size parameter as

$$\alpha_{\text{eff}}(t) = \frac{2 + \alpha(t/t_*)^\xi}{1 + (t/t_*)^\xi}, \qquad t > t_*, \qquad (4.14)$$

with the exponent $\xi \simeq 1$. A naive application of the results of Refs. [36–39] suggests that $\xi = 0.9$ in the radiation era. We will study a range of values of ξ.

B. Dark matter with friction

The discussion above provides all the ingredients needed to evaluate the dark matter density created by a string network in the presence of friction using Eq. (2.7). We do so here under the assumption that loop cusping is the dominant source of dark matter from the strings. Our results are presented in Fig. 5, where we show the dark matter density due to cosmic strings as a function of the symmetry-breaking VEV η. In making this plot, we have also set $\zeta = 10$, $p_c = 1$, $\epsilon_{\text{cusp}} = 1$, and we have again taken the freeze-out time to be $t_{\text{fo}} = 2 \times 10^{16}\ \text{GeV}^{-1}$. We have also fixed an overall prefactor in Eq. (4.12) [equal to $2\zeta/(2+\alpha)$) to ensure that the loop formation rate is continuous as t_i crosses t_*. The different curves in Fig. 5 correspond to different values of the scaling regime loop size parameter α and the exponent ξ appearing in Eq. (4.14).

The dark matter density curves in Fig. 5 show three distinct regions, with the transitions between them occurring around $\eta = 10^8$ GeV and $\eta = 10^{12}$ GeV. For values of η well above 10^{12} GeV, the curves coincide with those obtained in the absence of friction. Such large values of η imply a value of t_* that is much smaller than the freeze-out time t_{fo}, so that all loops generated while friction is relevant decay away before t_{fo}. On the other hand, when η lies below 10^{12} GeV, the most important contribution to the dark matter comes from loops formed while friction dominates the network evolution, $t_i < t_*$.

In the region $10^8\ \text{GeV} \lesssim \eta \lesssim 10^{12}$ GeV most of the dark matter is produced by loops that are formed in the

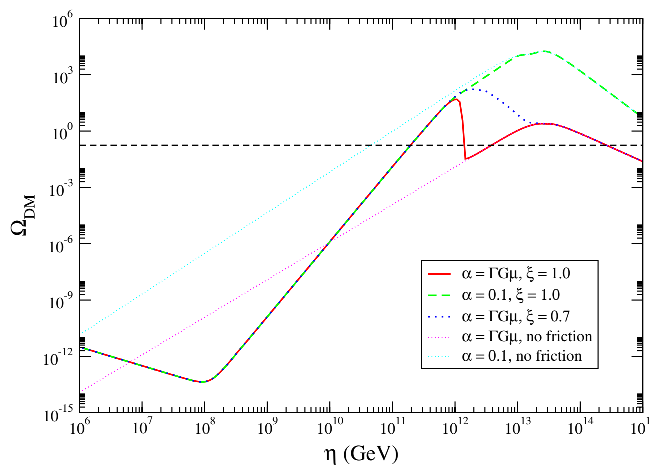

FIG. 5 (color online). Dark matter density due to loop cusping for $\epsilon_{\text{cusp}} = 1$, $p_c = 1$, and $t_{\text{fo}} = 2 \times 10^{16}\ \text{GeV}^{-1}$ as a function of the symmetry-breaking VEV η in the presence of friction. The various curves correspond to different values of the initial loop size parameter α. The black dashed horizontal line indicates the observed dark matter relic density.

friction era, with $t_i < t_*$. As a result, the dark matter density is independent of the scaling value of α for this range of η. The largest contribution to the dark matter in this region comes from loops that are also long-lived, with $t_i < t_{\rm fo}$. This enhances the amount of dark matter formed because, with the loop distribution function of Eq. (4.12), the integrand of the t_i integral in Eq. (2.7) is a rapidly decreasing function of t_i. Increasing η in this region leads to larger initial loop sizes that further extend the loop lifetime. However, the minimal value of t_i for which loops decay after $t_{\rm fo}$ decreases more slowly with increasing η rather than t_*. The sharp transition near $\eta = 10^{12}$ GeV occurs when these two quantities become equal. When this happens, increasing η further decreases the initial loop size [see Eq. (4.14)] for the dominant DM loops, and these loops go quickly from being long-lived to being short-lived, with the dominant loops being formed near $t_i = t_{\rm fo} > t_*$. The transition is more gradual when the exponent ξ in Eq. (4.14) is less than $\xi = 1.0$, which can be seen by comparing it with the curve for $\xi = 0.7$.

For $\eta \lesssim 10^8$ GeV, the initial loop size ℓ_f becomes small enough that the loops are short-lived, decaying away within a Hubble time. Thus, the majority of the dark matter produced for these smaller values of η comes from loops formed near the freeze-out time, $t_i \simeq t_{\rm fo}$, with initial size close to $\ell_f(t_{\rm fo})$. As η decreases below 10^8 GeV, we find that a larger fraction of the energy of each loop is lost to cusping, thereby increasing the dark matter density. Even so, the dark matter relic density generated by cosmic strings for $\eta \sim 10^3$ GeV is a negligibly small fraction of the observed value.

V. CONCLUSIONS

We have investigated the cold dark matter density created by the decays of loops of gauge cosmic strings. Dark matter is produced by string loops when they form cusps. At a cusp, a small portion of the string loop annihilates into its constituent fields, which can then cascade down to lighter states such as dark matter particles. Our results provide constraints on extensions of the gauge symmetry group of the standard model that give rise to cosmic strings in the early universe. The string loops that give decay to dark matter are themselves created continually by the network of long, horizon-length strings. We have studied the amount of dark matter generated when this network is in the *scaling* regime, and when its evolution is dominated by frictional forces. Both cases are physically relevant as the presence or absence of significant frictional interactions depends on the details of the symmetry breaking from which the strings originate.

The dark matter density generated by a string network in the scaling regime can potentially be enough to explain the observed relic density when the symmetry-breaking VEV η exceeds 10^{10} GeV. The amount of dark matter produced by this mechanism is greatest when the initial loop size approaches a significant fraction of the cosmological horizon, although much smaller initial loop sizes can also generate sizeable densities of dark matter. However, larger loops have cusp decay products that are highly boosted, and this can reduce the fraction of the loop energy that becomes cold dark matter. For values of η below 10^{10} GeV, the amount of cold dark matter created by a scaling string network is always well below the observed value.

String networks that are strongly influenced by frictional interactions can also generate a dark matter density that is equal to or greater than the observed value. This can occur if the symmetry-breaking scale is greater than about 10^{11} GeV. For very large values of the symmetry-breaking scale, above about 10^{13} GeV, frictional effects decouple well before the nonthermal dark matter is created; the frictional interactions do not alter the resulting dark matter density. When the symmetry-breaking scale is much below 10^{11} GeV, the contributions to the dark matter density in the presence of friction are very small. This contrasts with and is less constraining than the result of Ref. [8], in which the effects of cusping on the decays of loops were not included in computing the dark matter density.

We conclude from our investigation that the nonthermal cold dark matter relic density is safely below the observed value when the symmetry-breaking scale is less than $\eta \simeq 10^{10}$ GeV. For larger symmetry-breaking scales, the nonthermal dark matter density produced by decaying string loops can potentially be as large or larger than the observed value. Thus, our results also put constraints on new gauge symmetries broken at very high scales based on the requirement that they do not overproduce dark matter. These constraints are model-dependent, in that they depend on the effective branching fraction of the decaying string loops into dark matter, but they can be more severe than the constraints from the more traditional cosmic string signatures such as gravitational radiation and gravitational lensing [2,3]. For gauge symmetries broken at lower scales that could be probed by the LHC, we find that the contribution to the dark matter density from their corresponding scaling string loops is negligibly small.

ACKNOWLEDGMENTS

We thank Aaron Pierce, Joseph Polchinski, Jorge Rocha, and James Wells for helpful discussions and comments. We also thank Aaron Pierce and James Wells for suggestions and comments on the manuscript, and Nima Arkani-Hamed for pushing us to look more closely at the effect of large boosts from string cusps. This work was supported by the DOE, the Michigan Center for Theoretical Physics, and the National Science Foundation under Grant No. PHY05-51164. The authors would also like to thank the Kavli Institute for Theoretical Physics for their hospitality while this work was being completed.

[1] H. B. Nielsen and P. Olesen, Nucl. Phys. **B61**, 45 (1973).
[2] M. B. Hindmarsh and T. W. B. Kibble, Rep. Prog. Phys. **58**, 477 (1995).
[3] A. Vilenkin and E. P. S. Shellard, *Cosmic Strings and Other Topological Defects* (Cambridge University Press, Cambridge, 1994).
[4] T. W. B. Kibble, J. Phys. A **9**, 1387 (1976).
[5] R. Blumenhagen, M. Cvetic, P. Langacker, and G. Shiu, Annu. Rev. Nucl. Part. Sci. **55**, 71 (2005); P. Langacker, arXiv:hep-ph/9805486; P. Langacker and J. Wang, Phys. Rev. D **58**, 115010 (1998); J. R. Espinosa, Nucl. Phys. B, Proc. Suppl. **62**, 187 (1998); M. Cvetic and P. Langacker, Phys. Rev. D **54**, 3570 (1996); M. Cvetic and P. Langacker, Mod. Phys. Lett. A **11**, 1247 (1996).
[6] P. Langacker, N. Polonsky, and J. Wang, Phys. Rev. D**60**, 115005 (1999); D. Suematsu, Phys. Rev. D **59**, 055017 (1999).
[7] J. L. Hewett and T. G. Rizzo, Phys. Rep. **183**, 193 (1989); P. Langacker and J. Wang, Phys. Rev. D **58**, 115010 (1998); G. C. Cho, K. Hagiwara, and Y. Umeda, Nucl. Phys. **B531**, 65 (1998); **B555**, 651(E) (1999); S. F. King, S. Moretti, and R. Nevzorov, Phys. Rev. D **73**, 035009 (2006); Phys. Lett. B **634**, 278 (2006); R. Howl and S. F. King, J. High Energy Phys. 01 (2008) 030; P. Langacker, arXiv:0801.1345.
[8] R. Jeannerot, X. Zhang, and R. H. Brandenberger, J. High Energy Phys. 12 (1999) 003.
[9] T. Matsuda, J. Cosmol. Astropart. Phys. 04 (2006) 005.
[10] T. W. B. Kibble, Nucl. Phys. **B252**, 227 (1985); **B261**, 750 (E) (1985).
[11] D. P. Bennett and F. R. Bouchet, Phys. Rev. Lett. **60**, 257 (1988); Phys. Rev. D **41**, 2408 (1990).
[12] B. Allen and E. P. S. Shellard, Phys. Rev. Lett. **64**, 119 (1990).
[13] C. J. A. Martins and E. P. S. Shellard, Phys. Rev. D **53**, R575 (1996); **54**, 2535 (1996); **65**, 043514 (2002).
[14] G. R. Vincent, M. Hindmarsh, and M. Sakellariadou, Phys. Rev. D **56**, 637 (1997); G. Vincent, N. D. Antunes, and M. Hindmarsh, Phys. Rev. Lett. **80**, 2277 (1998).
[15] V. Vanchurin, K. D. Olum, and A. Vilenkin, Phys. Rev. D **74**, 063527 (2006).
[16] For reviews of dark matter, see G. Jungman, M. Kamionkowski, and K. Griest, Phys. Rep. **267**, 195 (1996); L. Bergstrom, Rep. Prog. Phys. **63**, 793 (2000); K. A. Olive, arXiv:astro-ph/0301505; G. Bertone, D. Hooper, and J. Silk, Phys. Rep. **405**, 279 (2005).
[17] E. Komatsu *et al.* (WMAP Collaboration), arXiv:0803.0547.
[18] For reviews, see for example H. E. Haber and G. L. Kane, Phys. Rep. **117**, 75 (1985); S. P. Martin, arXiv:hep-ph/9709356; D. J. H. Chung *et al.*, Phys. Rep. **407**, 1 (2005); M. A. Luty, arXiv:hep-th/0509029.
[19] T. Appelquist, H. C. Cheng, and B. A. Dobrescu, Phys. Rev. D **64**, 035002 (2001); G. Servant and T. M. P. Tait, Nucl. Phys. **B650**, 391 (2003); H. C. Cheng, J. L. Feng, and K. T. Matchev, Phys. Rev. Lett. **89**, 211301 (2002); F. Burnell and G. D. Kribs, Phys. Rev. D **73**, 015001 (2006); K. Kong and K. T. Matchev, J. High Energy Phys. 01 (2006) 038.
[20] H. C. P. Cheng and I. Low, J. High Energy Phys. 08 (2004) 061; J. Hubisz and P. Meade, Phys. Rev. D **71**, 035016 (2005); A. Birkedal, A. Noble, M. Perelstein, and A. Spray, Phys. Rev. D **74**, 035002 (2006).
[21] M. Srednicki and S. Theisen, Phys. Lett. B **189**, 397 (1987).
[22] S. Borsanyi and M. Hindmarsh, Phys. Rev. D **77**, 045022 (2008); M. Hindmarsh, S. Stuckey, and N. Bevis, arXiv:0812.1929.
[23] A. Vilenkin, Phys. Lett. B **107**, 47 (1981).
[24] R. H. Brandenberger, Nucl. Phys. **B293**, 812 (1987).
[25] T. W. B. Kibble and N. Turok, Phys. Lett. B **116**, 141 (1982).
[26] N. Turok, Nucl. Phys. **B242**, 520 (1984).
[27] J. J. Blanco-Pillado and K. D. Olum, Phys. Rev. D **59**, 063508 (1999); K. D. Olum and J. J. Blanco-Pillado, Phys. Rev. D **60**, 023503 (1999).
[28] T. Vachaspati and A. Vilenkin, Phys. Rev. D **31**, 3052 (1985).
[29] C. J. Burden, Phys. Lett. B **164**, 277 (1985).
[30] D. Garfinkle and T. Vachaspati, Phys. Rev. D **36**, 2229 (1987).
[31] J. M. Quashnock and D. N. Spergel, Phys. Rev. D **42**, 2505 (1990).
[32] V. Vanchurin, K. Olum, and A. Vilenkin, Phys. Rev. D **72**, 063514 (2005); V. Vanchurin, K. D. Olum, and A. Vilenkin, Phys. Rev. D **74**, 063527 (2006); K. D. Olum and V. Vanchurin, Phys. Rev. D **75**, 063521 (2007).
[33] C. Ringeval, M. Sakellariadou, and F. Bouchet, J. Cosmol. Astropart. Phys. 02 (2007) 023.
[34] C. J. A. Martins and E. P. S. Shellard, Phys. Rev. D **73**, 043515 (2006).
[35] X. Siemens and K. D. Olum, Nucl. Phys. **B611**, 125 (2001); **B645**, 367(E) (2002); X. Siemens, K. D. Olum, and A. Vilenkin, Phys. Rev. D **66**, 043501 (2002).
[36] J. Polchinski and J. V. Rocha, Phys. Rev. D **74**, 083504 (2006); **75**, 123503 (2007).
[37] J. Polchinski, arXiv:0707.0888.
[38] J. V. Rocha, Phys. Rev. Lett. **100**, 071601 (2008).
[39] F. Dubath, J. Polchinski, and J. V. Rocha, Phys. Rev. D **77**, 123528 (2008).
[40] V. Vanchurin, Phys. Rev. D **77**, 063532 (2008).
[41] Y. Cui, S. P. Martin, D. E. Morrissey, and J. D. Wells, Phys. Rev. D **77**, 043528 (2008).
[42] T. Damour and A. Vilenkin, Phys. Rev. Lett. **78**, 2288 (1997).
[43] M. Peloso and L. Sorbo, Nucl. Phys. **B649**, 88 (2003).
[44] T. Damour and A. Vilenkin, Phys. Rev. D **71**, 063510 (2005).
[45] X. Siemens, V. Mandic, and J. Creighton, Phys. Rev. Lett. **98**, 111101 (2007).
[46] M. Wyman, L. Pogosian, and I. Wasserman, Phys. Rev. D **72**, 023513 (2005); **73**, 089905(E) (2006).
[47] A. A. Fraisse, arXiv:astro-ph/0503402; J. Cosmol. Astropart. Phys. 03 (2007) 008.
[48] U. Seljak, A. Slosar, and P. McDonald, J. Cosmol. Astropart. Phys. 10 (2006) 014.
[49] L. Pogosian, I. Wasserman, and M. Wyman, arXiv:astro-ph/0604141.
[50] U. Seljak and A. Slosar, Phys. Rev. D **74**, 063523 (2006).
[51] E. Jeong and G. F. Smoot, Astrophys. J. **624**, 21 (2005); arXiv:astro-ph/0612706.
[52] S. Amsel, J. Berger, and R. H. Brandenberger, J. Cosmol.

Astropart. Phys. 04 (**2008**) 015; R.J. Danos and R.H. Brandenberger, arXiv:0811.2004.

[53] D.K. Hammond, Y. Wiaux, and P. Vandergheynst, arXiv:0811.1267.

[54] C.J. Hogan, Phys. Rev. D **74**, 043526 (2006); M.R. DePies and C.J. Hogan, Phys. Rev. D **75**, 125006 (2007).

[55] A. Vilenkin, Phys. Rev. D **43**, 1060 (1991).

[56] J. Garriga and M. Sakellariadou, Phys. Rev. D **48**, 2502 (1993).

[57] M.G. Alford and F. Wilczek, Phys. Rev. Lett. **62**, 1071 (1989).

[58] A.E. Everett, Phys. Rev. D **24**, 858 (1981).